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# Resonant states of the neutron-rich $\Lambda$ hypernucleus $^{7}_{\Lambda}$He

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The structure of the neutron-rich $\Lambda$ hypernucleus, $^{7}_{\Lambda}$He is studied within the framework of an $\alpha + \Lambda + n + n$ four-body cluster model. We predict second $3/2^+$ and $5/2^+$ states, corresponding to a $0s$ $\Lambda$ coupled to the second $2^+$ state of $^6$He, as narrow resonant states with widths of $\Gamma \sim 1$ MeV to be at 0.03 and 0.07 MeV with respect to the $\alpha + \Lambda + n + n$ threshold. From a separate estimate of the differential cross section for the $^7$Li$(\gamma, K^+)^{7}_{\Lambda}$He reaction, we suggest a possibility to observe these states at the Thomas Jefferson National Accelerator Facility (JLab) in the future. We also calculate the second $2^+$ state of $^6$He as a resonant state within the framework of an $\alpha + n + n$ three-body cluster model. Our result is 2.81 MeV with $\Gamma = 4.63$ MeV with respect to the $\alpha + n + n$ threshold. This energy position is $\sim$1 MeV higher, and with a much broader decay width, than the recent SPIRAL data. We suggest that an experiment at JLab to search for the second $3/2^+$ and $5/2^+$ states of $^{7}_{\Lambda}$He would provide an opportunity to confirm the second $2^+$ state of the core nucleus $^6$He.

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## I. INTRODUCTION

In 2013 the neutron-rich $\Lambda$ hypernucleus, $^{7}_{\Lambda}$He, was observed via the $(e,e'K^+)$ reaction, and an observed $\Lambda$ separation energy of $B_\Lambda = 5.68 \pm 0.03(\text{stat.}) \pm 0.25(\text{sys.})$ MeV was reported [1]. This observation stimulated us to study neutron-rich $\Lambda$ hypernuclei, because in light nuclei near the neutron drip line interesting phenomena concerning neutron halos have been observed. When a $\Lambda$ particle is added to such nuclei, it is expected that the resultant hypernuclei will become more stable against neutron decay due to the attraction of the $\Lambda N$ interaction and the fact that there is no Pauli exclusion effect between nucleons and a $\Lambda$ particle. This phenomenon is one of the "gluelike" roles of $\Lambda$ particles.

Before this measurement, we previously predicted in Refs. [2,3] the energy spectra of $^{7}_{\Lambda}$He in the bound-state region within the framework of a $^{5}_{\Lambda}\text{He} + n + n$ three-body model and an $\alpha + \Lambda + n + n$ four-body cluster model. The core nucleus $^6$He is known to be a typical neutron halo nucleus: the two-neutron separation energy is 0.975 MeV. The $\Lambda$ inclusion in the bound state of such a halo nucleus results in a more stable ground state of the hypernucleus. We predicted that the binding energy of the ground state is 5.36 MeV within the $\alpha + \Lambda + n + n$ four-body model which is consistent with the recent data in terms of the quoted error bar. For this ground state, we have another interesting insight related to the charge-symmetry breaking (CSB) components in the $\Lambda N$ interaction. It is considered that the most reliable evidence for CSB appears in the $\Lambda$-separation energies ($B_\Lambda$) of the $A = 4$ hypernuclei with $T = 1/2$ ($^{4}_{\Lambda}$H and $^{4}_{\Lambda}$He). In that system the CSB effects are associated with the separation-energy difference $\Delta_{\text{CSB}} = B_\Lambda(^{4}_{\Lambda}\text{He}) - B_\Lambda(^{4}_{\Lambda}\text{H})$, the experimental values of which are $0.35 \pm 0.06$ and $0.24 \pm 0.06$ MeV for the ground ($0^+$) and excited ($1^+$) states, respectively. It is also likely that CSB affects the binding energy of $^{7}_{\Lambda}$He, and the experimental research at the Thomas Jefferson National Accelerator Facility (JLab) on $^{7}_{\Lambda}$He was motivated by this question. Since a consistent understanding of the CSB in the $\Lambda N$ interaction has not yet been obtained, further experimental studies of $^{4}_{\Lambda}$H will be carried out in the near future at JLab and Mainz, and $^{4}_{\Lambda}$He at J-PARC.

