Scissors mode from a different perspective

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The scissors mode, a magnetic dipole excitation-mainly orbital, is usually discussed in terms of a transition from a $J = 0^+$ ground state to a $J = 1^+$ excited state. This is understandable because it follows from the way the experiment is performed, e.g., inelastic electron scattering. Here, however, we start with the excited 1^+ state and consider all possible transitions to $J = 0^+, 1^+$, and 2^+ states with final isospins. There is a larger transition to the 0_2^+ state than to ground. This has a much richer structure. We note that the "sum of sums" is independent of the interaction.

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I. INTRODUCTION

In a collective picture the scissors mode is an orbital magnetic dipole excitation in which the deformed proton symmetry axis vibrates against the corresponding axis of the neutrons. Some early discussions of this mode are presented by Richter's group in Bohle *et al.* [1,2]. In 2010 there was an extensive review of scissors modes by K. Heyde *et al.*, [3]. This has stimulated research by many different groups, both theoretical and experimental. It will not be practical to mention all of those that are referred to in the review article but we selected some that show the variety of approaches. These are listed in Refs. [4–17]. More recently, there has been work on $M1$ excitations by J. Beller *et al.* [18] in which the initial state has $I = 1^+$. This is of great relevance to the theme of the present work.

In all the experiments, which involve mainly inelastic electron scattering, one starts with the $J = 0^+$ ground state and considers excitations to the $J = 1^+$ states. The supporting calculations follow suit. However, since there are no practical constraints for theory, we will here start with the $J = 1^+$ scissors mode state and follow the various branches to which it can connect. Now we can go not only from $J = 1^+$ to $J = 0^+$ but also from $J = 1^+$ to $J = 2^+$ which gives a much richer spectrum.

This work can be regarded as an extension of previous work by the authors [19]. In that work the main focus was on selection rules with a $J = 0\,T = 1$ pairing interaction, i.e., why certain $B(M1)$'s vanish. In this work we will make quantitative comparisons of the nonvanishing strengths with different interactions. For example, there has been considerable work on $J_{\max}$ pairing by Zhao and Arima [20], Cederwall *et al.* [21], Xu *et al.* [22], Fu *et al.* [23], Zamick and Escuderos [24], and Hertz-Kintish and Zamick [25].

II. $B(M1)$ RESULTS FOR VARIOUS INTERACTIONS

We present results in Tables II–XXII, which are ^{44}Ti $I = 1$ to 0, ^{44}Ti $I = 1$ to 2, ^{46}Ti $I = 1$ to 0, and ^{46}Ti $I = 1$ to 2. As well as ^{44}Ti $I = 1$ to 1 and ^{46}Ti $I = 1$ to 1. There are four interactions used: $J = 0$ $(T = 1)$ pairing, Q.Q, MBZE [26], and $J_{\max}(T = 0)$ pairing. These are represented by eight numbers (seven independent), corresponding to two nucleons coupled to $J = 0$ to $J = 7$. They are listed in Table I.

TABLE I. Matrix elements for the interactions.

Interaction	$J = 0$	$J = 1$	$J = 2$	$J = 3$	$J = 4$	$J = 5$	$J = 6$	$J = 7$ (max)
$J = 0$ pairing	−2	0	0	0	0	0	0	0
Q.Q	0	0.4096	1.1471	2.0483	2.8677	3.2744	2.8677	1.1471
MBZE	0	0.6111	1.5863	1.4904	2.8153	1.5101	3.2420	0.6163
$J_{\max}$ pairing	0	0	0	0	0	0	0	− 2

TABLE II. Pairing $B(M1)$ ^{44}Ti $I = 1$ to $I = 0$.

State(v,T,t) $I = 0$	$I = 1$ Unshifted energy	210 1.500	411 2.250	411 2.250	Sum
000	0.000	2.6996	0	0	2.6996
020	0.750	8.0995	0	0	8.0995
400	2.250	1.9300	0.1117	2.8922	4.9339
400	2.250	0.8986	7.7693	1.9187	10.4966
	Sum	13.6277	7.7910	4.8109	26.2296

TABLE III. Pairing $B(M1)$ ^{44}Ti $I=1$ to $I=2$.

State(v,T,t) $I=2$	$I=1$ Unshifted energy	210 1.500	411 2.250	411 2.250	Sum
201	1.000	2.6015	0.4505	1.3857	4.4377
400	1.250	44.8541	3.7086	3.5942	52.1569
400	1.750	1.6209	6.3448	1.8866	9.8523
400	2.250	6.0571	4.7158	9.1692	19.9421
221	2.250	0	0	0	0
411	2.250	0	0	0	0
411	2.250	0	0	0	0
411	2.250	13.0086	2.2518	6.9270	22.1874
422	2.250	0.00005	21.4800	1.0920	22.5721
	Sum	68.1423	38.9515	24.0547	131.1483

TABLE IV. Pairing $B(M1)$ ^{46}Ti $I=1$ to $I=0$.

