

## Nucleon resonances in the $\gamma p \rightarrow \phi K^+ \Lambda$ reaction near threshold

Qi-Fang Lü,<sup>1</sup> Rong Wang,<sup>2,3</sup> Ju-Jun Xie,<sup>2,3,4,5</sup> Xu-Rong Chen,<sup>2,3</sup> and De-Min Li<sup>1,\*</sup>

<sup>1</sup>Department of Physics, Zhengzhou University, Zhengzhou, Henan 450001, China

<sup>2</sup>Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

<sup>3</sup>University of Chinese Academy of Sciences, Beijing 100049, China

<sup>4</sup>Research Center for Hadron and CSR Physics, Institute of Modern Physics of CAS and Lanzhou University, Lanzhou 730000, China

<sup>5</sup>State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

(Received 19 January 2015; published 20 March 2015)

We investigate the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction near threshold within an effective Lagrangian approach and the isobar model. Various nucleon resonances caused by the  $\pi$  and  $\eta$  meson exchanges and background contributions are considered. It is shown that the contribution from the  $N^*(1535)$  resonance caused by the  $\eta$  meson exchange plays the predominant role. Hence, this reaction provides a good new platform to study the  $N^*(1535)$  resonance. The predicted total cross section and specific features of the angular distributions can be tested by future experiments.

DOI: [10.1103/PhysRevC.91.035204](https://doi.org/10.1103/PhysRevC.91.035204)

PACS number(s): 13.75.-n, 14.20.Gk, 13.30.Eg

### I. INTRODUCTION

The study of nucleon resonances in meson production reaction is an interesting topic in light hadron physics. Among these resonances, the negative parity nucleon excited state,  $N^*(1535)$  (spin-parity  $J^P = \frac{1}{2}^-$ ), has been a controversial resonance for many years. In the traditional three-quark constituent model, it should be the lowest spatially excited nucleon state with one quark in a  $P$  wave [1,2]. However, the  $N^*(1440)$  ( $J^P = \frac{1}{2}^+$ ) has in fact a much lower mass, despite requiring two units of excitation energy. This is the long-standing mass inversion problem of the nucleon resonance spectrum.

Furthermore, the  $N^*(1535)$  resonance couples strongly to those strangeness channels [3]. Besides the  $\eta N$  channel [1], it also couples strongly to the  $\eta' N$  [4–6],  $\phi N$  [7,8], and  $K \Lambda$  [9,10] channels. Moreover, it is found that the  $N^*(1535)$  state is dynamically generated within a chiral unitary coupled-channel approach, with its mass, width, and branching ratios in fair agreement with the experimental results [11–14]. This approach shows that the couplings of the  $N^*(1535)$  resonance to the  $K \Sigma$ ,  $\eta N$ , and  $K \Lambda$  channels could be large compared to that for the  $\pi N$  channel.

The mass inversion problem could be understood if there were significant five-quark ( $uuds\bar{s}$ ) components in the wave function of the  $N^*(1535)$  resonance [15,16], which would also provide a natural explanation of the large couplings of the  $N^*(1535)$  resonance to the strangeness  $K \Lambda$ ,  $K \Sigma$ ,  $N \eta'$ , and  $N \phi$  channels. It would furthermore lead to an improvement in the description of the  $N^*(1535)$  helicity amplitudes [17,18]. In this paper, we wish to argue that the  $N^*(1535)$  resonance might play an important role in the associated strangeness production of the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction. Since this reaction needs to create two  $s\bar{s}$  quark pairs from the vacuum, its total cross sections should be small, which is why not much attention has been paid to it on either the theoretical or experimental sides.

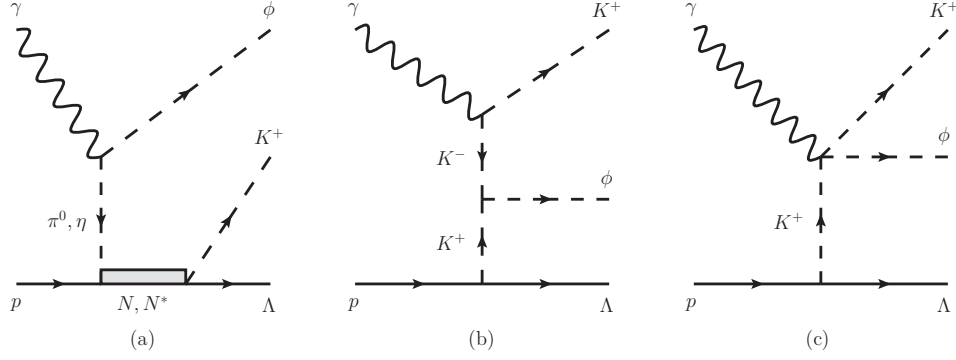
However, because of the weak interactions of  $\phi K^+$  and  $\phi \Lambda$ , the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction near threshold provides a good new platform to study the  $N^*(1535)$  resonance decaying to  $K^+ \Lambda$ .