The experimental study of $^{7}_{\Lambda}$He was performed again in 2009 with five times more statistics (JLab experiment E05-115). The preliminary result gives a more accurate binding energy of the ground state, and the first excited state ($3/2^+_1$ or $5/2^+_1$) was observed for the first time, which corresponds to the $2^+_1$ state of $^6$He core nucleus coupled with $\Lambda(0s)$ [4]. The observed $2^+_1$ state is located 0.827 MeV above the $\alpha + n + n$ breakup threshold with a decay width of $\Gamma = 0.113$ MeV [5]. The coupling of a $\Lambda$ to this state leads to bound states of $^{7}_{\Lambda}$He. A high-resolution spectroscopy experiment of $\Lambda$ hypernuclei using an electron beam is a powerful tool for producing bound and resonant hypernuclear states. Then, we have the following question: Is there the possibility of having other new hypernuclear states in $^{7}_{\Lambda}$He? To answer this question, it is necessary to look at the energy spectra of the $^6$He core nucleus before studying $^{7}_{\Lambda}$He. The observed data of the $^6$He [5] reported a $0_1^+$ ground state as a bound state and the $2_1^+$ resonant state with $E_x = 1.797$ MeV, $\Gamma = 0.113$ MeV. To search for the second $2^+$ state, some experiments were performed. For example, the charge-exchange reaction, $^6$Li$(t,{}^3$He$)^6$He, was employed to explore for excited states above the first 2+ state [6]. However, clear evidence of a second 2+ state was not obtained. In 2012, in Ref. [7], the transfer reaction experiment $p(^8$He$,t)^6$He shows an indication of a second $2^+$ state of $^6$He as a resonant state at $E_x = 2.6 \pm 0.3$ ($\Gamma = 1.6 \pm 0.4$) MeV.

Theoretically, many authors studied energy spectra of $^6$He with various approaches [8–11]. Among them, for instance, one of the present authors (T.M.) and collaborators studied the energy spectra of $^6$He within the framework of the cluster-orbital shell model (COSM) for the $\alpha + n + n$ three-body system using the complex scaling method (CSM), which is a powerful method of obtaining resonant energies and decay widths accurately. They reproduced the energies of the observed ground state and the first $2^+$ state very well. Moreover, they obtained a second $2^+$ state at 2.52 MeV above the $\alpha + n + n$ threshold with a width of $\Gamma = 3.87$ MeV.

When a $\Lambda$ particle is added to such a resonant state, due to a gluelike role of $\Lambda$ particle, it is likely to result in narrower resonant states of $3/2_2^+$ and $5/2_2^+$ of $^{7}_{\Lambda}$He. The prediction of energies of second $3/2^+$ and $5/2^+$ states and decay widths would encourage further experimental investigation of $^{7}_{\Lambda}$He at JLab. With this aim, we discuss resonant states for $^6$He and $^{7}_{\Lambda}$He using CSM within the framework of $\alpha + n + n$ and $\alpha + \Lambda + n + n$ three- and four-body cluster models, respectively.

This article is organized as follows: In Sec II, the method and interactions used in the three-body and four-body calculations for $^6$He and $^{7}_{\Lambda}$H are described. The numerical results and the corresponding discussions are presented in Sec. III. A summary is given in Sec. IV.

## II. MODEL AND INTERACTIONS

The models employed in this article are the same as those in our previous work [3]. We employ the $\alpha + n + n$ three-body model for $^6$He and the $\alpha + \Lambda + n + n$ four-body model for $^{7}_{\Lambda}$He; as for the Jacobi-coordinate sets, see Fig. 3 in Ref. [2] and Fig. 1 in Ref. [3], respectively.

The Hamiltonian for $^6$He is written as

$$H = T + V_{NN} + \sum_{i=1}^{2}\left[V_{\alpha N_i} + V_{\alpha N_i}^{\rm Pauli}\right] \quad , \qquad (1)$$

and that for $^{7}_{\Lambda}$He is

$$H = T + V_{NN} + V_{\Lambda\alpha} + \sum_{i=1}^{2}\left[V_{\Lambda N_i} + V_{\alpha N_i} + V_{\alpha N_i}^{\rm Pauli}\right]. \qquad (2)$$

Here, $T$ is the kinetic-energy operator and $V_{\alpha N_i}^{\rm Pauli}$ stands for the Pauli principle between the two valence neutrons and the neutrons in the $\alpha$ cluster. The two-body $N$-$N$ (AV8, $T = 1$), $\alpha$-$N$, $\alpha$-$\Lambda$, and $\Lambda$-$N$ interactions are chosen so as to reproduce accurately the observed properties of all the subsystems composed of $NN(T{=}1)$, $\alpha N$, $\alpha\Lambda$, $\alpha NN$, and $\alpha\Lambda N$; the details are described in Ref. [3]. Here, we do not include any charge-symmetry breaking interaction for the $\Lambda N$ part.