State(v,T,t) $I=0$	$I=1$ Unshifted energy	220 1.7500	411 2.0000	411 2.0000	421 2.5000	421 2.5000	611 2.7500	611 2.7500	Sum
010	0	1.0799	0	0	0	0	0	0	1.7099
030	1.2500	9.7200	0	0	0	0	0	0	9.7200
410	2.2500	2.4344	2.8794	0.0491	0.5611	0.4150	0	0	6.3390
410	2.7500	0.3947	0.7573	5.7648	0.1157	2.0588	0	0	6.3390
611	2.7500	0	1.0423	0.0987	3.1539	0.2640	2.3989	0.6317	9.0913
611	2.7500	0	0.0049	0.1721	0.0858	0.4450	0.0001	1.7267	7.5895
	Sum	13.6290	4.6839	6.0847	3.9165	3.1828	2.3990	2.3584	36.2543

TABLE V. Pairing $B(M1)$ ^{46}Ti $I=1$ to $I=2$.

State(v,T,t) $I=2$	$I=1$ Unshifted energy	220 1.7500	411 2.0000	411 2.0000	421 2.500	421 2.500	611 2.7500	611 2.7500	Sum
211	1.0000	1.3712	0.9874	0.3326	0.0005	0.0019	0	0	2.6936
211	1.0000	0.1715	0.4367	0.1472	0.0813	0.3238	0	0	1.1605
221	1.5000	2.5716	2.2323	0.7524	0.0222	0.0883	0	0	5.6668
412	1.5000	0	0.0916	1.5360	0.0607	0.4819	0	0	2.1702
411	2.0000	0	0.0847	0.0914	0.5024	0.0261	0.4364	0.0065	1.1475
411	2.0000	0	0.0041	0.0186	0.0014	0.0668	1.5191	0.0152	1.6244
422	2.0000	0	0.2746	4.6069	0.1821	1.4454	0	0	6.5090
410	2.2500	12.1303	0.0646	1.6850	0.0832	0.5004	0	0	14.4635
410	2.2500	2.9785	3.5617	0.1189	0.6431	0.5838	0	0	7.8860
410	2.2500	5.3986	0.4668	2.4445	0.0273	0.9432	0	0	9.2804
231	2.2500	2.0572	0	0	1.1354	4.5230	0	0	7.7156
421	2.5000	0	0.1804	0.0338	0.6237	0.0188	0.6123	0.0630	1.5320
421	2.5000	0	0.0862	0.2962	0.8883	0.2597	5.2534	0.0019	6.7857
611	2.7500	0	2.1377	0.2523	6.7325	0.4370	2.3618	0.0555	11.9768
611	2.7500	0	0.2654	0.0135	0.4044	0.4321	0.1597	0.8390	2.1141
611	2.7500	0	0.0367	0.1344	0.0050	0.5082	7.1099	1.4178	9.2120
611	2.7500	0	0.0375	0.0024	0.1070	0.0127	0.0873	0.0461	0.2930
611	2.7500	0	0.1215	1.3291	0.3483	4.0036	0.00007	5.7321	11.5347
	Sum	26.6789	11.0699	11.8488	14.6567	13.7952	17.5400	8.1771	103.7666

TABLE VI. Q.Q $B(M1)$ ^{44}Ti $I = 1$ to $I = 0$.

$I = 0$	$I = 1$ Unshifted energy	1_1 3.3648*	1_2 6.3405*	1_3 9.5620*	Sum
0_1	0.0000	1.3174	0.0015	0.0007	1.3196
0_2	3.6031**	1.8021	6.1454	0.1535	8.1010
0_3	7.5748	0.1833	9.0414	0.9530	10.1777
0_4	10.9236	0.0414	0.0577	6.5332	6.6323
	Sum	3.3442	15.2460	7.6404	26.2306

TABLE VII. Q.Q $B(M1)$ ^{44}Ti $I = 1$ to $I = 2$.

$I = 2$	$I = 1$ Unshifted energy	1_1 3.3648*	1_2 6.3405*	1_3 9.5620*	Sum
2_1	0.9655	2.4898	0.0111	0.0016	2.5025
2_2	3.6015*	0	0	0	0
2_5	4.7502*	0.1735	20.6912	1.3251	22.1898
2_3	6.4691	13.7051	8.2795	1.2061	23.1907
2_6	7.5695*	0	0	0	0
2_8	7.6179**	0.1271	0.8452	21.6001	22.5724
2_4	7.7501	0.0545	46.3395	0.6632	47.0572
2_9	9.7351*	0	0	0	0
2_7	10.4893	0.1723	0.0767	13.4096	13.6586
	Sum	16.7223	76.2432	38.2057	131.1711

TABLE VIII. Q.Q $B(M1)$ ^{46}Ti $I = 1$ to $I = 0$.

$I = 0$	$I = 1$ Unshifted energy	1_1 4.3546	1_2 8.1095	1_3 8.7081*	1_4 10.4611*	1_5 10.5846	1_6 10.8481	1_7 11.6407*	Sum
0_1	0.0000	1.3901	0.0006	0.0038	0	0	0	0.0002	1.3947
0_2	6.4642	2.6505	0.0897	2.2242	0.0003	0.3008	0.0015	0.0161	5.2831
0_3	7.9741	0.1986	5.0379	0.3734	0.1137	0.0310	0.2988	0.00004	6.0534
0_6	9.7237**	0	0	6.8081	0	0	0	2.9145	9.7226
0_4	10.7392	0.0191	0.4441	0.2985	2.9265	0.1054	4.0417	0.8731	8.7084
0_5	12.5438	0.000099	0.0097	0.0013	0.0020	0.8983	0.0079	4.1814	5.1007
	Sum	4.2584	5.5820	9.7093	3.0425	1.3355	4.3499	7.9853	36.2629

TABLE IX. Q.Q $B(M1)$ ^{46}Ti $I = 1$ to $I = 2$.

