Properties of ^{15}Be($5/2^+$)

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(Received 21 January 2015; published 11 March 2015)

A simple $(sd)^3$ shell-model calculation has previously worked extremely well in predicting absolute energies of the lowest $5/2^+$ state in ^{19}O, ^{17}C, and ^{13}Be. Here, I apply the same model to ^{15}Be. When combined with a recent experimental result, the analysis produces tight constraints on the s and d single-particle energies in ^{13}Be.

DOI: 10.1103/PhysRevC.91.034314 PACS number(s): 21.10.Dr, 21.10.Tg, 21.60.Cs, 27.20.+n

I. INTRODUCTION

Even after many years, the neutron-rich Be nuclei still present an exciting field of study. They exhibit a wide range of exotic features. In the 0^+ ground state (g.s.) of ^{12}Be, about 68% of the structure corresponds to two neutrons in the sd shell with the remainder having the normal p-shell character [1–5].

In ^{13}Be, states with one and three neutrons in the sd shell should exist at reasonably low excitation [6]. Three separate experiments [7–9] have reported an s-wave resonance near threshold. A recent experiment [10] found a d-wave resonance at 2.39(5) MeV. The inclusion of two d states in their analysis lowers the first one to about 2.0 MeV. However, a recent theoretical paper [11] finds the g.s. to be either $3/2^+$ or $5/2^+$. Two more recent experiments [12,13] and a recent analysis [14] all found the lowest s state near (or just below) 0.5 MeV. Aksyutina *et al.* [12] suggested a d state near 2 MeV. Randisi *et al.* [13] had two d states at 0.85 and 2.35 MeV.

For ^{14}Be, the first mass measurement was made with a pion-induced double-charge-exchange experiment ^{14}C(π^-,π^+), which gave a mass excess of 40.10(16) MeV [15]. The latest mass evaluation [16] lists 39.95(13) MeV, which corresponds to $E_{2n} = -1.27(13)$ MeV. Only the g.s. is bound. The first 2^+ is at $E_x = 1.54$ MeV [17] and is thus unbound by about 0.27 MeV. A second 2^+ state has been suggested [18] at an excitation energy of 3.54(15) MeV [$E_{2n} = 2.28(9)$ MeV]. Some disagreement exists [18,19] concerning the major configurations of the first two 2^+ states.

Little is known about ^{15}Be other than the fact that it is unbound. Its g.s. could have $J^\pi = 1/2^+$, $3/2^+$, or $5/2^+$. Failure to observe any ^{14}Be + n events that follow two-proton removal from ^{17}C [20] was taken to be evidence that the lowest $3/2^+$ state of ^{15}Be is unbound by more than 1.54 MeV for $1n$ decay. [The $3/2^+$ state of ^{15}Be should be preferentially populated in $2p$ removal from the $3/2^+$ g.s. of ^{17}C, and its structure is such that it should decay strongly to the 2^+ of ^{14}Be and only very weakly to the 0^+ g.s.] A recent experiment [21] used the reaction ^{14}Be(d,p) (in reverse kinematics) to populate a $5/2^+$ state. Its decay was observed by detecting ^{14}Be + n in coincidence. The energy and width of this $5/2^+$ resonance were reported as 1.8(1) MeV and 575(200) keV, respectively.

Reference [20] suggested ^{16}Be as a good candidate to be a simultaneous $2n$ emitter if it is bound to ^{15}Be + n. Indeed, a recent paper [22] claims to have observed this decay. It remains to be seen whether that interpretation survives close scrutiny.

II. $(sd)^3$ STATES IN $A + 3n$ NUCLEI

Lawson [23] used a simple model to calculate energies of $(sd)^3$ states in ^{19}O. The model assumed the three neutrons occupied the $2s_{1/2}$ and $1d_{5/2}$ orbitals (abbreviated s and d here) with $1d_{3/2}$ (called d' here) neglected. Lawson gave simple expressions for the Hamiltonian matrix elements for all the states in this space: one $1/2^+$, two $3/2^+$, three $5/2^+$, one $7/2^+$, and two $9/2^+$. I have applied this model to ^{19}O and other nuclei [24]. For nucleus $A + 3$, I use as s and d single-particle energies (spe's) (Table I) the $1/2^+$ and $5/2^+$ energies in nucleus $A + 1$, where A is a p-shell core. For two-body matrix elements, I use ones from an earlier treatment of ^{18}O [25] in which two-nucleon and cluster components were separately identified for nine low-lying positive-parity states.