The total wave functions for $^6$He and $^{7}_{\Lambda}$He are described as a sum of amplitudes of the rearrangement channels within the $LS$ coupling scheme, respectively:

$$\begin{aligned}\Psi_{JM}(^6\mathrm{He}) = &\sum_{c=1}^{3}\sum_{n,N}\sum_{l,L}\sum_{S} C_{nlNLSI}^{(c)} \\ &\times \mathcal{A}_N\big[\Phi(\alpha)\big[\chi_{\frac12}(N_1)\chi_{\frac12}(N_2)\big]_S \\ &\times \big[\phi_{nl}^{(c)}(\mathbf{r}_c)\psi_{NL}^{(c)}(\mathbf{R}_c)\big]_I\big]_{JM}. \qquad (3)\end{aligned}$$

$$\begin{aligned}\Psi_{JM}\big(^{7}_{\Lambda}\mathrm{He}\big) = &\sum_{c=1}^{9}\sum_{n,N,\nu}\sum_{l,L,\lambda}\sum_{S,\Sigma,I,K} C_{nlNL\nu\lambda S\Sigma IK}^{(c)} \\ &\times \mathcal{A}_N\big[\Phi(\alpha)\big[\chi_{\frac12}(\Lambda)\big[\chi_{\frac12}(N_1)\chi_{\frac12}(N_2)\big]_S\big]_\Sigma \\ &\times \big[\big[\phi_{nl}^{(c)}(\mathbf{r}_c)\psi_{NL}^{(c)}(\mathbf{R}_c)\big]_I\xi_{\nu\lambda}^{(c)}(\boldsymbol{\rho}_c)\big]_K\big]_{JM}. \qquad (4)\end{aligned}$$

Here the operator $\mathcal{A}_N$ stands for antisymmetrization between the two valence neutrons. $\chi_{\frac12}(N_i)$ and $\chi_{\frac12}(\Lambda)$ are the spin functions of the $i$th nucleon and $\Lambda$ particle. $\Phi^{(\alpha)}$ is the wave function of the $\alpha$ cluster having the $(0s)^4$ configuration. As for the spatial basis functions $\phi_{nlm}(\mathbf{r}_c)$, $\psi_{NLM}(\mathbf{R}_c)$, and $\xi_{\nu\lambda\mu}^{(c)}(\boldsymbol{\rho}_c)$, we took the Gaussian basis functions with the ranges in a geometric progression; details can be found in Refs. [3,12].

In this work, we focus on the resonant states of $^6$He and $^{7}_{\Lambda}$He. We then employ the CSM [13–17]. The CSM and its application to nuclear physics problems are extensively reviewed in Refs. [18,19] and references therein. Using the CSM, one can directly obtain the energy $E_r$ and the decay width $\Gamma$ of the $\alpha nn$ and $\alpha\Lambda nn$ systems by solving the eigenvalue problem for the complex scaled Schrödinger equation with a scaling angle $\theta$,

$$[H_{\alpha nn(\alpha\Lambda nn)}(\theta) - E(\theta)]\Psi_{\alpha nn(\alpha\Lambda nn)}(\theta) = 0, \qquad (5)$$

where the boundary condition of the many-body outgoing wave is automatically satisfied for the resonance, giving $E = E_r - i\Gamma/2$, which is, in principle, independent of $\theta$. The complex scaled Hamiltonian $H_{\alpha nn(\alpha\Lambda nn)}(\theta)$ is obtained by making the radial coordinate transformation with the common angle of $\theta$

$$r_c \to r_c\, e^{i\theta}, \quad R_c \to R_c\, e^{i\theta}, \quad \rho_c \to \rho_c\, e^{i\theta} \qquad (6)$$

in the Hamiltonian $H_{\alpha nn(\alpha\Lambda nn)}$ of the $\alpha nn$ ($\alpha\Lambda nn$) system. The nonresonant continuum states are obtained on the $2\theta$-rotated line in the complex energy plane.