$I = 2$	$I = 1$ Unshifted energy	1_1 4.3546	1_2 8.1095	1_3 8.7081*	1_4 10.4611*	1_5 10.5846	1_6 10.8481	1_7 11.6407*	Sum
2_1	0.8630	0.6034	0.0445	0.0011	0.0030	0	0.0001	0.0002	0.6523
2_2	3.5162	1.4891	0.3974	0.0151	0.0182	0.0005	0.0050	0.0017	1.9270
2_3	4.2764	1.8998	0.3245	0.1051	0.0023	0.0045	0.0495	0.0017	2.3874
2_4	6.2720	0.1660	0.0521	0.1476	0.0395	0.0276	0.1994	0.0026	0.6348
2_5	7.2633	0.0486	0.5793	0.2012	0.3204	0.0569	0.0156	0.0226	1.2446
2_{14}	7.3607*	2.3182	0.0359	1.9267	0.0346	0.4418	0.0221	0.0027	4.7820
2_6	7.7478	1.1676	0.5558	10.2928	0.0683	0.2179	0.0103	0.0004	12.3131
2_7	8.5830	0.0120	5.2029	0.0168	2.4499	0.2874	0.0202	0	7.9892
2_{15}	9.6011*	0.0372	3.2940	0.5506	0.5938	0.3825	3.3463	0.0549	8.2593
2_8	9.6672	0.0428	0.0018	5.5599	0.2733	2.7290	0.1841	2.1744	10.9653
2_{16}	9.8751*	0.1340	0.0460	0.0143	0.8130	4.0584	0.0469	0.0597	5.1723
2_9	10.5511	0.0154	0.0132	0.0092	4.2207	3.1012	0.1246	0.1755	7.6598
2_{18}	10.8708**	0	0	0.2478	4.0121	0	0	3.4565	7.7164
2_{10}	11.2619	0.0079	0.0528	1.0658	0.0045	3.7622	0.0004	4.0527	8.9463
2_{11}	11.3626	0	0.0340	0.0075	0.1386	0.3283	10.7713	0.1488	11.4285
2_{17}	12.1399*	0.0005	0.00004	0.0345	0.5456	0.1533	0.0738	1.4694	2.27714
2_{12}	12.4314	0.0004	0.0009	0.0935	0.4677	1.4645	0.0974	6.3983	8.5227
2_{13}	12.8660	0	0.0084	0.0033	0.1017	0.0005	0.0112	0.7700	0.8951
	Sum	7.9429	10.6435	20.2928	14.1072	17.0165	14.9782	18.7921	103.7732

TABLE X. MBZE $B(M1)$ ^{44}Ti $I = 1$ to $I = 0$.

$I = 0$	$I = 1$ Unshifted energy	1_1 5.66864*	1_2 7.58685*	1_3 9.72619*	Sum
0_1	0.00000	1.18248	0.17056	0	1.35304
0_2	5.58610	0.13111	5.29543	0.05642	5.48296
0_3	8.28402**	1.95508	6.07014	0.07579	8.10101
0_4	8.7875	0.17022	1.73958	9.38455	11.29435
	Sum	3.43889	13.27571	9.51676	26.2314

TABLE XI. MBZE $B(M1)$ ^{44}Ti $I = 1$ to $I = 2$.

$I = 2$	$I = 1$ Unshifted energy	1_1 5.66864*	1_2 7.58685*	1_3 9.72619*	Sum
2_1	1.16313	1.34744	0.42560	0.04716	1.82020
2_2	4.95650	12.97910	1.30523	0.27252	14.55685
2_3	5.23665*	0	0	0	0
2_4	7.81197	0	0	0	0
2_5	7.82336	1.09707	37.57634	12.68482	51.35823
2_6	7.96963	1.53883	10.26440	6.83864	18.64187
2_7	9.26771*	0	0	0	0
2_8	9.87032**	0.09840	16.68741	5.40033	22.18614
2_9	11.88190**	0.11349	0.11945	22.34037	22.57331
	Sum	17.19433	66.37843	47.58384	131.15657

TABLE XII. MBZE $B(M1)$ ^{46}Ti $I = 1$ to $I = 0$.

$I = 0$	$I = 1$ Unshifted energy	1_1 3.65521	1_2 6.05887	1_3 7.78516	1_4 8.73868	1_5 9.46213*	1_6 10.61597*	1_7 11.36444*	Sum
0_1	0.00000	0.55962	0.01755	0.00592	0.07044	0.13076	0.01481	0.00998	0.80908
0_2	4.62474	2.47374	0.18000	0.29729	0.51637	0.99056	0.14637	0.06077	4.6651
0_3	6.27338	0.67490	4.31054	0.11501	0.15137	0.78449	0.00119	0.07790	6.1154
0_4	7.89321	0.12817	0.46911	1.37366	0.36023	0.58752	2.16170	0.08942	5.16981
0_5	9.31823	0.00351	0.18194	3.21205	0.38400	0.04369	0.32570	5.58837	9.73926
0_6	13.20357**	0	0	0	0	4.6687	1.79917	3.25315	9.76218
	Sum	3.83994	5.15914	5.00393	1.52357	7.20572	4.44894	9.07959	36.26083

TABLE XIII. MBZE $B(M1)$ ^{46}Ti $I = 1$ to $I = 2$.

