Structure of ^{11}Li(g.s.) and an excited $3/2^-$ state

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(Received 28 March 2014; revised manuscript received 18 November 2014; published 12 January 2015)

New measurements of reaction cross sections for scattering of ^{11}Li from H and C have provided a value for the radius of the neutron density distribution. By using the sensitivity of calculated radii to neutron configuration, I find that the s^2 fraction required to reproduce the new radius is $P(s^2) = 0.33^{+0.03}_{-0.05}$, in agreement with an earlier estimate of 0.33(6). Calculations of relative cross sections for the reaction ^{9}Li(t,p) (in reverse kinematics) suggest a method to observe an expected excited $3/2^-$ state and to independently determine $P(s^2)$.

DOI: 10.1103/PhysRevC.91.017303 PACS number(s): 21.10.Gv, 21.60.Cs, 27.20.+n

I. Introduction. The magnitude of the $(sd)^2$ component in the ground state (g.s.) of ^{11}Li is still an open question. Knowledge of this quantity is important for calculating several properties of ^{11}Li. It is especially crucial in computing the energy of the mirror nucleus ^{11}O [1]. It might seem reasonable that the majority of the ^{11}Li(g.s.) wave function is just a $p_{3/2}$ proton hole in ^{12}Be(g.s.) for which the $(sd)^2$ component is about 68% [2,3]. Yet, estimates for this quantity in ^{11}Li range from 0 to 1 with several in the range of 0.2–0.5 (Refs. [4–10] and references therein).

The customary description of ^{11}Li uses a two-component model in which the g.s. is a linear combination of two basis states. One of the basis states is the normal p-shell g.s. of ^{11}Li. In a proper treatment, it is not merely two $p_{1/2}$ neutrons coupled to the g.s. of ^{9}Li. Antisymmetrization within the p shell requires also the presence of both $p_{1/2}$ and $p_{3/2}$ neutrons coupled to excited-core states that have $J^\pi_{\rm core} = 1/2^-$ to $7/2^-$. The other basis state is usually taken to be two sd-shell neutrons coupled to the ^{11}Li g.s. Because the last two neutrons are in a different major shell, antisymmetry does not require the use of excited ^{9}Li cores. Such cores are probably present in this second basis state to some extent, but their contribution is expected to be small, and they are usually neglected.

Sherr and I had earlier used the sensitivity of a calculation of matter radius to the assumed configuration to estimate the s^2 fraction $P(s^2)$ in ^{11}Li(g.s.) [10]. A simple expression has long been used [11–16] to relate the square of the matter radius $R_{\rm m}^2$ of a neutron-rich nucleus to that of a core $R_{\rm c}^2$ and $R_{\rm v}^2$, the expectation value of r^2 computed with the wave function of the last neutron, assumed to be a single-particle (sp) neutron radial wave function calculated with a Woods-Saxon potential well that has $r_0,a = 1.25,0.65$ fm. The well depth was adjusted to reproduce the separation energy. We used the $2n$ procedure in which the matter radius is computed from the expression,

$$R_{\rm m}^2 = \left[(A-2)/A\right]\left(R_{\rm c}^2 + 2R_{\rm v}^2/A\right).$$

This equation contains the standard center-of-mass correction [11,12].

This $2n$ procedure was previously applied to the nuclei of ^{6,8}He, ^{9}Be, ^{11}Li, ^{14}Be, ^{17}B, and ^{22}C in Ref. [14] and to ^{14}Be, ^{17,19}B, and ^{11}Li in Ref. [10]. It was later proposed by Bhagwat *et al.* [16] for use in a much more sophisticated model. The $2n$ procedure and the assumption of $B_{\rm n} = B_{2\rm n}/2$ has become a common feature of work in this field [12,16–21]. To make a 0^+ state, the last two neutrons must be identical. So, to have them share the binding energy equally is reasonable. The $2n$ equation is identical to that of Ref. [16] but slightly different from that of Ref. [19]. It is a special case of the generalized expression in Ref. [12]. The $2n$ wave function is a sum of product wave functions.