A great advantage of the CSM is that a resonant state is described by an $L^2$-integrable wave function. Therefore, the Gaussian basis functions mentioned above have been successfully used in calculations of the CSM [18].

## III. RESULTS AND DISCUSSION

In Fig. 1, we show the distributions of eigenenergies $[E_2^+(\theta) = E_r - i\Gamma/2]$ of the complex-scaled Hamiltonian $H_{\alpha nn}(\theta)$ for the $J^\pi = 2^+$ states of the $^6$He nucleus at $\theta = 28^\circ$. The eigenvalues of the $^4$He$\,+\,n + n$ three-body and

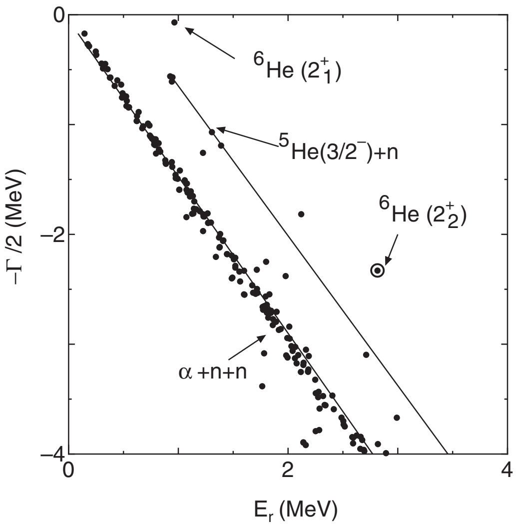


FIG. 1. The $2_1^+$ and $2_2^+$ energy eigenvalue distributions of the complex scaled Hamiltonian of $^6$He with $\theta = 28^\circ$. The energy is measured with respect to the $\alpha + n + n$ threshold. The two solid lines are the $\alpha + n + n$ and $^5\mathrm{He}(3/2^-) + n$ thresholds.

$^5\mathrm{He}(3/2^-) + n$ two-body continuum states appear reasonably along the lines which are rotated from the real axis by $2\theta$. We find two $2^+$ resonance poles at $E_\mathrm{r} = 0.96$ MeV with $\Gamma = 0.14$ MeV and $E_r = 2.81$ MeV with $\Gamma = 4.63$ MeV, respectively; the latter pole is stable against the change of $\theta$ from 25° to 33°. In addition, we find the $1^+$ state to be at $E_\mathrm{r} = 3.00$ MeV with $\Gamma = 5.22$ MeV. In Fig. 2, we summarize the energy spectra of $^6$He together with experimental data confirmed so far. The calculated energies of the $0^+$ ground state and the first $2^+$ excited state are in good agreement with the data. It is interesting that we obtain the second $2^+$ state at 2.81 MeV above the $\alpha + n + n$ threshold with $\Gamma = 4.63$ MeV. Recently, the SPIRAL experiment suggested the existence of the $2_2^+$ state at 1.63 MeV above the $\alpha + n + n$ threshold with $\Gamma = 1.6$ MeV [7]. For comparison, we note that here the $2_2^+$ state is calculated to be ∼1 MeV higher energy with a much broader width.

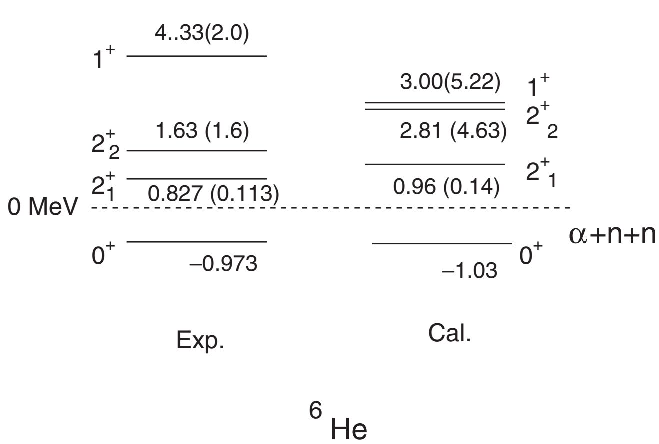


FIG. 2. Calculated energy spectra of $^6$He together with the experimental data. The energies in MeV are measured with respect to the $\alpha + n + n$ threshold. The values in parentheses are the decay widths Γ in MeV.