I ignore the $d_{3/2}$ orbital throughout. With that restriction, within the $(sd)^3$ space, there are three $5/2^+$ states—linear combinations of the three configurations d^3, d_2^2s, and ds_0^2, where s stands for $2s_{1/2}$ and d stands for $1d_{5/2}$. I have previously calculated energies and wave functions for these three $5/2^+$ states in three nuclei ^{19}O, ^{17}C, and ^{13}Be [24] in the spirit of Lawson [23] by assuming a configuration of $(sd)^3$ coupled to the ground states of ^{16}O, ^{14}C, and ^{10}Be, respectively. Single-particle energies were taken from ^{17}O, ^{15}C, and ^{11}Be. In all three cases, the sd-shell occupancy in the cores is small, and I ignored it. The resulting $3n$ energies are absolute.

One remarkable feature of these $(sd)^3$ calculations was the excellent agreement between calculated absolute energies of the lowest $5/2^+$ states and the known energies of their experimental counterparts. In ^{19}O and ^{17}C, the calculations missed the $5/2^+$ energy by about 100 and 50 keV, respectively. For ^{13}Be, the lowest $5/2^+$ state had a calculated energy of 1.8 MeV—reasonably close to the lowest known d state near 2.0 MeV. Energies of $1/2^+$ and $3/2^+$ states were in poorer agreement in all three nuclei. Thus, I would expect that the energy prediction of the first $5/2^+$ state in ^{15}Be should be reasonably reliable. This calculation is discussed in the next section.

III. CALCULATIONS FOR ^{15}Be

The configuration of the lowest states in ^{15}Be is expected to be three neutrons in the sd shell coupled to a p-shell ^{12}Be g.s.(^{12}Be$_{1p}$). A second set of states with five neutrons in the sd shell coupled to the p-shell g.s. of ^{10}Be is likely to lie considerably higher. Very early shell-model calculations [26] obtained a g.s. J^π of $5/2^+$ for ^{15}Be with a $3/2^+$ state nearby

TABLE I. Input energies (MeV) from core +1n nuclei.

Core	E_n(g.s.)	E_x		E_n	
		$1/2^+$	$5/2^+$	$1/2^+$	$5/2^+$
^{16}O	−4.144	0.871	0	−3.273	−4.144
^{14}C	−1.218	0	0.740	−1.218	−0.478
^{12}Be	E_s			E_s	$E_s + 2.3$

(at 0.07 MeV) with no $1/2^+$ listed. However, for ^{13}Be they have a $1/2^-$g.s. with $5/2^+$ at 0.05 MeV and a $1/2^+$ state at 1.55 MeV. By analogy with ^{17}C, which is dominated by the structure $^{14}\mathrm{C} \otimes (sd)^3$, others [20] have suggested that the lowest state in ^{15}Be will be $3/2^+$. All these states are unbound.

Of course, coupling three sd-shell neutrons to the physical g.s. of ^{12}Be would do violence to the Pauli principle, but coupling to $^{12}\mathrm{Be}_{1p}$ has no such problem. If E_s and E_d, respectively, are the s and d spe's relative to the physical g.s. of ^{12}Be, then relative to a pure p-shell ^{12}Be(g.s.), the spe's are $E_s' = E_s - E_0$ and $E_d' = E_d - E_0$, where E_0 is the energy of $^{12}\mathrm{Be}_{1p}$(g.s.) relative to $^{12}\mathrm{Be}_{\mathrm{phys}}$(g.s.). The well-established wave function [3] for $^{12}\mathrm{Be}_{\mathrm{phys}}$ (g.s.) has 68% of the configuration $^{10}\mathrm{Be}_{1p} \otimes (sd)^2$ and 32% of $^{12}\mathrm{Be}_{1p}$, with the excited 0^+ state at 2.24 MeV [27] having the orthogonal configuration. With these two wave functions, E_0 would be 1.52 MeV, but it will turn out that the final results do not depend on E_0. I previously estimated $E_d - E_s$ in ^{13}Be [24] to be about 2.3 MeV. I arrived at that value by considering the trends of the lowest $1/2^+$ and $5/2^+$ states in $N = 9$ and in $Z = 4$ nuclei. I treat E_s as an unknown parameter to be determined later. For any expected value of E_s, $E_s - E_0$ will be negative so that the s state is bound relative to $^{12}\mathrm{Be}_{1p}$(g.s.).

