Energy resolution with the Lorentz integral transform

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A brief outline of the Lorentz integral transform (LIT) method is given. The method is well established and allows treatment of reactions into the many-body continuum with bound-state-like techniques. The energy resolution that can be achieved is studied by means of a simple two-body reaction. From the discussion it will become clear that the LIT method is an approach with a controlled resolution and that there is no principle problem to even resolve narrow resonances in the many-body continuum. As an example, the isoscalar monopole resonance of ^{4}He is considered. The importance of the choice of a proper basis for the expansion of the LIT states is pointed out. By employing such a basis, a width of 180(70) keV is found for the ^{4}He isoscalar monopole resonance when using a simple central nucleon-nucleon potential model.

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I. INTRODUCTION

In recent years, ab initio approaches have gained increasing importance in the calculation of bound-state and reaction observables of nuclear systems with a nucleon number $A \geqslant 4$ (for a comparison of various techniques, see Refs. [1,2]). A particularly challenging aspect in such calculations is the proper treatment of resonances. Recently, via the EIHH (effective interaction hyperspherical harmonic) expansion technique [3], the Lorentz integral transform (LIT) method [4,5] was applied to calculate the isoscalar monopole strength of the ^{4}He(e,e') reaction with realistic nuclear forces, with special attention to the role of the low-lying 0^+ resonance of ^{4}He [6]. While the resonance strength could be obtained, it was not possible to determine the width of the resonance. The origin of this deficiency arises from the low density of LIT states in the resonance region, but it was not really clear why the density happens to be so low. One may think of various possible reasons: (i) the LIT method is based on bound-state techniques and might be incapable of describing the shape of narrow resonances, (ii) the used expansion in hyperspherical harmonics (HH) was not extended to a sufficiently large basis, or (iii) the HH expansion itself is not able to have a sufficient density of LIT states in the resonance region. The first possibility can be safely ruled out. In fact, in Ref. [7] it was shown that for a fictitious two-nucleon system with a narrow resonance in the $^3P_1$ partial wave, exact information about the shape of the resonance could be obtained with the LIT method. Thus the aim of the present paper is to study which of the two remaining reasons is responsible for the partial failure of the LIT method in Ref. [6] and how one can improve the calculation to get precise information on the resonance width.

To clarify the matter, first a classical two-body problem, namely deuteron photodisintegration in unretarded dipole approximation, is discussed. It is shown how the density of LIT states can be increased in this case. In a next step, it is considered how one can achieve such an increase also for the above-mentioned isoscalar monopole strength of the ^{4}He(e,e') reaction. In order to do so, a central NN potential model [Trento (TN) potential] is used that had been previously employed in the very first LIT applications for the calculation of the electromagnetic breakup of ^{4}He by electrons and real photons [8,9]. It is interesting to note that in Ref. [8] one already finds a clear signal of the 0^+ resonance in the ^{4}He longitudinal response function; however, at that early stage of LIT applications the resonance was not studied in greater detail.

The paper is organized as follows. Section II contains a brief description of the LIT method. In Sec. III the question of the density of the LIT states is addressed and illustrated for the above-mentioned two-body case. A LIT calculation for the isoscalar monopole strength of ^{4}He(e,e') is discussed in Sec. IV, where it is shown how an HH calculation can be modified in order to determine the width of narrow resonances. Finally, a summary is given in Sec. V.

II. LORENTZ INTEGRAL TRANSFORM (LIT)

Over the years the LIT approach [4] has been applied to a variety of inelastic electroweak reactions. A rather large number of applications can be found in the review articles [1,5].

The LIT of a function $R(E)$ is defined as follows:

$$L(\sigma) = \int dE\, \mathcal{L}(E,\sigma)\, R(E), \tag{1}$$

where the kernel $\mathcal{L}$ is a Lorentzian,

$$\mathcal{L}(E,\sigma) = \frac{1}{(E-\sigma_R)^2 + \sigma_I^2} \tag{2}$$

($\sigma = \sigma_R + i\sigma_I$); the parameter σ_I controls the width of the Lorentzian. Because of the adjustable width, and unlike many other integral transforms, the LIT is a transform with a controlled resolution. However, aiming at a higher resolution by reducing σ_I might make it necessary to increase the precision of the calculation. This point is discussed in greater detail in Sec. III.

For inclusive reactions the LIT $L(\sigma)$ is calculated by solving an equation of the form

$$(H-\sigma)\,\tilde{\Psi} = S\,, \tag{3}$$

where H is the Hamiltonian of the system under consideration and S is an asymptotically vanishing source term related to the operator inducing the specific reaction. The solution

$\tilde{\Psi}$ is localized. This a very important property, since it allows determination of $\tilde{\Psi}$ with bound-state methods, even for reactions where the many-body continuum is involved.

Having calculated $\tilde{\Psi}$, one obtains the LIT from the following expression:

$$L(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle \,. \tag{4}$$

The response function $R(E)$ is determined from the calculated $L(\sigma)$ by inverting the transform. A general discussion of the inversion and details about various inversion methods are given in Refs. [5,7,10,11].

