Parametrization of the statistical rate function for select superallowed transitions

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(Received 1 December 2014; published 26 January 2015)

We present a parametrization of the statistical rate function, f, for 20 superallowed $0^+ \to 0^+$ nuclear β transitions between $T = 1$ analog states, and for 18 superallowed "mirror" transitions between analog $T = 1/2$ states. All these transitions are of interest in the determination of V_{ud}. Although most of the transition Q_{EC} values have been measured, their precision will undoubtedly be improved in the future. Our parametrization allows a user to easily calculate the corresponding new f value to high precision ($\pm 0.01\%$) without complicated computing.

DOI: 10.1103/PhysRevC.91.015501 PACS number(s): 23.40.−s

I. INTRODUCTION

Precise measurements of nuclear β decay provide a valuable window into the electroweak standard model. In particular, superallowed $0^+ \to 0^+$ transitions between $T = 1$ analog states are used to set a limit on the presence of scalar interactions and to determine V_{ud}, the upper left element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and a key contributor to the most demanding available test of the unitarity of that matrix. While these transitions currently lead to the most precise determination of V_{ud}, mirror transitions between $T = 1/2$ analog states are becoming of interest as a means of confirming V_{ud} via a different experimental approach. To be useful, not only must the Q_{EC} value for each of these transitions be measured very precisely but the statistical rate function, f, which uses the Q_{EC} as input, must be calculated with equivalent precision.

Because there is no widely available means for calculating f to the required level of precision, we have devised a simple parametrization that reproduces the results of our full code for energies spanning a small range around the currently known Q_{EC} values for both types of superallowed transition. Together, these should provide a convenient resource for experimentalists to use in future to obtain high-precision f values from improved Q_{EC}-value measurements for these transitions.

Our goal in what follows is to parametrize f and present tables of the parameters for the two sets of transitions: (1) the 20 superallowed $0^+ \to 0^+$ nuclear β transitions between $T = 1$ analog states, whose properties have been surveyed in Refs. [1,2]; and (2) the 18 superallowed "mirror" β transitions between the analog $T = 1/2$ states surveyed in Ref. [3]. For each transition, we have computed f for 100 values of Q_{EC} taken over a range of ± 60 keV around the transition Q_{EC} value[1] and fitted these results to determine the coefficients in our parametrization. Our aim in fitting these 100 values is to achieve an accuracy of 0.01%, nearly a factor of 10 more precise than is currently required.

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[1]For ^{70}Br the Q_{EC} value is less precisely known, so the Q_{EC} value range for fitting was extended to ± 600 keV.

II. PARAMETRIZATION OF THE STATISTICAL RATE FUNCTION

To achieve 0.01% accuracy, the electron wave function must be determined with great precision. In our detailed evaluation of f [4], we accomplished this by solving the Dirac equation for the emerging electron moving in the Coulomb field of the nuclear charge distribution. The full expression for the computation of f is

$$f = \xi R(W_0) \int_1^{W_0} pW(W_0 - W)^2 F(Z,W) f_1(W) \times Q(Z,W) r(Z,W) dW, \quad (1)$$

where W is the electron total energy in electron rest-mass units, W_0 is the maximum value of W, $p = (W^2 - 1)^{1/2}$ is the electron momentum, Z is the charge number of the daughter nucleus (positive for electron emission, negative for positron emission), $F(Z,W)$ is the Fermi function, and $f_1(W)$ is the shape-correction function as defined by Holstein [5] (but with kinematic recoil corrections omitted).

Further, $Q(Z,W)$ is a screening correction for which we use the analytic prescription of Rose [6] (see Eq. (A44) in Ref. [4]), and $r(Z,W)$ is an atomic overlap correction described in Ref. [1]. The kinematic recoil corrections that Holstein includes in $f_1(W)$ are here written as $R(W_0)$. The expression for $R(W_0)$ is derived in the Appendix, with the result that

$$R(W_0) \simeq 1 - \frac{3W_0}{2M_A}, \quad (2)$$

where M_A is the average of the initial and final nuclear masses expressed in electron-mass units. Last, for allowed transitions it is customary to remove the leading nuclear matrix element from the definition of f. Thus we have introduced ξ in Eq. (1), where $\xi = 1/|\mathcal{M}_F|^2$ for superallowed Fermi transitions, $\mathcal{M}_F$ being the Fermi matrix element. For mixed Fermi and Gamow-Teller transitions, $\xi = 1/[\mathcal{M}_F^2 + g_A^2 \mathcal{M}_{GT}^2]$ with $\mathcal{M}_{GT}$ being the Gamow-Teller matrix element and g_A the axial-vector coupling constant.