In the $^{12}\mathrm{Be}_{1p} \otimes (sd)^3$ space, the diagonal matrix elements of the Hamiltonian will all contain a term $-3E_0$. The eigenvalues from this calculation can then be transformed back to ones relative to the physical $^{12}\mathrm{Be(g.s.)} \otimes (sd)^3$ by adding $3E_0$ to each eigenvalue. The final results will thus be independent of E_0 and will be energies relative to $^{12}\mathrm{Be}_{\mathrm{phys}}$ (g.s.) + $3n$.

The dominant feature of nuclei just below ^{16}O is the rapid decrease of the energy of the $2s_{1/2}$ orbital with decreasing mass. In ^{17}O, it is 0.87 MeV [28] above the $d_{5/2}$; in ^{15}C it is 0.74 MeV [29] below, and in ^{13}Be it is about 2.3 MeV [10,24] below. In ^{19}O the $5/2^+$ state is predominantly of the configuration $(d_{5/2})^3$, whereas the $1/2^+$ is nearly pure $(d_{5/2})^2_0(2s_{1/2})$. In ^{17}C the $5/2^+$ is much less pure—with approximately equal components of d^3 and ds^2 (Table II), but the $1/2^+$ is still close to single particle.

TABLE II. Configuration intensities for the first $5/2^+$ states in relevant nuclei.

Nucleus	d^3	d^2s	ds^2
^{19}O	0.89	Small	0.11
^{17}C	0.53	Very small	0.47
^{15}Be, $E_d - E_s = 2.3$ MeV	0.14	0.01	0.85
^{15}Be, $E_s = 0.50$, $E_d = 1.88$ MeV	0.59	Very small	0.41

TABLE III. Results (MeV) for the lowest $5/2^+$ states in core +3n nuclei.

Final nucleus	E_{3n}(g.s.)	$E_x(5/2^+)$	E_{3n}(calc)	E_{3n}(expt.)
^{19}O	−16.14	0.0	−16.04	−16.14
^{17}C	−6.20	0.33	−5.814	−5.87
^{15}Be, $E_d - E_s = 2.3$ MeV	Unknown		$3E_s + 0.358$	0.53(16)
^{15}Be, $E_s = 0.50$, $E_d = 1.88$ MeV	Unknown		0.53	0.53(16)

In the next three subsections, I present results for the lowest $5/2^+$ state of ^{15}Be for three different assumptions about spe's.

A. $E_d - E_s = 2.3$ MeV, E_s to be determined

With $E_d - E_s = 2.3\ MeV$ in ^{13}Be [24], the wave function of the lowest $5/2^+$ state in ^{15}Be is as listed in Table II. Keeping $E_d - E_s$ fixed will cause all ^{15}Be eigenvalues to contain a term $3E_s$. Then, equating the calculated energy of the lowest $5/2^+$ state (Table III) to the experimental value of $E_{3n} =$ 0.53(16) MeV [E_n (^{15}Be) = 1.8(1) MeV; E_{2n} (^{14}Be(g.s.)) = −1.27(13) MeV] produces a value of $E_s = 0.06(6)$ MeV. Recall that several early experiments [7–9] suggested an s state near threshold. However, more recent work [12–14] places it near 0.5 MeV, a fact that leads to the next subsection.

B. $E_s = 0.50$ MeV, E_d to be determined

If I set $E_s = 0.50$ MeV, I can compute the $5/2^+$ eigenvalue for various values of E_d. Then, requiring this calculated E_{3n} to be equal to 0.53(16) MeV establishes $E_d = 1.88(10)$ MeV—close to the lowest known d state at 2 MeV. Future experiments should be able to determine whether this state is primarily single particle or $(sd)^3$.

