Dynamical charge fluctuations in the hadronic medium

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Dynamical charge fluctuations have been studied in ultrarelativistic heavy-ion collisions by using hadronic model simulations, such as Ultrarelativistic Quantum Molecular Dynamics (UrQMD) and Heavy Ion Jet Interaction Generator (HIJING). The evolution of fluctuations has been calculated at different time steps during the collision as well as at different observation windows in pseudorapidity ($\triangle\eta$). The final state effects on the fluctuations have been investigated by varying $\triangle\eta$ and the time steps with the aim of obtaining an optimum observation window for capturing maximum fluctuations. It is found that $\triangle\eta$ between 2.0 and 3.5 gives the best coverage for the fluctuations studies. The results of these model calculations for Au + Au collisions at $\sqrt{s_{\rm NN}} =$ 7.7 to 200 GeV and for Pb + Pb collisions at 2.76 TeV are presented and compared to the available experimental data from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC).

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I. INTRODUCTION

The primary goal of heavy-ion collisions at ultrarelativistic energies is to explore the signatures of the deconfined state of matter, the quark-gluon-plasma (QGP). Dedicated experiments at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) and the Large Hadron Collider (LHC) at CERN have been setup for studying the QGP matter at the high temperatures (T) and low baryon chemical potentials ($\mu_{\rm B}$). Several signatures for studying the phase transition from hadronic matter to QGP have been proposed and also studied in dedicated experiments at BNL and CERN for the last few decades. Event-by-event fluctuations of conserved charges in limited phase space have been widely accepted as one of the most tantalizing signals of the QGP formation and also for the search of the QCD critical point [1–7]. With their large coverage, the Solenoidal Tracker (STAR) experiment at RHIC [8] and the A Large Ion Collider Experiment (ALICE) at LHC [9] are ideally suited for the detailed study of the QGP matter on an event-by-event basis. The dynamical charge fluctuations have been reported by these experiments [10–15]. Recent results from ALICE have shown a significant reduction in the ratio of charge fluctuations per entropy at the LHC energy [12], confirming to the QGP formation in heavy-ion collisions.

Event-by-event fluctuations of conserved quantities such as net electric charge and net baryon number act as distinct signals for the transition from hadronic (confined) phase to QGP (deconfined) phase. The amount of charge fluctuations is proportional to the squares of the charges present in the system, which depend on the state from which the charges originate. The system passing through a QGP phase has quarks as the charge carriers whereas for a hadron gas (HG) the charge carriers are the charged hadrons. Thus the charge fluctuations in case of QGP with fractional charges should be significantly lower than the HG where the charges are integral. Due to the differences in degrees of freedom of the two phases, QGP and HG, the magnitude of the charge fluctuations are very different. It is estimated that for the QGP, charge fluctuations are much smaller than the HG [3,6]. Here the question aries whether these primordial fluctuations, either from a QGP or from an HG, survive during the course of the evolution of the system [16–19]. The fluctuations observed at the freeze-out depend crucially on the equation of state of the system and final state effects. Nonequilibrium studies at the early partonic stage show that large charge fluctuations survive if they are accompanied by large temperature fluctuations at freeze-out [20]. In reality, the measurement of charge fluctuations depends on the observation window, which is to be properly chosen so that the majority of the fluctuations are captured without being affected by the conservation limits [17–19].

We studied the event-by-event dynamical net-charge fluctuations originating from the purely hadronic state using Ultrarelativistic Quantum Molecular Dynamics (UrQMD) [21,22] and Heavy Ion Jet Interaction Generator (HIJING) [23] event generators at different times during the evolution of the hadronic interaction. The dynamical charge fluctuations were estimated at different time steps and by varying the pseudorapidity window ($\triangle\eta$) of the measurement. The main focus is to understand the effect of final state effects that diffuse the charge fluctuations at different time and $\triangle\eta$ window. Assuming hadronization and freeze-out occur roughly at 5 and 30 fm/c, respectively, we have calculated the fluctuations of the system at 5, 30 fm/c, and at a much later time of 100 fm/c, where all possible interactions must seize.

