

Reexamination of the neutron-to-proton-ratio puzzle in intermediate-energy heavy-ion collisions

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(Received 3 February 2015; revised manuscript received 2 March 2015; published 14 April 2015)

Incorporating a newly improved isospin- and momentum-dependent interaction in the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model IBUU11, we have investigated relative effects of the density dependence of nuclear symmetry energy $E_{\text{sym}}(\rho)$ and the neutron-proton effective mass splitting $m_n^* - m_p^*$ on the neutron-to-proton ratio of free nucleons and those in light clusters. It is found that the $m_n^* - m_p^*$ has a relatively stronger effect than the $E_{\text{sym}}(\rho)$ and the assumption of $m_n^* \leq m_p^*$ leads to a higher neutron-to-proton ratio. Moreover, this finding is independent of the in-medium nucleon-nucleon cross sections used. However, results of our calculations using the $E_{\text{sym}}(\rho)$ and $m_n^* - m_p^*$ both within their current uncertainty ranges are all too low compared to the recent National Superconducting Cyclotron Laboratory double neutron-to-proton-ratio data from central $^{124}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ collisions at 50 and 120 MeV/u, thus calling for new mechanisms to explain the puzzlingly high neutron-to-proton ratio observed in the experiments.

DOI: 10.1103/PhysRevC.91.047601

PACS number(s): 25.70.-z, 24.10.Lx, 21.30.Fe

To pin down the density dependence of nuclear symmetry energy $E_{\text{sym}}(\rho)$ has long been a major challenge in both nuclear physics and astrophysics. While much progress has been made in the past decade, many interesting issues remain to be resolved [1–4]. A larger symmetry energy generally corresponds to a more repulsive (attractive) underlying symmetry potential U_{sym} for neutrons (protons) in neutron-rich nuclear matter as they are linearly proportional to each other according to the Hugenholtz-Van Hove theorem [5] or the Bruckner theory [6,7], see, e.g., Refs. [8–10] for the explicit relationship between $E_{\text{sym}}(\rho)$ and U_{sym} . On the other hand, the in-medium nucleon effective mass describes to the first order effects due to the nonlocality of the underlying nuclear interactions and the Pauli exchange effects in many-fermion systems [11]. It can be calculated from the momentum dependence of the single-particle potential in nonrelativistic models or the Schrödinger-equivalent potential in relativistic models. The nucleon effective mass is related to many interesting problems in both nuclear physics and astrophysics [12–15]. It has further been found that the neutrons and protons may have different effective masses in neutron-rich matter due to the momentum dependence of the symmetry (isovector) potential. However, calculations within different models using various interactions, e.g., the Brueckner-Hartree-Fock approach [16], the relativistic mean-field model [17], and the Skyrme-Hartree-Fock calculation [18,19], predict rather different values for the neutron-proton effective mass splitting $m_{n-p}^* \equiv m_n^* - m_p^*$. Thus, currently there is no consensus as to whether the m_{n-p}^* is negative, zero, or positive. However, the value of m_{n-p}^* affects significantly isospin-sensitive observables in

heavy-ion collisions [20–27] as well as thermal and transport properties of neutron-rich matter [28–30]. It also has important ramifications in astrophysics [31]. For instance, the equilibrium neutron-to-proton ratio in primordial nucleosynthesis is determined by $(n/p)_{\text{eq}} = e^{-m_{n-p}^*/T}$ for the early (≥ 1 ms) universe when the temperature T was high (≥ 3 MeV) [32].