The calculation for the second $2^+$ state of $^6$He has been already performed in Ref. [11] within the COSM combined with the CSM. They predicted this state to be at $E_r = 2.52$ MeV with $\Gamma = 3.87$ MeV using the Minnesota $NN$ interaction. Although the AV8 $NN$ interaction is employed in the present work, we remark that both theoretical calculations give essentially the same results with respect to the resonance energy and the decay width.

Now we discuss the results for the hypernucleus $^{7}_{\Lambda}$He. First, the calculated energy of the ground state ($J^\pi = 1/2^+$) is $E = -6.39$ MeV which corresponds to a Λ-separation energy of $B_\Lambda = E(^6\mathrm{He}) - E(^{7}_{\Lambda}\mathrm{He}) = 5.36$ MeV. This energy is the same as in Ref. [3] in the case of no CSB component in the $\Lambda N$ interaction. The JLab E01-011 experiment measured the $^7\mathrm{Li}(e,e'K^+)^{7}_{\Lambda}$He reaction spectrum and deduced [1] for the first time the Λ-separation energy of the $^{7}_{\Lambda}$He ground state to be $B_\Lambda^{\mathrm{expt}} = 5.68 \pm 0.03(\mathrm{stat.}) \pm 0.25(\mathrm{sys.})$ MeV. The theoretical prediction of $B_\Lambda$ is compatible with this value within the experimental error.

Second, in the present calculation, the first $3/2^+$ and $5/2^+$ states are obtained as bound states at $E = -4.73$ MeV and $E = -4.65$ MeV, respectively, with respect to the $\alpha + n + n + \Lambda$ four-body breakup threshold; these values are the same as in Ref. [3] in the case of no CSB component in the $\Lambda N$ interaction. It is noted that a recent analysis of the JLab E05-115 experiment [4] also reports an excited-state peak at $B_\Lambda^{\mathrm{expt}} = 3.65 \pm 0.20(\mathrm{stat.}) \pm 0.11(\mathrm{sys.})$ MeV. This peak can be well attributed to the present prediction of the $3/2_1^+$ and $5/2_1^+$ states, because the average theoretical Λ-separation energy for these states, $B_\Lambda = 3.66$ MeV, is in very good agreement with the experimental value. The agreement is confirmed also by the comparison of production cross sections for the ground and the excited states, since they are consistent with the theoretical estimate as will be shown below.

Third, we search for the pole positions of the second $3/2^+$ and $5/2^+$ state of $^{7}_{\Lambda}$He within the CSM. In Fig. 3, we show the distribution of complex eigenvalues of the $5/2_2^+$ states at $\theta = 15^\circ$ where the energy $E_r$ is measured from the $\alpha + \Lambda + n + n$ four-body breakup threshold. We find the resonance pole of the $5/2_2^+$ state at $E_r = 0.07$ MeV with $\Gamma/2 = 0.51$ MeV. This state is isolated from the continuum states of the $^{6}_{\Lambda}\mathrm{He} + n$ and $^{5}_{\Lambda}\mathrm{He} + n + n$ configurations and is stable against the change of the rotation angle, $\theta = 10^\circ - 18^\circ$. For the $3/2_2^+$ state, we find the resonance pole at $E_\mathrm{r} = 0.03$ with $\Gamma/2 = 0.56$ MeV. Figure 4 summarizes the theoretical level structure of $^{7}_{\Lambda}$He together with that of $^6$He. In Fig. 4, we see a small energy splitting for the second $3/2^+$–$5/2^+$ doublet states which is similar to the splitting of the first $3/2^+$-$5/2^+$ doublet. This is caused basically by the small spin-spin $\Lambda N$ interaction. The $1^+$ state is obtained at $E_r = 3.00$ MeV with a decay width $\Gamma = 5.22$ MeV, very close to that of the $2_2^+$ level, as shown in Fig. 4. However, we did not find any particular influence of the $1^+$ state on the $3/2_2^+$ energy, probably because they both have large decay widths. One naturally expects to have the third $3/2^+$ state and the second $1/2^+$ state in this region of excitation. However, it was difficult to distinguish these states

