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Constraints on energies of $^{10}He(0^+)$ and $^{9}He(1/2^+)$

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I have used the relationship between computed energies in ^{10}He and single-particle energies in ^{9}He to provide limits on the $s_{1/2}$ energy. The absence of any bound states in ^{10}He requires $E_s > 1$ MeV, contradicting all the experiments that have reported an s state near threshold. The present analysis supports the view that the variation of ^{10}He "ground-state" (g.s.) energies determined in various reactions is caused by the presence of two overlapping 0^+ resonances. Results of the two simplest reactions—proton knockout and (t, p)—have been used to extract the g.s. and excited 0^+ energies as a function of the mixing parameter b^2 between the p-shell and the $(sd)^2$ basis states.

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I. INTRODUCTION

A very simple model [1] has been extremely successful in predicting the absolute energies of $(sd)^2$ 0^+ states in light nuclei. The model space consists of two neutrons in the $2s_{1/2}$ or $1d_{5/2}$ orbitals coupled to a p-shell ground-state (g.s.) core. For nucleus $A + 2$, the model takes energies of $1/2^+$ and $5/2^+$ states in nucleus $A + 1$ as single-particle energies (spe's) for s and d, respectively. The two-body residual matrix elements (2BME's) are taken from work on ^{18}O [2] and are assumed to be the same in the various nuclei. The calculated absolute energies of the first $(sd)^2$ 0^+ state agree with the experimental energies of the states with this dominant configuration to within about 300 keV in ^{10,12}Be and ^{14,16}C [1]. The signs of the differences between experimental and calculated energies are easily understood on the basis of mixing between $(sd)^2$ and predominantly p-shell 0^+ states, given the approximate location of the latter. In ^{14,16}C, where they are known, the energies of the second $(sd)^2$ 0^+ states are also well reproduced. I recently applied the same procedure to estimate the single-particle energies in ^{13}Be, given the known $2n$ separation energy of ^{14}Be [3]. That work was successful in reproducing experimental energies. Here, I apply the same model to the 0^+ states of ^{10}He and the spe's in ^{9}He for both of which various reported g.s. energies vary considerably. First, I briefly discuss what is known about the low-lying states in these two nuclei.

II. ^{9}He

For a calculation of $(sd)^2$ states in ^{10}He, we need energies of $1/2^+$ and $5/2^+$ states in ^{9}He. Several different values have been suggested for the $1/2^+$ energy, varying from ~0 to 2.3 MeV. One experiment suggested a $5/2^+$ state above $E_n = 4.2$ MeV, others near 4.9 MeV. [E_n is measured relative to the ^{8}He + n threshold as are s and d single-particle energies E_s and E_d mentioned later.] A recent ^{8}He(d, p) reaction (in reverse kinematics) located it at 3.42(78) MeV. Results from several experiments [4–12] are listed in Table I. The $1/2^+$ and $5/2^+$ energies are plotted in Fig. 1. All the experiments agree that ^{9}He has no bound states. It can be noted that most of the experiments have an s state near threshold. It will turn out that the ^{10}He results are very insensitive to the energy chosen for the $5/2^+$ state. I have performed the analysis for two values: 4.2 and 4.9 MeV. The aim will be to try to pin down the $1/2^+$ energy.

III. ^{10}He

The experimental situation with regard to the apparent g.s. of ^{10}He is summarized in Table II and Figs. 2 and 3. The experimental energies [13–19] appear to be divided into two distinct groupings—one from the ^{8}He(t, p) reaction and the other from everything else with those from (t, p) being higher. A number of explanations have been offered for the different results. Earlier, Grigorenko and Zhukov [20] suggested that the energy measured in proton removal from ^{11}Li might be lower than in other reactions because of initial-state interactions in ^{11}Li. I suggested [21] the differences might arise from the presence of two overlapping 0^+ states, populated with different strengths in different reactions. I estimated the relative strengths to be expected in the different reactions. Results were presented in terms of the mixing between the two basis states—the p-shell one and one with the structure ^{8}He $x(sd)^2$. Recently, Sharov *et al.* [22] made a similar suggestion but with overlapping 0^+, 1^-, and 2^+ states. The (t, p) experiment [17] found separate energies for 0^+, 1^-, and 2^+ resonances, with (naturally) the 1^- and 2^+ above the 0^+. That 0^+ energy was about 0.7 MeV higher than the average of the "g.s." energies in all other reactions [13–15,19]. If the variations in the latter were the result of overlapping resonances of different J^π's, they should have been larger than the 0^+ energy. Thus, it is unlikely that the observed variation in energy is the result of overlapping resonances of different J^π's.