The couplings of the  $N^*(1535)$  resonance to the  $K \Lambda$  and  $\eta N$  channels and the ratio of  $g_{K \Lambda N^*(1535)}$  to  $g_{\eta N N^*(1535)}$ ,  $R \equiv |g_{K \Lambda N^*(1535)}|/|g_{\eta N N^*(1535)}|$ , have been intensively studied within various theoretical approaches. By analyzing the  $J/\psi \rightarrow \bar{p} K^+ \Lambda$  and  $J/\psi \rightarrow \bar{p} \eta p$  experimental data, Ref. [9] gives  $R = 1.3 \pm 0.3$ . From the latest and largest photoproduction database by using the isobar model, Ref. [19] gives  $R = 0.460 \pm 0.172$ . From the  $J/\psi$  decays within the chiral unitary approach, Ref. [20] gives  $R = 0.5 \sim 0.7$ . Based on the partial wave analysis of kaon photonproduction, Ref. [21] gives  $R = 0.42 \sim 0.73$ . The result of the  $s$ -wave  $\pi N$  scattering analysis within a unitarized chiral effective Lagrangian indicates  $|g_{K \Lambda N^*(1535)}|^2 > |g_{\eta N N^*(1535)}|^2$  [22]. The coupled-channel calculation predicts  $R = 0.8 \sim 2.6$  [23]. Study on the partial decay widths of the  $N^*(1535)$  resonance to the pseudoscalar mesons and octet baryons within a chiral constituent quark model shows  $R = 0.85 \pm 0.06$  [24]. In a very recent analysis of the  $\pi^- p \rightarrow K^0 \Lambda$  reaction, a value of  $R = 0.71 \pm 0.10$  is obtained [25]. Obviously, theoretical predictions on  $R$  are not completely consistent with each other, thus it is still worth studying the coupling constant  $g_{K \Lambda N^*(1535)}$  in different ways.

In the present work, we investigate the role of nucleon resonances in the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction near threshold in the framework of an effective Lagrangian approach and the isobar model. Initial interaction between incoming photons and protons is modeled by an effective Lagrangian which is based on the exchanges of the  $\pi$ ,  $\eta$ , and kaon mesons. The  $K^+ \Lambda$  production proceeds via the excitation of the  $N^*(1535)$ ,  $N^*(1650)$ ,  $N^*(1710)$ , and  $N^*(1720)$  intermediate nucleon resonances which have appreciable branching ratios for the decay into the  $K^+ \Lambda$  channel.

This article is organized as follows. In the next section, we will present the formalism and ingredients necessary for our calculations, then numerical results and discussions are given in Sec. III. A short summary is given in the last section.

\*lidm@zzu.edu.cn

FIG. 1. Feynman diagrams for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction.

## II. FORMALISM AND INGREDIENTS

We study the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction near threshold within an effective Lagrangian approach and the isobar model, which has been extensively applied to the study of scattering processes [26–37]. The basic tree level Feynman diagrams for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction are depicted in Fig. 1. It is assumed that the  $K \Lambda$  final states are produced by the decay of the intermediate nucleon resonances as the result of the  $\pi$  and  $\eta$  mesons exchanges [Fig. 1(a)]. Moreover, the background contributions including the  $s$ -channel nucleon pole,  $t$ -channel  $K$  exchange [Fig. 1(b)], and contact term [Fig. 1(c)] are also considered.

To compute the amplitudes of these diagrams shown in Fig. 1, the effective Lagrangian densities for the relevant interaction vertexes are needed. We use the commonly employed Lagrangian densities for  $\pi NN$ ,  $\eta NN$ , and  $K \Lambda N$  as follows [38]:

$$\mathcal{L}_{\pi NN} = -\frac{g_{\pi NN}}{2m_N} \bar{N} \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} N, \quad (1)$$

$$\mathcal{L}_{\eta NN} = -\frac{g_{\eta NN}}{2m_N} \bar{N} \gamma_5 \gamma_\mu \partial^\mu \eta N, \quad (2)$$

$$\mathcal{L}_{K \Lambda N} = -\frac{g_{K \Lambda N}}{m_N + m_\Lambda} \bar{N} \gamma_5 \gamma_\mu \partial^\mu K \Lambda + \text{H.c.} \quad (3)$$

The coupling constants in the above Lagrangian densities are taken as [7,31,35]:  $g_{\pi NN} = 13.45$ ,  $g_{K \Lambda N} = -13.98$ , and  $g_{\eta NN} = 2.24$ .

For the  $\phi$  meson and photon couplings, we take the interaction Lagrangian densities used in Refs. [37,39],

$$\mathcal{L}_{\phi\gamma\pi} = \frac{e}{m_\phi} g_{\phi\gamma\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \phi_\nu \partial_\alpha A_\beta \pi, \quad (4)$$

$$\mathcal{L}_{\phi\gamma\eta} = \frac{e}{m_\phi} g_{\phi\gamma\eta} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \phi_\nu \partial_\alpha A_\beta \eta, \quad (5)$$

$$\mathcal{L}_{\gamma KK} = -ie(\partial^\mu K^- K^+ - \partial^\mu K^+ K^-) A_\mu, \quad (6)$$

$$\mathcal{L}_{\phi KK} = -ig_{\phi KK}(\partial^\mu K^- K^+ - \partial^\mu K^+ K^-) \phi_\mu, \quad (7)$$

where  $e = \sqrt{4\pi\alpha}$  ( $\alpha = 1/137.036$  is the fine-structure constant), and  $A_\mu$  is the photon field.

The interaction Lagrangian densities involving nucleon resonances ( $\equiv R$ ) are taken from Ref. [40],

$$\mathcal{L}_{\pi NR} = ig_{\pi NR} \bar{R} \vec{\tau} \cdot \vec{\pi} N + \text{H.c.}, \quad (8)$$

$$\mathcal{L}_{\eta NR} = ig_{\eta NR} \bar{R} \eta N + \text{H.c.}, \quad (9)$$

$$\mathcal{L}_{K \Lambda R} = ig_{K \Lambda R} \bar{R} K \Lambda + \text{H.c.}, \quad (10)$$

for the  $J^P = \frac{1}{2}^-$  nucleon resonances  $N^*(1535)$  and  $N^*(1650)$ ,

$$\mathcal{L}_{\pi NR} = -\frac{g_{\pi NR}}{m_N + m_R} \bar{R} \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} N + \text{H.c.}, \quad (11)$$

