Light vector meson masses in strange hadronic matter: A QCD sum rule approach

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In the present work, the properties of the light vector mesons (ρ , ω , and ϕ) in strange hadronic matter are studied using the QCD sum rule approach. The in-medium masses of the vector mesons are calculated from the modifications of the light quark condensates and the gluon condensates in the hadronic medium. The light quark condensates in the hadronic matter are obtained from the values of the nonstrange and strange scalar fields of the explicit chiral symmetry breaking term in a chiral SU(3) model. The in-medium gluon condensate is calculated through the medium modification of a scalar dilaton field, which is introduced into the chiral SU(3) model to simulate the scale symmetry breaking of QCD. The mass of the ω meson is observed to have initially a drop with increase in density and then a rise due to the scattering with the baryons. The mass of the ρ meson is seen to drop with density due to decrease of the light quark condensates in the medium. The effects of isospin asymmetry and strangeness of the medium on the masses of the vector mesons are also studied in the present work. The ϕ meson is observed to have marginal drop in its mass in the nuclear medium. However, the strangeness of the medium is seen to lead to an appreciable increase in its mass arising due to scattering with the hyperons.

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I. INTRODUCTION

The study of the properties of hadrons in hot and dense matter is an important topic of research in strong interaction physics. The changes in the hadron properties in the medium affect the experimental observables from the hot and/or dense matter produced in the heavy ion collision experiments. The medium modifications of the properties of the light vector mesons [1] can affect the low mass dilepton spectra, the properties of the kaons and antikaons can show in their production as well as collective flow. The modifications of the properties of the charm mesons D and \overline{D} as well as the charmonium states can modify the yield of the open charm meson as well as charmonium states in the high energy nuclear collision experiments. The in-medium properties of the hadrons have been studied using various methods, namely the QCD sum rule (QSR) approach, the quark meson coupling (QMC) model, the effective hadronic models like quantum hadrodynamics (QHD) model [2] and chiral effective theories, as well as using the coupled channel approach. The hadron properties have also been studied in models like the Nambu-Jona Lasinio model [3–7], which describes the spontaneous chiral symmetry breaking of QCD leading to the properties of the pions which emerge as Goldstone modes. The AdS/CFT correspondence and the conjecture of gravity/gauge duality [8] in the recent years have provided a powerful method to study the strongly coupled gauge theories. Holography relates the quantum field theory (QFT) in d dimensions to quantum gravity in (d + 1) dimensions, with the gravitational description becoming classical when the QFT is strongly coupled. The method has been used extensively to investigate the hadron physics as well as strongly coupled quark gluon plasma. Using holography duality, many attempts have been made to study the hadrons [9].

In the present work, the medium modification of the masses of the light vector mesons $(\rho, \omega, \text{ and } \phi)$ in the strange hadronic matter, arising due to their interaction with the light quark condensates and the gluon condensates, are investigated, using the QCD sum rule approach [10-20]. The light quark condensates are calculated from the expectation values of the nonstrange and strange scalar fields of the explicit chiral symmetry breaking term in a chiral SU(3) model [21,22]. The gluon condensate in the hadronic medium is obtained from the medium modification of a scalar dilaton field introduced within the chiral SU(3) model through a scale symmetry breaking term in the Lagrangian density leading to the QCD trace anomaly. The chiral SU(3) model has been used to describe the hadronic properties in the vacuum as well as in nuclear matter [21], finite nuclei [22], and the bulk properties of (proto) neutron stars [23]. The vector mesons have also been studied within the model [24], arising due to their interactions with the nucleons in the medium. Within the chiral SU(3) model, the effects of the Dirac polarization effects on the in-medium masses of the vector mesons were found to be appreciable [24], similar to those observed within the quantum hadrodynamics (QHD) model [25]. The model has been used to study the medium modifications of kaons and antikaons in isospin asymmetric nuclear matter in Ref. [26] and in hyperonic matter in Ref. [27]. These studies have been done retaining the leading and next to leading contributions in the chiral perturbation expansion, with coefficients of the interactions as compatible with nuclear matter saturation properties as well as low energy kaon-nucleon scattering data [26,27]. The chiral effective model has also been generalized to SU(4) to derive the interactions of the charm mesons with the light hadrons to study the D mesons in asymmetric nuclear matter at zero temperature [28] and in the symmetric and asymmetric nuclear (hyperonic) matter at finite temperatures in Ref. [29] and Refs. [30,31]. Within the chiral effective approach, the D and \overline{D} mesons were observed to have mass drops of 77 and 27 MeV in symmetric nuclear matter at zero

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temperature at the nuclear matter saturation density ρ_0 [28]. The mass drop for the D meson in symmetric nuclear matter at density ρ_0 may be compared to the values of 60 MeV in the quark meson coupling model [32] and 50 MeV in the QCD sum rule approach [33]. The mass modifications of the charmonium states have also been calculated within the chiral effective model using OCD second-order Stark effect due to the medium modification of the color electric field squared, which can be expressed in terms of the scalar gluon condensate. The medium modification of the scalar gluon condensate is calculated from the modification of the dilaton field, which mimics the scale symetry breaking of QCD in the effective chiral model. The mass shifts for the charmonium states J/ψ , $\psi(3686)$ and $\psi(3770)$ in cold symmetric matter at the nuclear matter saturation density, were found to be -4.35, -59, and -78.5 MeV respectively, when the effects of the finite quark masses in the trace of energy momentum tensor in QCD are taken into account while evaluating the medium modification of the gluon condensate from the trace of the energy momentum tensor. In the limit of massless quarks in the trace of energy momentum tensor in QCD, these values turn out to be -9.3, -126.4, and -167.5 MeV. These values of the mass shifts may be compared with the values of -8, -100, and -140 MeV respectively, using the QCD second-order Stark effect in the linear density approximation [34]. In the linear density approximation, the density dependence of the quark and gluon condensates are calculated in the dilute gas approximation of nucleons [1], with additional contributions from the moments of the parton distribution functions in the nucleon [10,15,16]. Using the scalar as well as twist 2 gluon condensates as obtained from the dilaton field calculated in the chiral effective model [35] used in the present investigation, the mass shifts in the charmonium shifts have also been studied, within the framework of the QCD sum rule approach with operator product expansion up to mass dimension 4. The values of mass shifts of J/ψ and η_c were evaluated to be -4.48 and -5.21 MeV at the nuclear matter density, in symmetric nuclear matter at zero temperature [35]. These values may be compared with the values of -7 and -5 MeV in Ref. [36], calculated within the QCD sum rule approach, using the medium modification of the gluon condensate calculated in the linear density approximation. However, using the QCD sum rules and operator product expansion up to mass dimension 6 [37], the mass shift for J/ψ is obtained as -4 MeV. From the medium modified masses of the charmonium states as well as the D and \overline{D} mesons calculated within the chiral effective model, the partial decay widths of the charmonium states to $D\bar{D}$ pairs in the isospin asymmetric strange hadronic matter have been calculated [31] using the ${}^{3}P_{0}$ model [38]. In the hadronic medium, the masses of the charmonium state and the D and \overline{D} mesons can be such that the decay widths, which have the form of a polynomial multiplied by a Gaussian function, can vanish at certain densities. Such nodes have also been discussed in the literature [38]. In the present investigation, the properties of the light vector mesons are studied, using QCD sum rule approach, due to their interactions with the quark and gluon condensates in the isospin asymmetric strange hadronic medium, calculated in the chiral SU(3)model.

The outline of the paper is as follows: In Sec. II, the chiral SU(3) model used to calculate the quark and gluon condensates in the hadronic medium is briefly discussed. In the present work, the in-medium condensates as calculated in the chiral SU(3) model are taken as inputs for studying the in-medium masses of the light vector mesons using the OCD sum rule approach. The medium modifications of the quark and gluon condensates arise from the medium modification of the scalar fields of the explicit symmetry breaking term and of the scalar dilaton field introduced in the hadronic model to incorporate broken scale invariance of QCD. In Sec. III, the results for the medium modifications of the light vector mesons using a QCD sum rule approach are presented. In Sec. IV, the findings of the present investigation are summarized and compared with the existing results in the literature for the in-medium properties of the light vector mesons.