An alternative way to express the LIT is given by

$$L(\sigma) = -\frac{1}{\sigma_I} \mathrm{Im} \left(\langle S | \frac{1}{\sigma_R + i\sigma_I - H} | S \rangle \right) . \tag{5}$$

This reformulation is useful since it allows a direct application of the Lanczos algorithm for the determination of $L(\sigma)$ [12]. In fact, the calculations discussed in the following sections are performed using expansions on basis sets with a subsequent use of the Lanczos technique.

In order to calculate a specific reaction one has to specify the source term S in Eqs. (3) and (5). In the case of an inclusive electroweak reaction, response functions have the general form

$$R(E_f) = \int dE_f |\langle f | \theta | 0 \rangle|^2 \delta(E_f - E_0 - \omega) \,, \tag{6}$$

where $|0\rangle$ and $|f\rangle$ are ground-state and final-state wave functions of the system under consideration, while E_0 and E_f are the corresponding eigenenergies and $\omega = E_f - E_0$ is the energy transferred to the system. θ is the specific transition operator that induces the reaction. Note that unlike the normal convention, here the argument of the response function R is E_f and not ω.

The source term S for inclusive reactions has the following form:

$$|S\rangle = \theta |0\rangle \,. \tag{7}$$

A solution of the LIT equation (5) via an expansion on a basis with N basis functions can be understood as follows: One determines the spectrum of the Hamiltonian for this basis, thus finding N eigenstates ϕ_n with eigenenergies E_n. The energies E_n define the positions of the above-mentioned LIT states. Furthermore, the LIT solution assigns to any eigenenergy a Lorentzian with strength S_n and width σ_I. It should be noticed that the source term $|S\rangle$ solely affects the strength, leading to

$$S_n = |\langle \phi_n | \theta | 0 \rangle|^2 \,. \tag{8}$$

The LIT result then reads

$$L(\sigma) = \sum_{i=1}^{N} \frac{S_n}{(\sigma_R - E_n)^2 + \sigma_I^2} \,. \tag{9}$$

Note that this result is related to the so-called Lanczos response R_{Lnczs} by

$$R_{\mathrm{Lnczs}}(\omega, \sigma_I) = \frac{\sigma_I}{\pi} L(\sigma_R = E_0 + \omega, \sigma_I) \,. \tag{10}$$

In the limit $\sigma_I \to 0$ the Lanczos response is equal to the true response function R. However, one often calculates R_{Lnczs} for a small but finite σ_I value and identifies the Lanczos response with the true response, which in general is an uncontrolled approximation. In the LIT approach one does not make such an identification of transform and response function. A proper treatment requires an inversion (see discussion in Ref. [5]).

It is important to note that the definition of the LIT in Eq. (1) contains the full response function R with all breakup channels. For the calculation of the LIT one may use any complete localized A-body basis set. Automatically, for any such set, strength from all breakup channels is contained in the LIT. However, in principle, it can happen that in a specific energy interval of a given reaction one basis set is more advantageous than another one. In fact, such a case is discussed in Sec. IV.

III. A SIMPLE TWO-BODY PROBLEM

To better illustrate the LIT energy resolution, a simple two-body case is discussed, namely deuteron photodisintegration in unretarded dipole approximation. The corresponding cross section is given by

$$\sigma_{\mathrm{unret}}(\omega) = 4\pi^2 \, \alpha \, \omega \, R_{\mathrm{unret}}(E_f = E_0 + \omega) \,, \tag{11}$$

where ω denotes the photon energy and α is the fine structure constant. The transition operator for the response function R_{unret} is the dipole operator,

$$\theta = \sum_{i=1}^{2} z_i \, \frac{(1 + \tau_{i,z})}{2} \,, \tag{12}$$

where z_i and $\tau_{i,z}$ are the z components of the position vector and of the isospin operator of the ith nucleon, respectively. In the case of the deuteron, the dipole operator induces only transitions to the np final states $^3P_0$, $^3P_1$, and $^3P_2$-$^3F_2$. For simplicity in the following example, only transitions to the $^3P_1$ partial wave are considered. The ansatz for the corresponding $\tilde{\Psi}$ reads

$$|\tilde{\Psi}\rangle = R(r) \, |(l = 1, S = 1) j = 1\rangle \, |T = 1\rangle \,, \tag{13}$$

where r, l, S, j, and $T = 1$ is the relative distance, orbital angular momentum, total spin, total angular momentum, and isospin of the np pair, respectively. The resulting LIT equation can be easily solved by direct numerical methods or by expansions of $R(r)$ on a complete set. For nuclei with $A > 2$ very often HH expansions are used with separate hyperspherical and hyper-radial parts, where the latter is usually expanded in Laguerre polynomials $L_n^{(m)}$ times an exponential fall-off. Therefore a corresponding ansatz is made here for $R(r)$,

$$R(r) = r \sum_{n=0}^{N} c_n \, L_n^{(1)}(r/b) \, \exp\left(-\frac{r}{2b} \right) , \tag{14}$$

where c_n are the expansion coefficients and b is a parameter regulating the spatial extension of the basis.