I have computed the weighted and unweighted averages of all the energies in Table II, but with those from (t, p) excluded. These results are also listed in Table II. Most of the reactions used to populate the supposed g.s. of ^{10}He favor the p-shell structure. However, the (t, p) reaction strongly favors the $(sd)^2$ one [23]. The fact that the energy from (t, p) is larger than all the others probably indicates that the mixing is small, and the excited 0^+ state has more of the $(sd)^2$ component. This does not indicate that the excited 0^+ state (exc.) is stronger than the g.s. in (t, p), but only that the exc./g.s. ratio is larger in

TABLE I. Energies (relative to the ^{8}He + n threshold) and widths (both in MeV) of resonances in ^{9}He from the reactions indicated.

Label	Reaction	E_n	Width	J^π	Reference
1	^{9}Be(π^-,π^+)	1.13(10)	0.42(10)	$1/2^-$	[4]
		2.33(10)	0.42(10)	$1/2^+$	
		4.93(10)	0.50(10)	$5/2^+$ or $3/2^-$	
2	^{9}Be(^{13}C, ^{13}O) and	1.13	∼0.30	$1/2^-$	[5]
	^{9}Be(^{14}C, ^{14}O)	2.28	∼0.85	$1/2^+$ or $3/2^-$	
		4.93			
3	^{9}Be(^{14}C, ^{14}O)	1.27	0.10(6)	$1/2^-$	[6]
		2.37(10)	0.7(2)	$(3/2^-)$	
		4.30(10)	Narrow	$(5/2^+)$	
		5.25(10)	Narrow		
4	$2p$ knockout from ^{11}Be	(<0.2)		$1/2^+$	[7]
5	C(^{11}Be, ^{8}He + n)	<0.2		$1/2^+$	[8]
	C(^{14}B, ^{8}He + n)	∼0		$1/2^+$	
		∼1.3	∼1		
6	^{2}H(^{11}Li, ^{8}He + n)	(∼0)	Maybe not a true state		[9]
		1.33(8)	0.10 fixed	$1/2^-$	
		2.42(10)	0.70 fixed	$3/2^-$	
7	^{2}H(^{8}He, p)	∼0		$(1/2^+)$	[10]
		∼1.3		$(1/2^-)$	
		∼2.3			
8	^{2}H(^{8}He, p)	∼0		$1/2^+$	[11]
		2.0(2)	∼2	$1/2^-$	
		>4.2	>0.5	$5/2^+$	
9	^{2}H(^{8}He, p)	0.180(85)	0.18(16)	$1/2^+$	[12]
		1.235(115)	0.13(17)	$(1/2^-)$	
		3.42(78)	2.90(39)	$5/2^+$ or $3/2^+$	

(t, p) than in the other reactions. From Fig. 1 of Ref. [21], this condition is met for all $b^2 < 0.5$, where b^2 is the amount of the $(sd)^2$ configuration in the g.s.