II. HADRONIC CHIRAL SU(3) × SU(3) MODEL

In the present investigation, the values of the quark and gluon condensates in the hadronic medium are calculated within an effective chiral SU(3) model [22], which is used to study the in-medium properties of vector mesons. The model is based on the nonlinear realization of chiral symmetry [39–41] and broken scale invariance [21,22,24]. This model has been used successfully to describe nuclear matter, finite nuclei, hypernuclei, and neutron stars. The effective hadronic chiral Lagrangian density contains the following terms:

$$\mathcal{L} = \mathcal{L}_{kin} + \sum_{W = X, Y, V, A, u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{SB}.$$
 (1)

In Eq. (1), \mathcal{L}_{kin} is the kinetic energy term, \mathcal{L}_{BW} is the baryon-meson interaction term in which the baryon-spin-0 meson interaction term generates the vacuum baryon masses. \mathcal{L}_{vec} describes the dynamical mass generation of the vector mesons via couplings to the scalar mesons and contains additionally quartic self-interactions of the vector fields. \mathcal{L}_0 contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential. \mathcal{L}_{SB} describes the explicit chiral symmetry breaking.

To study the in-medium hadron properties using the chiral SU(3) model, the mean field approximation is used, where all the meson fields are treated as classical fields. In this approximation, only the scalar and the vector fields contribute to the baryon-meson interaction \mathcal{L}_{BW} since for all the other mesons, the expectation values are zero. The baryon-scalar meson coupling constants are fitted from the vacuum masses of the baryons. The parameters in the model [22,26] are chosen so as to decouple the strange vector field $\phi_{\mu} \sim \bar{s} \gamma_{\mu} s$ from the nucleon.

The concept of broken scale invariance leading to the trace anomaly in QCD, $\theta^{\mu}_{\mu} = \frac{\beta_{\text{QCD}}}{2g} G^a{}_{\mu\nu} G^{\mu\nu a}$, where $G^a_{\mu\nu}$ is the gluon field strength tensor of QCD, is simulated in the effective Lagrangian at tree level through the introduction of

the scale breaking terms [42,43]

$$\mathcal{L}_{\text{scale breaking}} = -\frac{1}{4} \chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3} \chi^4 \ln\left(\left(\frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0}\right) \left(\frac{\chi}{\chi_0}\right)^3\right).$$
(2)

The Lagrangian density corresponding to the dilaton field χ leads to the trace of the energy momentum tensor as [35,44]

$$\theta^{\mu}_{\mu} = \chi \frac{\partial \mathcal{L}}{\partial \chi} - 4\mathcal{L} = -(1-d)\chi^4.$$
(3)

The comparison of the trace of the energy momentum tensor arising from the trace anomaly of QCD with that of the present chiral model given by Eq. (3) gives the relation of the dilaton field to the scalar gluon condensate. In the limit of finite quark masses [45], one has

$$T^{\mu}_{\mu} = \sum_{i=u,d,s} m_i \bar{q}_i q_i + \left\langle \frac{\beta_{\text{QCD}}}{2g} G^a_{\mu\nu} G^{\mu\nu a} \right\rangle \equiv -(1-d)\chi^4, \quad (4)$$

where the first term of the energy-momentum tensor within the chiral SU(3) model is the negative of the explicit chiral symmetry breaking term \mathcal{L}_{SB} . In the mean field approximation, this chiral symmetry breaking term is given as

$$\mathcal{L}_{SB} = \operatorname{Tr}\left[\operatorname{diag}\left(-\frac{1}{2}m_{\pi}^{2}f_{\pi}(\sigma+\delta), -\frac{1}{2}m_{\pi}^{2}f_{\pi}(\sigma-\delta), \left(\sqrt{2}m_{k}^{2}f_{k}-\frac{1}{\sqrt{2}}m_{\pi}^{2}f_{\pi}\right)\zeta\right)\right].$$
(5)

In the above, the matrix, whose trace gives the Lagrangian density corresponding to the explicit chiral symmetry breaking in the chiral SU(3) model, has been explicitly written down. Comparing the above term with the explicit chiral symmetry breaking term of the Lagrangian density in QCD given as

$$\mathcal{L}_{SB}^{\text{QCD}} = -\operatorname{Tr}[\operatorname{diag}\left(m_u \bar{u} u, m_d \bar{d} d, m_s \bar{s} s\right)],\tag{6}$$

one obtains the nonstrange quark condensates $(\langle \bar{u}u \rangle \text{ and } \langle \bar{d}d \rangle)$ and the strange quark condensate $(\langle \bar{s}s \rangle)$ to be related to the scalar fields, σ , δ , and ζ as

$$m_u \langle \bar{u}u \rangle = \frac{1}{2} m_\pi^2 f_\pi(\sigma + \delta), \tag{7}$$

$$m_d \langle \bar{d}d \rangle = \frac{1}{2} m_\pi^2 f_\pi (\sigma - \delta), \qquad (8)$$

and

$$m_s \langle \bar{s}s \rangle = \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta.$$
(9)

The coupled equations of motion for the nonstrange scalar isoscalar field σ , scalar isovector field δ , the strange scalar field ζ , and the dilaton field χ , derived from the Lagrangian density, are solved to obtain the values of these fields in the strange hadronic medium.

The QCD β function occurring in the right hand side of Eq. (4), at one loop level, for N_c colors and N_f flavors, is given as

$$\beta_{\text{QCD}}(g) = -\frac{11N_c g^3}{48\pi^2} \left(1 - \frac{2N_f}{11N_c}\right) + O(g^5).$$
(10)

Using the one loop β function given by Eq. (10), for $N_c = 3$ and $N_f = 3$, the trace of the energy-momentum tensor in QCD is obtained as

$$\theta^{\mu}_{\mu} = -\frac{9}{8} \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu a} + \left(m_{\pi}^2 f_{\pi} \sigma + \left(\sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_{\pi}^2 f_{\pi} \right) \zeta \right), \quad (11)$$

where $\alpha_s = \frac{g^2}{4\pi}$. Using Eqs. (4) and (11), one obtains

$$\left\{ \frac{\alpha_s}{\pi} G^a{}_{\mu\nu} G^{a\,\mu\nu} \right\}$$

$$= \frac{8}{9} \left\{ (1-d)\chi^4 + \left[m_\pi^2 f_\pi \sigma + \left(\sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right] \right\}.$$
(12)

Hence the scalar gluon condensate of QCD ($\langle G^a_{\mu\nu}G^{\mu\nu a} \rangle$) is simulated by a scalar dilaton field in the present hadronic model. For the case of massless quarks, the scalar gluon condensate is proportional to the fourth power of the dilaton field, whereas for the case of finite masses of quarks, there are modifications arising from the scalar fields σ and ζ .

In the present work, the light quark condensates $\langle \bar{u}u \rangle$, $\langle \bar{d}d \rangle$, and $\langle \bar{s}s \rangle$ and the scalar gluon condensate $\langle \frac{\alpha_x}{\pi} G^a{}_{\mu\nu} G^{a\mu\nu} \rangle$ in the hadronic medium are calculated, using Eqs. (7)–(9), and (12) respectively, from the medium modifications of the scalar fields, σ , δ , ζ , and χ . These values of the quark and gluon condensates are then taken as inputs for studying the masses of the light vector mesons (ω , ρ , ϕ) in the strange hadronic matter using the QCD sum rule approach. In the next section, the QCD sum rule approach is described, which is used to study these in-medium vector meson masses in the isospin asymmetric strange hadronic medium.