This paper is organized as follows. The measure of dynamical charge fluctuations in heavy-ion collisions are discussed in Sec. II. In Sec. III, we present particle multiplicity distributions at $\Delta\eta = 1$, for different time steps for central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV, using UrQMD. The measure of dynamical charge fluctuations at different time steps and $\Delta\eta$ window are discussed in Sec. IV. The results of the calculations from hadronic models are presented in Sec. V, along with the experimental data from STAR and ALICE. The paper is summarized in Sec. VI.

II. NET-CHARGE FLUCTUATIONS

The Net-charge and the total charge of a system are denoted in terms of $Q = N_+ - N_-$ and $N_{ch} = N_+ + N_-$, where N_+ and N_- are the multiplicities of positively and negatively charged particles, respectively. The net-charge fluctuations can be expressed in terms of their ratio to entropy to take the volume term into account. Thus, one of the observables for net-charge fluctuations is [3]

$$D = 4\frac{\langle \delta Q^2 \rangle}{N_{ch}}, \tag{1}$$

where δQ^2 is the variance of the net charge. The value of D has been estimated by theoretical models for a QGP and a HG by taking various final state effects into account [3–6,17,18,24–26]. Early estimations put the value of D to be approximately four times smaller for a QGP compared to an HG. For an HG, resonance decays including those of neutral particles introduce additional correlation between charged particles, which reduces the value of D [17,18]. Present understandings put the value of D to be 1–1.5 for a QGP and 2.8 for an HG. In all cases, the signal gets diffused from hadronization time to freeze-out because of the final state interactions that need to be taken into account [17,18].

Net-charge fluctuations, measured in terms of D, have contributions from statistical as well as dynamical origins. It is a rather difficult task to estimate the dynamical component from the total fluctuations. A novel method of estimation of the dynamical fluctuations has been proposed, which takes into account the correlation strengths between ++, −−, and +− charged particle pairs [27]. The difference between the relative number of positively (N_+) and negatively (N_-) charged particles can be expressed in terms of its second moment as

$$\nu_{+-} = \left\langle \left(\frac{N_+}{\langle N_+ \rangle} - \frac{N_-}{\langle N_- \rangle} \right)^2 \right\rangle. \tag{2}$$

Here, the notation "⟨ ⟩" denotes the average over the ensemble of events. Assuming independent particle production mechanism, the value of ν_{+-} in the Poissonian limit can be expressed as

$$\nu_{+-,\mathrm{stat}} = \frac{1}{\langle N_+ \rangle} + \frac{1}{\langle N_- \rangle}. \tag{3}$$

The dynamical component is then evaluated as the difference between the two measured fluctuations, expressed as

$$\nu_{+-,\mathrm{dyn}} = \nu_{+-} - \nu_{+-,\mathrm{stat}}. \tag{4}$$

This can be expanded as

$$\nu_{+-,\mathrm{dyn}} = \frac{\langle N_+(N_+ - 1) \rangle}{\langle N_+ \rangle^2} + \frac{\langle N_-(N_- - 1) \rangle}{\langle N_- \rangle^2} - 2\frac{\langle N_- N_+ \rangle}{\langle N_+ \rangle \langle N_- \rangle}. \tag{5}$$

A stronger correlation between +− pairs compared to ++ and −− pairs yields a negative value of $\nu_{+-,\mathrm{dyn}}$.

It can be seen that the $\nu_{+-,\mathrm{dyn}}$ is related to the net-charge fluctuations D by

$$\langle N_{ch} \rangle \nu_{+-,\mathrm{dyn}} = D - 4. \tag{6}$$

By determining $\nu_{+-,\mathrm{dyn}}$ in the experiments, one can have access to net-charge fluctuations.