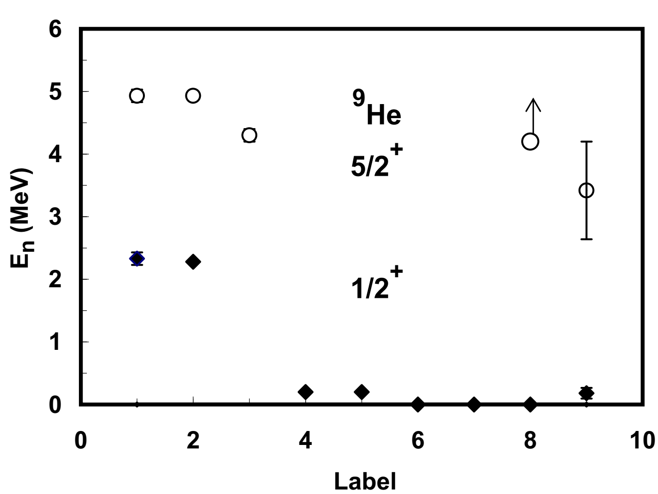

FIG. 1. (Color online) Experimental energies [4–12] of $1/2^+$ (closed) and $5/2^+$ (open) resonances in ^{9}He. Labels for various reactions are the same as in Table I.

IV. CALCULATIONS AND RESULTS

To begin, I assume $E_d = 4.2$ or 4.9 MeV. Later, I will leave E_s undetermined to be estimated from the analysis. But first, I compute the $(sd)^2$ eigenvalues for two sets of spe's—as listed in Table III. Set 1 is taken from the early rows in Table I, and set 2 is taken from later experiments. Then, with the 2BME's [2] that I always use, the energy of the first $(sd)^2$ 0^+ state in ^{10}He can be determined as also listed in Table III. It can be noted that, if the values of set 1 are appropriate, the g.s. of ^{10}He will be predominantly of p-shell character because the first $(sd)^2$ 0^+ state is calculated to be significantly higher than

TABLE II. Energy and width (both in MeV) of ^{10}He(g.s.) from various reactions.

Label	Reaction	E_{2n}	Γ	Reference
1	H(^{11}Li, $2p$)	1.7(3)(3)		[13]
2	^{2}H(^{11}Li, ^{3}He)	1.2(3)	<1.2	[14]
3	^{10}Be(^{14}C, ^{14}O)	1.07(7)	0.3(2)	[15]
4	^{14}Be − $2p2n$	1.60(25)	1.8(4)	[16]
5	^{3}H(^{8}He, p)	2.1(2)	∼2	[17]
6	^{3}H(^{8}He, p)	∼3		[18]
7	p knockout from ^{11}Li	1.42(10)	1.11(76)	[19]
8		1.54(11)	1.91(41)	
	Non-(t, p) weighted average	1.21(6)		
	Non-(t, p) unweighted average	1.40(23)		

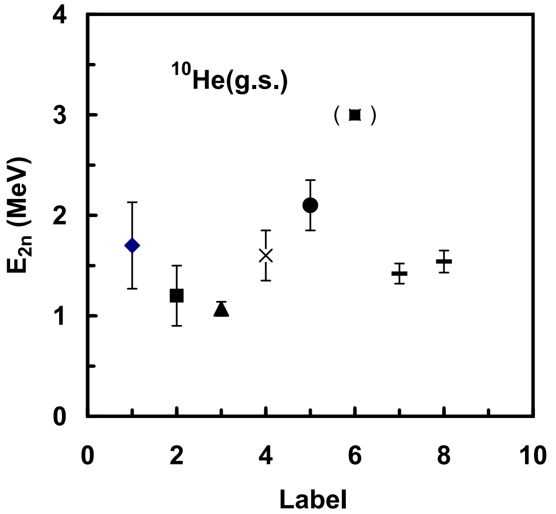

FIG. 2. (Color online) Experimental energies [13–19] of "g.s." resonance in ^{10}He. Labels for various reactions are the same as in Table II.

experimental values. Calculations with set 2 produce a 0^+ state bound by 1.74 MeV, which is clearly impossible because we know ^{10}He has no bound states.