III. QCD SUM RULE APPROACH

In the present section, the properties of the light vector mesons (ω, ρ, ϕ) in the nuclear medium are investigated, using the method of QCD sum rules. The in-medium masses of the vector mesons are computed from the medium modifications of the light quark condensates and the scalar gluon condensate calculated in the chiral effective model as described in the previous section. The current current correlation function for the vector meson $V(=\omega, \rho, \phi)$ is written as

$$\Pi_{\mu\nu} = i \int d^4x d^4y \langle 0|Tj^V_{\mu}(x)j^V_{\nu}(0)|0\rangle, \qquad (13)$$

where *T* is the time ordered product and J^V_{μ} is the current for the vector meson, $V = \rho, \omega, \phi$, given as $j^{\rho}_{\mu} = \frac{1}{2}(\bar{u}\gamma_{\mu}u - \bar{d}\gamma_{\mu}d)$, $j^{\omega}_{\mu} = \frac{1}{6}(\bar{u}\gamma_{\mu}u + \bar{d}\gamma_{\mu}d)$, and $j^{\phi}_{\mu} = -\frac{1}{3}(\bar{s}\gamma_{\mu}s)$. Current conservation gives the transverse tensor structure for the correlation function as

$$\Pi^{V}_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right)\Pi^{V}(q^2),$$
(14)

where

$$\Pi^{V}(q^{2}) = \frac{1}{3}g^{\mu\nu}\Pi^{V}_{\mu\nu}(q).$$
(15)

The correlation function $\Pi^V(q^2)$ in the large spacelike region $Q^2 = -q^2 \gg 1 \text{ GeV}^2$ for the light vector mesons (ω , ρ , and ϕ) can be written in terms of the operator product expansion (OPE) as [15,16]

$$12\pi^{2}\tilde{\Pi}^{V}(q^{2} = -Q^{2})$$

= $d_{V}\left[-c_{0}^{V}\ln\left(\frac{Q^{2}}{\mu^{2}}\right) + \frac{c_{1}^{V}}{Q^{2}} + \frac{c_{2}^{V}}{Q^{4}} + \frac{c_{3}^{V}}{Q^{6}} + \cdots\right],$ (16)

where $\tilde{\Pi}^{V}(q^{2} = -Q^{2}) = \frac{\Pi^{V}(q^{2} = -Q^{2})}{Q^{2}}$, $d_{V} = 3/2$, 1/6, and 1/3 for ρ , ω , and ϕ mesons respectively, and μ is a scale which we shall take as 1 GeV in the present investigation. The first term in the OPE given by Eq. (16) is the leading term calculated in the perturbative QCD. The subsequent terms in the OPE are suppressed as powers of $1/Q^2$. The coefficients $c_i^V(i = 1, 2, 3)$ of these terms contain the information of the nonperturbative effects of QCD in terms of the quark and gluon condensates, as well as of the Wilson coefficients [12,14]. This form of the OPE has been arrived at by treating the Wilson coefficients as medium independent, with all the medium effects incorporated into the quark and gluon condensates [12,14–17]. In an earlier work [18], accounting for the effect of temperature for calculating the short distance properties of the hadronic correlators was observed to lead to erroneous mixture of the short and long distance dynamics and temperature dependent Wilson coefficients. In Ref. [12], the QCD sum rule approach at finite temperatures was reformulated in a consistent manner, which separates the hard dynamics from the soft dynamics.

For the vector mesons, ρ and ω , containing the *u* and *d* quarks (antiquarks), these coefficients are given as [15]

$$c_0^{(\rho,\omega)} = 1 + \frac{\alpha_s(Q^2)}{\pi}, \quad c_1^{(\rho,\omega)} = -3(m_u^2 + m_d^2),$$
 (17)

$$c_2^{(\rho,\omega)} = \frac{\pi^2}{3} \left\langle \frac{\alpha_s}{\pi} G^{\mu\nu} G_{\mu\nu} \right\rangle + 4\pi^2 \langle m_u \bar{u}u + m_d \bar{d}d \rangle, \quad (18)$$

$$c_{3}^{(\rho,\omega)} = -4\pi^{3} \bigg[\langle \alpha_{s}(\bar{u}\gamma_{\mu}\gamma_{5}\lambda^{a}u \mp \bar{d}\gamma_{\mu}\gamma_{5}\lambda^{a}d)^{2} \rangle + \frac{2}{9} \\ \times \bigg\{ \alpha_{s}(\bar{u}\gamma_{\mu}\lambda^{a}u + \bar{d}\gamma_{\mu}\lambda^{a}d) \bigg(\sum_{q=u,d,s} \bar{q}\gamma^{\mu}\lambda^{a}q\bigg) \bigg\} \bigg].$$
(19)

In the above, $\alpha_S = 4\pi/[b \ln(Q^2/\Lambda_{QCD}^2)]$ is the running coupling constant, with $\Lambda_{OCD} = 140$ MeV and b = 11 - 100

 $(2/3)N_f = 9$. In Eq. (19), the " \mp " sign in the first term corresponds to the $\rho(\omega)$ meson.

For the ϕ meson, these coefficients are given as [15,46]

$$c_0^{\phi} = 1 + \frac{\alpha_s(Q^2)}{\pi}, \quad c_1^{\phi} = -6m_s^2$$
 (20)

$$c_2^{\phi} = \frac{\pi^2}{3} \left\langle \frac{\alpha_s}{\pi} G^{\mu\nu} G_{\mu\nu} \right\rangle + 8\pi^2 \langle m_s \bar{s}s \rangle, \tag{21}$$

$$c_{3}^{\phi} = -8\pi^{3} \bigg[2 \langle \alpha_{s} (\bar{s}\gamma_{\mu}\gamma_{5}\lambda^{a}s)^{2} \rangle + \frac{4}{9} \bigg\langle \alpha_{s} (\bar{s}\gamma_{\mu}\lambda^{a}s) \bigg(\sum_{q=u,d,s} \bar{q}\gamma^{\mu}\lambda^{a}q \bigg) \bigg\rangle \bigg].$$
(22)

After Borel transformation, the correlator for the vector meson given by Eq. (16) can be written as

$$12\pi^{2}\tilde{\Pi}^{V}(M^{2}) = d_{V} \left[c_{0}^{V}M^{2} + c_{1}^{V} + \frac{c_{2}^{V}}{M^{2}} + \frac{c_{3}^{V}}{2M^{4}} \right].$$
(23)

On the phenomenological side, the correlator function $\tilde{\Pi}^V(Q^2)$ can be written as

$$12\pi^2 \tilde{\Pi}_{\rm phen}^V(Q^2) = \int_0^\infty ds \, \frac{R_{\rm phen}^V(s)}{s+Q^2},\tag{24}$$

where $R_{\text{phen}}^V(s)$ is the spectral density proportional to the imaginary part of the correlator

$$R_{\rm phen}^V(s) = 12\pi \,{\rm Im}\Pi_{\rm phen}^V(s). \tag{25}$$

On Borel transformation, Eq. (24) reduces to

$$12\pi^2 \tilde{\Pi}^V(M^2) = \int_0^\infty ds e^{-s/M^2} R_{\rm phen}^V(s).$$
 (26)

Equating the correlation functions from the phenomenological side given by Eq. (26) to that from the operator product expansion given by Eq. (23), one obtains

$$\int_0^\infty ds e^{-s/M^2} R_{\text{phen}}^V(s) = d_V \left[c_0^V M^2 + c_1^V + \frac{c_2^V}{M^2} + \frac{c_3^V}{2M^4} \right].$$
(27)

The finite energy sum rules (FESR) for the vector mesons are derived from Eq. (27) by assuming that the spectral density separates to a resonance part $R_{\text{phen}}^{V(\text{res})}(s)$ with $s \leq s_0^V$ and a perturbative continuum as

$$R_{\rm phen}^{V}(s) = R_{\rm phen}^{V(\rm res)}(s)\theta(s_{0}^{V}-s) + d_{V}c_{0}^{V}\theta(s-s_{0}^{V}).$$
 (28)