$$\mathcal{L}_{\eta NR} = -\frac{g_{\eta NR}}{m_N + m_R} \bar{R} \gamma_5 \gamma_\mu \partial^\mu \eta N + \text{H.c.}, \quad (12)$$

$$\mathcal{L}_{K \Lambda R} = -\frac{g_{K \Lambda R}}{m_\Lambda + m_R} \bar{R} \gamma_5 \gamma_\mu \partial^\mu K \Lambda + \text{H.c.}, \quad (13)$$

for the  $J^P = \frac{1}{2}^+$  nucleon resonance  $N^*(1710)$ , and

$$\mathcal{L}_{\pi NR} = -\frac{g_{\pi NR}}{m_\pi} \bar{R}_\mu \vec{\tau} \cdot \partial^\mu \vec{\pi} N + \text{H.c.}, \quad (14)$$

$$\mathcal{L}_{\eta NR} = -\frac{g_{\eta NR}}{m_\eta} \bar{R}_\mu \partial^\mu \eta N + \text{H.c.}, \quad (15)$$

$$\mathcal{L}_{K \Lambda R} = -\frac{g_{K \Lambda R}}{m_K} \bar{R}_\mu \partial^\mu K \Lambda + \text{H.c.}, \quad (16)$$

for the  $J^P = \frac{3}{2}^+$  nucleon resonance  $N^*(1720)$ .

The coupling constants in the above Lagrangian densities can be determined from the partial decay widths,

$$\Gamma[\phi \rightarrow \pi\gamma] = \frac{e^2 g_{\phi\gamma\pi}^2 |\vec{p}_{\pi\gamma}|^3}{12\pi m_\phi^2}, \quad (17)$$

$$\Gamma[\phi \rightarrow \eta\gamma] = \frac{e^2 g_{\phi\gamma\eta}^2 |\vec{p}_{\eta\gamma}|^3}{12\pi m_\phi^2}, \quad (18)$$

$$\Gamma[\phi \rightarrow K^+ K^-] = \frac{g_{\phi KK}^2 |\vec{p}_{K^+ K^-}|^3}{6\pi m_\phi^2}, \quad (19)$$

for the  $\phi$  meson,

$$\Gamma[R \rightarrow N\pi] = \frac{3g_{\pi NR}^2 (E_N + m_N)}{4\pi m_R} |\vec{p}_{N\pi}|, \quad (20)$$

$$\Gamma[R \rightarrow N\eta] = \frac{g_{\eta NR}^2 (E_N + m_N)}{4\pi m_R} |\vec{p}_{N\eta}|, \quad (21)$$

$$\Gamma[R \rightarrow \Lambda K] = \frac{g_{K\Lambda R}^2 (E_\Lambda + m_\Lambda)}{4\pi m_R} |\vec{p}_{\Lambda K}|, \quad (22)$$

for the  $J^P = \frac{1}{2}^-$  nucleon resonances  $N^*(1535)$  and  $N^*(1650)$ ,

$$\Gamma[R \rightarrow N\pi] = \frac{3g_{\pi NR}^2 (E_N - m_N)}{4\pi m_R(m_R + m_N)} |\vec{p}_{N\pi}|, \quad (23)$$

$$\Gamma[R \rightarrow N\eta] = \frac{g_{\eta NR}^2 (E_N - m_N)}{4\pi m_R(m_R + m_N)} |\vec{p}_{N\eta}|, \quad (24)$$

$$\Gamma[R \rightarrow \Lambda K] = \frac{g_{K\Lambda R}^2 (E_\Lambda - m_\Lambda)}{4\pi m_R(m_R + m_\Lambda)} |\vec{p}_{\Lambda K}|, \quad (25)$$

for the  $J^P = \frac{1}{2}^+$  nucleon resonance  $N^*(1710)$ , and

$$\Gamma[R \rightarrow N\pi] = \frac{g_{\pi NR}^2 (E_N + m_N)}{4\pi m_R m_\pi^2} |\vec{p}_{N\pi}|^3, \quad (26)$$

$$\Gamma[R \rightarrow N\eta] = \frac{g_{\eta NR}^2 (E_N + m_N)}{12\pi m_R m_\eta^2} |\vec{p}_{N\eta}|^3, \quad (27)$$

$$\Gamma[R \rightarrow \Lambda K] = \frac{g_{K\Lambda R}^2 (E_\Lambda + m_\Lambda)}{12\pi m_R m_K^2} |\vec{p}_{\Lambda K}|^3, \quad (28)$$

for the  $J^P = \frac{3}{2}^+$  nucleon resonance  $N^*(1720)$ . Here

$$|\vec{p}_{f_1 f_2}| = \frac{\lambda^{\frac{1}{2}}(m_i^2, m_{f_1}^2, m_{f_2}^2)}{2m_i}, \quad (29)$$

where  $m_i$  denotes the mass of the decaying particle,  $m_{f_1}$  and  $m_{f_2}$  are the masses of two final particles, and  $\lambda$  is the Källén function with  $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ . The numerical results for the relevant coupling constants are listed in Table I. The coupling constant  $g_{K\Lambda N^*(1535)}$  will be discussed below.