TABLE I. Values of the coefficients a_0 and a_1 that yield the statistical rate function f_0 from Eq. (7), and coefficients b_0, b_1, b_2, and b_3 that yield the correction δ_S from Eq. (12). Coefficients a_2 and a_3 are held fixed at the values $a_2 = -2/15$ and $a_3 = 1/4$. The cases shown are the superallowed Fermi transitions between $T = 1$, $J^\pi = 0^+$ analog states surveyed in Refs. [1,2].

Parent nucleus	a_0	a_1	b_0(%)	b_1(%)	b_2(%)	b_3(%)
^{10}C	0.029 7225	– 0.143 1540	0.011 78	0.020 06	0.052 03	– 0.000 96
^{14}O	0.028 5463	– 0.141 7222	0.031 76	0.031 23	0.065 06	– 0.001 01
^{18}Ne	0.027 4005	– 0.139 8743	0.047 50	0.049 95	0.089 45	– 0.001 34
^{22}Mg	0.026 3237	– 0.137 4785	0.070 36	0.063 93	0.107 96	– 0.001 39
^{26}Si	0.025 3385	– 0.136 9946	0.103 06	0.078 27	0.128 56	– 0.001 46
^{30}S	0.024 2904	– 0.127 1838	0.149 05	0.095 29	0.154 13	– 0.001 61
^{34}Ar	0.023 3252	– 0.118 2701	0.153 36	0.118 96	0.184 78	– 0.001 84
^{38}Ca	0.022 3867	– 0.101 8182	0.173 01	0.135 58	0.206 74	– 0.001 86
^{42}Ti	0.021 6593	– 0.110 5386	0.156 25	0.152 93	0.233 80	– 0.001 96
^{26m}Al	0.025 7927	– 0.135 5697	0.098 13	0.072 08	0.120 37	– 0.001 49
^{34}Cl	0.023 8533	– 0.128 1700	0.167 59	0.105 98	0.169 11	– 0.001 73
^{38m}K	0.022 8360	– 0.109 0747	0.176 30	0.124 80	0.193 25	– 0.001 80
^{42}Sc	0.022 0302	– 0.108 2192	0.173 35	0.142 65	0.221 47	– 0.001 93
^{46}V	0.021 1437	– 0.089 4977	0.202 00	0.165 56	0.252 13	– 0.002 08
^{50}Mn	0.020 2722	– 0.059 7791	0.252 81	0.183 30	0.278 34	– 0.002 14
^{54}Co	0.019 5698	– 0.052 4836	0.328 12	0.197 57	0.303 37	– 0.002 17
^{62}Ga	0.018 1322	– 0.014 1676	0.493 42	0.244 18	0.369 15	– 0.002 43
^{66}As	0.017 3202	0.047 3840	0.572 18	0.272 60	0.407 70	– 0.002 61
^{70}Br	0.016 7829	0.041 7070	0.566 71	0.296 70	0.436 02	– 0.002 65
^{74}Rb	0.016 2385	0.044 6304	0.446 43	0.321 80	0.464 90	– 0.002 69

In order to parametrize f, it is convenient to factor it into two contributions:

$$f = f_0(1 + \delta_S), \quad (3)$$

$$f_0 = \int_1^{W_0} pW(W_0 - W)^2 F(Z,W)Q(Z,W)r(Z,W)dW, \quad (4)$$

$$\delta_S = (f - f_0)/f_0. \quad (5)$$

The purpose of this factorization is to place the role of the shape-correction function $f_1(W)$ entirely within the correction term δ_S, which is typically of the order of a few percent. The shape-correction function depends on nuclear matrix elements and differs for Fermi and Gamow-Teller transitions. This piece of the calculation is somewhat less certain since it is nuclear-structure dependent; however, being small, its accuracy is also less critical.

In the limit that $F(Z,W)Q(Z,W)r(Z,W) \to 1$, which occurs when $Z = 0$, the integral f_0 has an analytic value:

$$f_0(Z=0) = \tfrac{1}{30}W_0^4 p_0 - \tfrac{3}{20}W_0^2 p_0 - \tfrac{2}{15}p_0 + \tfrac{1}{4}W_0 \ln(W_0 + p_0), \quad (6)$$

with $p_0 = (W_0^2 - 1)^{1/2}$. This suggests a fitting function of the form

$$f_0 = a_0 W_0^4 p_0 + a_1 W_0^2 p_0 + a_2 p_0 + a_3 W_0 \ln(W_0 + p_0). \quad (7)$$

In fitting 100 values of f_0, we found that the four parameters a_0, a_1, a_2, and a_3 could not be uniquely determined with precision. Thus it was decided to fix a_2 and a_3 to their $Z = 0$ values, namely $a_2 = -2/15$ and $a_3 = 1/4$, and use the fitting process to determine a_0 and a_1. This procedure yielded the required precision for f_0. The resultant values of a_0 and a_1 are given in Table I for the $0^+ \to 0^+$ transitions, and in Table II for the $T = 1/2$ mirror transitions.