For the following LIT results of the $^3P_1$ channel, the AV18 NN potential [13] is used. First the Hamiltonian eigenvalues E_n, entering in Eq. (9), are studied for various basis sets. In the upper panel of Fig. 1 results are shown with 11 basis functions

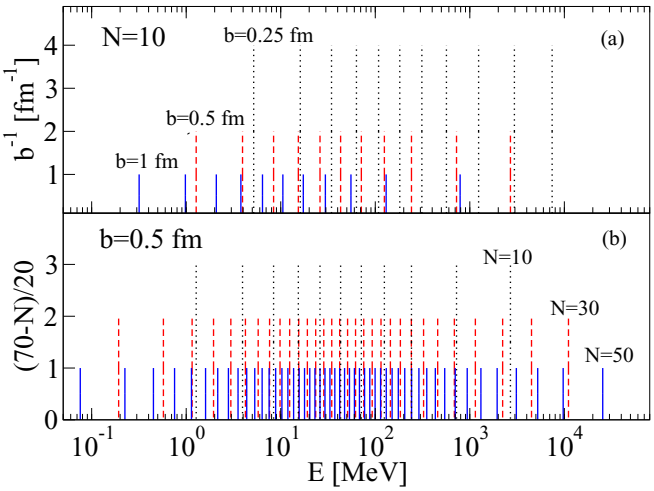

FIG. 1. (Color online) Spectrum of the Hamiltonian eigenenergies for the $^3P_1$ NN channel with the AV18 NN potential for the basis of Eqs. (13) and (14) with various values of N and b: $N = 10$ with $b = 1$, 0.5, and 0.25 fm in panel (a) and $b = 0.5$ fm with $N = 10$, 30, and 50 in panel (b).

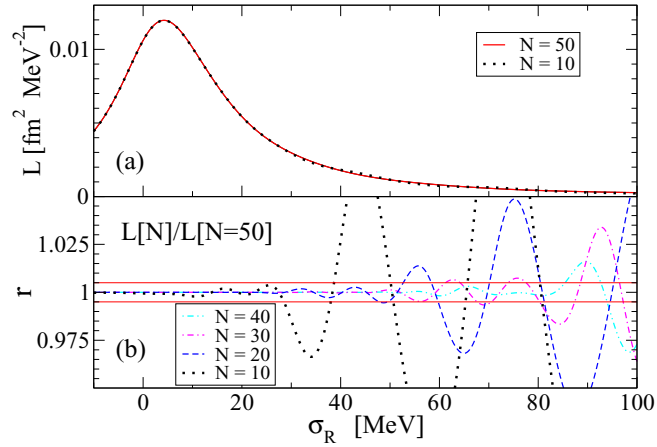

FIG. 2. (Color online) LIT $L(\sigma)$ of the $^3P_1$ channel with $\sigma_I = 10$ MeV and $b = 0.5$ fm. (a) $N = 10$ and 50; and (b) the ratio r of the LIT with $N = 10$, 20, 30, and 40 to the LIT with $N = 50$ (the values 0.995 and 1.005 are illustrated by full lines).

($N = 10$) and different values for the extension parameter b. One sees that a greater spatial extension of the basis functions leads to a shift of the spectrum to lower energies. As lowest (highest) eigenenergies one finds 5.17 (7424), 1.27 (2689), and 0.32 (790) MeV for $b = 0.25$, 0.5, and 1 fm, respectively. To obtain a higher density of LIT states one has to increase the number of the basis states N. This is illustrated in the lower panel of Fig. 1, where the cases with $N = 10$, 30, and 50 are shown for $b = 0.5$ fm. It is evident that the increase of basis functions leads not only to a higher density of LIT states but also to an extension of the eigenvalues E_n both to lower and higher energies. In fact as lowest (highest) values one has now 1.27 (2689), 0.19 (11056), and 0.076 (25457) for $N = 10$, 30, and 50, respectively. If one chooses the energy range up to pion threshold, one finds that in all three cases about two-thirds of the N LIT states are located therein.

After having discussed the energy distribution of the LIT states we come to the transform $L(\sigma)$ itself. First, in Fig. 2, the case with $\sigma_I = 10$ MeV and $b = 0.5$ fm is illustrated. The upper panel of the figure shows that an increase of the number of basis functions from 11 ($N = 10$) to 51 ($N = 50$) does not lead to noticeable differences. In other words, in this case, the LIT is already quite well converged with a rather small basis. This nice agreement of both results is not obvious, since Fig. 1 exhibits rather different positions and densities of the LIT states for both cases. This has to be interpreted as follows. A discretization of the continuum has no direct physical meaning and leads in principle to random results. On the contrary, one can use the discretization to calculate integral transforms, as for example the LIT, which leads in convergence to a unique result despite the randomness of the discrete eigenenergies.