Since the hadrons are not point-like particles, the form factors are also needed. For the exchanged mesons, we adopt the dipole form factor following that used in Refs. [7,41,42],

$$F_M(q_{\text{ex}}^2, M_{\text{ex}}) = \left( \frac{\Lambda_M^2 - M_{\text{ex}}^2}{\Lambda_M^2 - q_{\text{ex}}^2} \right)^2, \quad (30)$$

and for the exchanged baryons, we take the form factor employed in Refs. [43,44],

$$F_B(q_{\text{ex}}^2, M_{\text{ex}}) = \frac{\Lambda_B^4}{\Lambda_B^4 + (q_{\text{ex}}^2 - M_{\text{ex}}^2)^2}. \quad (31)$$

Here the  $q_{\text{ex}}$  and  $M_{\text{ex}}$  are the four-momentum and the mass of the exchanged hadron, respectively. In our present calculation, we use the cutoff parameters  $\Lambda_\pi = \Lambda_\eta = 1.3$  GeV for  $\pi$  and  $\eta$  mesons [7],  $\Lambda_K = 0.8$  GeV for the  $K$  meson [41,42], and  $\Lambda_N = \Lambda_{N^*(1535)} = \Lambda_{N^*(1650)} = \Lambda_{N^*(1710)} = \Lambda_{N^*(1720)} = 2.0$  GeV for baryons [7].

TABLE I. Relevant parameters used in the present calculation. The widths and branching ratios are taken from the Particle Data Group [1].

State	Width (MeV)	Decay channel	Adopted branching ratio	$g^2/4\pi$
$\phi$	4.26	$\pi\gamma$	$1.27 \times 10^{-3}$	$1.56 \times 10^{-3}$
		$\eta\gamma$	$1.31 \times 10^{-2}$	$4.01 \times 10^{-2}$
$N^*(1535)$	150	$K^+K^-$	0.49	1.59
		$N\pi$	0.45	$3.68 \times 10^{-2}$
$N^*(1650)$	150	$N\eta$	0.42	0.28
		$N\pi$	0.70	$5.22 \times 10^{-2}$
$N^*(1710)$	100	$N\eta$	0.10	$3.57 \times 10^{-2}$
		$\Lambda K$	0.07	$4.36 \times 10^{-2}$
		$N\pi$	0.13	$7.18 \times 10^{-2}$
		$N\eta$	0.20	0.97
$N^*(1720)$	250	$\Lambda K$	0.15	2.98
		$N\pi$	0.11	$2.04 \times 10^{-3}$
		$N\eta$	0.04	0.11
		$\Lambda K$	0.08	0.49

The propagators for exchanged  $\pi$ ,  $\eta$ , and  $K$  mesons used in our calculation are

$$G_{\pi,\eta,K}(q) = \frac{i}{q^2 - m_{\pi,\eta,K}^2}. \quad (32)$$

For the propagator of spin-1/2 baryon, we use

$$G_{\frac{1}{2}}(q) = \frac{i(\not{q} + M)}{q^2 - M^2 + iM\Gamma}. \quad (33)$$

For the propagator of spin-3/2 baryon, it can be taken as

$$G_{\frac{3}{2}}^{\mu\nu}(q) = \frac{i(\not{q} + M)P^{\mu\nu}(q)}{q^2 - M^2 + iM\Gamma}, \quad (34)$$

with

$$P^{\mu\nu}(q) = -g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3M}(\gamma^\mu q^\nu - \gamma^\nu q^\mu) + \frac{2}{3M^2}q^\mu q^\nu, \quad (35)$$

where  $q$ ,  $M$ , and  $\Gamma$  stand for the four-momentum, mass, and total width of the intermediate nucleon resonance, respectively.

From the above effective Lagrangian densities, the scattering amplitudes for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction can be obtained straightforwardly. For example, the amplitudes due to the  $\pi$  exchange can be written as

$$\begin{aligned} \mathcal{M}_N = & \frac{i g_{K\Lambda N} g_{\pi NN} g_{\phi\gamma\pi} F_\pi(q_\pi^2, m_\pi) F_N(q_N^2, m_N)}{2m_N(m_N + m_\Lambda)m_\phi} \\ & \times \bar{u}(p_5, s_5) \gamma_5 \not{p}_4 G_{1/2}(q_N) \gamma_5 \not{q}_\pi u(p_2, s_2) G_\pi(q_\pi) \\ & \times \epsilon^{\mu\nu\alpha\beta} p_{3\mu} \varepsilon_\nu^*(p_3, s_3) p_{1\alpha} \varepsilon_\beta(p_1, s_1), \end{aligned} \quad (36)$$

for the nucleon pole,

$$\begin{aligned} \mathcal{M}_R = & \frac{-i g_{K\Lambda R} g_{\pi NR} g_{\phi\gamma\pi} F_\pi(q_\pi^2, m_\pi) F_R(q_R^2, m_R)}{m_\phi} \\ & \times \bar{u}(p_5, s_5) G_{1/2}(q_R) u(p_2, s_2) G_\pi(q_\pi) \\ & \times \epsilon^{\mu\nu\alpha\beta} p_{3\mu} \varepsilon_\nu^*(p_3, s_3) p_{1\alpha} \varepsilon_\beta(p_1, s_1), \end{aligned} \quad (37)$$

for the  $J^P = \frac{1}{2}^-$  nucleon resonances  $N^*(1535)$  and  $N^*(1650)$ ,

$$\begin{aligned} \mathcal{M}_R = & \frac{i g_{K\Lambda R} g_{\pi N R} g_{\phi\gamma\pi} F_{\pi}(q_{\pi}^2, m_{\pi}) F_R(q_R^2, m_R)}{(m_N + m_R)(m_{\Lambda} + m_R) m_{\phi}} \\ & \times \bar{u}(p_5, s_5) \gamma_5 \not{p}_4 G_{1/2}(q_R) \gamma_5 \not{q}_{\pi} u(p_2, s_2) G_{\pi}(q_{\pi}) \\ & \times \epsilon^{\mu\nu\alpha\beta} p_{3\mu} \varepsilon_{\nu}^*(p_3, s_3) p_{1\alpha} \varepsilon_{\beta}(p_1, s_1), \end{aligned} \quad (38)$$