$I = 2$	$I = 1$ Unshifted energy	1_1 3.65521	1_2 6.05887	1_3 7.78516	1_4 8.73868	1_5 9.46213*	1_6 10.61597*	1_7 11.36444*	Sum
2_1	1.14826	0.20333	0.03800	0.00273	0.00135	0.00364	0.00619	0.02923	0.28447
2_2	2.49693	1.20214	0.46560	0.06665	0.00391	0.08832	0.13831	0.00351	1.96844
2_3	3.42179	1.73449	0.01221	0.22106	0.00967	0.00787	0.00670	0.03729	2.02929
2_4	4.88264*	0.24575	1.20038	0.26325	0.00453	0.28514	0.00118	0.00090	2.00566
2_5	5.15177	0.50943	0.76123	0.03848	0.24204	0.48245	0.17814	0.01875	2.23052
2_6	6.15814	0.64804	0.07549	0.36327	0.03414	0.02360	1.45758	0.11907	2.72119
2_7	6.79141	0.06966	3.30106	0.23216	0.05538	0.54470	1.07923	1.26442	6.54661
2_8	7.25799	0.46379	1.32288	0.00002	0.23771	8.01263	0.25422	0.30383	10.59508
2_9	7.53733	0.00342	0.03632	4.01174	0.55212	0.14699	0.45188	0.21191	5.41438
2_{10}	8.22517	0.19893	0.00043	0.08763	5.40760	4.16620	0.02142	1.45222	11.33443
2_{11}	8.25484*	1.62241	0.47339	0.53906	0.06593	2.14453	0.18933	0.24099	5.27564
2_{12}	8.49974	0.04708	0.19257	6.07608	1.84693	0.62106	1.35405	0.32188	10.45965
2_{13}	9.50002*	0.39393	1.53351	3.33583	1.81402	0.00121	0.00053	0.46185	7.54088
2_{14}	9.91064	0.01248	0.00940	0.07097	2.30873	0.02298	3.36939	0.03948	5.83343
2_{15}	10.18382	0.00954	0.06965	0.27111	0.94621	0.15988	1.72104	10.96380	14.14123
2_{16}	10.40254*	0.05283	0.56227	0.41358	2.90402	0.01365	2.05851	0.27184	6.27670
2_{17}	11.89813*	0.00532	0.00054	0.06234	0.61446	0.07426	0.44744	0.19571	1.40007
2_{18}	14.78987**	0	0	0	0	0.07617	0.00399	7.63472	7.71488
	Sum	7.42257	10.0549	16.05596	17.04875	16.87528	12.73913	23.57140	103.76801

TABLE XIV. $J_{\max}$ $B(M1)$ ^{44}Ti $I = 1$ to $I = 0$.

$I = 0$	$I = 1$ Unshifted energy	1_1 3.0851*	1_2 5.0769*	1_3 5.0769*	Sum
0_1	1.0758	1.3441	0	0	1.3441
0_2	5.0769	0.2309	4.6967	1.3398	6.2674
0_3	5.0769	0.2687	5.8869	4.3617	10.5173
0_4	5.0769**	1.1300	5.6054	4.3646	11.1000
	Sum	2.9737	16.189	7.0661	26.2288

TABLE XV. J_{max} $B(M1)$ ^{44}Ti $I = 1$ to $I = 2$.

$I = 2$	$I = 1$ Unshifted energy	1_1 3.0851*	1_2 5.0769*	1_3 5.0769*	Sum
2_1	1.0776	2.8055	0	.0010	2.8065
2_2	3.0518	10.7765	0.0698	4.6408	15.4871
2_3	3.0676*	0	0	0	0
2_4	5.0769	0.4151	54.3381	2.0636	56.8168
2_5	5.0769	0.0086	0.2783	10.9842	11.2711
2_6	5.0769*	0	0	0	0
2_7	5.0769*	0	0	0	0
2_8	5.0769**	0.68461	23.0100	8.3647	32.0593
2_9	5.0769**	0.1578	3.2658	9.2783	12.7019
	Sum	14.8682	80.9620	35.3326	131.1628

TABLE XVI. J_{max} $B(M1)$ ^{46}Ti $I = 1$ to $I = 0$.

$I = 0$	$I = 1$ Unshifted energy	1_1 2.4966	1_2 3.0668	1_3 4.8057	1_4 5.4724	1_5 5.1080*	1_6 5.6332*	1_7 7.0280*	Sum
0_1	1.0143	1.6533	0.0134	0.00005	0	0	0.0002	0	1.6670
0_2	2.4037	0.0905	2.8076	0.3393	0.0091	0.0183	0.0161	0.0002	2.2811
0_3	4.0284	1.8661	1.0119	0.0091	0.4326	0.0680	2.0710	0.0018	5.4605
0_4	4.9091	0.0136	0.5472	4.7113	0.2183	2.8754	0.0207	0.3620	8.7485
0_5	7.0280	0	0.0002	0.3270	1.4752	0.0037	0.1439	5.4312	7.3812
0_6	7.0280**	0	0	0	0	0.0956	3.7398	5.8845	9.7199
	Sum	3.6236	4.3803	5.3863	2.1352	3.0610	5.9916	11.6797	36.2577

TABLE XVII. J_{max} $B(M1)$ ^{46}Ti $I = 1$ to $I = 2$.

