Nonthermal nuclear reactions induced by fast α particles in the solar core

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Nonthermal nuclear effects triggered in the solar carbon-nitrogen-oxygen (CNO) cycle by fast α particles products of the *pp* chain reactions—are examined. The main attention is paid to 8.674-MeV α particles generated in the ⁷Li(*p*, α) α reaction. Nonthermal characteristics of these α particles and their influence on some nuclear processes are determined. It is found that the α -particle effective temperature is at a level of 1.1 MeV and exceeds the solar core temperature by 3 orders of magnitude. These fast particles are able to significantly enhance some endoergic (α , *p*) reactions neglected in standard solar model calculations. In particular, they can substantially affect the balance of the *p* + ¹⁷O $\rightleftharpoons \alpha$ + ¹⁴N reactions due to an appreciable increase of the reverse reaction rate. It is shown that in the region *R* = 0.08–0.25 *R*_☉ the reverse α + ¹⁴N reaction can block the forward *p* + ¹⁷O reaction, thus preventing closing of the CNO-II cycle, and increase the ¹⁷O abundance by a factor of 2–155 depending on *R*. This indicates that the fast α particles produced in the *pp* cycle can distort running of the CNO cycle, making it essentially different in the inner and outer core regions.

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The kinetics of nuclear reactions in high-temperature equilibrium or nearly equilibrium plasmas is commonly described within a thermal model of nuclear interaction between Maxwellian particles. In some cases, however, this traditional approach may miss specific effects arising from peculiarities of nuclear processes in plasmas. For example, nonthermal i' + jreactions induced by fast non-Maxwellian particles i' can to a certain degree contribute to the total i + j reaction rates. These particles are naturally produced in exoergic reactions and also created by recoil in close collisions of bulk plasma particles with some energetic reaction products. Nonthermal nuclear effects triggered by fast particles can manifest in both laboratory and astrophysical plasmas. For example, it was recently obtained [1] that energetic products of fusion processes in the primordial plasma can increase the rates of some reactions in big bang nucleosynthesis. The effects are particularly pronounced for endoergic reactions especially sensitive to the presence of fast particles in the matter. As a result, this can change the relation between forward and reverse reactions $i + j \rightleftharpoons k + l + Q$, causing its deviation from a standard law $\langle \sigma v \rangle_{kl} / \langle \sigma v \rangle_{ij} \propto \exp(-Q/T)$. Recently [2] such phenomenon was demonstrated for reactions in the primordial plasma.

In light of this, an important question arises regarding to what extent non-Maxwellian effects could affect reaction rates in stars and particularly in the sun. I should note that one type of such effects—possible *depletion* of ion distribution tails—was discussed earlier [3–7] in the context of the solar ⁸B neutrino problem. At the same time, another type of non-Maxwellian effects—possible *enhancement* of ion distribution tails—has still not attracted due attention. The effect is of certain interest as the solar core plasma is irradiated by MeV ions produced in reactions of the *pp* chain and the carbon-nitrogen-oxygen (CNO) cycle, which are two nuclear mechanisms of energy generation in main sequence stars [8,9]. These ions can

increase the population of high-energy tails of ion distribution functions and induce a number of nonthermal processes neglected in standard solar models (SSM).

It is reasonable to expect that the *pp* chain predominant in the sun is the main emitter of fast ions—mostly protons and α particles. In turn, the CNO cycle operating with a number of (p,α) and (α, p) reactions can serve as an appropriate object to examine *p*- and α -induced nonthermal effects. Given the continuing interest in the CNO cycle related to various aspects of energy and neutrino production, it is important to know quantitatively the possible impact of nonthermal effects on the CNO reaction rates. So far, it has not been determined.

Table I presents nuclear reactions of the pp chain and the CNO cycle, generating MeV protons and α -particles in the solar core plasma. I restrict consideration to reactions involving elements up to oxygen as processes with higher-Znuclei are suppressed at solar temperatures. Let us consider the CNO bicycle shown in Fig. 1 and identify reactions potentially sensitive to the presence of fast particles in the plasma. The number density of these particles is very low (see estimates below) because a combination of two factors-high density $(\rho \simeq 150 \mathrm{g/cm^3})$ and moderate temperature $(T \simeq 1.3 \mathrm{ keV})$ of the solar core-favors rapid particle thermalization. So one can expect that the reaction-produced protons have almost no influence on *exoergic* (p, γ) and (p, α) reactions of the CNO cycle. However, *endoergic* reverse (α, p) reactions neglected in the SSM due to strong suppression at keV temperatures are the most promising candidates to observe a signature of α -particle-induced nonthermal effects. As an interesting example, I focus on the following reactions:

$$p + {}^{17}\text{O} \rightleftharpoons \alpha + {}^{14}\text{N}, \quad Q = 1.191 \text{ MeV}.$$
 (1)

The forward (p,α) reaction closes the CNO-II cycle and is one of processes primarily determining the cycle fusion rates. On the other hand, however, the reverse (α, p) reaction has a low threshold $E_{\alpha,\text{thr}} = 1.531$ MeV, and being enhanced by fast α particles may suppress the forward process.