for the  $J^P = \frac{1}{2}^+$  nucleon resonance  $N^*(1710)$ , and

$$\begin{aligned} \mathcal{M}_R = & \frac{i g_{K\Lambda R} g_{\pi N R} g_{\phi\gamma\pi} F_{\pi}(q_{\pi}^2, m_{\pi}) F_R(q_R^2, m_R)}{m_{\pi} m_K m_{\phi}} \\ & \times \bar{u}(p_5, s_5) p_{4\mu} G_{3/2}^{\mu\nu}(q_R) q_{\pi\nu} u(p_2, s_2) G_{\pi}(q_{\pi}) \\ & \times \epsilon^{\mu\nu\alpha\beta} p_{3\mu} \varepsilon_{\nu}^*(p_3, s_3) p_{1\alpha} \varepsilon_{\beta}(p_1, s_1), \end{aligned} \quad (39)$$

for the  $J^P = \frac{3}{2}^+$  nucleon resonance  $N^*(1720)$ . Here  $p_1$ ,  $p_2$ ,  $p_3$ ,  $p_4$ , and  $p_5$  are the four-momenta of the photon, proton,  $\phi$ ,  $K^+$ , and  $\Lambda$ , respectively;  $s_1$ ,  $s_2$ ,  $s_3$ , and  $s_5$  are the spin projections of the photon, proton,  $\phi$ , and  $\Lambda$ , respectively.  $q_{\pi} = p_1 - p_3$  is the four-momentum for the exchanged  $\pi$  meson, and  $q_R = p_4 + p_5$  is the four-momentum for the intermediate nucleon resonance. The amplitudes due to the  $\eta$  exchange are similar to those due to the  $\pi$  exchange, and can be obtained by replacing  $\pi$  with  $\eta$  in the above equations.

The amplitudes due to the  $K$  exchange can be written as

$$\begin{aligned} \mathcal{M}_K = & \frac{e g_{K\Lambda N} g_{\phi K K}}{m_N + m_{\Lambda}} F_K(q_{K^-}^2, m_{K^-}) F_K(q_{K^+}^2, m_{K^+}) \\ & \times \bar{u}(p_5, s_5) \gamma_5 \not{q}_{K^+} u(p_2, s_2) G_K(q_{K^+}) \\ & \times (q_{K^-} - q_{K^+}) \cdot \varepsilon^*(p_3, s_3) G_K(q_{K^-}) \\ & \times (p_4 - q_{K^-}) \cdot \varepsilon(p_1, s_1), \end{aligned} \quad (40)$$

where  $q_{K^-} = p_1 - p_4$  is the four-momentum for the exchanged  $K^-$  meson, and  $q_{K^+} = p_2 - p_5$  is the four-momentum for the exchanged  $K^+$  meson.

The contact term is required to keep the full amplitude gauge invariant, and can be written as

$$\begin{aligned} \mathcal{M}_c = & i \frac{e g_{K\Lambda N} g_{\phi K K}}{m_N + m_{\Lambda}} F_K(q_{K^-}^2, m_{K^-}) F_K(q_{K^+}^2, m_{K^+}) \\ & \times \bar{u}(p_5, s_5) \gamma_5 \not{q}_{K^+} u(p_2, s_2) G_K(q_{K^+}) \varepsilon_{\mu}^*(p_3, s_3) \\ & \times \left[ g^{\mu\nu} + \frac{2(p_4 + q_{K^+})^{\mu} p_4^{\nu}}{t - m_K^2} \right] \varepsilon_{\nu}(p_1, s_1). \end{aligned} \quad (41)$$

Then the calculations of the differential and total cross sections for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction are straightforward,

$$\begin{aligned} d\sigma(\gamma p \rightarrow \phi K^+ \Lambda) = & \frac{1}{8E_{\gamma}} \sum_{s_i} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \frac{m_{\Lambda} d^3 p_5}{E_5} \\ & \times \delta^4(p_1 + p_2 - p_3 - p_4 - p_5), \end{aligned} \quad (42)$$

where  $E_3$ ,  $E_4$ , and  $E_5$  are the energy of the  $\phi$ ,  $K^+$ , and  $\Lambda$ , respectively, and  $E_{\gamma}$  is the photon energy at the laboratory frame. Since the relative phases between various amplitudes

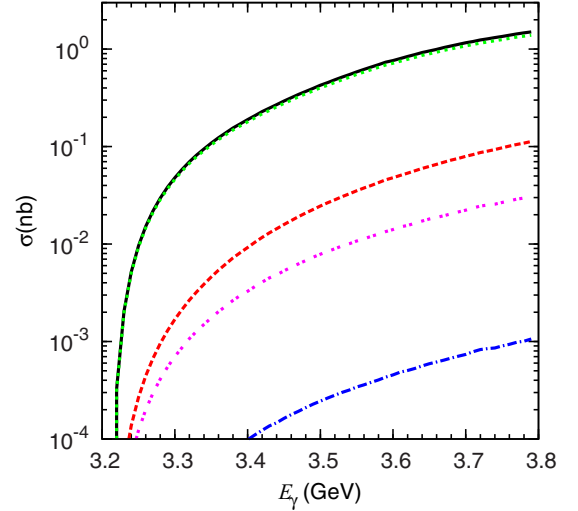


FIG. 2. (Color online) Total cross sections for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction as a function of the beam energy  $E_{\gamma}$ . The red-dashed and green-dotted lines stand for the contributions from the  $\pi$  and  $\eta$ , respectively. The blue-dashed-dotted line denotes the contribution from the  $K$  and contact term. The total contribution from the  $\pi$ ,  $\eta$ ,  $K$ , and the contact term is shown by the solid line. The contribution from the  $K$  and contact term with  $\Lambda_K = 1.0$  GeV is shown by the pink-dotted-dotted curve.