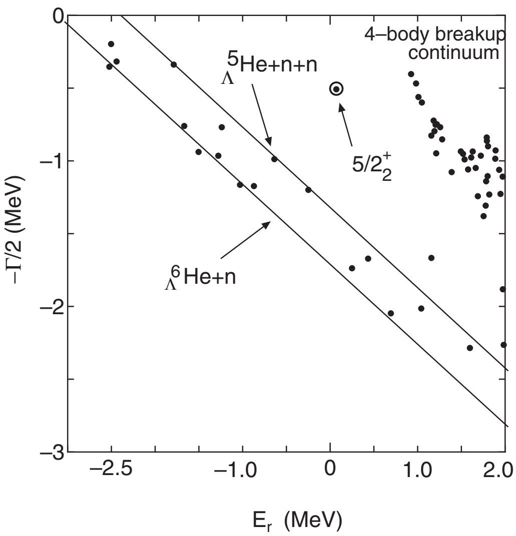


FIG. 3. The $5/2^+_2$ eigenvalue distributions of the complex scaled Hamiltonian with $\theta = 15^\circ$. The energy is measured with respect to the $\alpha + \Lambda + n + n$ breakup threshold. The three solid lines are the $^6_\Lambda\mathrm{He} + n$, $^5_\Lambda\mathrm{He} + n + n$, and $^5\mathrm{He}(3/2^-) + n + \Lambda$ thresholds.

since these states were embedded into the four-body breakup continuum states.

Here we emphasize that when the $\Lambda$ particle is added, the responses of two $2^+$ core states are so different that the energy spacing between the centroids of the first doublet ($3/2^+_1, 5/2^+_1$) and the second doublet ($3/2^+_2$, $5/2^+_2$) in $^7_\Lambda$He becomes quite large ($\sim$4.7 MeV) in comparison with the 1.85 MeV spacing of the two $2^+$ states. This effect is attributed to the difference in size (spatial structure) of the two $2^+$ wave functions in $^6$He. In fact the decay width of the $2^+_1$ resonant state is very small (0.11 MeV), and the state is compact. On the other hand, the decay width of the $2^+_2$ state is considerably large (4.63 MeV). Therefore it is expected that the radial extent of the wave function for the $2^+_2$ state is much larger than that of the $2^+_1$ state. When a $\Lambda$ particle is added to such states having different characters, the energy gain, $\Lambda$ separation energy, in the compact state is much larger than that in the dilute state. A similar phenomenon has been pointed out in Ref. [20], namely, that the $\Lambda$ separation energy of the compact shell-like ground state, $1/2^+_1$ in $^{13}_\Lambda$C, is much larger than that in a well-developed clustering state such as the $1/2^+_2$ state. If the predicted second doublet states ($3/2^+_2$ and $5/2^+_2$) are confirmed in a future experiment, then we can see the clear state-dependent response to the addition of a $\Lambda$ particle.

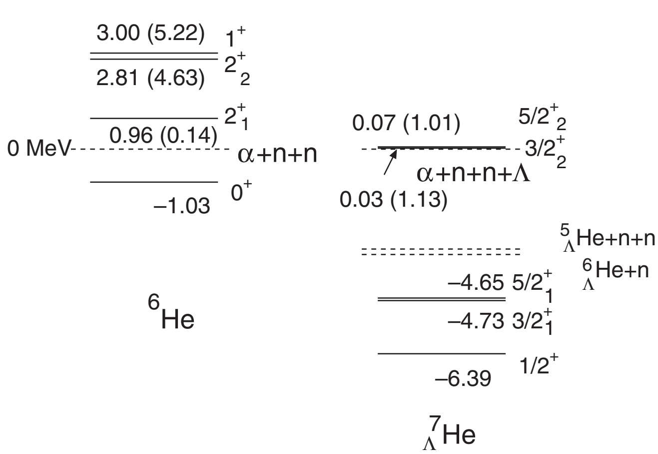


FIG. 4. Calculated energy levels of $^6$He and $^7_\Lambda$He. The level energies in MeV are measured with respect to the $\alpha + n + n$ and $\alpha + \Lambda + n + n$ breakup thresholds. The values in parentheses are decay widths $\Gamma$ in MeV.

Next, it is interesting to investigate how we can confirm experimentally the energy positions of the $3/2^+_2$ and $5/2^+_2$ states predicted here in $^7_\Lambda$He. In view of the successful E05-115 experiment [1,4] which already shows the ground-state and the excited-state peaks, the unique possibility is to perform a dedicated $^7\mathrm{Li}(e,e'K^+)^7_\Lambda\mathrm{He}$ reaction experiment with better energy resolution in the future. For the sake of an experimental feasibility study, we give here brief estimates of the $^7\mathrm{Li}(\gamma,K^+)^7_\Lambda\mathrm{He}$ reaction cross sections in DWIA on the basis of the COSM results of the proton pickup spectroscopic amplitudes $S^{1/2}$ which are ready to be used. As for the $^7$Li target state with strong binding, we use the ordinary shell-model wave function of the maximum symmetric $|p^3(30)_{L=1}(T = 1/2, S = 1/2); J = 3/2^-\rangle$ which corresponds to the $\alpha - t$ cluster configuration in the lowest approximation. However, one should be careful in treating weakly bound or unbound states in $^6$He.