Recently, analyses of the free neutron-to-proton double ratio from central $^{124}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ collisions at 50 and 120 MeV/u at the National Superconducting Cyclotron Laboratory at Michigan State University (NSCL/MSU) seem to indicate that protons have a slightly larger effective mass than neutrons based on comparisons with calculations within an improved quantum molecular dynamics model using Skyrme interactions [33]. The preferred Skyrme interaction SLy4 [34] is being widely used in describing the ground state properties and excitations of neutron-rich nuclei. However, the applicability of Skyrme interactions is restricted, in general, by the nuclear structure calculations and the small amplitude nuclear motions due to the incorrect energy behavior of nucleon-nucleus isovector optical potential in comparison with that extracted from experiments [10,35–41]. This situation clearly calls for more theoretical studies with different transport models and examinations of various model ingredients. In fact, it was known that the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model using a momentum-dependent symmetry potential corresponding to $m_n^* > m_p^*$ [42] underpredicts the old NSCL double neutron-to-proton data [43] no matter how the density-dependence of symmetry energy and the in-medium nucleon-nucleon cross sections are adjusted. Thus, to understand the puzzlingly high neutron-to-proton double ratio, the relative effects of the symmetry energy and the neutron-proton effective mass splitting should be studied within the same model.

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We have recently incorporated a newly improved isospin- and momentum-dependent interaction (ImMDI) [29] based on the Gogny force in the IBUU11 transport model. The original Gogny interaction is known to give an asymptotic value of the isoscalar potential at saturation density less than that extracted from optical model analyses of nucleon-nucleus scattering data. We removed this drawback and introduced a new parameter to adjust easily the values of the m_{n-p}^* besides those varying the magnitude and density dependence of the $E_{\text{sym}}(\rho)$. In this Brief Report, we revisit the neutron-to-proton puzzle. We find that indeed the $m_n^* - m_p^*$ has a relatively stronger effect than the $E_{\text{sym}}(\rho)$ and the assumption of $m_n^* \leq m_p^*$ leads to a higher neutron-to-proton ratio. However, the puzzle remains, because our calculations using the $E_{\text{sym}}(\rho)$ and $m_n^* - m_p^*$ both within their current uncertainty ranges still underpredict significantly the NSCL/MSU data.

The ImMDI interaction is developed from the MDI interaction, which has a similar functional form as the Gogny effective interaction while replacing the Gaussian-type finite-range term with a Yukawa form [44,45]. The potential energy density in asymmetric nuclear matter from the MDI interaction or the ImMDI interaction is expressed as [44]

$$V(\rho, \delta) = \frac{A_u \rho_n \rho_p}{\rho_0} + \frac{A_l}{2\rho_0} (\rho_n^2 + \rho_p^2) + \frac{B}{\sigma + 1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} \times (1 - x\delta^2) + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \times \int \int d^3 p d^3 p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}. \quad (1)$$

In the above, ρ_n and ρ_p are respectively the neutron and proton density, and $\rho = \rho_n + \rho_p$ is the total density. ρ_0 is the saturation density, and $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry. $\tau = 1(-1)$ denoting neutrons (protons) is the isospin index, and $f_\tau(\vec{r}, \vec{p})$ is the phase-space distribution function. The single-particle potential from the mean-field approximation depends on the density ρ and isospin asymmetry δ of the nuclear medium as well as the isospin τ and momentum \vec{p} of the nucleon [44]

$$U_\tau(\rho, \delta, \vec{p}) = A_u \frac{\rho^{-\tau}}{\rho_0} + A_l \frac{\rho^\tau}{\rho_0} + B \left(\frac{\rho}{\rho_0} \right)^\sigma (1 - x\delta^2) - 4\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \delta \rho_{-\tau} + \frac{2C_{\tau, \tau}}{\rho_0} \int d^3 p' \frac{f_\tau(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} + \frac{2C_{\tau, -\tau}}{\rho_0} \int d^3 p' \frac{f_{-\tau}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}. \quad (2)$$