In order to find differences between the two results of Fig. 2(a), one has to present them in a different way, as is done in Fig. 2(b). There one sees that the agreement is extremely good up to about 30 MeV and thus one can consider the LIT for $\sigma_I = 10$ MeV to be already converged in this energy range with only very few basis functions. Beyond 30 MeV the LIT is not yet converged with $N = 10$ and starts oscillating about the LIT with $N = 50$ with differences becoming even greater than 5%. Because of the regular oscillations about the more precise result, it should not lead to serious inversion problems as long the response does not contain any specific structure at higher energies. If one wants to check the existence of such structures, one should improve the precision of the calculation by enlarging the basis. In fact, with larger N values of 20, 30, and 40, one finds an increased high-precision range with relative differences compared to the case with $N = 50$ of less than 0.5% up to about 50, 60, and 85 MeV, respectively. However, in order to search for possible structures it is not the proper strategy to just enlarge the basis. One has to consider that a resonance with a width considerably smaller than σ_I is smoothed out in the LIT such that the details of the shape are hidden in tiny contributions to the transform. To disentangle the details one should reduce σ_I, increasing in this way the energy resolution of the LIT.

The situation for smaller values of σ_I is illustrated in Fig. 3, where LIT results are shown up to about pion threshold for $\sigma_I = 0.1$, 1, 2.5, and 10 MeV and for various values of N. For the smallest value, $N = 10$ [Fig. 3(a)], one finds isolated Lorentzian peaks. In the case of $\sigma_I = 0.1$ MeV they appear already at low energy, for $\sigma_I = 1$ MeV at somewhat higher energy, and for $\sigma_I = 2.5$ MeV at even higher energy. Since the density of LIT states grows with growing N (see other panels of Fig. 3) isolated peaks are pushed to higher energies if N is increased. In addition one notes that any decrease of σ_I, i.e., any increase of the resolution, shrinks the convergence range for a given value of N. For the smallest σ_I value of 0.1 MeV a rather strong oscillatory behavior is still present at very low energies even with $N = 50$. In order to get a smooth and converged LIT also in this case one would need to increase N considerably.

For a reliable inversion of $L(\sigma)$ the transform has to be sufficiently converged for a given σ_I. In particular, isolated

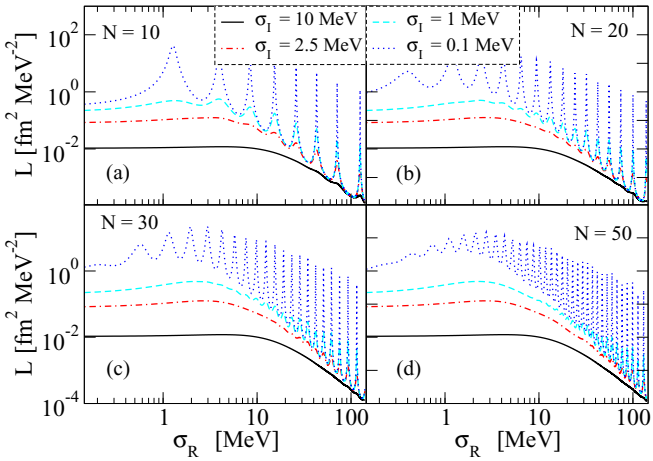

FIG. 3. (Color online) LIT $L(\sigma)$ of the $^3P_1$ channel with $\sigma_I =$ 0.1, 1, 2.5, and 10 MeV in all four panels ($b = 0.5$ fm), but different N values in the various panels: 10 (a), 20 (b), 30 (c), and 50 (d).

peaks of single Lorentzians should not appear; i.e., for any σ_R value one should have a significant contribution from more than one Lorentzian. Of course, for the present two-body case one could increase N further without greater problems, but in a general many-body calculation this might not be easily possible. On the other hand, it is not necessary to work with a single σ_I value only. Considering again the LIT results of Fig. 3 with $N = 50$ one sees that for $\sigma_I = 1$ MeV a rather converged result is obtained up to about 10 MeV. Thus structures of a relatively small width would leave a visible signal in the transform in this energy range. In fact, one can use this LIT in the low-energy range and combine it with a LIT with a larger σ_I for higher energies. In general one can proceed as follows: one defines a new transform T in an interval $[\sigma_0,\sigma_M]$ by combining M transforms $L_m = L(\sigma_R,\sigma_{I,m})$ such that only the LIT with the specific $\sigma_I = \sigma_{I,m}$ enters significantly in the energy range $[\sigma_{R,m-1},\sigma_{R,m}]$:

$$T = L_1 f_1 + \sum_{m=2}^{M-1} a_m L_m (1 - f_{m-1}) f_m + a_M L_M (1 - f_{M-1}) . \tag{15}$$