The magnitude of net-charge fluctuations is limited by the global conservation of charged particles [27]. Considering the effect of global charge conservation, the dynamical fluctuations need to be corrected by a factor of $\nu_{+-,\mathrm{dyn}} = -4/\langle N_{4\pi} \rangle$, where $\langle N_{4\pi} \rangle$ is the average of the total number of charged particles produced over full phase space. The corrected value of $\nu_{+-,\mathrm{dyn}}$ after considering the global charge conservation and finite acceptance is

$$\nu^{\mathrm{corr}}_{+-,\mathrm{dyn}} = \nu_{+-,\mathrm{dyn}} + \frac{4}{N_{4\pi}}. \tag{7}$$

The modified value of the net-charge fluctuations turns out to be

$$D = \langle N_{ch} \rangle \nu^{\mathrm{corr}}_{+-,\mathrm{dyn}} + 4. \tag{8}$$

In the rest of the article we will evaluate $\langle N_{ch} \rangle \nu^{\mathrm{corr}}_{+-,\mathrm{dyn}}$ and D for different center-of-mass energies.

III. MULTIPLICITY DISTRIBUTIONS AT DIFFERENT TIME STEPS

To understand the evolution of multiplicity distributions of different particle species at different time steps, we used UrQMD model simulations for Au + Au collisions corresponding to RHIC energies. The UrQMD model simulates the microscopic transport of a covariant propagation of quarks and diquarks with hadronic degrees of freedom. The formation of hadrons is introduced by the color string fragmentation. Various resonances and their decay along with rescattering among hadrons have been incorporated during the evolution [22]. This model helps to explore the evolution of conserved charge fluctuations and their distribution at different time steps in the hadronic medium.

In the present study, the UrQMD model has been used to simulate Au + Au collisions at various collision energies. The event-by-event distributions of differently charged particle and antiparticle species are estimated at time 5, 30, and 100 fm/c after the collision. Multiplicity distributions within $|\eta| < 1.0$ and the transverse momentum range of $0.2 < p_T < 5.0$ GeV/c are presented in Fig. 1 for central (0–5% centrality) Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV Multiplicity distributions of charged particles (N_+ and N_-), pions (π^+ and π^-), kaons (K^+ and K^-), and protons (p and $\bar{p}$) are shown for the three time steps. The distributions shift to the right as the system evolves with time in going from 5 to 30 and