$I = 2$	$I = 1$ Unshifted energy	1_1 2.4966	1_2 3.0668	1_3 4.8057	1_4 5.1080*	1_5 5.4724	1_6 5.6332*	1_7 7.0280*	Sum
2_1	1.0281	1.0052	0.0013	0.0004	0.0008	0.00008	0.00007	0.00001	1.0079
2_2	1.7145	1.4089	1.8389	0	0.0055	0.0002	0.0036	0.00008	3.2572
2_3	2.4212	0.0064	0.2088	0.0189	0.0048	0.0114	0.0079	0.0011	0.2593
2_4	2.7178	0.0639	1.4115	0.0500	0.0011	0.0679	0.0153	0.0019	1.6116
2_5	3.1507	0.0964	1.5097	0.8884	0.0500	0.0149	0.1374	0.0035	2.7003
2_6	3.7368	0.0392	2.2568	0.0312	4.6084	0.1493	0.0380	0.0006	7.1235
2_7	3.9423	0.1232	0.0827	0.0085	0.7506	0.6391	0.1956	0.0019	1.8016
2_{12}	4.0692*	2.6849	0.0001	0.0410	0.0132	2.1248	0.7631	0.0010	5.6281
2_8	4.1408	1.2421	0.0179	0.1064	0.0300	0.5087	4.4675	0.0108	6.3834
2_9	4.6429	0.00008	0.1848	8.8388	0.8913	0.0559	0.0605	0.0219	10.0533
2_{13}	4.8658*	0.0005	0.9376	4.6612	2.0247	0.0229	0.0774	0.1122	7.8365
2_{10}	5.2300	0.0003	0.4780	0.1015	0.4972	7.5926	0.4851	0.0425	9.1972
2_{16}	5.4124*	0.0017	0.2989	0.1136	0.1323	3.1652	1.1913	0.5122	5.4152
2_{11}	5.5500	0.0010	0.1784	0.8460	0.4731	3.0674	7.7698	3.0295	15.3652
2_{14}	7.0280	0	0.00005	0.0058	0.4008	0.1698	0.0383	1.3172	1.9320
2_{15}	7.0280	0.00001	0.0010	0.3228	0.1072	0.5371	0.0262	13.8722	14.8665
2_{17}	7.0280*	0	0.0007	0.3348	0.3573	0.0048	0.0049	0.9106	1.6131
2_{18}	7.0280**	0	0	0	1.5404	0	0.6822	5.4929	7.7155
	Sum	6.6738	9.4072	16.3693	11.8887	18.1321	15.9642	25.3321	103.7670

TABLE XVIII. All interactions $B(M1)$ ^{44}Ti $I = 1$ to $I = 1$.

$I = 1$	$I = 1$ Unshifted energy	1_1 –	1_2 –	1_3 –	Sum
1_1	–	0.1466	0	0	0.1466
1_2	–	0	0.1466	0	0.1466
1_3	–	0	0	0.1466	0.1466
	Sum	0.1466	0.1466	0.1466	0.4398

TABLE XIX. Pairing $B(M1)$ ^{46}Ti $I = 1$ to $I = 1$.

$I = 1$	$I = 1$ Unshifted energy	1_1 1.7500	1_2 2.0000	1_3 2.0000	1_4 2.5000	1_5 2.5000	1_6 2.7500	1_7 2.7500	Sum
1_5	1.7500	0.1466	0	0	0	0	0	0	0.1466
1_1	2.0000	0	0.6726	0.0066	0.5047	0.0883	1.4888	0.00001	2.76104
1_2	2.0000	0	0.0066	0.0328	0.1744	0.7997	0.3788	0.5271	1.9194
1_6	2.5000	0	0.5047	0.1744	1.0309	1.6374	1.3985	0.4689	5.2148
1_7	2.5000	0	0.0883	0.7997	1.6374	0.3968	4.2042	1.1124	8.2388
1_3	2.7500	0	1.4888	0.3788	1.3985	4.2042	6.5643	1.0992	15.1338
1_4	2.7500	0	0.00001	0.52713	0.4689	1.1124	1.0992	0.2795	3.48714
	Sum	0.1466	2.76104	1.9194	5.2148	8.2388	15.1338	3.48714	36.9016

TABLE XX. Q.Q $B(M1)$ ^{46}Ti $I = 1$ to $I = 1$.

$I = 1$	$I = 1$ Unshifted energy	1_1 4.3546	1_2 8.1095	1_3 8.7081*	1_4 10.4611*	1_5 10.5846	1_6 10.8481	1_7 11.6407*	Sum
1_1	4.3546	0.1475	0.00001	0.0003	0.0016	0.0023	0.0012	0.0006	0.15351
1_2	8.1095	0.00001	0.1417	0.0026	0.0084	0.0320	0.0075	0.0060	0.1982
1_5	8.7081*	0.0003	0.0026	0.4191	0.3493	0.6084	0.2609	0.1632	1.8038
1_6	10.4611*	0.0016	0.0084	0.3493	0.7721	4.4337	1.3977	0.8162	7.7790
1_3	10.5846	0.0023	0.0320	0.6084	4.4337	6.9384	3.3583	1.4211	16.7942
1_4	10.8481	0.0012	0.0075	0.2609	1.3977	3.3583	0.5204	0.6096	6.1556
1_7	11.6407*	0.0006	0.0060	0.1632	0.8162	1.4211	0.6096	1.0009	4.0176
	Sum	0.15351	0.1982	1.8038	7.7790	16.7942	6.1556	4.0176	36.9019

TABLE XXI. MBZE $B(M1)$ ^{46}Ti $I = 1$ to $I = 1$.