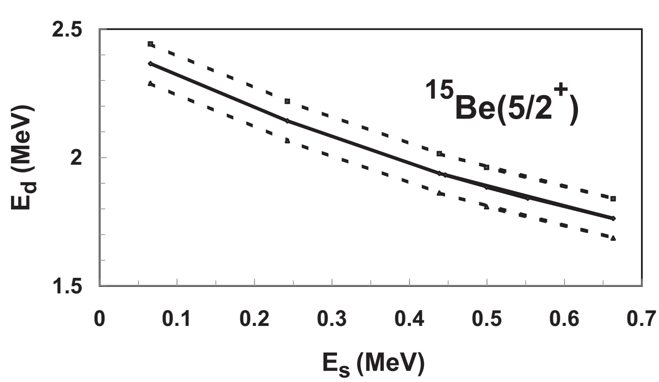

FIG. 1. Relationship (with an uncertainty band) between the single-particle energies E_s and E_d to reproduce the absolute energy [$E_n = 1.8(1)$, $E_{3n} = 0.53(16)$ MeV] of the lowest $5/2^+$ state in ^{15}Be.

TABLE IV. Calculated and measured widths (keV) of ^{15}Be($5/2^+$).

Source	S	Γ_{sp}	$\Gamma_{calc} = S\Gamma_{sp}$	$\Gamma_{expt.}$
Reference [21]	0.44	405	178	575(200)
Present paper	0.9	430	390	

C. E_s and E_d both variable

If I vary both E_s and E_d, requiring the lowest $5/2^+$ eigenvalue to match the experimental value provides a relationship between E_s and E_d as illustrated in Fig. 1. Here, I plot values of E_d vs E_s with uncertainty bands that produce $E_{3n} = 0.53(16)$ MeV. The fact that ^{13}Be has no bound states requires $E_s > 0$, which results in an upper limit on E_d of about 2.3 MeV. A lower limit is provided by the fact that all experiments have found E_s less than about 0.7 MeV. Any pair of values within this band will reproduce the experimental $5/2^+$ energy.

D. Width of the $5/2^+$ resonance

As mentioned in the Introduction, the measured width of the $5/2^+$ resonance was 575(200) keV [21]. Even with the large uncertainty, this width is much larger than expected. Combining the published spectroscopic factor $S = 0.44$ and the single-particle width $\Gamma_{sp} = 405$ keV [21] produces an expected width $\Gamma_{calc} = 178$ keV. The experimental width is thus about 2σ larger than the calculated value. I estimate a slightly larger sp width of 430 keV but a much larger spectroscopic factor $S \sim 0.9$. Even so, the observed width is still larger than expected (Table IV). The extra width could arise from decays of the $5/2^+$ state to the first 2^+ state of ^{14}Be. If the 2^+ configuration is primarily ds as I suggested [19], rather than dd as suggested elsewhere [18], the $5/2^+$ state would have a strong $\ell = 0$ branch to the 2^+ state. [The d^2s component in the lowest $5/2^+$ state is tiny for any value of $E_d - E_s$.] This branch might be observable as $^{12}\text{Be} + 3n$ coincidences because the 2^+ is unbound.

IV. SUMMARY

A simple $(sd)^3$ shell-model calculation has previously proven quite successful in reproducing the absolute energies of the lowest $5/2^+$ state in several $A + 3n$ nuclei, where A is a p-shell core. Here, I have applied the same model to ^{15}Be. Requiring the calculated $5/2^+$ energy to agree with the experimental value of $E_n = 1.8(1)$ MeV [$E_{3n} = 0.53(16)$ MeV] provides tight constraints on the s and d spe's. In particular, if $E_d - E_s$ is about 2.3 MeV as previously suggested, the analysis requires $E_s = 0.06(6)$ MeV. If, instead, I use $E_s = 0.50$ MeV as recently claimed [12–14], the result is $E_d = 1.88(1)$ MeV. For other values of the spe's, I have presented, in graphical form, the relationship between E_s and E_d that reproduces the experimental energy. I have also computed the expected width of this $5/2^+$ resonance, and I suggest a possible source of the extra width.

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