Quartet structure in atomic nuclei

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A quartet is a tightly bound cluster of two protons and two neutrons, similar to the α particle but occasionally with nonzero spin and/or isospin. If the interaction between two quartets is weak in given states, such states are concluded to be well represented by a quartet structure. In this paper, we study the quartet structure of eight valence nucleons in two cases. The first is a single j shell, demonstrating that the so-called stretch scheme [M. Danos and V. Gillet, Phys. Rev. Lett. **17**, 703 (1966)] is very good for low-spin states with a quadruple-quadruple interaction, and is reasonably good under realistic interactions. The second case is the ground state of ^{92}Pd in the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell with the JUN45 effective interaction. We show that the quartet correlation is essential in the ground state of ^{92}Pd.

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The α particle (namely, ^{4}He) is a very tightly bound system of two protons and two neutrons. Therefore α correlation in low-lying states of atomic nuclei has been conjectured and studied extensively [1]. One of the most famous examples is the Hoyle state [2] (weak coupling of the α particle and ^{8}Be in the ^{12}C nucleus). The work in Ref. [3] suggests that the Hoyle state has a deformed shape accompanied by a rotational band. Similar structure was also studied in ^{8}Be, ^{16}O, ^{20}Ne, and ^{24}Mg.

There has been much work done towards quartet correlations in $N = Z$ nuclei [4–11]. In the literature a cluster with two protons and two neutrons coupled to a given spin and isospin is usually called a quartet. It is a tightly bound cluster, similar to the α particle but occasionally with nonzero spin and/or isospin. If the interaction between two quartets is weak in given states, such states are concluded to be well represented by quartet structure. Among various efforts along this line, here we mention work involving antisymmetrized molecular dynamics (AMD) [12–14] and "quartet truncation" models [6,15–19]. In AMD, the wave function is constructed by using multicenter configurations, similar to the idea in molecular physics. Not only the quartet structure but also other types of clustering structures are taken into account in the configuration space of the AMD. The quartet truncation models are based on the quartet truncation scheme of the nuclear shell model. In the quartet truncation models, the building blocks of wave functions are quartets. It has been shown that pairing correlation energies in the full shell model space and those in the quartet-truncated space are very close to each other under a simple phenomenological interaction [15–19]. The connection between the AMD and the quartet (or many-body cluster) truncation scheme is discussed in Refs. [20,21].

In this paper we study structures consisting of two quartets (namely, eight valence nucleons). The framework we use here is a quartet truncation model similar to that in Refs. [6,15]. Our formalism is based on the nucleon-pair approximation of the shell model with isospin symmetry [22–24]. The building blocks of this model are collective nucleon pairs with good quantum numbers of both spins and isospins, and a quartet is written in terms of two such pairs, viz.,

$$\mathbb{Q}^{(q)\dagger} = \sum_{r_1 r_2} x(r_1 r_2 q)(A^{(r_1)\dagger} \times A^{(r_2)\dagger})^{(q)}, \tag{1}$$

where

$$A^{(r_i)\dagger} = \sum_{j_1 j_2} y(j_1 j_2 r_i)(a_{j_1}{}^\dagger \times a_{j_2}{}^\dagger)^{(r_i)} \tag{2}$$

denotes a collective coupled pair with spin J_{r_i} and isospin T_{r_i}; (r_i) is short for (J_{r_i}, T_{r_i}), and (q) is short for (J_q, T_q); $y(abr_i)$ is called the pair structure coefficient, and $x(r_1 r_2 q)$ is called the quartet structure coefficient. For a state with spin I and isospin T for eight valence nucleons, the configuration of our quartet-truncated model is

$$(\mathbb{Q}^{(q_1)\dagger} \times \mathbb{Q}^{(q_2)\dagger})^{(I,T)}|0\rangle. \tag{3}$$

The matrix elements of overlaps, one-body and two-body operators were derived in Ref. [24], based on the Wick theorem of coupled clusters raised by Chen *et al.* [25].

Let us begin with the so-called stretch scheme [26,27] for a single j shell. In the stretch scheme the wave function of four valence protons and four valence neutrons is

$$\Psi_{\text{stretch}} = (\mathbb{Q}^{(4j-2,0)\dagger} \times \mathbb{Q}^{(4j-2,0)\dagger})^{(I,0)}|0\rangle. \tag{4}$$

Here

$$\mathbb{Q}^{(4j-2,0)\dagger} = (A^{(2j,0)\dagger} \times A^{(2j,0)\dagger})^{(4j-2,0)}$$

is called the stretch quartet. References [6,26,27] suggested that the stretch scheme provides a good description for low-lying rotational spectra in a large-j shell.