Three types of $2n$ correlations are included, implicitly or explicitly, in these calculations. Correlations that arise from configuration mixing are included because I use a configuration-mixed wave function. Any additional spatial correlations that might change the energy of the state are included because I use experimental separation energies. Finally, because the last two neutrons have $J^\pi = 0^+$, they must both occupy the same nlj orbital in each component of the wave function. Additional correlations of other possible types are not included.

In that analysis [10], one very large value of $R_{\rm m}$ [22] was excluded from our weighted average. With a weighted average of $R_{\rm m} = 3.41(8)$fm from three other values [23–25], we deduced a value of $P(s^2) = 0.33(6)$ [10]. This result assumed the remainder of the wave function was p shell, but inclusion of the d^2 configuration (with the d^2/s^2 ratio as in ^{12}Be [2]) gave a very similar number 0.34. Because these two different values of the d^2/s^2 ratio gave virtually identical values of $P(s^2)$, it follows that any ratio between these two will also give the same $P(s^2)$. A recent estimate [26] of $P(d^2) = 0.11(2)$ [when combined with the above value of $P(s^2)$] is approximately equal to my d^2/s^2 ratio in ^{12}Be [2].

This value of $P(s^2) = 0.33$ in ^{11}Li is surprisingly small, especially considering the situation in ^{12}Be. Furthermore, a potential-model prediction [1] for the mass of ^{11}O(g.s.) agrees with estimates [27] from the isobaric multiplet mass equation (IMME) only for $P(s^2) > 0.59$. In an experiment that involved angular correlations after ^{11}Li breakup, Simon *et al.* [28] concluded a value of $P(s^2) = 0.45(10)$—in between the two possibilities but consistent with the value of 0.33(6) deduced in Ref. [10] to better than one standard error.

A two-component model of the ^{11}Li g.s. requires the presence of an excited $3/2^-$ state whose wave function is the orthogonal linear combination of the same two components. Here, I discuss the g.s. and expectations for the presently unknown excited $3/2^-$ state, and I suggest a possible experiment to independently extract the wave-function components.

TABLE I. Radii (fm) for ^{9}Li and ^{11}Li.

Quantity	^{9}Li	^{11}Li
R_m	2.32(2)[a]	$3.34^{+0.04}_{-0.08}$[b]
R_n	2.43(3)[b]	$3.68^{+0.07}_{-0.10}$[b]

[a]Reference [30].
[b]Reference [29].

II. Ground state. A recent experiment [29] measured reaction cross sections for scattering of ^{11}Li from targets of H and C at energies near 31 and 41 MeV/nucleon. With a Glauber model in the optical limit, they extracted density functions for neutron, matter, and proton distributions. They assumed simple algebraic forms with adjustable parameters. For neutrons (and hence also for matter), they used a two-component density with a Gaussian shape in the interior and a Yukawa tail. They introduced a variable cutoff radius for the point at which the two components intersect. They also extracted root-mean-square (rms) radii for neutron, matter, and proton distributions. Relevant values [29,30] are listed in Table I.

Earlier (as stated above), we treated ^{11}Li as a $2n$ halo nucleus and computed matter radii for various sp configurations. For a given wave function, our procedure is exact if the density of ^{11}Li is considered to be the sum of the density of ^{9}Li and the density of the valence neutrons. Our only parameter was $P(s^2)$, the fraction of the s^2 component in ^{11}Li(g.s.). Our weighted average of several reported matter radii was 3.41(8) fm. With this average, we concluded that the g.s. configuration corresponded to $P(s^2) = 0.33(6)$ [10].

The new value of R_m for ^{11}Li is $3.34^{+0.04}_{-0.08}$ fm [29]. With this value and our procedure, I find $P(s^2) = 0.29^{+0.02}_{-0.04}$, not very different from our earlier estimate but slightly smaller. (In the new calculations, I am using 2.32 fm [30], rather than 2.30 as in Ref. [10] for the ^{9}Li core radius.)