I turn now to calculations with E_s allowed to vary. The resulting values of E_s are plotted in Fig. 4 vs the calculated $(sd)^2$ energy. As ^{10}He has no bound states, this result requires $E_s > 1$ MeV in ^{9}He, higher than suggested by many experiments. This result would appear to contradict all the experiments that found an s state near threshold. And, of course, this $(sd)^2$ 0^+ state must be somewhat higher because mixing with the "normal" p-shell 0^+ state will lower the lower of the two mixed states. It seems reasonable to require E_{2n} $[(sd)^2_{0+}] > 1.4$ MeV, i.e., higher than the probable g.s. energy. This limit results in $E_s >\sim 1.8$ MeV. It can be seen that results are not very different for the two different chosen values of E_d. The nature of the physical g.s. of ^{10}He will depend critically on

TABLE III. Results for ^{10}He(0^+) for different ^{9}He energies (all energies in MeV).

Label	E_s	E_d	$E_{2n}(sd)^2$	Conclusion
Set 1	2.30	4.93	2.42	g.s. not $(sd)^2$
Set 2	0.10	4.20	−1.74	Impossible

the location of the normal p-shell g.s. If it is above the $(sd)^2$ state, the physical g.s. will be mostly $(sd)^2$, whereas if the p-shell g.s. is below the $(sd)^2$ 0^+ state, the physical g.s. will be mostly p-shell. Reference [20] had suggested the p-shell 0^+ to be at about 2.0–2.3 MeV and the "alternate" s^2 g.s. to be at $E_{2n} < 0.25$ MeV. As noted elsewhere [21,23], these energies, with any appreciable mixing, would put the mixed g.s. below threshold—where no states exist.

If the $1/2^+$ state in ^{9}He is above 1 (or 1.8) MeV, then what are the s-wave structures reported near threshold in several experiments? They must be caused by the $\ell = 0$ component of the true multi-body continuum background.

I have investigated the expected energies of the two mixed states as a function of E_s. For definiteness, I took the p-shell 0^+ state to be in the middle of the range suggested by Ref. [20], viz. 2.15 MeV. For illustrative purposes, I took the mixing matrix element V to be the same as in ^{12}Be, which has the same number of neutrons. Given wave-function amplitudes of α,β in a two-state model (where α,β are the amplitudes of the basis states in the mixed states), an observed energy difference of E, V is given by the expression $V = \alpha\beta E$. With my favorite wave functions [24] for ^{12}Be, the result is $V = 1.05$ MeV. Because the s^2/d^2 ratio will be different in ^{12}Be and ^{10}He, this V is not rigorously correct, but it should serve for present purposes. This approach requires no experimental input. Results are plotted in Fig. 5. Below the p-shell/$(sd)^2$ crossing, the g.s. will contain more of the $(sd)^2$ configuration, whereas above this crossing, the opposite is true. With these results combined with the result that the energy from (t, p) is

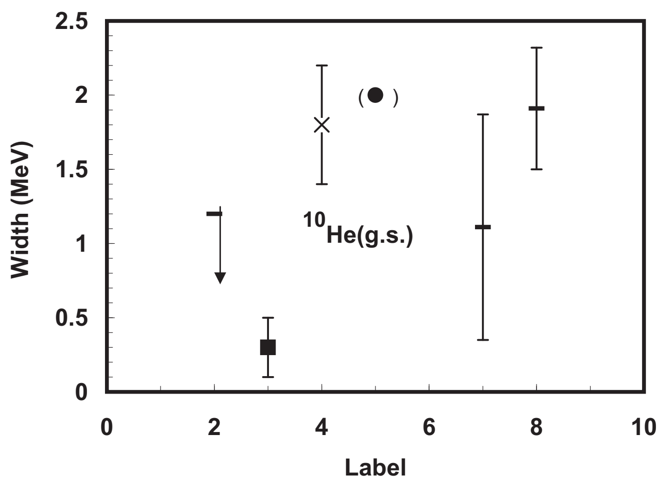

FIG. 3. Experimental widths of "g.s." resonance in ^{10}He. Labels for various reactions are the same as in Table II.