The low-lying states of $^6$He have been studied extensively by the COSM with $\alpha$ core in Ref. [11] which treats the many-body resonances in the complex scaling method (CSM). Here we remark that the present three-body cluster model and the COSM treatment give essentially the same physical properties of nuclei. As explained in Refs. [21,22], the choice of the relative coordinates in the model space of core+$n$+$n$ is different but the COSM gives consistent results with the core + $n$ + $n$ three-cluster model for the structures such as $^6$He and $^{11}$Li [11]. It is also confirmed that the COSM and Gamow shell model (GSM) give the same results for the $^6$He structure for the energy eigenvalues, configuration mixing, and the density distribution of halo neutrons [19,23]. Instead of using the four-body cluster model wave functions, we make use of the proton pickup spectroscopic amplitudes $S^{1/2}$ derived in the COSM framework, and correspondingly we assume the simplified weak-coupling wave functions consisting of the $^6\mathrm{He}(0^+,2^+_1,2^+_2)$ solutions and $s^\Lambda_{1/2}$ in the brief estimates of the cross sections.

One may refer to Table II of Ref. [11] for the $^6\mathrm{He}(0^+)$ bound-state wave function, while the two $2^+$ resonance states of $^6$He have the following structures, respectively, showing the dominant components symbolically.

$$
\begin{aligned}
(2^+_1) &= \sqrt{0.898 + i0.013}\left[p^2_{3/2}\right] \\
&\quad + \sqrt{0.089 - i0.013}\left[p_{3/2}p_{1/2}\right], \\
(2^+_2) &= \sqrt{0.089 - i0.023}\left[p^2_{3/2}\right] \\
&\quad - \sqrt{0.889 + i0.024}\left[p_{3/2}p_{1/2}\right].
\end{aligned}
$$

Here the amplitude of each COSM component is complex, reflecting the calculated decay width. It is also notable that these wave functions are much different from those of the usual SU(3)($\lambda\mu$) prescription like $|p^2(20)_{L=2}(T{=}1,S=0); J=2_1^+\rangle$ and $|p^2(01)_{L=1}(T=1,S=1); J=2_2^+\rangle$, because the $(p_{3/2}^2)$ component is smaller than $(p_{3/2}p_{1/2})$ in $J=2_1^+$ in this prescription; the two components are almost equally admixed in conventional shell-model calculations [24]. The essential reason for the difference is attributed to the fact that in the COSM treatment the experimental spin-orbit splitting for the unbound $p$-state neutron on $\alpha$ is properly taken into account through a realistic $\alpha N$ potential [22,25] consistent with the phase-shift analysis. It should be also noted that in the present three- and four-body calculations, the same $\alpha N$ potential is employed. For readers' reference, we remark that in COSM the $(p_{3/2}p_{1/2})_{J=2}$ single-channel energy is estimated to be 1.28 MeV higher than the $(p_{3/2}^2)_{J=2}$ channel. It is also noted that the proton-pickup spectroscopic factors from the $^7$Li($3/2^-_{\rm g.s.}$) leading to the $^6$He states are calculated to be $C^2S=0.559$ ($0^+$), 0.257 ($2_1^+$), and 0.097 ($2_2^+$), by neglecting the small imaginary components. If one uses the conventional shell-model wave functions with an unrealistically small spin-orbit splitting, the $S$ factor leading to the $2_2^+$ state is almost vanishing.