Now we investigate the stretch scheme for the lowest $T = 0$ states of eight nucleons in a single $j = 9/2$ shell. We first assume a schematic quadruple-quadruple interaction which approximately reproduces the SU(3) rotational

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states.

$$H_{\rm rot} = -(Q_\pi + Q_\nu)\cdot(Q_\pi + Q_\nu), \tag{5}$$

where Q is the quadrupole operator,

$$Q = \sum_{j_1 j_2} q(j_1 j_2)(a_{j_1}{}^\dagger \times \tilde{a}_{j_2})^{(21)}, \quad q(j_1 j_2) = (-)^{l_1+l_2+1}\frac{[1+(-)^{l_1+l_2}]\hat{j}_1}{\sqrt{16\pi}} C^{j_2\frac{1}{2}}_{j_1\frac{1}{2}20}\langle Nl_1|r^2|Nl_2\rangle.$$

Here $C^{j_2\frac{1}{2}}_{j_1\frac{1}{2}20}$ is the Clebsch-Gordan coefficient, and the matrix element of r^2 [28] is

$$\langle Nl_1|r^2|Nl_2\rangle = \begin{cases} (N+3/2){r_0}^2, & \text{if} \quad l_1 = l_2, \\ -\sqrt{(N+l_2+2\pm 1)(N-l_2+1\mp 1)}{r_0}^2, & \text{if} \quad l_1 = l_2 \pm 2, \end{cases}$$

where ${r_0}^2$ is equal to $\hbar/(m_N\omega_0)$, m_N is the mass of a nucleon, and ω_0 is the harmonic oscillator frequency.

We denote our calculated wave function in the stretch scheme $\Psi_{\rm stretch}$, and that in the full shell model configuration $\Psi_{\rm SM}$. The overlap between the stretch scheme wave function and the shell model wave function is denoted by $\langle\Psi_{\rm stretch}|\Psi_{\rm SM}\rangle^2$. The calculated results are shown in Figs. 1 and 2. One sees that the stretch scheme well reproduces the rotational feature of the yrast states with $I = 0$–6, but deteriorates for $I = 8$–20. This fact can be understood from two perspectives. One perspective is that the stretch scheme represents an ideal rotor for the states with small I (see the caption of Fig. 3 in Ref. [27]), but this picture is not good as I increases. The second perspective is that the quadruple-quadruple interaction generates rotational structures only for states with small values of I in a single j shell [29]. For the states with $I = 22$ and 24, the stretch scheme works well again. This is not surprising, because the state of $I = 24$ is unique and the two pictures are equivalent.

We now study the stretch scheme of the same system but with a more realistic interaction. In Ref. [30] Van Isacker studied two quartet-truncated wave functions for $T = 0$ states of ^{48}Cr, in the model space of four valence protons and four valence neutrons in the single $f_{7/2}$ shell. The first quartet-truncated wave function is the same as given in Eq. (4), and the second is the so-called seniority-like quartet-truncated wave function $\Psi_{\rm seniority}$, i.e., one quartet with both spin and isospin zero, and the other with spin I and isospin zero. Namely,

$$\Psi_{\rm seniority} = (\mathbb{Q}^{(0,0)^\dagger} \times \mathbb{Q}^{(I,0)^\dagger})^{(I,0)}|0\rangle, \tag{6}$$

where

$$\mathbb{Q}^{(0,0)^\dagger} = (A^{(2j,0)^\dagger} \times A^{(2j,0)^\dagger})^{(0,0)},$$