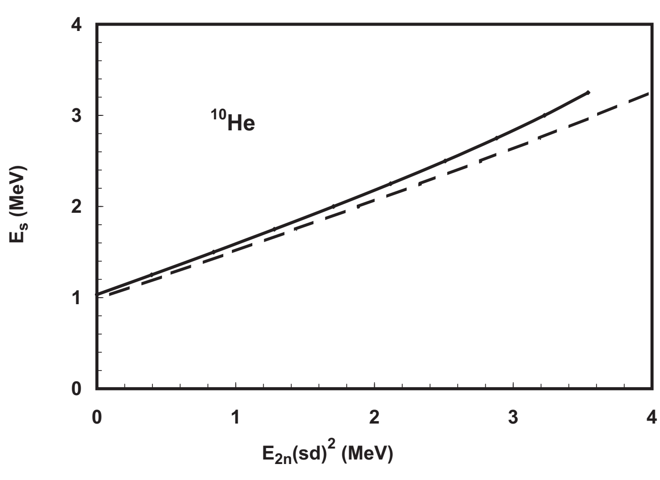

FIG. 4. Relationship between the assumed energy of $1/2^+$ resonance in ^{9}He and the computed energy of the $(sd)^2$ 0^+ state in ^{10}He for $E_d = 4.20$ (solid line) and 4.93 MeV (dashed line).

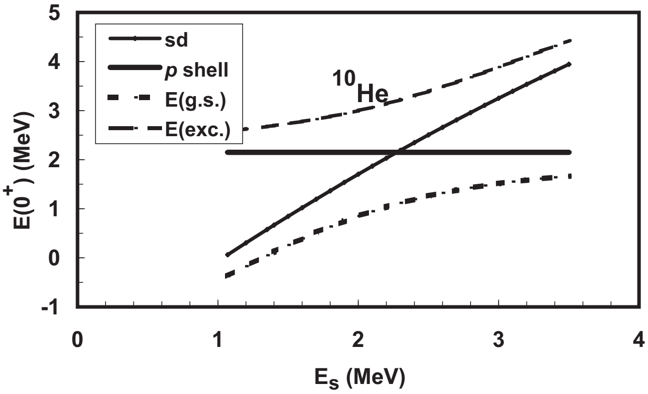

FIG. 5. In ^{10}He, plotted vs E_s in ^{9}He are the energies of the p-shell 0^+ state (Ref. [20]) (wide solid line), the $(sd)^2$ 0^+ state computed herein (thin solid curve), and the resulting energies of the g.s. (short-dashed curve) and excited 0^+ state (long-dashed curve), assuming a mixing matrix element of $V = 1.05$ MeV (see text).

higher than from all other reactions, one might conclude that a lower p-shell 0^+ energy is more appropriate.

Another approach is to attempt to fit the observed energies from two different reactions by assuming the predicted relative cross sections. This procedure assumes nothing about the mixing potential but only that a two-state model suffices. If I use centroid energies of $E_{2n} = 1.42(10)$ from p removal from ^{11}Li and $E_{2n} = 2.1(2)$ MeV from (t, p), then with my earlier predicted exc./g.s. ratios [21], it is possible to compute the energies of the two physical states. This procedure is depicted in Fig. 6. It can be seen that the g.s. energy is relatively constant, whereas the excited 0^+ energy changes rapidly with

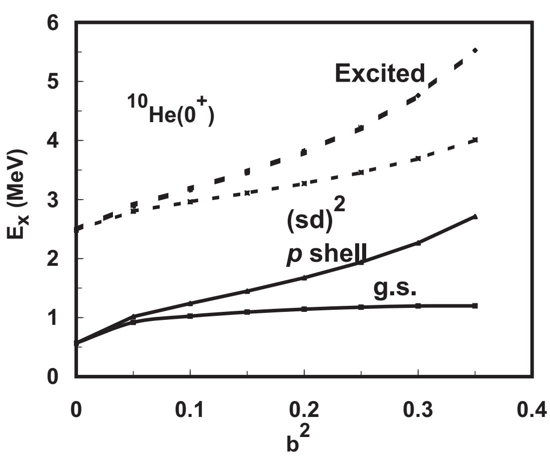

FIG. 6. Plotted vs b^2 [the amount of $(sd)^2$ in ^{10}He(g.s.)] are the energies of the g.s. (lower solid curve) and excited 0^+ state (upper dashed curve) that are required to fit results of energy measurements in proton knockout from ^{11}Li and in the reaction ^{8}He(t, p)—assuming the experiments measure the centroid of two overlapping 0^+ resonances. The upper solid curve is the resulting energy of the p-shell basis state, and the lower dashed curve is the energy of the $(sd)^2$ 0^+ basis state. This fit makes no assumption about the mixing, but only that a two-state model suffices.