The function f_m in Eq. (15) is a smooth cutoff from 1 to 0 at $\sigma_R = \sigma_{R,m}$, for example, $f_m(\sigma_R \leqslant \sigma_{R,m}) = 1$ and $f_m(\sigma_R > \sigma_{R,m}) = \exp(-[(\sigma_R - \sigma_{R,m})/\Delta]^{2n})$, where one has to take reasonable values for the parameters Δ and n. The coefficients a_m in Eq. (15) can be chosen such that size of the transform does not change too drastically from one energy range to the next. Thus, for the case of the LITs of Fig. 3 with $N = 50$, one could set $\sigma_{I,1} = 1$, $\sigma_{I,2} = 2.5$, and $\sigma_{I,3} = 10$ MeV with $\sigma_{R,1} = 10$ and $\sigma_{R,2} = 30$ MeV. In order to improve the precision in the threshold region even further one could include the case with $\sigma_I = 0.1$ MeV, but one would need to further increase N. Alternatively, keeping $N = 50$, one could check the convergence behavior of a LIT with a somewhat larger σ_I, for example, $\sigma_I = 0.25$ MeV.

The discussion above shows that the LIT is an approach with a controlled resolution. In an actual calculation one should check which lowest σ_I leads in a specific energy range to a sufficiently converged and smooth LIT without a single LIT state sticking out. Structures which are considerably smaller than such a σ_I value cannot be resolved by the inversion. A helpful criterion is given in Ref. [7] (see discussion of Fig. 7 in Ref. [7]).

IV. ^{4}He ISOSCALAR MONOPOLE RESPONSE FUNCTION

The isoscalar monopole response function $M(q, E_f = E_0 + \omega)$ can be determined in inclusive electron scattering (q and ω represent momentum and energy transferred by the virtual photon to the nucleus). In this case the transition operator θ of Eq. (6) becomes q dependent and takes the form

$$\theta(q) = \frac{G_E^s(q^2)}{2} \sum_{i=1}^{A} j_0(q r_i) , \tag{16}$$

where $G_E^s(q^2)$ is the nucleon isoscalar electric form factor, $\boldsymbol{r}_i$ is the position of nucleon i, and j_0 is the spherical Bessel function of 0th order.

Experimental investigations of the ^{4}He(e,e') reaction in Refs. [14–16] revealed a 0^+ resonance located less than 1 MeV above the ^{4}He breakup threshold with quite a narrow width of about 250 keV. It is interesting to note that very recently a EIHH-LIT calculation was carried out, where it is pointed out that the resonance might be interpreted as a breathing mode [17]. Unfortunately, this and the preceding EIHH-LIT calculations [6] were only able to determine a resonance strength but not a resonance width since the density of LIT states in the resonance region was too low. On the other hand, with the experience made in the previous section it seems to be easy to increase the density of LIT states also in the region of the ^{4}He isoscalar monopole resonance, namely by increasing the number of HH basis states. Unfortunately with an HH expansion this does not work very well in the energy region below the three-body breakup threshold. To illustrate this, the first LIT application [8], already mentioned in the introduction, is considered in the following: The calculation was carried out for inclusive electron scattering off ^{4}He and used a correlated HH (CHH) basis, where NN short-range correlations are introduced to accelerate the convergence of the HH expansion. The NN interaction (TN potential, Coulomb force included) consisted of a central spin-dependent interaction active only in even NN partial waves. In Fig. 4 we show an unpublished result for the LIT of the response function M from this calculation. One sees that there is only an isolated LIT state in the resonance region. In this specific case one finds the next LIT state only about 0.1 MeV below the four-body breakup threshold at $\sigma_R = 0$ MeV. On the contrary, for positive values of σ_R one has quite a high density of LIT states. An increase of the number of hyperspherical and/or hyper-radial basis states does not change the general picture of having only very few LIT states for negative σ_R values, as in fact it was the case in the LIT-EIHH calculations [6,17]. In addition it was checked for the present work that an increase of the parameter b in the hyper-radial functions ($\sim L_n^{(8)}(\rho/b)\exp(-\rho/2b)$) in the four-body CHH calculation of Ref. [8] does not lead to any significant change concerning the low-energy density of LIT states.

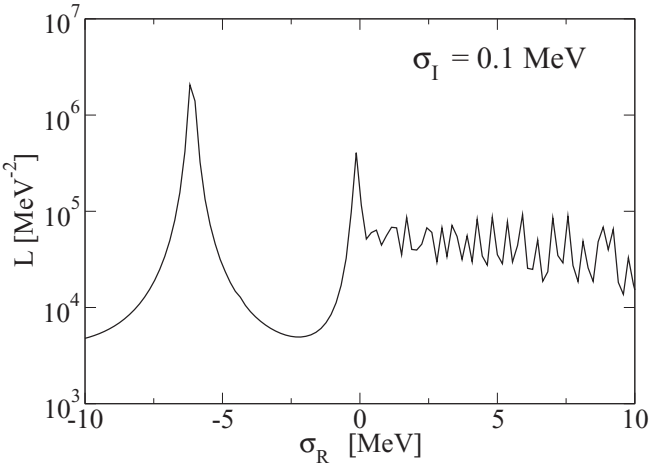

FIG. 4. LIT $L(\sigma)$ of $M(q,\omega)$ at $q = 300$ MeV/c (TN potential) of ^{4}He with four-body CHH basis from Ref. [8].