are not known, the interference terms between these parts are ignored in the present calculation.<sup>1</sup>

### III. NUMERICAL RESULTS AND DISCUSSIONS

With the formalism and ingredients given above, the total cross section versus the beam energy  $E_{\gamma}$  for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction is calculated by using a Monte Carlo multiparticle phase space integration program.<sup>2</sup>

The roles of various meson exchange processes in describing the total cross section are shown in Fig. 2, where one can see that the  $\eta$  meson exchange plays a predominant role, while contributions from the  $\pi$  exchange,  $K$  exchange, and contact term are small. This behavior does not vary much with  $R \equiv |g_{K\Lambda N^*(1535)}/g_{\eta N N^*(1535)}|$ . The overwhelming  $\eta$  exchange contribution, compared with that from the  $\pi$  exchange, can be easily understood since the value of the coupling constant  $g_{\phi\gamma\eta}^2$  is 26 times larger than the one of  $g_{\phi\gamma\pi}^2$ . Hence, this reaction

<sup>1</sup>In effective Lagrangian approaches, the relative phases between different amplitudes are not fixed. We should generally introduce a relative phase between different amplitudes as a free parameter. However, we do not have experimental information for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction, and we will see in the following that the magnitudes of the contributions from different processes in the energy region what we considered are much different, hence the effect of the interference term should be small, and we ignore them in the present work.

<sup>2</sup>In our calculations, we take  $g_{K\Lambda N^*(1535)}^2 = 3.52$ . Because the value of  $R$  varies in a wide range as mentioned in the Introduction, we take  $R = 1$  here for simplicity.  $R = 1$  leads to  $g_{K\Lambda N^*(1535)}^2 = R^2 g_{\eta N N^*(1535)}^2 = 3.52$  based on Table I.

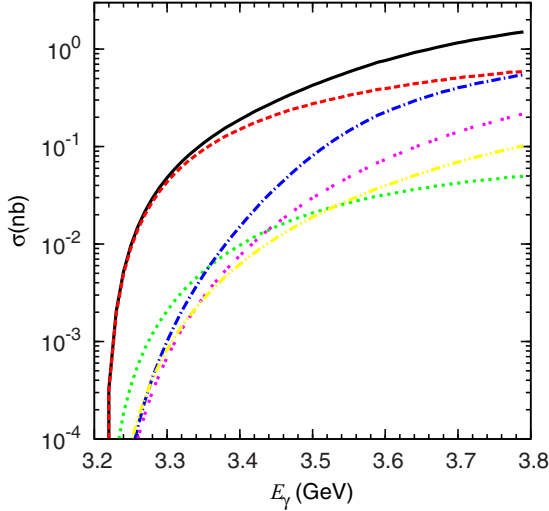


FIG. 3. (Color online) Total cross sections vs the beam energy  $E_\gamma$  for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction. The red-dashed, green-dotted, blue-dashed-dotted, and pink-dotted-dotted, and yellow-dotted-dotted-dashed lines stand for contributions from the  $N^*(1535)$ ,  $N^*(1650)$ ,  $N^*(1710)$ ,  $N^*(1720)$ , and background, respectively. Their sum is shown by the solid line.

provides a good platform for studying the nucleon resonances that couple strongly to the  $\eta N$  and  $K \Lambda$  channels.

It is worthy mentioning that the contribution from the  $K$  exchange and contact term is small, but it increases rapidly and depends much on the value of the cutoff parameter  $\Lambda_K$ . To see how much it depends on the cutoff parameter, we also show in Fig. 2 the theoretical result with  $\Lambda_K = 1.0$  GeV for comparison.

The relative importance of the contributions of each intermediate resonance to the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction is demonstrated in Fig. 3, where the contributions of the  $N^*(1535)$ ,  $N^*(1650)$ ,  $N^*(1710)$ ,  $N^*(1720)$ , and background are shown by red-dashed, green-dotted, blue-dashed-dotted, and pink-dotted-dotted and yellow-dotted-dotted-dashed curves, respectively. Their total contribution is depicted by the solid line. It is clear that the  $N^*(1535)$  resonance gives the dominant contribution from the reaction threshold to  $E_\gamma$  around 3.4 GeV, while the other resonances and background give the minor contribution. When the beam energy  $E_\gamma$  is above 3.4 GeV, the contributions from other processes are also important.

The contribution from the  $N^*(1535)$  resonance be proportional to  $R^2$ . If  $R$  varies in the range of about 0.5 ~ 2.6 as mentioned in Sec. I, from Fig. 3 one can see that near the threshold of the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction, the contribution from the  $N^*(1535)$  resonance remains dominant.

When the  $\eta$  exchange is dominant as shown in Fig. 2, these facts, i.e., the  $N^*(1535)$  resonance couples to the  $\eta N$  and  $K \Lambda$  channels in the  $S$ -wave, the  $N^*(1710)$  and  $N^*(1720)$  resonances couple to the  $\eta N$  and  $K \Lambda$  channels in  $P$ -wave, and the  $N^*(1650)$  couples weaker to the  $\eta N$  channel, could account for the fact that the contribution from the  $N^*(1535)$  resonance is dominant while the contributions from the  $N^*(1650)$ ,  $N^*(1710)$ , and  $N^*(1720)$  are suppressed at the beam energy close to the reaction threshold.