$I = 1$	$I = 1$ Unshifted energy	1_1 3.65521	1_2 6.05887	1_3 7.78516	1_4 8.73868	1_5 9.46213*	1_6 10.61597*	1_7 11.36444*	Sum
1_1	3.65521	0.20475	0.00235	0.03278	0.17333	0.04688	0.00029	0.06086	0.52124
1_2	6.05887	0.00235	0.12924	0.02634	0.19778	0.01634	0.06996	0.00190	0.44391
1_3	7.78516	0.03278	0.02634	0.35892	3.47085	0.20043	1.73496	0.19646	6.02074
1_4	8.73868	0.17333	0.19778	3.47085	6.05178	1.84761	0.33556	4.24016	16.31707
1_5	9.46213*	0.04688	0.01634	0.20043	1.84761	1.09868	0.02403	0.46457	3.69854
1_6	10.61597*	0.00029	0.06996	1.73496	0.33556	0.02403	0.52859	1.02258	3.71597
1_7	11.36444*	0.06086	0.00190	0.19646	4.24016	0.46457	1.02258	0.20025	6.18678
	Sum	0.52124	0.44391	6.02074	16.31707	3.69854	3.71597	6.18678	36.90425

TABLE XXII. $J_{\max}$ $B(M1)$ ^{46}Ti $I=1$ to $I=1$.

$I=1$	$I=1$ Unshifted energy	1_1 2.4966	1_2 3.0668	1_3 4.8057	1_4 5.1080*	1_5 5.4724	1_6 5.6332*	1_7 7.0280*	Sum
1_1	2.4966	0.1456	0.0004	0.0022	0.0022	0.0003	0.0001	0	0.1508
1_2	3.0668	0.0004	0.0098	0.5060	0.4823	0.1481	0.0067	0.0005	1.2150
1_3	4.8057	0.0022	0.5060	1.4629	2.6995	0.0218	0.3418	0.0659	5.1001
1_5	5.1080*	0.0022	0.4823	2.6995	1.6023	0.2780	0.1540	0.0105	5.2288
1_4	5.4724	0.0003	0.1481	0.0218	0.2780	11.5766	3.0925	1.7817	16.8990
1_6	5.6332*	0.0001	0.0067	0.3418	0.1540	3.0925	1.2682	0.4129	5.2762
1_7	7.0280*	0	0.0005	0.0659	0.0105	1.7817	0.4129	0.8250	3.0965
	Sum	0.1508	1.2150	5.1001	5.2288	16.8990	5.2762	3.0965	36.9051

In some cases, in order to remove degeneracies with schematic interactions we add −1.00 MeV to all the odd-J, $T=0$ matrix elements. This has the effect of shifting the energies by an amount $T(T+1)$. If we did not do this, then states of different isospins would be degenerate and arbitrary mixtures of these states would appear in the computer output. This trick pushes up states of higher isospin to higher energies, but the energies of lower isospin receive a smaller shift or remain unchanged. We call these new energies shifted. These higher isospin states in ^{44}Ti are indicated with a star (∗) for $T=1$ and two stars (∗∗) for $T=2$. Similarly, higher isospin states for ^{46}Ti are given one star (∗) for $T=2$ and two stars (∗∗) for $T=3$. We give the seniority, isospin, and reduced isospin for the pairing interactions so we do not use the star notation for labeling the states. We also present the results in Fig. 1. All $B(M1)$'s are in units of $(\mu_N)^2$.

We here repeat the expressions for the $B(M1)$'s given by Harper and Zamick [19]:

$$B(M1)=\frac{3}{4\pi}\frac{2I_f+1}{2I_i+1}\left[g_{j_p}X_1+(-1)^{I_f-I_i}g_{j_n}X_2\right]^2. \quad (1)$$

$$\text{Here, } g_j=g_l\pm\left\{\frac{g_s-g_l}{2l+1}\right\}, \quad (2)$$

$$g_{s_p}=5.586,\quad g_{l_p}=1, \quad (3)$$

$$g_{s_n}=-3.826,\quad g_{l_n}=0. \quad (4)$$

For the case I_f not equal to I_i we find

$$X_1=(-1)^{I_f-I_i+1}X_2, \quad (5)$$

$$B(M1)=\frac{3}{4\pi}\frac{2I_f+1}{2I_i+1}(g_{jp}-g_{jn})^2X_1^2. \quad (6)$$

III. SELECTION RULES FOR THE PAIRING INTERACTION

In a previous work [19] we commented on selection rules for vanishing $B(M1)$'s with a $J=0\,(T=1)$ pairing interaction. The basis states were written as (v,T,t) seniority, isospin, and reduced isospin. We briefly repeat the selection rules here and refer to Tables II–V. For the $J=0\,(T=1)$ pairing interaction we previously found the following:

(a) Transitions with $\Delta T=2$ or more are forbidden.

(b) For N=Z nuclei, $T=1$ to $T=1$ $M1$ transitions are zero.

(c) $\Delta v=4$ $M1$ transitions are forbidden.

(d) Transitions in which both v and t change are forbidden.