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TABLE I. Nuclear reactions generating MeV particles in the solar core.

| Cycle | | Reaction | Q (MeV) | $E_{\alpha,0}$ (MeV) |
|-------|-----|--|---------|----------------------|
| pp | Ι | 3 He(3 He,2 <i>p</i>) α | 12.860 | ≲4.3 |
| | II | $^{7}\text{Li}(p,\alpha)\alpha$ | 17.348 | 8.674 |
| | III | ${}^{8}\mathrm{B}(\beta^{+}){}^{8}\mathrm{Be}^{*}[3.03] \rightarrow 2\alpha$ | 18.072 | 1.561 ^a |
| | III | ${}^{8}B(\beta^{+}){}^{8}Be^{*}[16.626] \rightarrow 2\alpha$ | 18.072 | 8.359 ^a |
| CNO | Ι | $^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}$ | 4.966 | 3.725 |
| | Π | ${}^{17}{\rm O}(p,\alpha){}^{14}{\rm N}$ | 1.191 | 0.926 |
| | III | $^{18}\mathrm{O}(p,\alpha)^{15}\mathrm{N}$ | 3.980 | 3.142 |

 ${}^{a}E_{\alpha,0}$ may change within the half-width of the ${}^{8}Be^{*}$ state being considered.

To examine this conjecture, one needs to evaluate the flux of fast α particles produced in the reactions of Table I. The emission rate of α particles in a reaction $i + j \rightarrow \alpha + \cdots$ is

$$R_{\alpha,ij} = N_{\alpha} \times R_{ij}, \quad R_{ij} = (1 + \delta_{ij})^{-1} n_i n_j \langle \sigma v \rangle_{ij}.$$
(2)

In Eq. (2), N_{α} is the number of α particles produced per pair of (ij), and R_{ij} is the reaction rate depending on the particle number density $n_x (x = i, j)$ and the reactivity $\langle \sigma v \rangle_{ij}$. The latter is given by

$$\langle \sigma v \rangle_{ij} = \frac{1}{n_i n_j} \int f_i(\mathbf{v}_i) f_j(\mathbf{v}_j) \sigma(|\mathbf{v}_i - \mathbf{v}_j|) |\mathbf{v}_i - \mathbf{v}_j| \, d\mathbf{v}_i \, d\mathbf{v}_j,$$
(3)

where $f_x(\mathbf{v}_x)$ is the density-normalized velocity distribution function of species *x*, and σ is the reaction cross section. Maxwellian reactivities provided by the NACRE Collaboration [10] and element density and temperature profiles [11] obtained by running the MESA code [12] were used in these calculations. The reactivity enhancement $\langle \sigma v \rangle_{ij}^{\text{solar}} / \langle \sigma v \rangle_{ij} =$ f_{ij} in the solar core due to electron screening was also taken into account in a weak-screening approximation [13] properly describing the screening effects for reactions with $Z_i Z_j \leq 10$ [14] (for details, see a review in Ref. [15]).

Figure 2 shows the emission rates $R_{\alpha,ij}$ of MeV α particles in the solar core at $R < 0.3 R_{\odot}$. These particles are



FIG. 1. (Color online) The CNO bicycle. The dashed arrows present reverse reactions neglected in the SSM network.



FIG. 2. (Color online) The emission rates $R_{\alpha,ij}$ of fast α particles in the solar core plasma.

predominantly produced in the ³He(³He,2*p*) α and ⁷Li(*p*, α) α reactions of the *pp* cycle. In this work, I focus on nonthermal effects triggered by the ⁷Li(*p*, α) α reaction products. This reaction generates the most energetic α particles with an energy $E_{\alpha,0} = 8.674$ MeV and simultaneously provides the highest particle flux at $R \leq 0.06 R_{\odot}$.