Comparing with the MDI interaction, the ImMDI interaction has been improved mainly in two aspects [29]. First, the high-momentum part of the nucleon isoscalar mean-field potential has been refitted to reproduce the optical potential extracted from the proton-nucleus scattering experimental data up to the nucleon kinetic energy of about 1 GeV. Second, besides the x parameter, which was previously used to mimic the density dependence of the symmetry energy by adjusting

the relative contributions of different spin-isospin channels of the density-dependent interaction, another two parameters y and z are introduced to vary respectively the isospin splitting of the nucleon effective mass and the value of the symmetry energy at saturation density. The parameters y and z enter the functional form through

$$A_l(x, y) = A_0 + y + x \frac{2B}{\sigma + 1}, \quad (3)$$

$$A_u(x, y) = A_0 - y - x \frac{2B}{\sigma + 1}, \quad (4)$$

$$C_{\tau, \tau}(y, z) = C_{l0} - \frac{2(y - 2z)p_{f0}^2}{\Lambda^2 \ln[(4p_{f0}^2 + \Lambda^2)/\Lambda^2]}, \quad (5)$$

$$C_{\tau, -\tau}(y, z) = C_{u0} + \frac{2(y - 2z)p_{f0}^2}{\Lambda^2 \ln[(4p_{f0}^2 + \Lambda^2)/\Lambda^2]}, \quad (6)$$

where $p_{f0} = \hbar(3\pi^2\rho_0/2)^{1/3}$ is the nucleon Fermi momentum in symmetric matter at saturation density. The values of A_0 , C_{u0} , C_{l0} , B , σ , and Λ are fixed by six empirical constraints at $x = 0$, $y = 0$, and $z = 0$, i.e., the saturation density $\rho_0 = 0.16 \text{ fm}^{-3}$, the binding energy $E_0(\rho_0) = -16 \text{ MeV}$, the incompressibility $K_0 = 230 \text{ MeV}$, the isoscalar effective mass $m_{s,0}^* = 0.7m$, the symmetry energy $E_{\text{sym}}(\rho_0) = 32.5 \text{ MeV}$, and the isoscalar potential at infinitely large momentum $U_{0,\infty} = 75 \text{ MeV}$; and the values of the corresponding parameters are $A_0 = -66.963 \text{ MeV}$, $C_{u0} = -99.7017 \text{ MeV}$, $C_{l0} = -60.4860 \text{ MeV}$, $B = 141.963 \text{ MeV}$, $\sigma = 1.26521$, and $\Lambda = 2.42401 p_{f0}$.

The ImMDI interaction provides us with more flexibility to investigate the detailed isovector properties of the nuclear interaction. In the present work, we set $z = 0$ and vary the values of x and y to study the effects of the symmetry energy and the neutron-proton effective mass splitting. The nucleon effective mass is defined as

$$\frac{m_\tau^*}{m} = \left(1 + \frac{m}{p} \frac{dU_\tau}{dp} \right)^{-1}, \quad (7)$$

and it generally depends on the density and isospin asymmetry of the medium as well as the isospin and momentum of the nucleon. The density dependence of the symmetry energy E_{sym} and the relative neutron-proton effective mass splitting are displayed in Fig. 1 with different values of x and y . As discussed and shown in Ref. [29], x affects only E_{sym} while y affects both E_{sym} and the m_{n-p}^* . In the following, we select several special sets of parameters to examine the relative effects of E_{sym} and m_{n-p}^* . With $(x = 0, y = -115 \text{ MeV})$ and $(x = 1, y = 115 \text{ MeV})$, the symmetry energy is almost the same while the relative neutron-proton effective mass splittings are opposite in sign. On the other hand, with $(x = 0, y = -115 \text{ MeV})$ and $(x = 1, y = -115 \text{ MeV})$ the relative neutron-proton effective mass splitting is the same while the latter gives a softer symmetry energy. We can thus study the effect of the isospin splitting of the nucleon effective mass by comparing the results from the former two parameter sets while investigating that of the symmetry energy by comparing the results from the latter two sets. We note that the current uncertainty range of the slope parameter L of E_{sym} is about