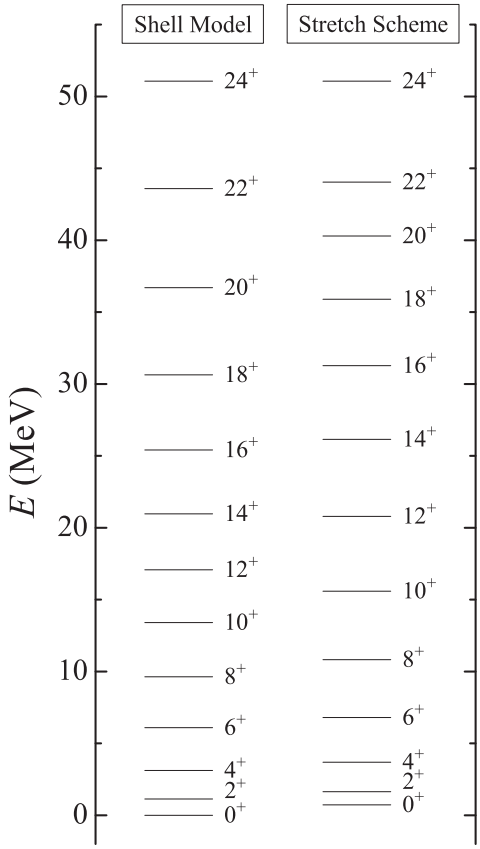

FIG. 1. Yrast $T = 0$ states for eight nucleons in the single $j = 9/2$ shell under the quadruple-quadruple interaction [see Eq. (5)]. All the level energies are plotted as relative energies with respect to the 0^+ state energy obtained by the shell model.

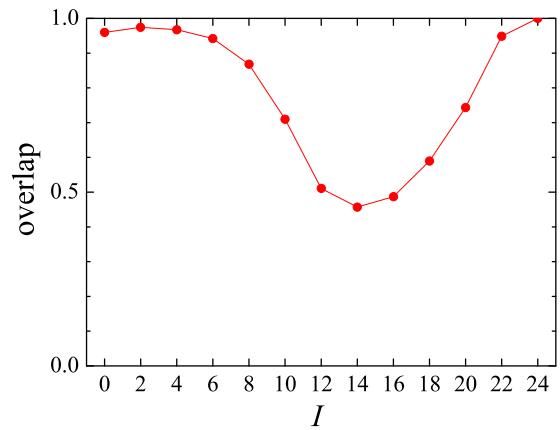

FIG. 2. (Color online) Overlap between the stretch scheme wave function and the shell model wave function for the yrast $T = 0$ states of eight nucleons in a single $j = 9/2$ shell under the quadruple-quadruple interaction [see Eq. (5)].

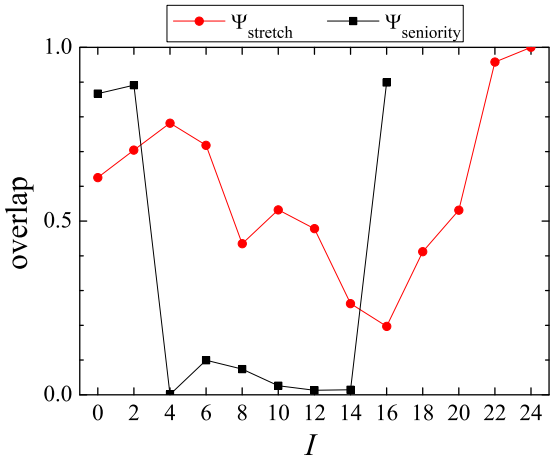

FIG. 3. (Color online) Overlap between the quartet-truncated wave functions and the shell model wave function for the yrast $T = 0$ states of eight nucleons in the $g_{9/2}$ shell under a realistic interaction taken from Ref. [32]. The stretch scheme wave function Ψ_{stretch} is given by Eq. (4), and the seniority-like quartet-truncated wave function $\Psi_{\text{seniority}}$ is given by Eq. (6).

$$\mathbb{Q}^{(I,0)\dagger} = (A^{(2j,0)\dagger} \times A^{(2j,0)\dagger})^{(I,0)}.$$

He showed that the seniority-like quartet-truncated wave function is more appropriate than the stretch quartet-truncated wave function for the yrast states with $I = 0$ and 2, but is not relevant for the yrast state with $I = 4$. In Ref. [31] Neergård presented similar results for the ground state of ^{48}Cr in the $f_{7/2}$ shell and for the ground state of ^{88}Ru and ^{92}Pd in the $g_{9/2}$ shell, and showed further that the seniority-like quartet truncation is better in the $f_{7/2}$ shell than in the $g_{9/2}$ shell.