For $M > \sqrt{s_0^V}$, the exponential function in the integral of the left hand side of Eq. (27) can be expanded in powers of s/M^2 for $s < s_0^V$. The left hand side of Eq. (27) is then

obtained as

$$\int_{0}^{\infty} e^{-s/M^{2}} R_{\text{phen}}^{V}(s)$$

$$= \int_{0}^{s_{0}^{V}} ds R_{\text{phen}}^{V(\text{res})}(s) - \frac{1}{M^{2}} \int_{0}^{s_{0}^{V}} ds s R_{\text{phen}}^{V(\text{res})}(s)$$

$$+ \frac{1}{2M^{4}} \int_{0}^{s_{0}^{V}} ds s^{2} R_{\text{phen}}^{V(\text{res})}(s)$$

$$+ d_{V} c_{0} M^{2} \left(1 - \frac{s_{0}^{V}}{M^{2}} + \frac{(s_{0}^{V})^{2}}{2M^{4}} + \frac{(s_{0}^{V})^{3}}{6M^{6}} - \cdots\right). \quad (29)$$

Equating the powers in $1/M^2$ in the Borel transformations of the spectral functions, given by Eqs. (28) and (29), the finite energy sum rules (FESR) are obtained as

$$\int_{0}^{s_{0}^{V}} ds R_{\text{phen}}^{V(\text{res})} = d_{V} \left(c_{0}^{V} s_{0}^{V} + c_{1}^{V} \right), \tag{30}$$

$$\int_{0}^{s_{0}^{V}} ds \, s \, R_{\text{phen}}^{V(\text{res})} = d_{V} \left(\frac{\left(s_{0}^{V}\right)^{2} c_{0}^{V}}{2} - c_{2}^{V} \right), \qquad (31)$$

$$\int_{0}^{s_{0}^{V}} ds s^{2} R_{\text{phen}}^{V(\text{res})} = d_{V} \left(\frac{\left(s_{0}^{V}\right)^{3}}{3} c_{0}^{V} + c_{3}^{V} \right), \qquad (32)$$

To evaluate c_3^V for the vector mesons ρ , ω , and ϕ , given by Eqs. (19) and (22), the factorization method [47]

$$\langle (\bar{q}_i \gamma_\mu \gamma_5 \lambda^a q_j)^2 \rangle = -\langle (\bar{q}_i \gamma_\mu \lambda^a q_j)^2 \rangle = \delta_{ij} \frac{16}{9} \kappa_i \langle \bar{q}_i q_i \rangle^2 \quad (33)$$

is used. In the above, $q_i = u, d, s$ for i = 1, 2, 3 and κ_i is introduced to parametrize the deviation from exact factorization ($\kappa_i = 1$). Using Eq. (33), the four quark condensate for the $\omega(\rho)$ meson given by Eq. (19) becomes

$$c_{3}^{(\rho,\omega)} = -\alpha_{s}\pi^{3} \times \frac{448}{81}\kappa_{q}(\langle \bar{u}u \rangle^{2} + \langle \bar{d}d \rangle^{2}), \qquad (34)$$

where $\kappa_u \simeq \kappa_d = \kappa_q$ has been assumed.

For the ϕ meson, using Eqs. (22) and (33), one obtains the four quark condensate c_3^{ϕ} as given by [46]

$$c_3^{\phi} = -8\pi^3 \times \frac{224}{81} \alpha_s \kappa_s \langle \bar{s}s \rangle^2. \tag{35}$$

Using a simple ansatz for the spectral function $R_{\text{phen}}^V(s)$ as [15,16]

$$R_{\rm phen}^{V}(s) = F_V \delta(s - m_V^2) + d_V c_0^V \theta(s - s_0^V), \qquad (36)$$

the finite energy sum rules for vacuum given by Eqs. (30)–(32), can be written as

$$F_V = d_V (c_0^V s_0^V + c_1^V), \tag{37}$$

$$F_V m_V^2 = d_V \left(\frac{(s_0^V)^2 c_0^V}{2} - c_2^V \right), \tag{38}$$

$$F_V m_V^4 = d_V \left(\frac{\left(s_0^V\right)^3}{3} c_0^V + c_3^V \right).$$
(39)

Using Eqs. (37) and (38), the values of F_V and s_0^V are determined by assuming the values of c_0^V , with $Q^2 = s_0$ ($\alpha_s(Q^2 \simeq 1 \text{ GeV}^2) = 0.5$) and c_1^V as calculated in the chiral SU(3) model. These values are assumed in Eq. (39) to find the

vacuum value of the four quark condensate c_3^V and hence the value of κ_i .

At finite densities, there is contribution to the spectral function for the vector mesons, due to scattering from the baryons, and Eq. (27) is modified to

$$\int_{0}^{\infty} ds e^{-s/M^{2}} R_{\text{phen}}^{V}(s) + 12\pi^{2} \Pi^{V}(0)$$
$$= d_{V} \left[c_{0}^{V} M^{2} + c_{1}^{V} + \frac{c_{2}^{*V}}{M^{2}} + \frac{c_{3}^{*V}}{2M^{4}} \right], \qquad (40)$$

where, in the nuclear medium, $\Pi^V(0) = \frac{\rho_B}{4M_N}$ for $V = \omega, \rho$ and vanishes for the ϕ meson [11,15,48,49]. However, in the presence of hyperons in the hadronic medium, the contribution due to the scattering of the ω and ρ vector mesons from the baryons is modified to

$$\Pi^{V}(0) = \frac{1}{4} \sum_{i} \left(\frac{g_{Vi}}{g_{VN}}\right)^2 \frac{\rho_i}{M_i},\tag{41}$$

where g_{Vi} is the coupling strength of the vector meson Vwith the *i*th baryon $(i = N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0})$; ρ_i and M_i are the number density and mass of the *i*th baryon. For the ω meson, $\frac{g_{\omega i}}{g_{\omega N}} = (1, \frac{2}{3}, \frac{2}{3}, \frac{1}{3})$ for $i = N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$ respectively. For the ρ meson, the ratio $\frac{g_{\rho i}}{g_{\rho N}} = (1,0,2,1)$ for $i = (N,\Lambda,\Sigma^{\pm,0},\Xi^{-,0})$. In the nuclear medium, the contribution for the ϕ meson due to scattering from nucleons vanishes, since the ϕ meson-nucleon coupling strength is zero. In the strange hadronic matter, the contribution is, however, nonzero due to the presence of the hyperons in the medium. For the ϕ meson, $\frac{g_{\phi i}}{g_{\phi \Lambda}} = (1,1,2)$ for $i = (\Lambda, \Sigma^{\pm,0}, \Xi^{-,0})$. In Eq. (40), the coefficients c_0^V and c_1^V , given by Eqs. (17) and (20), are medium independent. However, the coefficients c_2^{*V} and c_3^{*V} , given by Eqs. (18), (19), (21), and (22), are expressed in terms of the in-medium quark and gluon condensates.

At finite densities, the finite energy sum rules (FESR) for vacuum given by Eqs. (37)–(39) are modified to

$$F_V^* = d_V \left(c_0^V s_0^{*V} + c_1^V \right) - 12\pi^2 \Pi^V(0), \qquad (42)$$

$$F_V^* m_V^{*\,2} = d_V \left(\frac{\left(s^{*V}_0\right)^2 c_0^V}{2} - c^{*V}_2 \right),\tag{43}$$

$$F_V^* m_V^{*\,4} = d_V \left(\frac{\left(s_0^{*\,V}\right)^3}{3} c_0^V + c_3^{*\,V} \right). \tag{44}$$

These equations are solved to obtain the medium dependent mass m_V^* , the scale s_0^{*V} , and F_V^* , by using the coefficient κ of the four quark condensate for the vector mesons, as determined from the FESRs in vacuum.