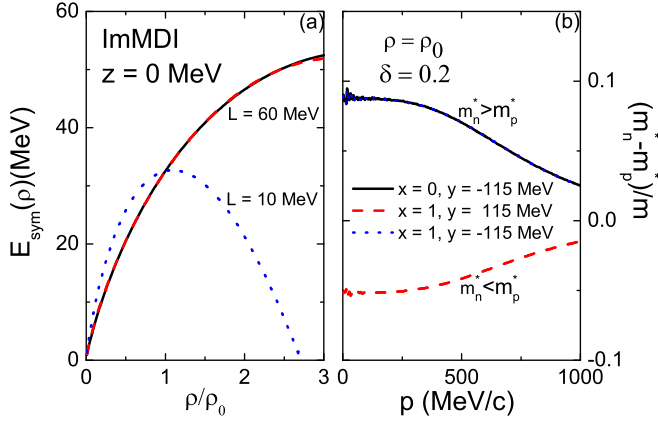


FIG. 1. (Color online) Density dependence of the symmetry energy (a) and momentum dependence of the relative neutron-proton effective mass splitting in nuclear matter of the density $\rho = \rho_0$ and isospin asymmetry $\delta = 0.2$ (b) from the ImMDI interaction with different values of x and y .

50 \pm 20 MeV [46], which is not quite different from the chosen range (10, 60) MeV in the present study.

The ImMDI interaction with parameter sets described above was implemented in the IBUU11 model [4]. In our calculation, 200 test particles per nucleon are used and about 20 000 events are generated for each beam energy and impact parameter. The initial density distribution is generated from the Skyrme-Hartree-Fock calculation using the MSL0 force [47], and the initial nucleon momenta are generated from local Thomas-Fermi approximation.

The neutron-to-proton ratio in collisions induced by neutron-rich nuclei was first used as a probe of the symmetry energy [48]. Later, the double neutron-to-proton ratio of nucleon emission for two collision systems with isotopes of different total isospin asymmetries was introduced to reduce systematically the influence of the Coulomb force and the poor efficiency of detecting neutrons [42,49]. To explore the effects of both the symmetry energy and the isospin splitting of the nucleon effective mass on the double neutron-to-proton ratio within the ImMDI and IBUU framework, we generate events for $^{112}\text{Sn}+^{112}\text{Sn}$ and $^{124}\text{Sn}+^{124}\text{Sn}$ collisions at beam energies of 120 and 50 MeV/u. Similar to the treatment in Ref. [48], we stop the evolution at $t = 150$ fm/c when the interaction becomes negligible, and identify nucleons and clusters based on the final nucleon phase-space distribution, i.e., two nucleons are within one cluster if their spatial distance is closer than $\Delta r = 3$ fm and their momentum distance is smaller than $\Delta p = 300$ MeV/c. We notice that our final results are not sensitive to the variation of these coalescence parameters within about 30% of the above values.

The coalescence invariant yield is constructed by considering both free nucleons and those bound in light clusters including deuterons, tritons, ^3He , and ^4He , and the angular gate is chosen to be $70^\circ < \theta_{\text{c.m.}} < 110^\circ$, as in the experimental analysis. The impact parameter is set to be $b = 2$ fm to mimic the centrality in the experiments. The double neutron-to-proton ratio $\text{DR}(n/p)$ in $^{124}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ collisions is