The $^7{\rm Li}(\gamma,K^+)^7_\Lambda{\rm He}$ differential cross sections are estimated at $E_\gamma^{\rm lab}=1.5$ GeV and $\theta_K^{\rm lab}=7^\circ$ corresponding the E05-115 experimental kinematics. The calculated results are

$$d\sigma/d\Omega(1/2_G^+) = 49.0 \text{ nb/sr}, \tag{7}$$

$$d\sigma/d\Omega(3/2_1^+ + 5/2_1^+) = 10.0 + 11.6 = 21.6 \text{ nb/sr}, \tag{8}$$

$$d\sigma/d\Omega(3/2_2^+ + 5/2_2^+) = 3.4 + 4.3 = 7.7 \text{ nb/sr}. \tag{9}$$

Here the two doublet strengths are summed up, as they are degenerate in energies. If one chooses $\theta_K^{\rm lab}=2^\circ$, then the differential cross sections corresponding to Eqs. (7)–(9) are estimated to be 73.8, 32.7, and 11.6 nb/sr, respectively. We remark that the relative strength for the ground state and the degenerate excited states (49.0 vs 21.6 nb/sr) is in very good agreement with the ground and second peaks observed in the JLab E05-115 experiment [1,4]. Thus we can surely expect that the $3/2_2^+$ and $5/2_2^+$ states should appear as the third narrow peak having a differential cross section of about 40% of that for the second peak. If this peak position is identified in a future experiment, it will open an important window toward the study of characteristic structures of neutron-rich hypernuclei.

As for the excited states of $^6$He, we note that the energy position of the second $2_2^+$ state is calculated to be $\sim$1 MeV higher, with much broader width, than the SPIRAL experimental data [7] that provide the $2_2^+$ state energy for the first time. In view of the theory vs experiment discrepancies in energy and width for this $2_2^+$ resonance state, we suggest new experiments to search for this state in $^6$He for reconfirmation. We emphasize that such experiments to study the typical neutron-halo nuclear excited states, combined with a dedicated $^7{\rm Li}(e,e'K^+)^7_\Lambda{\rm He}$ experiment, could provide a new possibility for the spectroscopy of neutron-rich hypernuclei.

## IV. SUMMARY

Motivated by the recent data on $^7_\Lambda$He obtained at JLab, we calculated resonant states of $3/2_2^+$ and $5/2_2^+$ within the framework of an $\alpha+n+n+\Lambda$ four-body cluster model using CSM. The resonant $2_2^+$ state of the core nucleus $^6$He was calculated within the framework of the $\alpha+n+n$ three-body cluster model. All the two-body interactions among subunits for $^6$He and $^7_\Lambda$He are the same as in Ref. [3], that is, the interactions are chosen to reproduce the binding energies of all subsystems composed of two and three subunits. Then, the energy we obtained for the second $2^+$ state was 2.81 MeV with $\Gamma=4.63$ MeV for $^6$He. This calculated energy is located above by $\sim$1 MeV and narrower than the recent data from SPIRAL [7]. For $^7_\Lambda$He, we predicted the energies of the second $3/2^+$ and $5/2^+$ resonant states to be 0.03 and 0.07 MeV with $\Gamma=\sim$1 MeV, which have narrower widths than the corresponding state of the $^6$He core nucleus, $2_2^+$, due to the gluelike role of the $\Lambda$ particle.

To encourage future experiments at JLab, we present brief estimates for the $^7{\rm Li}(\gamma,K^+)^7_\Lambda{\rm He}$ reaction cross sections by making use of the spectroscopic amplitudes derived from the COSM framework. The calculated cross sections are 3.4 and 4.3 nb+sr for $3/2_2^+$ and $5/2_2^+$, respectively, if the E05-115 experimental kinematics is assumed. Thus we predict that the first doublet peak ($3/2_1^+$ and $5/2_1^+$) should have the strength of about 40% of the ground-state peak, while the second doublet should form the third peak having about 15% which seems to be a feasible strength for a future experiment. For further reconfirmation, the wave functions within the four-body cluster model framework for $\alpha+n+n+p$ and $\alpha+\Lambda+n+n$ will be also applied to the cross-section estimates and the results will be discussed in a forthcoming paper.

In conclusion we have shown the importance of measuring the third peak consisting of the $3/2_2^+$ and $5/2_2^+$ resonant states of $^7_\Lambda$He together with the experimental confirmation of the second $2^+$ state energy and width of the core nucleus $^6$He. The predicted large changes of binding energies and widths before and after the $\Lambda$ addition are interesting aspects to be investigated in neutron-rich hypernuclei. Thus, we suggest that measurements should be performed to confirm the second $2^+$ resonant state of $^6$He on the one hand and, on the other hand, to find the second $3/2^+$ and $5/2^+$ doublet states in a dedicated $^7{\rm Li}(e,e'K^+)^7_\Lambda{\rm He}$ experiment.

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