IV. RESULTS AND DISCUSSIONS

In this section, the effects of density on the scalar gluon condensate and the light quark condensates, arising due to the modifications of the dilaton field χ and the scalar isoscalar fields σ and ζ calculated in the chiral SU(3) model, are first investigated. The values of the scalar and dilaton fields in the isospin asymmetric strange hadronic matter are



FIG. 1. (Color online) The quark condensates $(-m_q \langle \bar{q}q \rangle)^{1/4}$ (q = u,d) and $(-m_s \langle \bar{s}s \rangle)^{1/4}$, in units of MeV, are plotted as functions of density for isospin asymmetric hadronic matter (for $f_s = 0, 0.3$ and 0.5) in (b) and (d), and compared with the isospin symmetric case, shown in (a) and (c).

obtained by solving the coupled equations of these fields in the mean field approximation. The nonstrange and strange quark condensates, $\langle \bar{q}q \rangle$ (q = u,d) and $\langle \bar{s}s \rangle$, as well as the scalar gluon condensate, $\langle \frac{\bar{\alpha}_s}{\pi} G^a{}_{\mu\nu} G^{a\mu\nu} \rangle$, are calculated from the in-medium values of the fields σ , ζ , and χ , by using Eqs. (7), (8), (9), and (12), respectively. The values of the current quark masses are taken as $m_u = 4$ MeV, $m_d = 7$ MeV and $m_s = 150$ MeV in the present investigation. In Fig. 1, the density dependence of the quantities $(-m_q \langle \bar{q}q \rangle)^{1/4} (q = u, d)$, $(-m_s \langle \bar{s}s \rangle)^{1/4}$ are shown for given isospin asymmetry and strangeness of the hadronic medium. For the isospin symmetric situation ($\eta = 0$), the quantity $(-m_q \langle \bar{q}q \rangle)^{1/4}$ is identical for uand d quarks, for a given value of f_s . It is also seen that the effect from the strangeness fraction is very small. For the isospin asymmetric situation, the quantities $(-m_u \langle \bar{u}u \rangle)^{1/4}$ and $(-m_d \langle \bar{d}d \rangle)^{1/4}$ are no longer identical, and their difference is due to the nonzero value of the isoscalar scalar field δ , as can be seen from Eqs. (7) and (8). For the *u* quark, there is seen to be smaller drop with density as compared to the d quark due to the negative value of the isoscalar scalar field δ in the medium. For the isospin symmetric nuclear matter, the value of the quantity $(-m_q \langle \bar{q}q \rangle)^{1/4}$ for q = u, d changes from the vacuum value of 95.8 MeV to 85.7 MeV at the nuclear matter saturation density. This corresponds to a drop of the quantity $(-m_q \langle \bar{q}q \rangle)$ for q = u,d by about 36% at the nuclear matter saturation density from its vacuum value. At densities of $3\rho_0$ and $4\rho_0$, this quantity is modified to $(74.7 \text{ MeV})^4$ and $(72 \text{ MeV})^4$ respectively, which correspond to a drop of about 63% and 69% from its vacuum value. As can be seen from Eqs. (7) and (8), in the isospin symmetric matter, $m_u \langle \bar{u}u \rangle = m_d \langle \bar{u}u \rangle$.

In addition, if $m_u = m_d$, one has $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ in isospin symmetric matter. For the case of $m_u \neq m_d$, the vacuum value of the quantity $(-m_q \langle \bar{q}q \rangle)^{1/4}$ for q = u,d corresponds to the mass averaged nonstrange quark condensate, $\langle \bar{\psi}_q \psi_q \rangle =$ $(m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle)/\bar{m_q}$ with $\bar{m_q} = (m_u + m_d)/2$. For $m_u =$ 4 MeV, $m_d = 7$ MeV, and $f_{\pi} = 93$ MeV, as chosen in the present investigation, the value of $\langle \bar{\psi}_q \psi_q \rangle$ turns out to be $(-m_{\pi}^2 f_{\pi}^2)/\bar{m_q} = (-248 \text{ MeV})^3$. One might note that for the case of $m_u = m_d$, $\langle \bar{\psi}_q \psi_q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$. In Refs. [15,16], the value of the chiral condensates for the *u* as well as *d* quarks were taken to be $(-245 \text{ MeV})^3$. The Nambu-Jona-Lasinio model [3] describes the feature of the spontaneous chiral symmetry breaking of QCD leading to nonzero values of the quark-antiquark condensates. The value of the nonstrange quark condensate in vacuum in the chiral effective model as used in the present investigation may be compared with the values in the Nambu-Jona-Lasinio model of $(-248 \text{ MeV})^3$ in Ref. [3] and $(-264 \text{ MeV})^3$ in Ref. [7]. The density dependence of this light nonstrange quark condensate was studied in the linear density approximation and the value at ρ_0 was observed to be $(-216 \text{ MeV})^3$, assuming the value of the nucleon σ term as 45 MeV. This may be compared to the value in the present investigation of $(-214 \text{ MeV})^3$. The drop in the nonstrange quark condensate from its vacuum value was calculated to be 25-50% [45] at the nuclear matter saturation density, taking the nucleon σ term to be lying within the range 30–60 MeV. In Ref. [10], the nonstrange light quark condensate was observed to have a drop of about 20-30% from its vacuum value, at the normal nuclear matter density. The drop of the nonstrange condensate in the medium is the dominant contribution to the modification of the ω and ρ mesons in the medium. The quantity $(-m_s \langle \bar{s}s \rangle)^{1/4}$ for given isospin symmetric $(\eta = 0)$ and isospin asymmetric (with $\eta = 0.5$) situations are shown in subplots (c) and (d) respectively. The vacuum value of $(-m_s \langle \bar{s}s \rangle)^{1/4}$ is about 258 MeV, which may be compared with the value of 210 MeV in Ref. [10]. With the current quark mass of the s quark m_s as 150 MeV as chosen in the present investigation, the vacuum value of the strange quark condensate turns out to be $\langle \bar{s}s \rangle = (-309 \text{ MeV})^3$, which may be compared to the value of $(-311 \text{ MeV})^3$ [7] and $(-258 \text{ MeV})^3$ [3] in the Nambu-Jona-Lasinio model. In the present investigation, for the symmetric nuclear matter, the quantity $(-m_s \langle \bar{s}s \rangle)^{1/4}$ changes from the vacuum value of 258 MeV to 252, 249.1, and 248.7 MeV at densities of ρ_0 , $3\rho_0$, and $4\rho_0$ respectively, which correspond to about 9%, 13.1%, and 13.7% drop in the quantity, $8\pi^2 \langle m_s \bar{s}s \rangle$ occurring in c_2^{ϕ} in the finite energy sum rule for the ϕ meson given by Eq. (21). Figure 2 shows the quartic root of the scalar gluon condensate, $\langle \frac{\alpha_s}{\pi} G^a{}_{\mu\nu} G^{a\mu\nu} \rangle^{1/4}$, as a function of the baryon density in units of the nuclear matter saturation density, for isospin symmetric ($\eta = 0$) as well as asymmetric hadronic medium (with $\eta = 0.5$) for typical values of the strangeness fraction. The value of the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G^a{}_{\mu\nu} G^{a\mu\nu} \rangle$ for isospin symmetric nuclear matter is observed to be modified from the vacuum value of $(373 \text{ MeV})^4$ to $(371.3 \text{ MeV})^4$, $(364.2 \text{ MeV})^4$, and $(361.9 \text{ MeV})^4$ at densities of ρ_0 , $3\rho_0$, and $4\rho_0$ respectively, which correspond to about 1.8%, 9.1%, and 11.4% drop in the medium from its vacuum value. The vacuum



FIG. 2. (Color online) The quantity $\langle \frac{\alpha_s}{\pi} G^a{}_{\mu\nu} G^{a\mu\nu} \rangle^{1/4}$ in MeV plotted as a function of the baryon density in units of the nuclear matter saturation density. This is plotted for isospin asymmetric hadronic matter (for strangeness fraction, $f_s = 0$, 0.3, 0.5 and isospin asymmetric parameter, $\eta = 0.5$) in (b) and compared with the symmetric matter ($\eta = 0$) in (a).