At first sight it is not understandable why an increase of the HH basis states does not have a significant effect on the density of LIT states in the energy range below the three-body breakup threshold. On the other hand one has to consider that the dynamical variable for a two-body break-up, i.e., the relative vector $\mathbf{r}'_4 = \mathbf{r}_4 - \sum_{i=1}^{3} \mathbf{r}_i/3$ between the free nucleon and the bound three-body system, does not appear explicitly in the HH formalism. Therefore it might be helpful to use a different basis, where $\mathbf{r}'_4$ is taken into account as variable explicitly. In the following, the four-body calculation is switched to such a basis. The new basis states consist of a product of a three-body CHH basis state for the first three nucleons times a single-particle basis state for the fourth nucleon. The CHH basis state is then again a product of a hyperspherical basis state $\mathcal{Y}_{[K\alpha]}(\Omega)$ times a hyper-radial basis state $R_n(\rho)$, where the former depends on the grand angle Ω and is characterized by the grand-angular quantum number K and a set of other quantum numbers $[\alpha]$, whereas the latter is proportional to $L_n^{(5)}(\rho/b)\exp(-\rho/2b)$ (for a detailed definition of the HH basis, see, for example, Ref. [3]).

The single-particle states are defined as follows:

$$|\Phi(\mathbf{r}'_4)\rangle = \sum_{l_4 m_{s_4} m_{t_4}} \phi_{l_4}(r'_4)\,|l_4 m_{l_4}\rangle\,|s_4 m_{s_4}\rangle\,|t_4 m_{t_4}\rangle\,, \qquad (17)$$

where l_4, $s_4 = 1/2$, and $t_4 = 1/2$ are the orbital angular momentum, spin, and isospin quantum numbers, respectively, and m_{l_4}, m_{s_4}, and m_{t_4} denote the corresponding projections. The radial wave function $\phi_{l_4}(r'_4)$ is given by

$$\phi_{l_4}(r'_4) = (r'_4)^{l_4} \prod_{i=1}^{3} f(r_{i4}) \sum_{n_4=0}^{N_4-1} c_{n_4} L_{n_4}^{(2)}(r'_4/b_4) \exp\left(\frac{-r'_4}{2b_4}\right), \qquad (18)$$

where $f(r_{i4}) = f(|\mathbf{r}_i - \mathbf{r}_4|)$ is the NN correlation function (same correlation as in CHH basis). The CHH basis provides antisymmetric states for the first three nucleons, whereas the product of the CHH basis states and the single-particle states has to be antisymmetrized in order to have totally antisymmetric basis states for all four particles. In order to calculate the ^{4}He ground state or the transitions induced by the action of the operator $\theta(q)$ of Eq. (16) on the ^{4}He ground state one has to couple the CHH basis states with those of Eq. (17) to a total angular momentum L equal to zero, and also the total spin (isospin) wave function of the four nucleons has to be coupled to a total spin S (isospin T) equal to zero.

TABLE I. Convergence of ^{4}He binding energy BE with the TN potential (Coulomb force included): l_4 denotes the orbital angular momentum of the single-particle motion [see Eqs. (17) and (18)], K_3 is the grand-angular quantum number of the three-body CHH states, "sym" and "mixed" indicate that the CHH state is symmetric and mixed symmetric, respectively (note that for $l_4 > 0$ two symmetric CHH states and only the lowest mixed symmetric state are taken).

l_4	K_3	CHH symmetry	No. CHH states	BE (MeV)
0	0	sym	1	28.66
0	4	sym	1	30.11
0	6	sym	1	30.56
0	8	sym	1	30.67
0	10	sym	1	30.77
0	12	sym	2	30.82
0	14	sym	1	30.85
0	2	mixed	1	31.28
0	4	mixed	1	31.32
1	1,3,5	sym and mixed	3	31.39
2	2,4	sym and mixed	3	31.41

The calculations described in the following proceed in the same way as in Refs. [8,9], namely by using a nine-dimensional Monte Carlo integration for the evaluation of the various Hamiltonian and norm matrix elements. Such an approach leads to reliable results (see the benchmark test in Ref. [18]).

In Table I the convergence of the ^{4}He binding energy is illustrated. It is evident that the dominant contribution comes from the basis functions where the three-body CHH basis is in a symmetric state and the relative angular momentum l_4 of the single-particle motion is equal to zero. Mixed symmetric CHH states together with $l_4 = 0$ enhance the binding energy by about 0.5 MeV. The contribution of states with $l_4 > 0$ is very small and leads to a further increase of about 0.1 MeV (antisymmetric CHH states have been neglected). The final result of 31.41(5) MeV agrees very well with the converged result of the four-body CHH calculation of 31.40(5) MeV.