Here we perform calculations similar to those in Refs. [30,31] but for yrast $T = 0$ states of ^{92}Pd in terms of four valence proton holes and four valence neutron holes in the single $g_{9/2}$ shell. The realistic interaction we take here is based on the optimized interaction for the $g_{9/2}$ shell in Ref. [32]. Our calculated results are shown in Figs. 3 and 4, where one sees that the stretch scheme is reasonably good for the yrast $I = 0$–6 states, with overlaps $\langle\Psi_{\text{stretch}}|\Psi_{\text{SM}}\rangle^2$ ranging from 0.6 to 0.8. The stretch scheme is not good for $I = 8$, 10, 12, 18, and 20, with $\langle\Psi_{\text{stretch}}|\Psi_{\text{SM}}\rangle^2$ ranging from 0.4 to 0.5. This picture is not applicable to $I = 14$ and 16 (as $\langle\Psi_{\text{stretch}}|\Psi_{\text{SM}}\rangle^2 < 0.3$), but is recovered for $I = 22$ and 24. This overall tendency of $\langle\Psi_{\text{stretch}}|\Psi_{\text{SM}}\rangle^2$ versus I is very similar to the result with the quadruple-quadruple interaction shown in Fig. 2. For the ground state, the overlap between the stretch scheme wave function and the shell model wave function is equal to 0.62, while in Ref. [33] Qi obtained 0.81 with the interaction derived from Ref. [34]. The pattern of the seniority-like quartet-truncated wave function is even sharper. The seniority-like quartet-truncated wave function is superior to the stretch scheme for the states with $I = 0$, 2, and 16 ($\langle\Psi_{\text{seniority}}|\Psi_{\text{SM}}\rangle^2 > 0.86$), but is almost irrelevant for other states.

Now we switch to the case of sophisticated multi-j shells. We take the ground state of ^{92}Pd in the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell with the JUN45 effective interaction [35], in both the shell

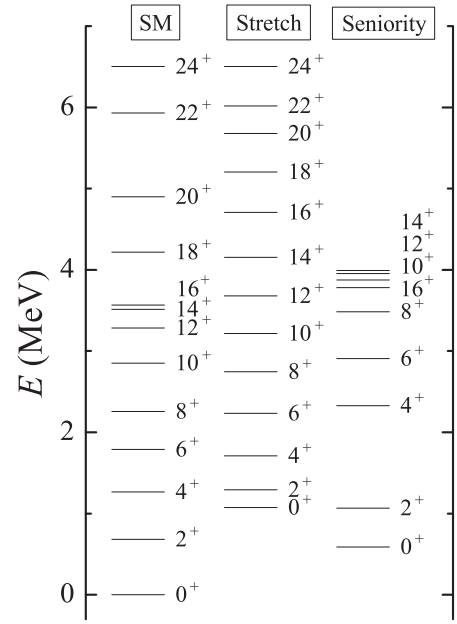

FIG. 4. Yrast $T = 0$ states for eight nucleons in the $g_{9/2}$ shell under a realistic interaction taken from Ref. [32]. All the level energies are plotted as relative energies with respect to the 0^+ state energy obtained by the shell model. "SM" represents the shell model. "Stretch" represents the stretch scheme defined by Eq. (4). "Seniority" represents the seniority-like quartet truncation defined by Eq. (6).

model and the quartet truncation model. We take two quartet-truncated wave functions as follows:

$$\Psi_1 = (\mathbb{Q}^{(0,0)\dagger} \times \mathbb{Q}^{(0,0)\dagger})|0\rangle,$$
$$\Psi_2 = (\mathbb{Q}^{(0,0)\dagger} \times \mathbb{Q}'^{(0,0)\dagger})|0\rangle, \tag{7}$$

where

$$\mathbb{Q}^{(0,0)\dagger} = \sum_{j_1 j_2 j_3 j_4 r} x(j_1 j_2 j_3 j_4 r) \times (A^{(r)}(j_1 j_2)^\dagger \times A^{(r)}(j_3 j_4)^\dagger)^{(0,0)},$$
$$\mathbb{Q}'^{(0,0)\dagger} = \sum_{j_1 j_2 j_3 j_4 r} x'(j_1 j_2 j_3 j_4 r) \times (A^{(r)}(j_1 j_2)^\dagger \times A^{(r)}(j_3 j_4)^\dagger)^{(0,0)}.$$