Here we discuss case (d) in more detail than we did in [19]. In say, ^{46}Ti we break the six nucleons into three pairs. We cannot have an $M1$ transition involving only a pair of identical particles; we must consider a neutron-proton pair. The only way to change seniority is to create or destroy a $J=0\,T=1$ pair. The reduced isospin excludes $J=0\,T=1$ pairs. If the $M1$ operator acts on a $J=0\,(T=1)$ pair it creates a $J=1\,(T=0)$ pair. Since this new pair has $T=0$ it will not change the reduced isospin. Alternately, if we act on a $J=1\,(T=0)$ pair we note that because it has $T=0$, it does not contribute to the reduced isospin. The $M1$ operator will change this to a $J=0\,(T=1)$ pair and such pairs are excluded from the reduced isospin set. Hence, if we change seniority we cannot change the reduced isospin. These arguments of course also explain (c), why v cannot change by more than two units.

In Tables IV and V, we show using the $J=0$ pairing interaction the ^{46}Ti transitions from 1^+ to 0^+ states and 1^+ to 2^+ states respectively. We find an abundance of confirmations of rule (d). We note in Table IV that all transitions from the $J=0^+$ (220) configuration to 1^+ states except for (220) vanish. These latter 1^+ states have configurations (411), (611), and (421). In Table V we see that $B(M1)$'s from $J=2^+$ (410) to 1^+ (611) vanish.

In Table V we also see that $\Delta v=4$ $B(M1)$'s are zero, e.g., (211) to (611). Note that the $\Delta T=2$ transitions from the last 2^+ state (231) to $J=1^+\,T=1$ states all vanish. This selection rule is the easiest to understand, i.e., in terms of the Wigner-Eckart theorem.

IV. SELECTION RULES FOR THE Q.Q AND $J_{\max}$ $T=0$ INTERACTIONS

We find also some vanishing $B(M1)$'s when the quadrupole-quadrupole interaction Q.Q is employed. Some are not surprising like the $T=1$ to $T=1$ transitions in $N=Z$ ^{44}Ti shown in Table VII. Likewise, the $\Delta T=2$ transitions in Table IX from the 2_{18} $T=3$ state in ^{46}Ti to all $T=1$, $J=1^+$ states. However, the vanishing $B(M1)$'s in the top line of Table VIII involving $J=0^+$ and $J=1^+$ states in ^{46}Ti are hard to explain and we will not attempt to do so here. The vanishings are from the lowest 0^+ state to two $T=1$ states and one $T=2$

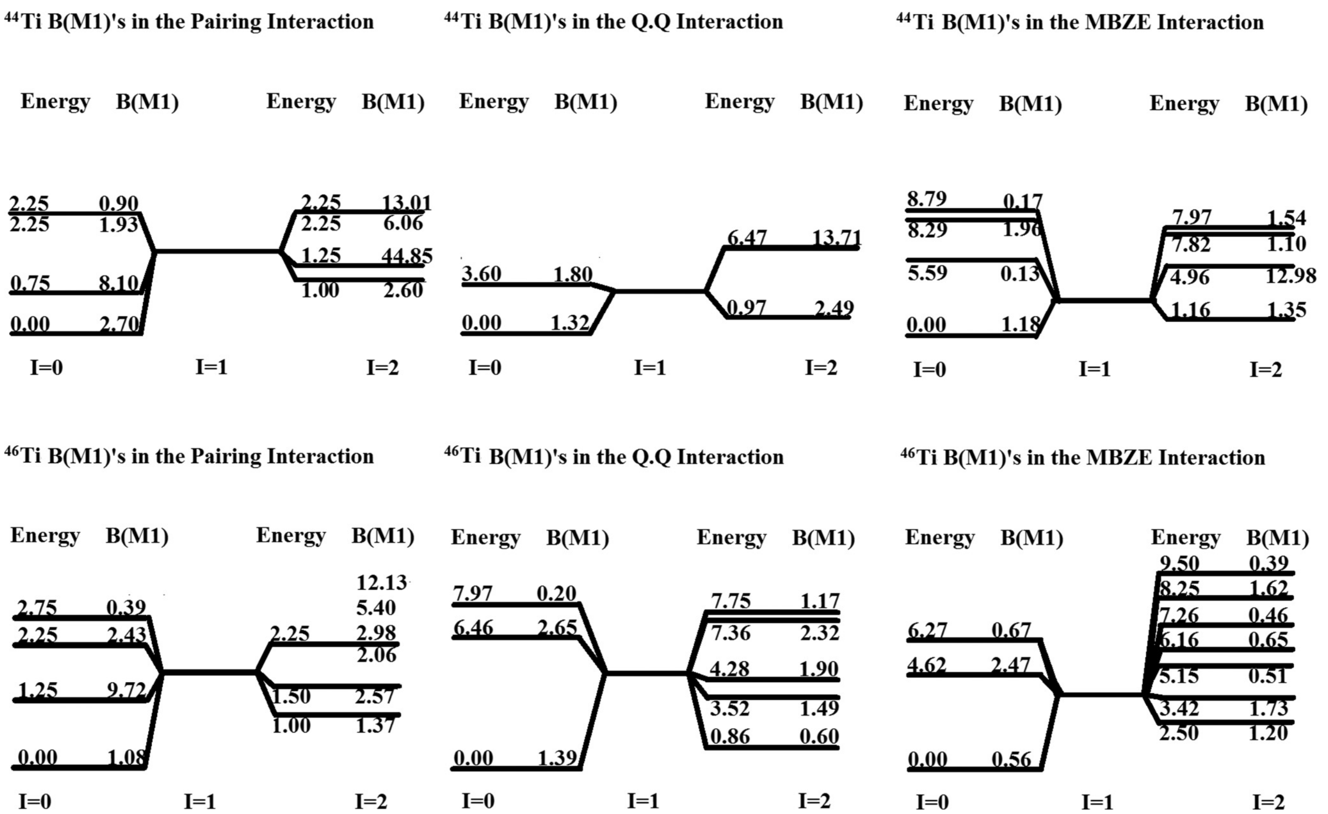

FIG. 1. Strong $B(M1)$ diagrams.

state. But we have nonvanishings to other $T=1$ and $T=2$ states, so there is no simple connection with isospin.