value of the scalar gluon condensate of $(373 \text{ MeV})^4$ may be compared to the value of (350 MeV)⁴ [12] and (330 MeV)⁴ in vacuum [15,19,46,47]. In the linear density approximation [15], the value of the scalar gluon condensate was observed to be modified to $(324 \text{ MeV})^4$ at ρ_0 , which is about 7% drop from its vacuum value. The drop of this quantity was observed to be similar (about 5% in Ref. [45] and about 8% in Ref. [10]). One might note here that in the present investigation, the quark and gluon condensates are calculated for a given baryon density, strangeness fraction, and isospin asymmetry of the hadronic medium. These condensates, for isospin symmetric nuclear matter, in the limit of low densities, reduce to the values estimated in the linear density approximation. The present model thus is more general as the condensates are evaluated at high densities as well, when the linear density approximation is no longer valid, but the hadrons are still the relevant degrees of freedom. However, at still higher densities, the model ceases to be valid when the hadrons no longer exist as the degrees of freedom, but the system undergoes a transition to quark matter. In the present work, it is observed that the light nonstrange quark condensate has a larger drop in the hadronic medium as compared to the decrease in the strange quark condensate as well as the scalar gluon condensate in the medium, similar to that seen in the linear density approximation in Ref. [10]. This is observed as a much smaller drop of the mass of the ϕ meson in the medium as compared to the mass shifts of the ω and ρ mesons. Using the QCD sum rules, the effects of isospin asymmetry as well as strangeness of the medium on the masses of the vector mesons, arising from the in-medium quark and gluon condensates, are investigated in the present work. As has already been mentioned, using the vacuum values of the vector meson mass and the quark and gluon condensates, the finite energy sum rules (FESR) for the vector mesons in vacuum given by Eqs. (37)–(39) are solved to obtain the values for s_0^V , F_V , and the coefficient of the four quark condensate $\kappa_{q(s)}$. The vacuum value of the scale, s_0^V , which separates the resonance part from the continuum part is obtained as 1.3, 1.27, and 1.6 GeV² and the value of F_V is obtained as 0.242, 0.258, and 0.55 GeV² for the for ω , ρ , and ϕ mesons respectively. The value of the coefficient of the four quark condensate is obtained as 7.788, 7.236, and -1.21 for the ω , ρ , and ϕ mesons, which are then used to obtain the medium dependent mass m_V^* , the scale s_0^{*V} , and F_V^* for the vector mesons, by solving the FESRs in the medium given by Eqs. (42)-(44).

In Fig. 3, the density dependence of the mass of the ω meson is shown for the cases of isospin symmetric ($\eta = 0$) as well as the asymmetric matter for given values of the strangeness fraction f_s . There is seen to be initially a drop in the ω -meson mass with increase in density. However, as the density is further increased, the mass of the ω meson is observed to increase with density. This behavior can be understood from Eqs. (42) and (43), which yield the expression for the mass squared of the vector meson as

$$m_V^{*2} = \frac{\left(\frac{(s_0^{*V})^2 c_0^V}{2} - c_2^{*V}\right)}{\left(c_0^V s_0^{*V} + c_1^V\right) - 12\pi^2 [\Pi^V(0)/d_V]}.$$
 (45)

The contribution of c_1^V is negligible for the ρ and ω mesons, due to the small values of the masses of the *u* and *d* quarks. At low densities, the contribution from the scattering of the vector mesons from baryons, given by the last term in the denominator of (45), is negligible and the mass drop of the ω meson mainly arises due to the drop of the light quark condensates in the medium, given by the second term, c_2^{*V} , in the numerator which comes with a negative sign. As seen in Fig. 2, the modification of the scalar gluon condensate of the term c_2^{*V} is much smaller than that of the light quark condensate. However, at higher baryon densities, the last term in the denominator, the so-called Landau scattering term, becomes important for the ω meson. This leads to an increase in the mass of the ω vector meson with density, as can be observed in Fig. 3. The denominator becomes negative above a certain value of density, when there does not exist any solution for the mass of the ω meson, since m_V^{*2} becomes negative. For the case of nuclear matter, the mass of the ω meson remains very similar in the isospin symmetric as well as isospin asymmetric cases. This is because the modification of the ω meson at low densities is mainly due to the quark condensates in the combination $(m_u \bar{u} u + m_d \bar{d} d)$, which depends only on the value of σ [as seen from equations (7) and (8)], and, σ is marginally different for the symmetric and



FIG. 3. (Color online) The mass of ω meson plotted as a function of the baryon density in units of nuclear saturation density, for the isospin asymmetric strange hadronic matter (for strangeness fraction, $f_s = 0, 0.3, 0.5$, and isospin asymmetric parameter, $\eta = 0.5$) in (b) and compared with the symmetric matter ($\eta = 0$) shown in (a).

asymmetric cases. At higher densities, the effect of the Landau scattering term becomes important. However, there is still observed to be a very small difference between the $\eta = 0$ and $\eta = 0.5$ cases of nuclear matter, since the dependence of this term on the proton and neutron densities is in the form (ρ_p + ρ_n), which is the same for the two cases at a given density. With the inclusion of hyperons in the medium, the contribution of the scattering term in the denominator of Eq. (45) becomes smaller in magnitude due to the smaller values of the baryon- ω meson coupling strengths for the hyperons as compared to the nucleons. However, the trend of the initial mass drop followed by an increase at higher densities is still seen to be the case for the mass of the ω meson. However, the density above which the ω mass is observed to increase with density is seen to be higher for the finite strangeness fraction in the hadronic medium, since the contribution from the Landau scattering term is smaller for the case of hyperonic matter as compared to nuclear matter. For the ρ meson, the contribution from the Landau scattering term remains small as compared to the contribution from the light quark condensate in the medium, due to the factor $(1/d_V)$ in this term, which makes the contribution of the Landau scattering term to be nine times smaller than that of the ω meson, as



FIG. 4. (Color online) The mass of ρ meson plotted as a function of the baryon density in units of nuclear saturation density, for the isospin asymmetric strange hadronic matter (for strangeness fraction, $f_s = 0, 0.3, 0.5$, and isospin asymmetric parameter, $\eta = 0.5$) in (b) and compared with the symmetric matter ($\eta = 0$) shown in (a).

 $(1/d_{\rho})/(1/d_{\omega}) = 9$. This is observed as a monotonic decrease of the mass of the ρ meson with density in Fig. 4. The value of the in-medium ρ mass at the nuclear matter saturation density of 622 MeV may be compared with the value of 670 MeV obtained using QCD sum rule approach [16] with the linear density approximation for the quark and gluon condensates. At the nuclear matter saturation density, the drop in the ρ meson mass of about 19% drop from its vacuum value of 770 MeV, as calculated in the present investigation, may be compared with a drop of about 18% using QCD sum rule approach using the linear density approximation in Ref. [10], assuming the value of the nucleon σ term as 45 MeV. The uncertainty in the value of the nucleon σ term, however, could give an error of about 30% in the mass shift of the ρ meson [10]. A similar drop of the ω meson mass as the ρ meson was also calculated as arising from the drop in the nonstrange light quark condensate in the medium [10]. In an improved OCD sum rule calculation [13], the in-medium ρ meson mass was studied, by accounting for the interaction of the ρ meson with the pions, with the pions as modified due to Δ -hole polarization in the medium. This was observed to give smaller values for the ρ meson mass in the nuclear medium, with a value of about 530 MeV at nuclear matter



FIG. 5. (Color online) The mass of ϕ meson plotted as a function of the baryon density in units of nuclear saturation density, for the isospin asymmetric strange hadronic matter (for strangeness fraction, $f_s = 0$, 0.3, 0.5, and isospin asymmetric parameter, $\eta = 0.5$) in subplot (b) and compared with the symmetric matter ($\eta = 0$) shown in (a).