For the calculation of the LIT of the ^{4}He isoscalar monopole response function $M(q,\omega)$ hyper-radial and radial basis states are chosen with a rather large extension in space. This should lead to a sufficiently high density of LIT states at low energy (see Fig. 1). The following choice is made: $b = 4$ fm (hyper-radial CHH states) and $b_4 = 1.33$ fm (radial single-particle states). For the hyper-radial part, 15 basis functions are taken, whereas the number of the radial single-particle basis states is kept variable and denoted by N_4. Since the main interest of this investigation is concentrated on the low-energy part of $M(q,\omega)$ only those three-body CHH basis states which lead to a significant contribution to the ^{4}He binding energy (see Table I) are taken into account: symmetric CHH states up to $K_3 = 6$ and the mixed symmetric CHH state

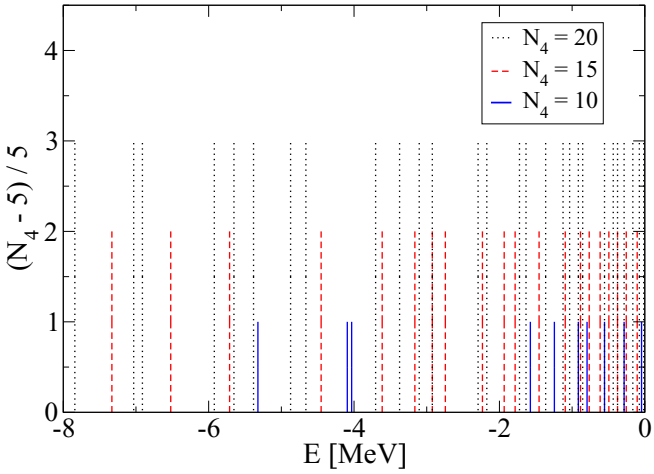

FIG. 5. (Color online) Spectrum of the Hamiltonian eigenenergies for the ^{4}He($J^\pi = 0^+$) states with the TN NN potential for the basis described in the text with N_4 basis functions for the radial single-particle basis.

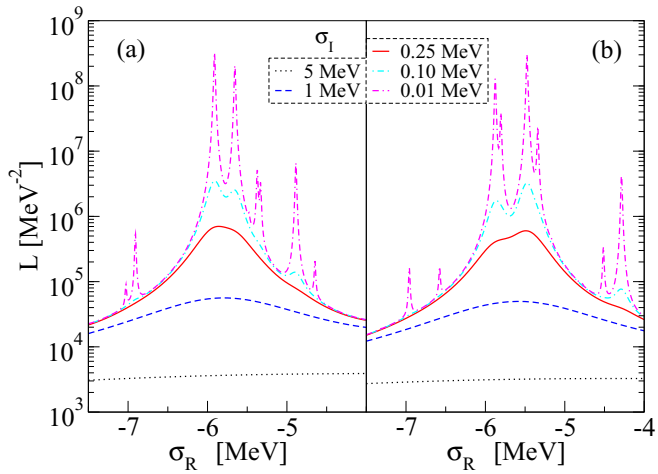

FIG. 6. (Color online) LIT $L(\sigma)$ of $M(q,\omega)$ at $q = 250$ MeV/c (TN potential) of ^{4}He with $\sigma_I = 0.01$, 0.1, 0.25, 1, and 5 MeV calculated with the new basis states [three-body CHH basis times single-particle basis of Eqs. (17) and (18)]: in panel (a) the single-particle basis with 20 and in panel (b) with 21 states (see text).

with $K_3 = 2$. All the remaining states contribute only with 0.42 MeV to the ^{4}He binding energy. Thus it is reasonable to expect also a shift of the position of the ^{4}He 0^+ resonance by about 0.5 MeV towards higher energies with respect to the four-body CHH calculation of Fig. 4, leading to a value of about -5.9 MeV. To take into account only four hyperspherical three-body CHH states reduces the numerical effort of the calculation quite a bit, but even with such a reduced basis the calculation requires considerable numerical effort (note that due to the rather weak fall of the hyper-radial/radial basis functions the relevant integration volume becomes much larger than for the bound-state calculation).

In Fig. 5 the energy distribution of LIT states is shown up to the four-body breakup threshold with three different N_4 values. As has been anticipated, the density of LIT states increases with a growing number of radial single-particle states. Due to the interplay of CHH basis states and radial single-particle basis states the pattern is not as regular as in the two-body example of Fig 1.

It is obvious that a further increase of N_4 would lead to an even higher density of low-energy LIT states, but, unfortunately, the precision of the nine-dimensional Monte Carlo integration is not sufficiently high and a solution with $N_4 > 20$ becomes problematic. In this respect it is a drawback to work with a correlated HH basis because one loses the orthogonality. Nonetheless, as shown in the following, the present calculation allows a determination of the width of the ^{4}He isoscalar monopole resonance.

In Fig. 6 the LIT of $M(q,\omega)$ is shown for $N_4 = 20$ and $N_4 = 22$. In the latter case the basis function with $n_4 = 20$ [see Eq. (18)] is dropped because it makes the numerical solution of the corresponding eigenvalue problem problematic (precision of Hamiltonian and norm matrices is not sufficiently high). The figure illustrates that with increasing resolution, i.e., with decreasing σ_I, the resonance becomes more pronounced against the background. One also finds that the anticipated peak position of about -5.9 MeV is roughly confirmed.