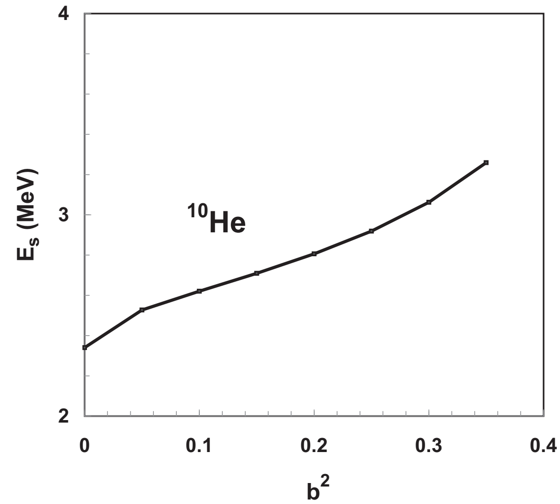

FIG. 7. By combining the dependence of $E(sd)^2$ on b^2 from Fig. 6 with the dependence of E_s on $E(sd)^2$ from Fig. 4, this plot displays vs b^2 the $1/2^+$ energy in ^{9}He that will fit the ^{10}He "g.s." energies measured in p knockout and (t, p).

the mixing parameter. For larger mixing, this approach is unreliable because in the arithmetic, the denominator in one part of the expression becomes very small. In any case, the fact that the energy from (t, p) is larger than from other reactions indicates that the mixing parameter is small.

I can produce the dependence of E_s on the mixing parameter by using the earlier relationship (Fig. 4) between E_s and $E(sd)^2$ combined with the dependence of $E(sd)^2$ on b^2 from Fig. 6. The result is plotted in Fig. 7. If the suggestion [21] of overlapping 0^+ states being populated with different relative strengths in different experiments is correct, it might be possible to perform one of the reactions with sufficient resolution to observe a two-peak structure. It might even be possible to see the expected interference dip between the two 0^+ resonances. Such an experiment would also allow a determination of b^2. The best candidate is probably ^{8}He(t, p) because the exc./g.s. ratio (for small mixing) is predicted to be much larger (but still near unity) in that reaction than in any other.

V. SUMMARY

To summarize, I have used the relationship between computed energies in ^{10}He and single-particle energies in

TABLE IV. Summary.

Condition	Conclusion
^{10}He has no bound states	$E_s > 1$ MeV
E(g.s.) = 1.4 MeV	$E_s > 1.8$ MeV
Calculated $E_{2n}(sd)^2$	E_s from Fig. 4
Centroids of overlapping 0^+	E(g.s.) < 1.4, E(excited) > 2.1 MeV
E(g.s.) from p knockout and (t, p)	Energies from Fig. 6, E_s from Fig. 7

^{9}He to provide limits on the $s_{1/2}$ energy. The absence of any bound states in ^{10}He requires $E_s > 1$ MeV. This conclusion contradicts all the experiments that reported an s state near threshold in ^{9}He. The assumption that the non-(t, p) reactions measure the g.s. energy results in a limit $E_s > 1.8$ MeV. The present analysis supports the view that the variation in ground-state energies determined in various reactions is caused by the presence of two overlapping 0^+ resonances. Results of the two simplest reactions—proton knockout and (t, p)—have been used to extract the g.s. and excited 0^+ energies as a function of the mixing parameter b^2 between the p-shell and the $(sd)^2$ basis states. This information was then used to determine E_s as a function of b^2. These conclusions are listed in Table IV.

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