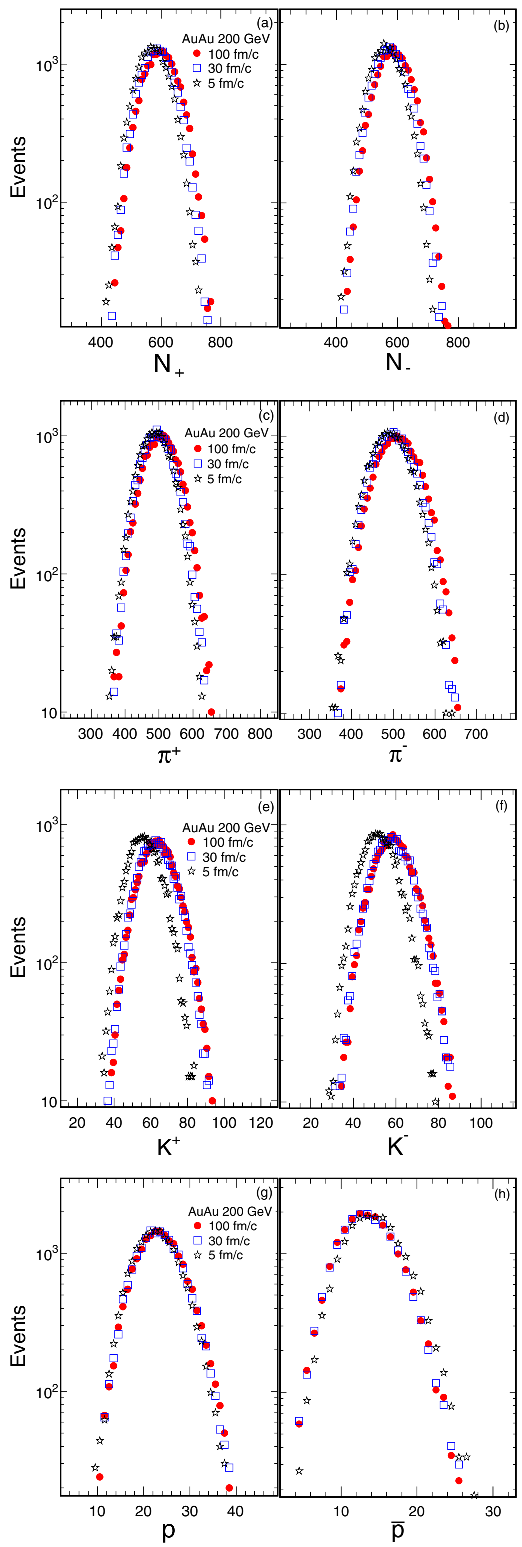

FIG. 1. (Color online) Multiplicity distributions for Au + Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV within $|\eta| < 0.5$ and $0.2 < p_T < 5.0$ GeV/c at different time steps 5, 30, and 100 fm/c for (a) positively charged particles, (b) negatively charged particles, (c) π^+, (d) π^-, (e) K^+, (f) K^-, (g) p and (h) $\bar{p}$.

100 fm/c. The shifts for the N_+ and N_-, pions and kaons are quite appreciable, whereas protons and antiprotons are less affected. The multiplicity distributions of pions and kaons mainly contribute the change in total positively and negatively charged multiplicity distribution. The shift of the kaon multiplicity distributions after 5 fm/c can occur because the kaon production from meson-meson and baryon-meson interactions, as implemented in the UrQMD model, dominate during this time. Additional change at higher multiplicity may be due to rescattering and resonance decays in a given phase space. Because of their higher masses, the distributions for protons and antiprotons compared to those of the pions and kaons, are less affected during the evolution of the system. The proton number is expected to diffuse more slowly because of rescattering [5]. This change of multiplicity distributions is maximum in the larger pseudorapidity window.

Due to the final state effects, the change of the shape of multiplicity distributions may affect various event-by-event observables. The fluctuations of multiplicity distributions diffuse at different time scales in heavy-ion collisions, in the rapidity space. Hence, it is expected that different fluctuation's measures may be affected differently with the time evolution in a given phase space. In the next sections, we present the dynamical charge fluctuations measures at different time steps for Au + Au collisions at 200 GeV using the UrQMD model.

IV. FLUCTUATIONS AS A FUNCTION OF $\Delta\eta$

The evolutions of $\nu^{\rm corr}_{+-,\rm dyn}$ are studied by using UrQMD by varying different $\Delta\eta$ windows for different time steps. The main goal of this exercise is to understand the evolution of fluctuations through a purely hadronic medium as well as to find an optimum coverage where most of the fluctuations can be measured. This information helps to understand the evolution of fluctuations through a purely hadronic medium, as charge fluctuations are supposed to be diffused with the increase in the $\Delta\eta$ window. The total charge of a system is conserved leading to vanishing net-charge fluctuations for full coverage. At the same time, studying fluctuations in a very small $\Delta\eta$ window may not be ideal for capturing most of the initial fluctuations. An optimum coverage is to be obtained by taking these into account.

To obtain the optimum value of fluctuations, taking all effects into account, we considered the $\Delta\eta$ range from 0.2 to 10.0. The fluctuations are calculated for central (0–5%) collisions. To avoid the dependence on the central bin width, the value of $\nu_{+-,\rm dyn}$ is determined using the unit bin method. In this method, the value of $\nu_{+-,\rm dyn}(m)$ for each multiplicity is calculated and then averaged over the width of a particular centrality with the weights corresponding to relative cross section. The weighted average for $\nu_{+-,\rm dyn}$ are calculated as

$$\nu_{+-,\rm dyn}(m_{\rm min} \leqslant m < M_{\rm max}) = \frac{\sum \nu_{+-,\rm dyn}(m)p(m)}{\sum p(m).} \qquad (9)$$

Here, $p\ (m)$ is the weight of a particular centrality m. Finally, the corrected values of $\nu_{+-,\rm dyn}$ have been obtained using Eq. (7).