To examine the nonthermal reverse (α, p) reaction, Eq. (1), I use a formalism of in-flight reaction probability. According to it, the probability $W_{\alpha^{14}N}$ for an α particle to undergo the nonthermal $\alpha + {}^{14}N \rightarrow p + {}^{17}O$ reaction while slowing in the plasma from the energy $E_{\alpha,0}$ down to the reaction threshold $E_{\alpha,\text{thr}}$ is

$$W_{\alpha^{14}N}(E_{\alpha,0} \to E_{\alpha,\text{thr}})$$

$$= 1 - \exp\left[\int_{E_{\alpha,\text{thr}}}^{E_{\alpha,0}} \left(\frac{2E_{\alpha}}{m_{\alpha}}\right)^{1/2} \times \frac{n_{14}N\sigma(E_{\alpha})}{\langle dE_{\alpha}/dt \rangle_{\text{Coul}} + \langle dE_{\alpha}/dt \rangle_{\text{NES}}} dE_{\alpha}\right], \quad (4)$$

where $n_{^{14}N}$ is the ^{14}N number density, σ is the reaction cross section, and $\langle dE_{\alpha}/dt \rangle_{L}$ is the average rate of α -particle energy loss via Coulomb elastic scattering off background charged particles (L = Coul) and via nuclear elastic scattering off ambient nuclei (L = NES). In this study, the Coulomb process plays a main role. The rate $\langle dE_{\alpha}/dt \rangle_{Coul}$ can properly be described in a binary-collision model with a Debye cutoff [16]

$$\left\langle \frac{dE_{\alpha}}{dt} \right\rangle_{\text{Coul}} = \sum_{j} -\frac{4\pi e^4 (Z_{\alpha} Z_j)^2}{(2m_j T_j)^{1/2}} n_j \Lambda_j \frac{\Psi(x_j)}{x_j},\tag{5}$$

$$\Psi(x_j) = \operatorname{erf}(x_j) - \frac{2}{\pi^{1/2}} \left(1 + \frac{m_j}{m_\alpha} \right) x_j \exp\left(-x_j^2\right), \quad (6)$$

where $x_j = [m_j E_\alpha/(m_\alpha T_j)]^{1/2}$ and the summation is taken over bulk charged species j (electrons and ions). In these equations, n_j and T_j are the number density and the temperature of plasma species j, while m_l and Z_l are the mass and the charge number of particle $l(=j,\alpha)$. I assume that the bulk species have the same temperatures $T_j = T$. The Coulomb logarithm Λ_j is treated in classical and quantum-mechanical approximations [16]. In turn, the energy loss via NES off ambient nuclei i is given by the expression

$$\left\langle \frac{dE_{\alpha}}{dt} \right\rangle_{\text{NES}} = -\sum_{i} \left(\frac{2E_{\alpha}}{m_{\alpha}} \right)^{1/2} n_{i} E_{\alpha} \left(1 - \frac{3T}{2E_{\alpha}} \right)$$
$$\times \frac{4\pi m_{\alpha} m_{i}}{(m_{\alpha} + m_{i})^{2}} \int_{b}^{1} \sigma(E_{\alpha}, \mu) (1 - \mu) d\mu$$
(7)

following from a formula [17] in which the effect of plasma thermal motion is allowed for within a first-order correction [18]. In Eq. (7), $\sigma(E_{\alpha},\mu)$ is the differential cross section for $\alpha - i$ NES (allowing for Coulomb-nuclear interference), μ is the cosine of scattering angle in the center-of-mass frame, and b = -1 (for $i \neq \alpha$) or 0 (for $i = \alpha$).

To realize the level of nonthermal reaction, I convert its probability to the corresponding reaction rate. Once $W_{\alpha^{14}N}$ is determined, the rate of nonthermal reaction $R_{\alpha^{14}N,nonth}$ can be evaluated as

$$R_{\alpha^{14}N,nonth} = W_{\alpha^{14}N} \times R_{\alpha,p^{7}Li},$$
(8)

where $R_{\alpha,p^7\text{Li}}$ is the emission rate of fast α particles in the ⁷Li(*p*, α) α reaction. Equation (8) can be rewritten in the form of Eq. (2)

$$R_{\alpha^{14}N,nonth} = n_{\alpha,nonth} n_{14N} \langle \sigma v \rangle_{\alpha^{14}N,nonth}, \qquad (9)$$

where $\langle \sigma v \rangle_{\alpha^{14}N,nonth}$ is the effective nonthermal reactivity. It can be reduced to

$$\langle \sigma v \rangle_{\alpha^{14}\text{N,nonth}} = \frac{W_{\alpha^{14}\text{N}}}{n^{14}\text{N}} \left[\int_{E_{\alpha,\text{thr}}}^{E_{\alpha,0}} -dE_{\alpha}/\langle dE_{\alpha}/dt \rangle \right]^{-1}. (10)$$