saturation density. In the present investigation, the effects of the strangeness fraction as well as isospin asymmetry of the medium are seen to be small on the ρ meson mass. In Fig. 5, the mass of ϕ meson is plotted as a function of the baryon density in units of nuclear matter saturation density for isospin symmetric and asymmetric cases for typical values of the strangeness fraction. Due to the larger value of the strange quark mass as compared to the u(d) quark masses, the contribution from c_1^{ϕ} is no longer negligible as was the case for the $\omega(\rho)$ meson. The dominant contribution to the mass modification of the ϕ meson is from the in-medium modification of the strange quark condensate of the coefficient c_2^{ϕ} in the nuclear medium. This is because the ϕ meson has no contribution from the scattering term in nuclear matter, since the nucleon- ϕ meson coupling is zero. The strange quark condensate as well as the scalar gluon condensate have very small effects from isospin asymmetry, leading to the modifications of the ϕ meson mass to be very similar in the isospin symmetric and asymmetric nuclear matter. The values of the ϕ meson mass in isospin symmetric (asymmetric) nuclear matter at densities of ρ_0 and $4\rho_0$ are about 1001.5 (1001.8) and 999 (998.4) MeV

respectively, which correspond to about 1.8% and 2% of mass shifts from the vacuum value, at these densities. For the ϕ meson, a drop of about 1.5–3% was predicted at density ρ_0 in Ref. [10], however, with uncertainty of about 30% arising due to the uncertainty in the value of the strangeness fraction of the nucleon. For nonzero f_s , due to the hyperons in the medium, there are contributions from the Landau scattering term, which leads to an increase in the mass of the ϕ meson at higher values of the densities. For nuclear matter, the mass of the ϕ meson does not have a contribution from the scattering term and since the in-medium modifications of both the strange quark condensate as well as the scalar gluon condensate are small and occur with opposite signs in the coefficient $c_{2}^{*\phi}$, the mass of ϕ meson is observed to have negligible change with density, the value being modified from the vacuum value of 1020 MeV to about 998 MeV (999 MeV) at densities of $3\rho_0$ ($4\rho_0$). For the case of isospin symmetric hyperonic matter, there is seen to be an increase in the mass of the ϕ meson at low densities, due to scattering from the Ξ^- and Ξ^0 , whose number densities are equal for this $\eta = 0$ case. The Σ^+ , Σ^- , and Σ^0 (with equal number densities) start appearing at around $3\rho_0$, when the number densities of the Ξ^- and Ξ^0 show a downward trend with density. It is the overall contributions from the hyperons to the scattering term which leads to the observed increase in the mass of the ϕ meson in the strange hadronic medium with $f_s = 0.3$ and 0.5, shown in Fig. 5. For the isospin asymmetric hyperonic matter, there is contribution from the Σ^+ and Ξ^0 for $\eta = 0.5$ situation (but not from $\Sigma^{0,-}$ and Ξ^{-}), which is seen as a smaller increase of the ϕ mass at high densities as compared to the isospin symmetric hyperonic matter.

The density dependence of the scale s_0^{*V} , which separates the resonance part from the perturbative continuum, is shown in Fig. 6 for the ω , ρ , and ϕ vector mesons. For isospin symmetric nuclear matter, for the ω meson, the vacuum value of 1.3 GeV² is modified to about 1.086 GeV² and 1.375 GeV² at densities of ρ_0 and $2\rho_0$ respectively. The dependence of s_0^{*V} on density as an initial drop followed by an increase is similar to that of the density dependence of the mass of the ω meson. This can be understood in the following way. From the medium dependent FESRs, we obtain the expression for the scale s_0^{*V} as

$$s_{0}^{*V} = m_{V}^{*2} \left(1 + \frac{2}{m_{V}^{*4} c_{0}^{V}} \times \left\{ c_{1}^{V} m_{V}^{*}^{2} + c_{2}^{*V} - [12\pi^{2}\Pi(0)/d_{V}] \right\} \right)^{1/2}.$$
 (46)

The value of the second term, within the square root, is found to be small compared to 1. At higher densities, the second term still remains small compared to 1, due to the canceling effect of the contributions from the quark condensate and the Landau scattering term. This is seen as the density dependence of $s_0^{*\omega}$ to have first a drop and then an increase with density as found for the mass of the ω meson. The dependence of the scale s_0^{*V} for the ρ meson is observed to be a monotonic drop with increase in density, due to the negligible contribution from the Landau damping term compared to the contribution from the light quark condensate. In the case of ϕ meson, the



FIG. 6. (Color online) The density dependence of s_0^{*V} for the vector mesons (ω , ρ , and ϕ) in the strange hadronic matter is shown for the isospin symmetric ($\eta = 0$) and isospin asymmetric (with $\eta = 0.5$) cases for values of $f_s = 0, 0.3$, and 0.5.

effect of the scattering term is zero for the nuclear matter case, when $s_{0}^{*\phi}$ is observed to have a small drop due to the marginal drop of the strange condensate and the gluon condensate in the medium. For the hyperonic matter, there is observed to be an increase in $s_0^{*\phi}$ due to the scattering from the hyperons, which is observed to be larger for the isospin symmetric case as compared to the isospin asymmetric situation. In Fig. 7, the value of F_V^* is plotted as a function of density. From the first finite energy sum rule given by Eq. (42), due to the small masses of the u and d quarks, the term c_1^V is negligible for the ω and ρ mesons. At low densities, the value of F_V^* turns out to be proportional to s_0^{*V} , since the contribution from the Landau scattering term is small. At higher densities, there is contribution from the Landau scattering term, which modifies the behavior of F_V^* to a slower change with density for the ω meson. For the ρ meson, this is approximately proportional to $s_{0}^{*\rho}$ as the Landau term has negligible contribution. For the ϕ meson, the scattering from the hyperons leads to an increase of F_{ϕ}^* at higher densities. In Fig. 8, the quartic quark condensates, $c_3^{\bar{V}}$ for the ω , ρ , and ϕ mesons, given by Eqs. (34) and (35) are plotted as functions of density, for the isospin symmetric and asymmetric nuclear (hyperonic) matter. For the ρ and ω mesons, the values of κ calculated from the vacuum FESRs are found to be 7.236 and 7.788, which yield very similar



FIG. 7. (Color online) The density dependence of F_V^* for the vector mesons (ω , ρ , and ϕ) in the strange hadronic matter is shown for the isospin symmetric ($\eta = 0$) and isospin asymmetric (with $\eta = 0.5$) cases for values of $f_s = 0, 0.3$, and 0.5.

values for the four quark condensate for the ω and ρ mesons, shown in Fig. 8. The vacuum FESRs for the ϕ meson yield the four quark condensate to be negative, with the value of κ as -1.21. There is seen to be a large effect from the strangeness fraction of the medium on c_3^{ϕ} , since the strange condensate has appreciable effect from f_s , as can be seen from Fig. 1.

V. SUMMARY

In the present investigation, using the QCD sum rules, the effect of density on the masses of the light vector mesons $(\omega, \rho, \text{ and } \phi)$ is studied from the light quark condensates and gluon condensates in the medium calculated within a chiral SU(3) model. The effects of the isospin asymmetry as well as the strangeness of the medium on the modifications of these masses have also been investigated. The light quark condensates $(\langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle)$ in the isospin asymmetric strange hadronic medium are calculated from the values of the nonstrange and strange scalar mesons σ and ζ and the isoscalar scalar meson δ of the explicit symmetry breaking term of the chiral SU(3) model. The scalar gluon condensate is calculated from a scalar dilaton field, which is introduced in the chiral SU(3) model to mimic the scale symmetry breaking of QCD. For low densities ($\rho_B \leq \rho_0$), the drop in the light quark condensates (nonstrange as well as strange) and the



FIG. 8. (Color online) The density dependence of the four quark condensate for the cases of the ω , ρ , and ϕ mesons is shown for the isospin symmetric ($\eta = 0$) and asymmetric (with $\eta = 0.5$) hadronic matter for the values of the strangeness fraction, $f_s = 0, 0.3$, and 0.5.

scalar gluon condensate in the hadronic medium, as calculated within the chiral SU(3) model, turn out to be similar to the values estimated in the linear density approximation. Within the chiral SU(3) model, one obtains the medium modification of the quark and gluon condensates at higher densities as well. However, the applicability of the model ceases to be valid at still higher densities ($\rho_B \ge 3-4\rho_0$), when the hadrons no longer exist as the degrees of freedom and quarks become the relevant degrees of freedom. The fact that the in-medium modifications of the quark and gluon condensates calculated within the chiral SU(3) model, in the limiting case of low densities, are similar to those obtained in the linear density approximation, the mass shifts of the vector mesons (ρ , ω , and ϕ mesons) in the hadronic medium at low densities evaluated in the present investigation also turn out to be similar to the values calculated using QCD sum rules in the linear density approximation. One might note here that using the in-medium modifications of the gluon condensates obtained within the chiral SU(3) model, the masses of the charmonium states J/ψ

and η_c have been calculated earlier using the QCD sum rule approach [35], which, at low densities, are observed to be similar to those evaluated in the linear density approximation [36]. Also, within the model, the masses of the charmonium states have been studied using the QCD Stark effect, from the medium modification of the square of the color electric field, which are related to the scalar gluon condensate [31] and the results, at low densities, are observed to be similar to those obtained in the linear density approximation [34].