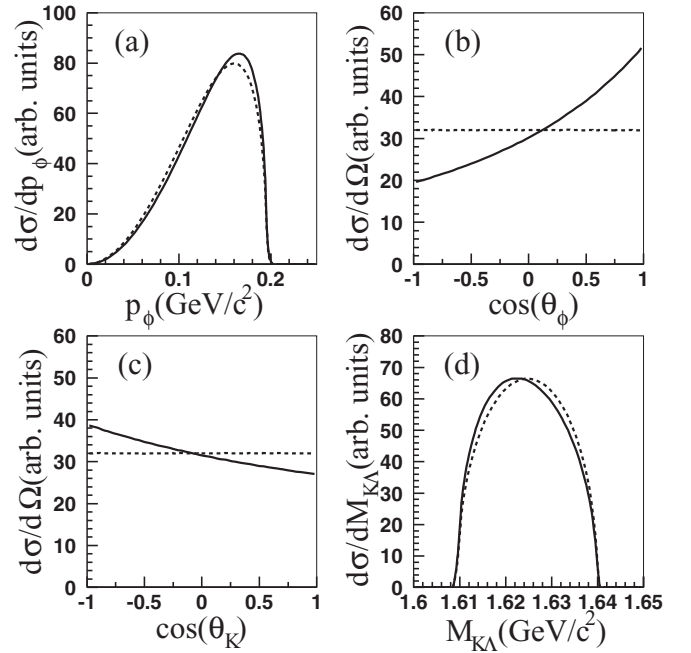


FIG. 4. Differential distributions for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction at the beam energy  $E_\gamma = 3.3$  GeV. (a) The momentum distribution of the final  $\phi$  meson; (b) the angular distribution of the final  $\phi$  meson; (c) the angular distribution of the final  $K^+$  meson; (d) the invariant mass distribution of the final  $K^+ \Lambda$  pair. The solid curves stand for our theoretical predictions while the dashed lines represent the pure phase space distributions.

In addition to the total cross section, the differential distributions for the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction are also calculated. The corresponding momentum of the  $\phi$  meson, the angular distributions of the  $\phi$  and  $K^+$  mesons, and the  $K \Lambda$  invariant mass spectrum at beam energies  $E_\gamma = 3.3$  and 3.7 GeV are shown in Figs. 4 and 5, respectively. The dashed lines are pure phase space distributions and the solid lines stand for our theoretical results.

From Figs. 4 and 5, one can see that the difference between the momentum distribution of the  $\phi$  meson and phase space distribution is slight at  $E_\gamma = 3.3$  GeV but apparent at  $E_\gamma = 3.7$  GeV. Also, the specific features of the angular distributions of the  $\phi$  and  $K^+$  mesons, i.e., the forward contribution for the  $\phi$  meson and the backward contribution for the  $K^+$  meson, exist at both beam energies. For the  $K \Lambda$  invariant mass spectrum, the enhancement near threshold is from the contribution of the  $N^*(1535)$  resonance, and the other bump appearing at  $E_\gamma = 3.7$  GeV results from the  $N^*(1710)$  resonance.

#### IV. SUMMARY

In this work, we have studied the  $\gamma p \rightarrow \phi K^+ \Lambda$  reaction near threshold within an effective Lagrangian approach. In addition to the background contributions from the  $s$ -channel nucleon pole,  $K$  exchange, and contact term, the intermediate nucleon resonances due to the  $\pi$  and  $\eta$  mesons exchanges are also investigated. The total and differential cross sections are predicted. Our results show that the contribution from the

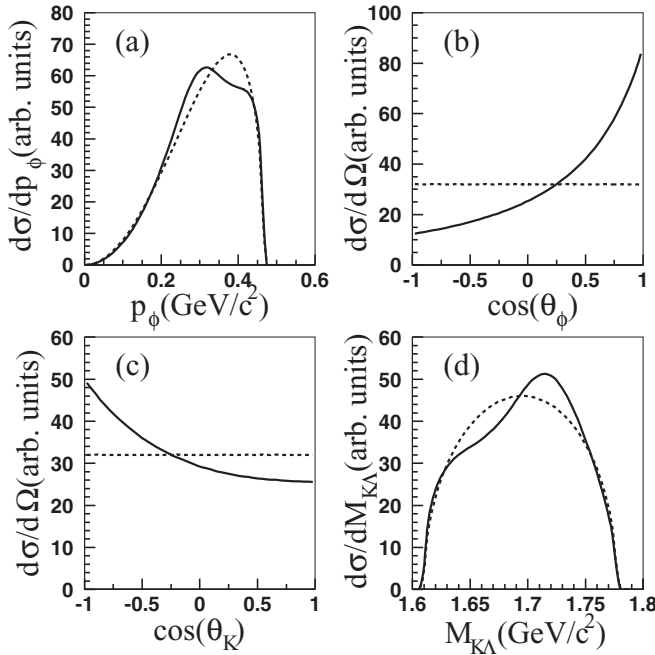


FIG. 5. As in Fig. 4 but at the beam energy  $E_\gamma = 3.7$  GeV.

$N^*(1535)$  resonance due to the  $\eta$  exchange plays the dominant role near threshold, while other resonances and background contributions are small and can be ignored. Thus, this reaction provides a good chance to study the coupling of  $K \Lambda N^*(1535)$  interaction. It is also found that the  $\phi$  meson has the forward angular distribution, while the  $K^+$  meson gives the backward angular contribution. These specific features of the angular distributions, together with the total cross section which is in the magnitude of 0.2 nb at photon energy  $E_\gamma = 3.3 \sim 3.4$  GeV, can be tested by future experiments. An experiment with a precision about 0.1 nb will be enough to check our model. The future experiments in the JLab 12 GeV upgrade with large luminosity promise to reach such a requirement.

#### ACKNOWLEDGMENTS

We thank Xu Cao for useful discussions. This work is partly supported by the Ministry of Science and Technology of China (2014CB845406), the National Natural Science Foundation of China under Grants No. 11105126, No. 11475227, No. 11375024, and No. 11175220, the One Hundred Person Project of The Chinese Academy of Sciences (Y101020BR0), and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry.