The wave function Ψ_1 is constructed by two identical quartets ($\mathbb{Q}$), and Ψ_2 is constructed by two different quartets ($\mathbb{Q}$ and $\mathbb{Q}'$). For $\mathbb{Q}$ the quartet structure coefficient $x(j_1 j_2 j_3 j_4 r)$ is determined by using the ground-state wave function of ^{96}Cd; for $\mathbb{Q}'$ the quartet structure coefficient $x'(j_1 j_2 j_3 j_4 r)$ is obtained by minimizing the energy of Ψ_2 under the effective interaction. The calculated overlap $\langle\Psi_1|\Psi_{\text{SM}}\rangle^2 = 0.78$, which is smaller than the overlap of 0.98 obtained in the single $g_{9/2}$ shell calculation [33]. The energy difference between the binding energy obtained by the shell model and that obtained by Ψ_1 is equal to 1.414 MeV. The overlap $\langle\Psi_2|\Psi_{\text{SM}}\rangle^2 = 0.92$, and the energy difference between the binding energy obtained by the shell model and that obtained by Ψ_2 drops to 0.531 MeV. Therefore the ground state of

^{92}Pd is well described by two quartets with spin and isospin zero.

A quartet is regarded as an tightly bound cluster of four nucleons [5]. To see whether or not this is true, we calculate the binding energies of the quartets $\mathbb{Q}$ and $\mathbb{Q}'$. Namely, we calculate two-body interactions between four nucleons in the quartets, denoted by $\langle\mathbb{Q}|V|\mathbb{Q}\rangle$ and $\langle\mathbb{Q}'|V|\mathbb{Q}'\rangle$, where V is the two-body interaction in the JUN45 effective interaction. The calculated binding energies are as follows:

$$B(\mathbb{Q}) = -\langle\mathbb{Q}|V|\mathbb{Q}\rangle = 10.949 \text{ MeV},$$
$$B(\mathbb{Q}') = -\langle\mathbb{Q}'|V|\mathbb{Q}'\rangle = 11.411 \text{ MeV}.$$

On the other hand, we evaluate the two-body interaction between two quartets $\mathbb{Q}$ and $\mathbb{Q}'$ as follows:

$$\langle\Psi_2|V|\Psi_2\rangle - \langle\mathbb{Q}|V|\mathbb{Q}\rangle - \langle\mathbb{Q}'|V|\mathbb{Q}'\rangle = -1.191 \text{ MeV}.$$

The quartets $\mathbb{Q}$ and $\mathbb{Q}'$ are tightly bound clusters, while the interaction between them is weak and attractive in ^{92}Pd.

In the shell model calculations, there is an empirical scaling factor for two-body matrix elements (TBME) in effective interactions. For example, in the shell model calculations for nuclei in the *sd* shell with the USD interaction [36], one uses the same set of TBME over the entire region of the *sd* shell with the form $\text{TBME}(A) = \text{TBME}(A = 18) \times (A/18)^{-0.3}$. The same scaling factor is used in the GXPF1 interaction [37] and the JUN45 interaction [35]. This scaling factor is a result of the change of the single-particle radial wave function as a function of mass number A [38]. The binding energy of the α particle is $B_\alpha = 28.296$ MeV, and we evaluate the binding energy of the quartet in ^{92}Pd simply by using the product of B_α and the scaling factor,

$$B_\alpha\left(\tfrac{92}{4}\right)^{-0.3} = 11.046 \text{ MeV}.$$

Interestingly, this value is close to 10.949 and 11.411 MeV obtained above.

To summarize, in this paper we study quartet correlations in isospin $T = 0$ states of eight-nucleon systems. We investigate the validity of the stretch scheme raised by Danos and Gillet [26] for eight nucleons in a single $j = 9/2$ shell with both the schematic quadruple-quadruple interaction and the realistic interaction, by calculating overlaps between quartet-truncated wave functions and shell model wave functions. The quartet correlation in the ground state of ^{92}Pd is studied in the $p_{1/2}p_{3/2}f_{5/2}g_{9/2}$ shell under the JUN45 effective interaction.

Our single $j = 9/2$ shell calculation result shows that the stretch scheme is good for the yrast $T = 0$ states when the total spin of the state I is close to zero or close to the maximum spin $I_{\max} = 8j - 12$, with both the quadruple-quadruple interaction and the realistic interaction. The seniority-like quartet-truncated wave function, constructed by two quartets, one of which has spin zero and isospin zero and the other of which has spin I and isospin zero, is very good for the states with $I = 0$, 2, and 16, but is almost irrelevant for other states. The multi-j shell calculation result shows that the quartet truncation scheme with spin zero and isospin zero can reasonably well reproduce the ground state of ^{92}Pd. We demonstrate that the quartet is a tightly bound cluster, while the interaction between two quartets is weak.

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