Figure 2 shows both the uncorrected ($\langle N_{\rm ch}\rangle \nu_{+-,\rm dyn}$) and corrected ($\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$) values of fluctuations as a function of the $\Delta\eta$ window for central (0–5%) Au + Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV, obtained from UrQMD. The results

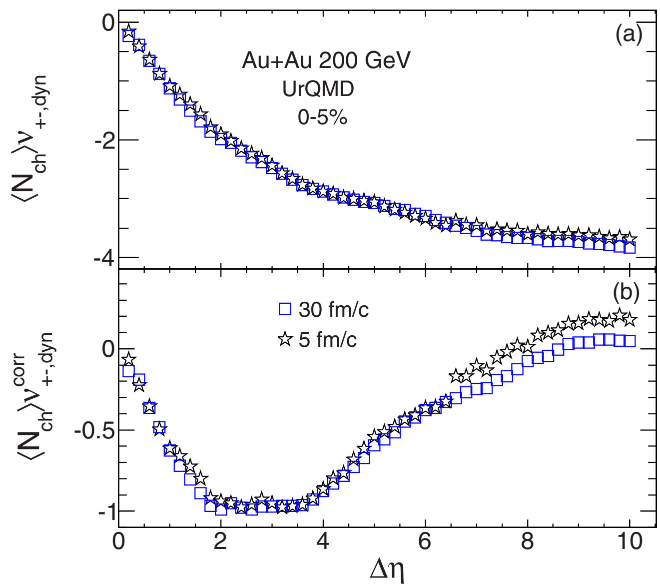

FIG. 2. (Color online) The values of $\langle N_{\rm ch}\rangle \nu_{+-,\rm dyn}$ (upper panel) and $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ (lower panel), plotted as functions of $\Delta\eta$ window using UrQMD model at two different time steps for central (0–5%) Au + Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV.

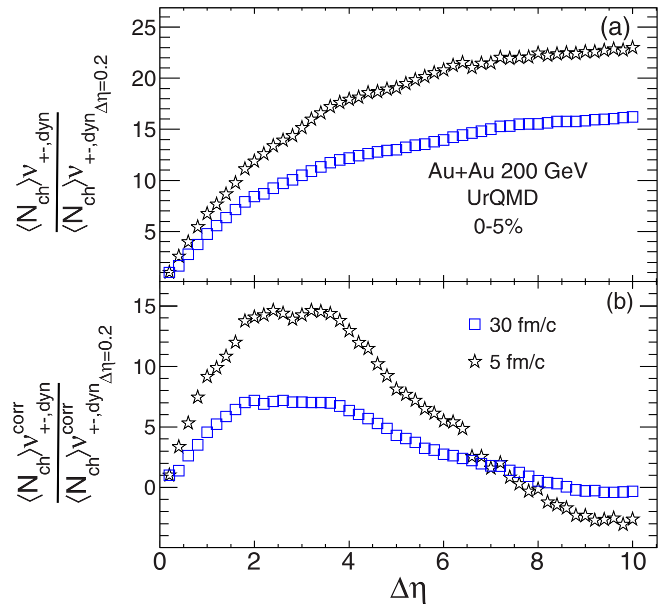

FIG. 3. (Color online) The ratios of $\langle N_{\rm ch}\rangle \nu_{+-,\rm dyn}$ (upper panel) and $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ (lower panel), with respect to their are normalized values at the smallest $\Delta\eta$ of 0.2 for central Au + Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV. The ratios are plotted as a function of $\Delta\eta$ for two different time steps, 5 and 30 fm/c.