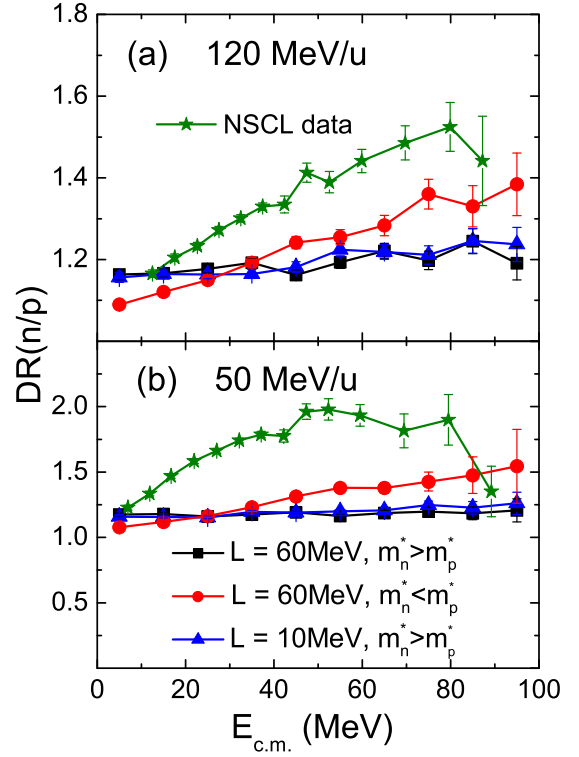


FIG. 2. (Color online) Coalescence invariant double neutron-to-proton ratios $\text{DR}(n/p)$ in $^{124}\text{Sn}+^{124}\text{Sn}$ collisions to $^{112}\text{Sn}+^{112}\text{Sn}$ collisions as a function of nucleon center-of-mass energy at beam energies of (a) 120 and (b) 50 MeV/u with the impact parameter $b = 2$ fm and the angular gate $70^\circ < \theta_{\text{c.m.}} < 110^\circ$. The NSCL data are from Ref. [33].

defined as

$$\text{DR}(n/p) = \frac{[Y(n)/Y(p)]_{^{124}\text{Sn}+^{124}\text{Sn}}}{[Y(n)/Y(p)]_{^{112}\text{Sn}+^{112}\text{Sn}}}. \quad (8)$$

Since the yield neutron-to-proton ratio $Y(n)/Y(p)$ in $^{124}\text{Sn}+^{124}\text{Sn}$ collisions is larger than that in $^{112}\text{Sn}+^{112}\text{Sn}$ collisions, the $\text{DR}(n/p)$ is always larger than unity. Figure 2 shows the $\text{DR}(n/p)$ as a function of nucleon center-of-mass energy at beam energies of 120 and 50 MeV/u. At 50 MeV/u, it is seen that the $\text{DR}(n/p)$ for high-energy nucleons is slightly larger for a softer symmetry energy ($L = 10$ MeV) consistent with the finding in Ref. [42]. It is noteworthy that a stiffer symmetry energy can lead to a larger $\text{DR}(n/p)$ at beam energies as high as 400 MeV/u, when the symmetry energy at suprasaturation densities becomes important. At the beam energy of 120 MeV/u shown in panel (a) of Fig. 2, the $\text{DR}(n/p)$ is rather insensitive to the stiffness of the symmetry energy. On the other hand, the neutron-proton effective mass splitting has a more appreciable effect on the $\text{DR}(n/p)$. It is seen that a negative m_{n-p}^* results in a larger $\text{DR}(n/p)$ at higher nucleon energies, consistent with our expectation. However, even for the two extreme cases considered here, the resulting values of the $\text{DR}(n/p)$ are still far below the NSCL data. We notice that in the above calculation the isospin-dependent in-medium nucleon-nucleon

scattering cross sections scaled by the nucleon effective mass [23] are used, and we have checked that the results do not change by much even if we use free nucleon-nucleon scattering cross sections. Thus, within the present IBUU11 transport model, the variation of neither the symmetry energy nor the isospin effective mass splitting within their current uncertainty ranges is able to explain the experimental data. This situation clearly calls for possibly new mechanisms and explanations to resolve the neutron-to-proton-ratio puzzle. It is thus interesting to note that several microscopic many-body theories and phenomenological models have predicted that the isospin dependence of short-range nucleon-nucleon correlations dominated by the tensor force can significantly reduce the kinetic part of the symmetry energy [50–54]. The potential part of the symmetry energy and thus the symmetry potential has to be enhanced accordingly to meet existing constraints on the symmetry energy at saturation density. This effect has been shown to enhance significantly the double neutron-to-proton ratio of free nucleons in transport model calculations without considering the neutron-proton effective mass splitting [55,56]. A comprehensive study considering the isospin dependence of short-range nucleon-nucleon correlations and the neutron-proton effective mass splitting in the IBUU11 model is planned.