The mass of the ω meson in the present investigation is observed initially to drop with increase in density in the hadronic matter. This is because the magnitudes of the light nonstrange quark condensates become smaller in the hadronic medium as compared to the values in vacuum. However, as the density is further increased, there is seen to be a rise in its mass, when the effect from the Landau term due to the scattering of the ω meson from the baryons becomes important. In the presence of hyperons, the increase in the mass of the ω meson occurs at a higher value of the density compared to the case of nuclear matter. This is because the contribution from the Landau term becomes less with inclusion of hyperons due to smaller values of the coupling strengths of the ω meson with hyperons as compared to coupling strengths with the nucleons. The ρ meson mass is observed to drop monotonically with density dominantly from the drop in the light quark condensate in the medium, with negligible contribution from the Landau scattering term. The effect of isospin asymmetry is observed to be small on the masses of the ω and ρ mesons, as the dependence on the light quark condensates is through the combination $(m_u \bar{u} u + m_d \bar{d} d)$, which has marginal effect from the isospin asymmetry. For the ϕ meson, there is observed to be a drop in the mass in nuclear matter due to the modification of the strange quark condensate and scalar gluon condensate, because the contribution from the Landau term for the ϕ meson vanishes in the nuclear matter. The mass shift of the ϕ meson in the nuclear medium is seen to be small, of the order of 20 MeV at densities of $3-4\rho_0$. This is because the strange condensate as well as gluon condensate have very small modifications in the medium and occur with opposite signs in the coefficient $c_{2}^{*\phi}$. In the presence of hyperons, however, there is seen to be an increase in the mass of the ϕ meson with density due to the contribution from the Landau term arising from the scattering of the ϕ meson with the hyperons. The mass of the ϕ meson is observed to have a larger effect from the Landau scattering term for the isospin symmetric case as compared to the isospin asymmetric hyperonic matter.

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- [1] R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000).
- B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986);
 S. A. Chin, Ann. Phys. (N. Y.) 108, 301 (1977).

- [3] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
- [4] M. Buballa, Phys. Rep. 407, 205 (2005).
- [5] T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 221 (1994).
- [6] H. Mishra and S. P. Misra, Phys. Rev. D 48, 5376 (1993);
 H. Mishra and J. C. Parikh, Nucl. Phys. A 679, 597 (2001).
- [7] A. Mishra and H. Mishra, Phys. Rev. D 69, 014014 (2004); 71, 074023 (2005).
- [8] J. M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999); S. Gubser,
 I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B 428, 105 (1998); E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998).
- [9] J. Erlich, E. Katz, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005); A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006); T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005); G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. **94**, 201601 (2005); D. K. Hong, T. Inami, and H. U. Yee, Phys. Lett. B **646**, 165 (2007); K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D **75**, 086003 (2007); D. Li and M. Huang, J. High Energy Phys. 11 (2013) 088.
- [10] T. Hatsuda and S. H. Lee, Phys. Rev. C 46, R34 (1992).
- [11] T. Hatsuda, S. H. Lee, and H. Shiomi, Phys. Rev. C 52, 3364 (1995).
- [12] T. Hatsuda, Y. Koike, and S. H. Lee, Nucl. Phys. B 394, 221 (1993).
- [13] M. Asakawa and C. M. Ko, Nucl. Phys. A 560, 399 (1993).
- [14] S. Zschocke, O. P. Pavlenko, and B. Kämpfer, Eur. Phys. J. A 15, 529 (2002).
- [15] F. Klingl, N. Kaiser, and W. Weise, Nucl. Phys. A 624, 527 (1997).
- [16] Y. Kwon, M. Procura, and W. Weise, Phys. Rev. C 78, 055203 (2008).
- [17] Y. Kwon, C. Sasaki, and W. Weise, Phys. Rev. C 81, 065203 (2010).
- [18] C. Adami, T. Hatsuda, and I. Zahed, Phys. Rev. D 43, 921 (1991).
- [19] R. Thomas, T. Hilger, and B. Kaempfer, Nucl. Phys. A 795, 19 (2007).
- [20] A. K. Dutt-Mazumder, R. Hofmann, and M. Pospelov, Phys. Rev. C 63, 015204 (2000).
- [21] A. Mishra, K. Balazs, D. Zschiesche, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C 69, 024903 (2004).
- [22] P. Papazoglou, D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, Phys. Rev. C 59, 411 (1999).
- [23] A. Mishra, A. Kumar, S. Sanyal, V. Dexheimer, and S. Schramm, Eur. Phys. J. A 45, 169 (2010).
- [24] D. Zschiesche, A. Mishra, S. Schramm, H. Stöcker, and W. Greiner, Phys. Rev. C 70, 045202 (2004).

- [25] H. Shiomi and T. Hatsuda, Phys. Lett. B 334, 281 (1994);
 T. Hatsuda, H. Shiomi, and H. Kuwabara, Prog. Theor. Phys. 95, 1009 (1996); H.-C. Jean, J. Piekarewicz, and A. G. Williams, Phys. Rev. C 49, 1981 (1994); K. Saito, K. Tsushima, A. W. Thomas, and A. G. Williams, Phys. Lett. B 433, 243 (1998).
- [26] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich,
 S. Schramm, and H. Stöcker, Phys. Rev. C 70, 044904 (2004);
 A. Mishra and S. Schramm, *ibid.* 74, 064904 (2006).
- [27] A. Mishra, A. Kumar, S. Sanyal, and S. Schramm, Eur. Phys. J. A 41, 205 (2009).
- [28] A. Mishra and A. Mazumdar, Phys. Rev. C 79, 024908 (2009).
- [29] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich, S. Schramm, and H. Stöcker, Phys. Rev. C 69, 015202 (2004).
- [30] A. Kumar and A. Mishra, Phys. Rev. C 81, 065204 (2010).
- [31] A. Kumar and A. Mishra, Eur. Phys. J. A 47, 164 (2011).
- [32] K. Tsushima, D. H. Lu, A. W. Thomas, K. Saito, and R. H. Landau, Phys. Rev. C 59, 2824 (1999); A. Sibirtsev, K. Tsushima, and A. W. Thomas, Eur. Phys. J. A 6, 351 (1999).
- [33] A. Hayashigaki, Phys. Lett. B 487, 96 (2000).
- [34] S. H. Lee and C. M. Ko, Phys. Rev. C 67, 038202 (2003).
- [35] A. Kumar and A. Mishra, Phys. Rev. C 82, 045207 (2010).
- [36] F. Klingl, S. Kim, S. H. Lee, P. Morath, and W. Weise, Phys. Rev. Lett. 82, 3396 (1999).
- [37] S. Kim and S. H. Lee, Nucl. Phys. A 679, 517 (2001).
- [38] B. Friman, S. H. Lee, and T. Song, Phys. Lett. B 548, 153 (2002).
- [39] S. Weinberg, Phys. Rev. 166, 1568 (1968).
- [40] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969); C. G. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid*. 177, 2247 (1969).
- [41] W. A. Bardeen and B. W. Lee, Phys. Rev. 177, 2389 (1969).
- [42] J. Schechter, Phys. Rev. D 21, 3393 (1980).
- [43] J. Ellis, Nucl. Phys. B 22, 478 (1970); B. A. Campbell, J. Ellis, and K. A. Olive, *ibid.* 345, 57 (1990).
- [44] E. K. Heide, S. Rudaz, and P. J. Ellis, Nucl. Phys. A 571, 713 (1994).
- [45] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C 45, 1881 (1992).
- [46] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).
- [47] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 448 (1979).
- [48] A. I. Bochkarev and M. E. Shaposhnikov, Phys. Lett. B 145, 276 (1984); Nucl. Phys. B 268, 220 (1986).
- [49] W. Florkowski and W. Broniowski, Nucl. Phys. A 651, 397 (1999).