are presented for two time steps, 5 and 30 fm/c. The trends for the uncorrected and corrected values of fluctuations are observed to be very different. The upper panel of the figure shows that $\langle N_{\rm ch}\rangle \nu_{+-,\rm dyn}$ keep decreasing with the increase in $\Delta\eta$. This is unphysical, as the fluctuations should vanish for measurements at the full coverage. The nature of $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$, however, shows a different trend, where the values decrease up to $\Delta\eta$ values of 2 to 2.5, then remain constant till about $\Delta\eta = 3.5$, and then increase as per the expectations. The values of $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ tend to zero at the highest $\Delta\eta$ due to the global charge conservation. This decreasing trend of $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ up to $\Delta\eta \sim 2$ is due to the strengthening of multiplicity correlations with the increase in $\Delta\eta$.

The nature of the fluctuations, at two time steps, as a function of $\Delta\eta$ may be better understood by plotting the ratio of the fluctuations at different $\Delta\eta$ values with respect to a particular $\Delta\eta$ (normalizing with respect to smallest $\Delta\eta$). Figure 3 shows the ratios of $\langle N_{\rm ch}\rangle \nu_{+-,\rm dyn}$ and $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ with respect to their values at $\Delta\eta = 0.2$. From the upper panel of the figure, it is seen that the uncorrected normalized $\langle N_{\rm ch}\rangle \nu_{+-,\rm dyn}$ ratios increase monotonously with the increase in $\Delta\eta$. However, the corrected normalized $N_{ch}\nu^{\rm corr}_{+-,\rm dyn}$ values increase up to $\Delta\eta \sim 2$, then remain constant up to $\Delta\eta = 3.5$. As the hadronic system evolves, it encounters more and more rescattering and resonance decay as compared to a smaller pseudorapidity window. Within the $\Delta\eta$ range of 2.0 and 3.5, the diffusion of dynamical charge fluctuations may remain insensitive. Going beyond $\Delta\eta = 3.5$, the fluctuations decrease due to the dilution of correlations and the effect of global charge conservation. Near $\Delta\eta$ of 8.0, the fluctuations are close to zero. Going to higher $\Delta\eta$, the ratio goes below zero, indicating $\nu_{+-,\rm dyn}$ becomes positive. This can happen because +ve and −ve charged particles become uncorrelated, possibly because of the inclusion of spectator particles. In addition to the dependence of fluctuations on $\Delta\eta$, Fig. 3 also gives the time dependence of fluctuations for wide $\Delta\eta$ windows compared to a narrow bin of $\Delta\eta = 0.2$. It is observed that the fluctuations for wide $\Delta\eta$ compared to the corresponding narrow $\Delta\eta$ are more pronounced at a time of 5 fm/c, compared to the corresponding values at a later time of 30 fm/c.

From the present study, we conclude that the optimal coverages for observing the charge fluctuations are for $\Delta\eta = 2 - 3.5$ for $\sqrt{s_{\rm NN}} = 200$ GeV. For lower energies, the $\Delta\eta$ window will be somewhat lower. These values are in confirmation with earlier published results [17–19]. The $\Delta\eta$ dependence of charge fluctuations may give information about the properties of the hot and dense medium created in heavy-ion collisions [19].

V. COMPARISON OF MODEL CALCULATIONS TO EXPERIMENTAL DATA

Net-charge fluctuations have been measured by experiments at CERN-SPS, RHIC, and LHC. Recently, the ALICE experiment published the net-charge fluctuations for Pb + Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV [12]. The results from the STAR experiment at RHIC energies had been published earlier [10]. The measured values of net-charge fluctuations are presented in Fig. 4, where both $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ and D are plotted as a function of center-of-mass energy for Pb + Pb collisions at LHC and Au + Au collisions at RHIC. The STAR results are measured for $\Delta\eta = 1.0$ and the ALICE results are shown for both $\Delta\eta = 1.0$ and 1.6. The values of dynamical net-charge fluctuations, $\nu_{+-,\rm dyn}$, remain negative at all cases, which