Trusting the data, the apparent success of the ImQMD model with the SLy4 interaction and the failure of the IBUU11 model with the ImMDI interaction in describing the data requires further investigation. Generally speaking, different inputs and algorithms used in different transport models may lead to different predictions. We speculate that the different handling of cluster formation might contribute appreciably to the difference between the ImQMD and IBUU11 calculations. However, we honestly do not know at this time what are really the main causes because each model has several major inputs besides some technical differences. One possible way out of this unfortunate situation is to conduct multiobservable versus multi-input covariance analyses, which have been successfully utilized in several other areas of nuclear physics recently (see Ref. [57] for a topic review). In the covariant analyses, the correlation matrix among all observables and input parameters as well as their uncertainties can be calculated consistently and simultaneously. Such analyses with the IBUU11 model are underway. It will also be useful to perform such analyses with the ImQMD model. For example, the ImQMD calculations using the SLy4 and SkM* interactions predict an approximately 50% difference in the free neutron-proton ratio at 50 MeV/u. Where does this difference come from? It was attributed to the difference in the isospin effective mass splitting in Ref. [33]. In fact, at $\delta = 0.2$, m_n^*/m_n and m_p^*/m_p with SLy4 are respectively 0.68 and 0.71, while that with SkM* are respectively 0.82 and 0.76, with relative effective mass splitting -3% for SLy4 and $+6\%$ for SkM*, smaller

than that in the present calculation. However, these are not the only differences between the two interactions. In particular, the isoscalar effective mass is different by about 14%, and the curvature of the symmetry energy K_{sym} differs by 30%. Without examining sensitivities of observables to a particular input by varying it while fixing all others, it is hard to conclude what input is actually responsible for the observed change in any observable. Thus, multidimensional covariance analyses look promising although they are computationally extremely costly using transport models for nuclear reactions.

In summary, within the IBUU11 transport model using a newly improved isospin- and momentum-dependent interaction, we revisited but failed to resolve the neutron-to-proton-ratio puzzle in heavy-ion collisions at intermediate energies. Nevertheless, some interesting physics and useful lessons have been learned. We found that the neutron-proton effective mass splitting $m_n^* - m_p^*$ indeed has a relatively stronger effect than the symmetry energy $E_{\text{sym}}(\rho)$, and the assumption of $m_n^* \leq m_p^*$ leads to a higher neutron-to-proton ratio of free nucleons and those in light clusters. Using the $E_{\text{sym}}(\rho)$ and $m_n^* - m_p^*$ both within their current uncertainty ranges, with the IBUU11 model and the ImMDI interaction we are unable to reproduce the recent NSCL/MSU double neutron-to-proton-ratio data in central $^{124}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ collisions at 50 and 120 MeV/u. This situation clearly calls for new mechanisms to explain the puzzlingly high neutron-to-proton ratio observed in the experiments. Among the possible new physics origins, effects of the isospin-dependent short-range nucleon-nucleon correlation deserve special attention.

We thank W. G. Lynch, M. B. Tsang, Y. X. Zhang, and W. J. Guo for helpful communications, and C. Zhong for maintaining the high-quality performance of the computer facility. This work was supported in part by the Major State Basic Research Development Program (973 program) in China under Contracts No. 2015CB856904, No. 2014CB845401, and No. 2013CB834405; the National Natural Science Foundation of China under Grants No. 11475243, No. 11421505, No. 11275125, No. 11135011, and No. 11320101004; the “100-talent plan” of Shanghai Institute of Applied Physics under Grant No. Y290061011 from the Chinese Academy of Sciences; the “Shanghai Pujiang Program” under Grant No. 13PJ1410600, the “Shu Guang” project supported by Shanghai Municipal Education Commission and Shanghai Education Development Foundation; the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning; the Science and Technology Commission of Shanghai Municipality (11DZ2260700); the U.S. National Science Foundation Grant No. PHY-1068022; and the CUSTIPEN (China-U.S. Theory Institute for Physics with Exotic Nuclei) under DOE Grant No. DE-FG02-13ER42025.

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