There are no vanishings for the latter states except of course in the bottom row where we have the $\Delta T = 2$ selection rule. That is to say, the 0_6 state has $T=3$ and will not connect to $J=1^+, T=1$ states.

There are other peculiarities with the Q.Q interaction. As noted in [19], in ^{44}Ti there is a degenerate pair of $J=2^+$ states at 7.75 MeV: one has isospin $T=0$ and the other $T=2$. Likewise we find some hard to understand selection rules for the $J_{\max}$ ($T=0$) interaction. In Table XVI we find for ^{46} Ti, from the lowest 0^+ state there are vanishing $B(M1)$'s to one $T=1$ state and two $T=2$ states. As in the case with Q.Q this is hard to understand. Note that the sum of sums, because of orthonormality and completeness, is independent of the interaction used and has the value:

$$SS = \frac{3}{4\pi}\frac{2I_f+1}{2I_i+1}(g_p - g_n)^2 \sum_{J_p J_n} U(1, J_p I_f J_n; J_p I_i)^2 \times J_p(J_p+1). \quad (7)$$

V. NONMONOTONIC BEHAVIOR OF THE $B(M1)$ 1_1 TO 0_1 AS ONE SWITCHES FROM $J=0$ PAIRING TO $J_{\max}$ PAIRING

Let us focus on the 1_1^+ transitions. The conventional scissors mode excitation is from 0_1^+ to 1_1^+ which will be a factor of 3 larger than the reverse transition 1_1^+ to 0_1^+. With the Q.Q interaction we note, however, that there are even larger $B(M1)$'s to other states. In ^{46}Ti whereas the $B(M1)$ for 1_1 to 0_1 is 1.3901, from 1_1 to 0_2 it is 2.6505, almost twice as large. One possible explanation of this is that the 0_2 state is a double scissors mode excitation.

Let us however now focus on the 1_1^+ to 0_1^+ in ^{46}Ti, i.e., the conventional spin-scissors mode. Some values from the above tables are given in Table XXIII.

It is puzzling that Q.Q and MBZE are so different because there is a big overlap between their respective wave functions. To better understand this we now consider simple interactions which are mixtures of $J=0$ pairing and $J_{\max}$ pairing:

$$V = a\delta_{J=0} + b\delta_{J=7}. \quad (8)$$

We present the $B(M1)$ for selected values of (a,b) in Table XXIV.

We see a fairly complicated behavior: relatively large $B(M1)$'s at the two limits, $J=0$ pairing and $J_{\max}$ pairing. However, for equal $J=0$ and $J=J_{\max}=7$ pairing the value is much smaller 0.210 as compared with 1.080 and 1.641. We get a nonmonotonic behavior for this spin-scissors mode. We

TABLE XXIII. Comparison of 1_1^+ to 0_1^+ in ^{46}Ti.

Interaction	Table	$B(M1)$
$J=0$ pairing	VI	1.0799
Q.Q	VII	1.3901
MBZE	XII	0.55962

TABLE XXIV. $B(M1)$ for a mixture of pairing and J_{max}.

a	b	B(M1)	
–1	0	1.082	$J = 0$ pairing
–1.15	–1	0.210	Close to lowest $B(M1)$
–1	–1	0.260	Equal $J = 0$ and $J = J_{max}$ pairing
0	–1	1.641	J_{max} pairing

get the lowest possible $B(M1)$ for (a,b) close to $(-1.15, -1)$, i.e., $B(M1) = 0.210$. Going back to Q.Q and MBZE, evidently there is more $J = 0$ and $J = J_{max}$ interference in MBZE than there is in Q.Q.

VI. ADDITIONAL COMMENTS

We note that the $B(M1)$ from the lowest 1^+ to the lowest 0^+ (generally considered the scissors mode transition) is considerably smaller than the transition from this 1^+ to all 0^+ states. For example with MBZE (the most realistic interaction here) the respective numbers are 1.6533 and 3.6136. The respective numbers from 1^+ to 2^+ are 1.0252 and 6.6738.

Note in Table XIV that along the diagonal of the one-to-one "transitions" in ^{44}Ti the values of $B(M1)$ are all the same. Of course they are not real transitions, but they can be related to the magnetic moments. Note that for N=Z nuclei in the single j approximation the magnetic g factor is independent of the details of the wave function. As seen in the Appendix of [19] the value is

$$g = \frac{g_p + g_n}{2} = 0.55. \quad (9)$$

This explains why all the diagonal $B(M1)$'s are the same in ^{44}Ti. This is not the case in ^{46}Ti. The off-diagonal zeros in Table XV are due to the fact, as mentioned in [19], that in $N = Z$ nuclei, transitions from T to the same T (in this case $T = 1$) are forbidden.

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