- [1] K. A. Olive *et al.* (Particle Data Group), *Chin. Phys. C* **38**, 090001 (2014).
- [2] S. Capstick and N. Isgur, *Phys. Rev. D* **34**, 2809 (1986).
- [3] J. J. Xie and C. Wilkin, *Phys. Rev. C* **82**, 025210 (2010).
- [4] X. Cao and X. G. Lee, *Phys. Rev. C* **78**, 035207 (2008).
- [5] X. H. Zhong and Q. Zhao, *Phys. Rev. C* **84**, 065204 (2011).
- [6] F. Huang, H. Habermann, and K. Nakayama, *Phys. Rev. C* **87**, 054004 (2013).
- [7] J. J. Xie, B. S. Zou, and H. C. Chiang, *Phys. Rev. C* **77**, 015206 (2008).
- [8] J. P. Dai, P. N. Shen, J. J. Xie, and B. S. Zou, *Phys. Rev. D* **85**, 014011 (2012).
- [9] B. C. Liu and B. S. Zou, *Phys. Rev. Lett.* **96**, 042002 (2006).
- [10] B. C. Liu and B. S. Zou, *Phys. Rev. Lett.* **98**, 039102 (2007).
- [11] N. Kaiser, T. Waas, and W. Weise, *Nucl. Phys. A* **612**, 297 (1997).
- [12] T. Inoue, E. Oset, and M. J. Vicente Vacas, *Phys. Rev. C* **65**, 035204 (2002).
- [13] J. Nieves and E. R. Arriola, *Phys. Rev. D* **64**, 116008 (2001).
- [14] C. Garcia-Recio, M. F. M. Lutz, and J. Nieves, *Phys. Lett. B* **582**, 49 (2004).
- [15] B. S. Zou, *Nucl. Phys. A* **827**, 333c (2009).
- [16] B. S. Zou, *Nucl. Phys. A* **835**, 199 (2010).
- [17] C. S. An and B. S. Zou, *Eur. Phys. J. A* **39**, 195 (2009).
- [18] C. S. An, *Chin. Phys. C* **33**, 1393 (2009).
- [19] T. Mart, *Phys. Rev. C* **87**, 042201 (2013).
- [20] L. S. Geng, E. Oset, B. S. Zou, and M. Doring, *Phys. Rev. C* **79**, 025203 (2009).
- [21] A. V. Sarantsev, V. A. Nikonov, A. V. Anisovich, E. Klempt, and U. Thoma, *Eur. Phys. J. A* **25**, 441 (2005).
- [22] P. C. Bruns, M. Mai, and U. G. Meissner, *Phys. Lett. B* **697**, 254 (2011).
- [23] G. Penner and U. Mosel, *Phys. Rev. C* **66**, 055211 (2002).
- [24] C. S. An and B. Saghai, *Phys. Rev. C* **84**, 045204 (2011).
- [25] C.-Z. Wu, Q.-F. Lü, J.-J. Xie, and X.-R. Chen, *Commun. Theor. Phys.* **63**, 215 (2015).
- [26] K. Tsushima, A. Sibirtsev, A. W. Thomas, and G. Q. Li, *Phys. Rev. C* **59**, 369 (1999); **61**, 029903 (2000).
- [27] A. Sibirtsev, J. Haidenbauer, H.-W. Hammer, and S. Krewald, *Eur. Phys. J. A* **27**, 269 (2006).
- [28] B. C. Liu and B. S. Zou, *Commun. Theor. Phys.* **46**, 501 (2006).
- [29] O. V. Maxwell, *Phys. Rev. C* **85**, 034611 (2012).
- [30] B. C. Liu, *Phys. Rev. C* **86**, 015207 (2012).
- [31] J.-J. Xie, B.-C. Liu, and C.-S. An, *Phys. Rev. C* **88**, 015203 (2013).
- [32] B. C. Liu and J. J. Xie, *Phys. Rev. C* **85**, 038201 (2012).
- [33] B. C. Liu and J. J. Xie, *Phys. Rev. C* **86**, 055202 (2012).
- [34] Q. F. Lü, X. H. Liu, J. J. Xie, and D. M. Li, *Mod. Phys. Lett. A* **29**, 1450012 (2014).
- [35] Q. F. Lü, J. J. Xie, and D. M. Li, *Phys. Rev. C* **90**, 034002 (2014).
- [36] J. J. Xie, E. Wang, and J. Nieves, *Phys. Rev. C* **89**, 015203 (2014).
- [37] J. J. Xie and J. Nieves, *Phys. Rev. C* **82**, 045205 (2010).
- [38] W.-H. Liang, P.-N. Shen, B.-S. Zou, and A. Faessler, *Eur. Phys. J. A* **21**, 487 (2004).
- [39] H.-Y. Ryu, A. I. Titov, A. Hosaka, and H.-C. Kim, *Prog. Theor. Exp. Phys.* **2014**, 023D03 (2014).
- [40] B. S. Zou and F. Hussain, *Phys. Rev. C* **67**, 015204 (2003).
- [41] Y. Oh, C. M. Ko, and K. Nakayama, *Phys. Rev. C* **77**, 045204 (2008).
- [42] P. Gao, J. J. Wu, and B. S. Zou, *Phys. Rev. C* **81**, 055203 (2010).
- [43] T. Feuster and U. Mosel, *Phys. Rev. C* **58**, 457 (1998); **59**, 460 (1999).
- [44] V. Shklyar, H. Lenske, and U. Mosel, *Phys. Rev. C* **72**, 015210 (2005).