Moriguchi *et al.* [29] also provided a value for the rms radius of the neutron distribution in ^{11}Li: $R_n = 3.68^{+0.07}_{-0.10}$ fm. By following the prescription for matter radii, the corresponding equation for R_n is

$$R_n^2(11) = (6/8)\left[R_n^2(9) + 2R_v^2/8\right].$$

Of course, the radius of the valence neutron R_v is the same in the expressions for R_m and R_n. Because the weighting factors in R_n and R_m are different, this procedure for R_n will (in general) provide a slightly different value of $P(s^2)$ than that deduced from the matter radius. With the R_n mentioned above, the resulting $P(s^2)$ is $0.33^{+0.03}_{-0.05}$. These two new estimates of $P(s^2)$ are compared with our previous value in Table II and Fig. 1 where I also have plotted the weighted average of the three. The overall agreement is apparent.

Moriguchi *et al.* [29] state that their results suggest that the ^{9}Li core in ^{11}Li is excited. They could simply mean only that ^{9}Li is not a filled $p_{3/2}$ neutron orbital with $p_{1/2}$ empty. Of course, in a realistic p-shell calculation for ^{11}Li(g.s.), antisymmetry requires that several ^{9}Li core states should be included, which have $J^\pi = 1/2^-$ to $7/2^-$. This is usually not performed in treatments of ^{11}Li. Such additional core states are not required for the s^2 component if ^{9}Li has no $1s$ occupancy.

TABLE II. The s^2 fraction in ^{11}Li(g.s.) needed to reproduce various radii.

Label	To fit	$P(s^2)$
1	R_m (previous)[a]	0.33(6)[a]
2	R_m (new)[b]	$0.29^{+0.02}_{-0.04}$[c]
3	R_n (new)[b]	$0.33^{+0.03}_{-0.05}$[c]
4		$0.31^{+0.02}_{-0.03}$[c,d]

[a]Reference [10]. $R_m = 3.41(8)$ fm.
[b]Reference [29] and Table I.
[c]Present paper.
[d]Weighted average of the three values above.

Reference [29] also mentions the possibility of excitation of a $0s$ pair into the $0p$ shell.

III. Excited $3/2^-$ state. As stated in the Introduction, in a two-state model, the g.s. of ^{11}Li and an excited $3/2^-$ state will be orthonormal linear combinations of the same two basis states. At present, this excited $3/2^-$ state is unknown. If the matrix element that mixes these two $3/2^-$ states is similar to the value that is responsible for mixing the first two 0^+ states in ^{12}Be, then the excitation energy of the excited $3/2^-$ state in ^{11}Li should be in the range of 2.5–2.8 MeV for the region of $P(s^2)$ discussed within. Here, I suggest an experiment to find it and to independently determine the s^2 components of the two states.

In the (t,p) reaction in this mass region, the calculated cross section depends sensitively on the orbitals into which the two neutrons are transferred. The $2n$ transfer amplitude is a coherent sum of the amplitudes for each component in the wave function. For example, transfer into the $1s_{1/2}$ orbital produces a cross section [31] that is four to five times that for transfer into the $0p$ shell. In a two-state model, the $2n$ transfer amplitude will be constructive for the g.s. and destructive for the orthogonal excited state. This effect is so pronounced that in some cases the excited state can be very weak. For example,

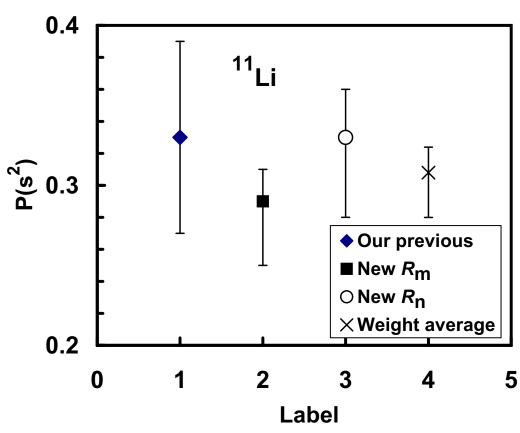

FIG. 1. (Color online) The fraction of the s^2 configuration in ^{11}Li(g.s.) required to reproduce various radii: filled symbols from matter radius—diamond (previous) and square (new); open symbol from R_n and x for weighted average. Labels correspond to those in Table II.

in the ^{10}Be(t,p) reaction [31], the excited 0^+ state was too weak to be observed. On the other hand, in the ^{9}Be(t,p) ^{11}Be reaction [32], the second $3/2^-$ state was stronger than the first one by more than a factor of 2—a result which indicates small mixing [33] in that case.