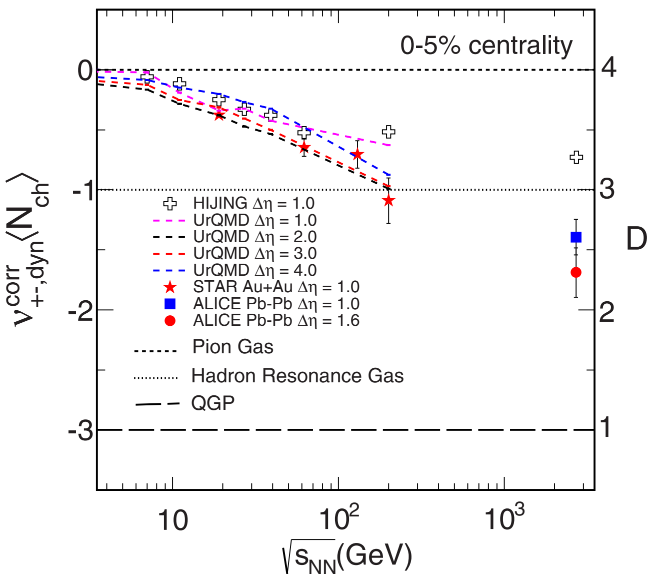

FIG. 4. (Color online) $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ (left axis) and corresponding values of D (right axis) as a function center-of-mass energy in Au + Au or Pb + Pb collisions from HIJING and UrQMD event generators for different $\Delta\eta$ windows. Estimations for fluctuations originating from pion gas, hadron resonance gas, and QGP are indicated.

implies the existence of a finite correlation between +ve and −ve particles. The fluctuations are observed to decrease as the center-of-mass energy increases.

We calculated the net-charge fluctuations for Au + Au collisions at $\sqrt{s_{\rm NN}} = 7.7$, 11.7, 19.0, 27, 39, 62, and 200 GeV, using two hadronic models, HIJING and UrQMD. The HIJING model is a perturbative-QCD-inspired model that contains jet and mini-jet formation mechanisms. On the other hand, UrQMD is a transport model that contains various resonance decays and elastic and inelastic interactions. The results are superimposed in Fig. 4. The HIJING calculations are performed at $\Delta\eta = 1.0$ for 0–5% central collisions. The UrQMD results are for time at 30 fm/c, and for a set of values at $\Delta\eta$ from 1.0 to 4.0. The values of D from these model calculations are within the pion gas and hadron resonance gas limits.

The net-charge fluctuations obtained from experimental measurements at RHIC energies for $\Delta\eta = 1.0$ are within the pion gas and hadron resonance gas (HRG) limits. Extending the $\Delta\eta$ range will be advantageous for better understanding the fluctuations. At $\sqrt{s_{\rm NN}} = 2.76$ TeV corresponding to the LHC energy, the result for $\Delta\eta = 1.0$ for the central collision is below the HRG limit. At $\Delta\eta = 1.6$, the fluctuations further decrease. The values of D being within the HRG limit and QGP imply that at the LHC energy the fluctuations have their origin in the QGP phase.

VI. SUMMARY

We study the dynamical charge fluctuations at different time steps using the UrQMD model for Au + Au collisions. The positively and negatively charged particle multiplicity distributions, at $\Delta\eta = 1.0$ for central collisions, change with time. It is found that contributions at different time steps for protons and antiprotons are less as compared to those of the pions and kaons. Dynamical fluctuations are studied using $\nu^{\rm corr}_{+-,\rm dyn}$, corrected for global charge fluctuations. The net-charge fluctuations, expressed in terms of D and $\langle N_{\rm ch}\rangle \nu^{\rm corr}_{+-,\rm dyn}$ are studied for a range of $\Delta\eta$, from a narrow window of 0.2 to the maximum of 10.0 for Au + Au collisions at $\sqrt{s_{\rm NN}} =$ 200 GeV. One of the major goals of the present study is to find an optimum $\Delta\eta$ window for which the maximum amount of charge fluctuations, originating from the early stages of the collision, can be captured. We find that with increasing $\Delta\eta$ window, the value of the fluctuations increase, indicating final state effects, such as resonance decay and rescattering. The value of D does not grow any more beyond $\Delta\eta = 2.0$. On the other hand, D remains constant till $\Delta\eta = 3.5$, and then decreases close to zero for $\Delta\eta = 10.0$. This observation confirms the charge conservation scenario. From this study, we can conclude that the optimum value of charge fluctuations are captured for $\Delta\eta = 2.0$–3.5.