In order to determine a resonance width the LIT has to be inverted. The procedure of the inversion in presence of a resonance is described in Ref. [7]. As rule of thumb one can say that the chosen σ_I should not be much larger than the resonance width. However, in principle, if the LIT is calculated with a very high precision one can choose also a considerably larger σ_I. In fact in the model study [7] it is nicely demonstrated that the obtained width is independent from the used σ_I over a very large range, and even with $\sigma_I = 5$ MeV the shape of a resonance with a width of 270 keV was exactly determined. In the present case the precision of the calculated LIT is not as high as in Ref. [7]. The actual inversion was made with two different σ_I values [see discussion of transform T introduced in Eq. (15)], $\sigma_{I,2} = 5$ MeV beyond -4 MeV, whereas $\sigma_{I,1}$ was varied in the range from 0.1 to 0.5 MeV in the low-energy region. The various results for the width were quite stable (maximal difference: 0.01 MeV) and lead to the following values: 120(10) keV ($N_4 = 20$) and 240 keV ($N_4 = 22$); the mean value amounts to 180(70) keV. The result lies in the same ballpark as the experimental value of 270(50) keV [14]. One could try to increase the theoretical precision of the determination of the width by a further increase of N_4. Because of the problem just mentioned, this would require a non-negligible effort in the present calculation. On the other hand, it is much more desirable to make such a calculation with a realistic nuclear force instead of having a very precise result for a simple NN interaction like the TN potential model.

For the inversion of the LIT it is assumed that there is only a single peak in the resonance region. In principle, from an increase of the precision of the calculation, one could also find out that the resonance has a more complicated structure, for example, a double peak. It is evident from Fig. 6 that in the present calculation one controls the resonance with a resolution of about 100 keV; thus it is not possible to resolve structures with an even smaller width. The situation is similar

to an experiment, where one works with a given resolution of the experimental apparatus.

V. SUMMARY

In this work first a brief outline of the LIT approach is given, a method that allows calculation of continuum observables with bound-state techniques. In a next step the energy resolution that can be obtained with the method is discussed. To this end a specific channel of the final state in deuteron photodisintegration is studied in unretarded dipole approximation. In order to do so the LIT equation is solved using a bound-state basis to calculate the relevant quantities. It is shown how the spatial extension and the number of basis states affect the calculation. The role of the width parameter σ_I of the LIT is discussed, especially which σ_I values should be chosen in a specific situation. The discussion makes clear that the LIT constitutes an approach with a controlled resolution. In particular it allows determination of the width of narrow resonances in the continuum. However, for LIT calculation with an arbitrary many-body bound-state basis, this is not always guaranteed. Usually narrow resonances of an A-body system in the continuum are located below the three-body breakup threshold. If one intends to study the resonance in a scattering state calculation it is mandatory to take into account the relevant *dynamical* variable, i.e., the relative vector between the two fragments. In spite of the LIT bound-state character this *dynamical* variable should also appear explicitly in a LIT calculation. It guarantees that the density of LIT states can be enhanced in the resonance region by increasing the number of those basis states that directly depend on the *dynamical* variable. On the contrary, if this variable is not included explicitly, as for example in an A-body HH calculation, it is difficult, maybe even impossible, to obtain detailed information about the resonance.

In a general case of a resonance with various open channels the resonance can have a *partial* decay width to all these channels, which then results in a *total* decay width. In this case it is sufficient to choose just one of the various possible *dynamical* variables. The only aim which has to be fulfilled is a sufficient density of LIT states. The LIT then by definition collects strength from all open channels and a determination of the width via inversion leads to the *total* width.

In order to illustrate the situation in greater detail the isoscalar monopole response function M of ^{4}He is considered as test case using a simple spin-dependent central NN interaction (Coulomb force is included). In fact, here one finds a narrow resonance in the continuum below the three-body breakup threshold. Obviously, in this case the *dynamical* variable is given by the relative vector of the free nucleon and the bound three-nucleon system. Using a proper basis as described above it is indeed found that the density of LIT states grows in the resonance region if the number of single-particle basis states is enhanced. It is shown that the LIT state density becomes sufficiently high to determinate the resonance width via an inversion of the transform. In fact, a width of 180(70) keV is found, a result that is not too far from the experimental value of 270(50) keV. It would be very interesting to perform such a calculation also with modern realistic nuclear forces, as have been used in the LIT-EIHH calculations of the response function M [6,17]; however, the effective interaction approach has to be a bit redesigned, since one would not have a pure HH basis.

The present approach is advantageous not only in the case of cross sections with narrow resonances, but also for nonresonant two-body breakup cross sections at very low energies. The possibility to increase the LIT state density allows work with smaller σ_I values, thus enhancing the energy resolution of the calculation. This might be particularly interesting in the case of astrophysical reactions.

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