Two-neutron transfer reactions of the type (p,t) [34,35] and (t,p) [36] have recently been performed (in reverse kinematics) for β-unstable nuclei. Here, I consider the reaction ^{9}Li(t,p) that would lead to the g.s. and excited $3/2^-$ state. Correlations are included to the extent mentioned in the Introduction. The dimensionless ratio of the cross sections exc./g.s. (where exc. refers to the excited $3/2^-$ state) is found to depend on $P(s^2)$ as depicted by the solid curve in Fig. 2. These are forward-angle ratios, calculated by assuming simultaneous $2n$ transfer. I expect the ratio to be the same for any combination of sequential and simultaneous transfer [37–39]. The solid vertical line just above $P = 0.3$ represents the value of $0.31^{+0.02}_{-0.03}$ extracted from the analysis above. The dashed vertical line near 0.59 is at the lower limit of P for which potential-model [1] and IMME [27] predictions for the ^{11}O(g.s.) mass agree. Results of such a reaction should provide a clear distinction between the two possibilities. If the present value from the analysis above is even approximately correct, the excited state should be about half as strong as the g.s. However, if the value is in the range required by consistency of potential-model and IMME predictions of the ^{11}O(g.s.) mass, then the excited state will probably be too weak to observe. I note in passing that a value in the latter region would be consistent with the result ($P \sim 0.68$) of weakly coupling a $p_{3/2}$ proton hole to the g.s. of ^{12}Be. In the ^{11}Li(p,t) ^{9}Li(g.s.) reaction, Tanihata *et al.* [34] found that values of $P(s^2) = 0.31$ or 0.45 gave better agreement with their data than did $P(s^2) = 0.03$.

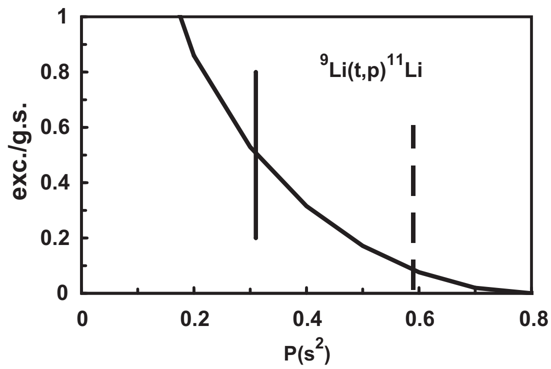

FIG. 2. For the reaction ^{9}Li(t,p) ^{11}Li, the expected cross-section ratio for the (unknown) excited $3/2^-$ state to the g.s. is plotted vs $P(s^2)$, the fractional s^2 intensity in ^{11}Li(g.s.). The solid vertical line just above $P = 0.3$ represents the value of $0.31^{+0.02}_{-0.03}$ extracted from the analysis herein. The dashed vertical line near 0.59 is at the lower limit of P for which potential-model [1] and IMME [27] predictions for the ^{11}O(g.s.) mass agree.

IV. Conclusions. To summarize, with new measurements [29] of reaction cross sections for ^{11}Li that provide new values of radii for matter and neutron distributions, the amount of the s^2 configuration needed to reproduce these radii is only about 31%. It is perhaps ironic that ^{11}Li, which initially became famous because of its large matter radius [23], seems to require such a small s^2 component to reproduce this radius. When combined with recent calculations [1] of the ^{11}O mass, the present result for $P(s^2)$ reinforces the prediction that ^{11}O is unbound to ^{9}C + $2p$ by about 4.49(11) MeV. Calculations of relative cross sections for the reaction ^{9}Li(t,p) (in reverse kinematics) suggest a method to observe the expected excited $3/2^-$ state and to independently determine $P(s^2)$.

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