It is useful to introduce one more quantity—the effective temperature $T_{\alpha,\text{nonth}}$ of nonthermal (non-Maxwellian) reaction-produced α particles. It can be estimated [19] by equating the pressure of these particles having a slowing-down distribution function to the pressure of Maxwellian α particles

$$\frac{1}{3} \langle m_{\alpha} v_{\alpha}^2 \rangle_{\text{nonth}} \simeq \frac{1}{3} \langle m_{\alpha} v_{\alpha}^2 \rangle_{\text{M}} = n_{\alpha,\text{nonth}} T_{\alpha,\text{nonth}}, \qquad (11)$$

where angular brackets denote folding $m_{\alpha}v_{\alpha}^2$ over the corresponding density-normalized distribution functions. The slowing-down distribution is obtained by solving a Fokker-Planck equation with a δ -function source term $S_{\alpha} \propto \delta(v_{\alpha} - v_{\alpha,0})$. Equation (11) gives

$$T_{\alpha,\text{nonth}} = \frac{2I_4(v_c/v_{\alpha,0})}{3I_2(v_c/v_{\alpha,0})} E_{\alpha,0}, \quad I_n(a) \equiv \int_0^1 \frac{x^n}{a^3 + x^3} \, dx,$$
(12)

where v_c is the crossover velocity [19].

Now let us consider the results of calculations. The probability of the nonthermal $\alpha + {}^{14}N \rightarrow p + {}^{17}O$ reaction, Eq. (4), calculated with its measured cross sections [20] is shown in Figs. 3 and 4 as a function of R/R_{\odot} and α -particle deceleration time, respectively. I remind readers that this reaction is induced by fast α particles born in the ${}^{7}\text{Li}(p,\alpha)\alpha$ process ($E_{\alpha,0} =$ 8.674 MeV). These particles rapidly slow down in the plasma. Our analysis shows that the particle thermalization time $\tau_{\alpha,\text{th}}$ and range $l_{\alpha,\text{th}}$ are at most 10^{-12} s and 10^{-5} m, respectively, while the particle number density $n_{\alpha,\text{nonth}} \simeq \tau_{\alpha,\text{th}} R_{\alpha,p^{7}\text{Li}}$ does



FIG. 3. (Color online) The probability of the nonthermal α + ¹⁴N \rightarrow *p* + ¹⁷O reaction induced by a 8.674-MeV α particle in the solar core plasma.

not exceed 10^2 m^{-3} . At the same time, however, the effective temperature $T_{\alpha,\text{nonth}}$ of these non-Maxwellian α particles, Eq. (12), proves to be remarkably high. Figure 5 shows $T_{\alpha,\text{nonth}}$ in a comparison with the solar core temperature *T*. As seen, the α -particle temperature $T_{\alpha,\text{nonth}}$ range is within 1.06–1.17 MeV and exceeds the core temperature *T* by about 3 orders of magnitude. So one may expect that these α particles can enhance the reverse $\alpha + {}^{14}\text{N}$ reaction in Eq. (1). Note that the thermal $\alpha + {}^{14}\text{N}$ reaction is dramatically suppressed at the core temperature—its Maxwellian reactivity $\langle \sigma v \rangle_{\alpha}{}^{14}\text{N}$ and rate $R_{\alpha}{}^{14}\text{N}$ are at most $10^{-420} \text{ cm}{}^3 \text{ s}{}^{-1}$ and $10^{-380} \text{ cm}{}^{-3} \text{ s}{}^{-1}$, i.e., fully negligible. It is not surprising therefore that thermal reverse reactions are not considered in the SSM calculations.

At the same time, however, the presence of MeV α particles in the plasma can crucially change the situation. I have obtained that the $\alpha + {}^{14}$ N reactivity $\langle \sigma v \rangle_{\alpha}{}^{14}$ N,nonth and rate $R_{\alpha}{}^{14}$ N,nonth, allowing for the contribution of 8.674-MeV α particles, become much higher than their Maxwellian estimates. Furthermore, they can even exceed the respective parameters of the forward $p + {}^{17}$ O reaction. These surprising results are



FIG. 4. (Color online) Time dynamics of the $\alpha + {}^{14}N \rightarrow p + {}^{17}O$ reaction probability during α -particle deceleration in the plasma.