The charge fluctuations, obtained from HIJING and UrQMD models, are compared to the experimental data at RHIC and LHC energies. It is observed that a value of $\Delta\eta$ around 2.0 is ideal at all energies for studying charge fluctuations. As expected, the results from the model calculations remain within the limit of pion gas and hadron resonance gas values for all energies.

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[1] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998).
[2] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Phys. Rev. D **60**, 114028 (1999).
[3] S. Jeon and V. Koch, Phys. Rev. Lett. **85**, 2076 (2000).
[4] S. Jeon and V. Koch, Phys. Rev. Lett. **83**, 5435 (1999).
[5] M. Asakawa, U. Heinz, and B. Mller, Phys. Rev. Lett. **85**, 2072 (2000).
[6] M. Bleicher, S. Jeon, and V. Koch, Phys. Rev. C **62**, 061902 (2000).
[7] C. Athanasiou, K. Rajagopal, and M. Stephanov, Phys. Rev. D **82**, 074008 (2010).
[8] J. Adams *et al.* (STAR Collaboration), Nucl. Phys. A **757**, 102 (2005).
[9] K. Aamodt *et al.* (ALICE Collaboration), J. Instrum. **3**, S08002 (2008).

[10] B. I. Abelev *et al.* (STAR Collaboration), Phys. Rev. C **79**, 024906 (2009).
[11] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **68**, 044905 (2003).
[12] B. Abelev *et al.* (ALICE Collaboration), Phys. Rev. Lett. **110**, 152301 (2013).
[13] L. Adamczyk *et al.* (STAR Collaboration), Phys. Rev. Lett. **113**, 092301 (2014).
[14] L. Adamczyk *et al.* (STAR Collaboration), Phys. Rev. Lett. **112**, 032302 (2014).
[15] N. R. Sahoo (for the STAR Collaboration), J. Phys. Conf. Ser. **535**, 012007 (2014), arXiv:1407.1554.
[16] B. Mohanty, J. Alam, and T. K. Nayak, Phys. Rev. C **67**, 024904 (2003).
[17] E. V. Shuryak and M. A. Stephanov, Phys. Rev. C **63**, 064903 (2001).
[18] M. Abdel Aziz and S. Gavin, Phys. Rev. C **70**, 034905 (2004).
[19] M. Sakaida, M. Asakawa, and M. Kitazawa, Phys. Rev. C **90**, 064911 (2014).
[20] M. Prakash, R. Rapp, J. Wambach, and I. Zahed, Phys. Rev. C **65**, 034906 (2002).
[21] S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998).
[22] M. Bleicher *et al.*, J. Phys. G **25**, 1859 (1999); S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 225 (1998).
[23] M. Gyulassy and X. N. Wang, Comput. Phys. Commun. **83**, 307 (1994); X. N. Wang and M. Gyulassy, Phys. Rev. D **44**, 3501 (1991).
[24] S. Jeon and V. Koch, in *Quark-Gluon Plasma 3*, edited by R. C. Hwa and X. N. Wang (World Scientific, Singapore, 2004), p. 430.
[25] Q. H. Zhang, V. Topor Pop, S. Jeon, and C. Gale, Phys. Rev. C **66**, 014909 (2002).
[26] L. Shi and S. Jeon, Phys. Rev. C **72**, 034904 (2005).
[27] C. Pruneau, S. Gavin, and S. Voloshin, Phys. Rev. C **66**, 044904 (2002).