FIG. 5. (Color online) The effective temperature $T_{\alpha,\text{nonth}}$ of non-Maxwellian α particles, products of the ⁷Li(p,α) α reaction, in comparison with the solar core temperature T.

presented in Fig. 6. Here, nuclear data from the NACRE compilation [10] and new experiments [21] are used to describe the $p + {}^{17}\text{O}$ reaction. Figure 6(a) gives a comparison of the reverse and forward reactivities $\langle \sigma v \rangle_{\alpha^{14}N,nonth}$ and $\langle \sigma v \rangle_{p^{17}O}$. It is seen that in the solar core $\langle \sigma v \rangle_{\alpha^{14}N,nonth} \gg \langle \sigma v \rangle_{p^{17}O}$. Figure 6(b) shows a comparison of the respective reaction rates $R_{\alpha^{14}N,nonth}$ and $R_{p^{17}O}$, the quantities most important for network calculations. As seen, the reverse $\alpha + {}^{14}N$ reaction rate can not only become comparable with the forward $p + {}^{17}\text{O}$ reaction rate at $R \simeq 0.08 R_{\odot}$ but also exceeds it by a factor of ~200 at $R = 0.25 R_{\odot}$. Thus, although the amount of fast α particles is small, they can drastically change the balance of the $p + {}^{17}\text{O} \rightleftharpoons \alpha + {}^{14}\text{N}$ reactions. In the outer core region $R \simeq 0.08 - 0.25 R_{\odot}$ the reverse reaction can block the forward one and thereby distort normal running of the CNO-II cycle. Indeed, the blocking prevents closing of the CNO-II cycle and redirects nuclear flow from ${}^{17}\text{O} \rightarrow {}^{14}\text{N}$ to ${}^{17}\text{O} \rightarrow {}^{18}\text{F}$, i.e., to the CNO-III cycle. This may affect the ¹⁷O abundance and the abundances of heavier elements like ¹⁸O and ¹⁹F produced in a sequence of reactions triggered by ¹⁷O. The SSM ¹⁷O number density n_{17O} employed in our calculations satisfies a balance equation (see a CNO multicycle diagram [22])

$$\frac{dn_{^{17}\text{O}}}{dt} = -n_p n_{^{17}\text{O}} \langle \sigma v \rangle_{p^{^{17}\text{O}} \to \alpha^{^{14}\text{N}}} - n_p n_{^{17}\text{O}} \langle \sigma v \rangle_{p^{^{17}\text{O}} \to \gamma^{^{18}\text{F}}} + \frac{n_{^{17}\text{F}}}{\tau} = 0$$
(13)

and equals $6.6 \times 10^{19} \text{ cm}^{-3}$ (at $0.1 R_{\odot}$), $5.3 \times 10^{18} \text{ cm}^{-3}$ (at $0.2 R_{\odot}$), and $3.1 \times 10^{18} \text{ cm}^{-3}$ (at $0.25 R_{\odot}$). To allow for the additional ¹⁷O production in the nonthermal ¹⁴N(α , p)¹⁷O



FIG. 6. (Color online) The comparison of the $p + {}^{17}\text{O} \rightleftharpoons \alpha + {}^{14}\text{N}$ reactions allowing for the contribution of fast α particles born in the ${}^{7}\text{Li}(p,\alpha)\alpha$ process. (a) The forward (p,α) and reverse (α, p) reactivities, $\langle \sigma v \rangle_{p^{17}\text{O}}$ and $\langle \sigma v \rangle_{\alpha^{14}\text{N,nonth}}$. (b) The forward (p,α) and reverse (α, p) reaction rates, $n_p n_{17_{O}} \langle \sigma v \rangle_{p^{17_{O}}}$ and $n_{\alpha,\text{nonth}} n_{14_{N}} \langle \sigma v \rangle_{\alpha^{14}\text{N,nonth}}$.

reaction, one needs to add a term $[n_{\alpha,\text{nonth}}n_{^{14}N}\langle\sigma v\rangle_{\alpha^{14}N,\text{nonth}}]$ in Eq. (13). This leads to an increase of the ^{17}O number density $n_{^{17}\text{O}} \rightarrow n_{^{17}\text{O}}^*$ and $n_{^{17}\text{O}}^*/n_{^{17}\text{O}}$ was estimated to be approximately 2 (at 0.1 R_{\odot}), 24 (at 0.2 R_{\odot}), and 155 (at 0.25 R_{\odot}). Therefore, the SSM ^{17}O abundance is likely to be underestimated in the outer core. Note that to a lesser degree the nonthermal effects discussed may also manifest for the $p + {}^{15}\text{N} \rightleftharpoons \alpha + {}^{12}\text{C}$ reactions of the CNO-I cycle (see Fig. 1). However, a lack of reliable nuclear data for the reverse $\alpha + {}^{12}\text{C}$ reaction makes it hardly possible to carry out a detailed analysis here.

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