We note that in Eq. (7) each successive term gives a smaller and smaller contribution to the total. Thus in deciding to fix a_2 and a_3 to their $Z = 0$ values, we have fixed the coefficients for the two smallest terms in the expression for f_0. This parametrization is not unique however. We could equally well have chosen to hold two different coefficients to their $Z = 0$ values and consequently have obtained another parametrization that would also produce f_0 values accurate to within 0.01%.

For the correction δ_S we start with the approximate expression

$$f_0\delta_S \simeq \int_1^{W_0} pW(W_0 - W)^2 F(Z,W) \times \left[\xi f_1(W) - 1 - \frac{3W_0}{2M_A}\right] dW \quad (8)$$

and write

$$\xi f_1(W) - 1 = B_0 + B_1 W + B_2/W + B_3 W^2. \quad (9)$$

The coefficients B_0, B_1, B_2, and B_3 are different for Fermi and Gamow-Teller transitions. This choice of parametrization is guided by the early work of Schopper [7] who used such a parametrization for the shape-correction function.

TABLE II. Values of the coefficients a_0 and a_1 that yield the statistical rate function f_0 from Eq. (7), and coefficients b_0, b_1, b_2, and b_3 that yield the correction δ_S from Eq. (12). For each case, the b coefficients in the first row correspond to the Fermi component, b_0^F etc., and those in the second row correspond to the Gamow-Teller component, b_0^{GT} etc. Coefficients a_2 and a_3 are held fixed at values $a_2 = -2/15$ and $a_3 = 1/4$. The cases shown are the mixed Fermi and Gamow-Teller transitions between mirror $T = 1/2$ states in odd-mass nuclei surveyed in Ref. [3].

Parent nucleus	a_0	a_1	b_0(%)	b_1(%)	b_2(%)	b_3(%)
^{11}C	0.029 7280	− 0.143 1964	0.012 27	0.020 03	0.050 30	− 0.000 93
			0.715 78	0.076 95	0.564 95	− 0.000 55
^{13}N	0.029 1054	− 0.142 0911	0.021 66	0.025 66	0.056 40	− 0.000 95
			0.319 15	0.040 52	0.364 91	− 0.000 26
^{15}N	0.028 5370	− 0.141 6159	0.045 52	0.033 30	0.068 46	− 0.001 17
			0.250 27	0.029 49	0.312 61	− 0.000 24
^{17}F	0.027 9301	− 0.140 0527	0.025 28	0.041 32	0.074 82	− 0.001 13
			1.479 63	0.065 56	0.676 35	− 0.000 74
^{19}Ne	0.027 3984	− 0.139 8186	0.055 21	0.049 98	0.089 14	− 0.001 35
			1.342 70	0.058 76	0.550 39	− 0.000 99
^{21}Na	0.026 8709	− 0.139 4429	0.059 22	0.058 59	0.099 92	− 0.001 42
			1.733 65	0.063 13	0.640 05	− 0.001 01
^{23}Mg	0.026 3324	− 0.137 8844	0.063 75	0.064 36	0.107 31	− 0.001 38
			1.913 34	0.063 48	0.646 97	− 0.000 93
^{25}Al	0.025 8123	− 0.136 4357	0.081 54	0.071 60	0.118 49	− 0.001 44
			2.347 66	0.072 37	0.737 37	− 0.000 90
^{27}Si	0.025 2815	− 0.133 0446	0.093 98	0.079 80	0.129 45	− 0.001 49
			2.751 64	0.071 92	0.790 82	− 0.000 86
^{29}P	0.024 7788	− 0.130 1483	0.111 10	0.087 17	0.139 82	− 0.001 51
			2.410 55	0.068 57	0.661 34	− 0.000 86
^{31}S	0.024 3506	− 0.132 8386	0.145 54	0.096 12	0.154 60	− 0.001 63
			2.174 79	0.077 99	0.608 83	− 0.000 92
^{33}Cl	0.023 9077	− 0.133 3333	0.152 09	0.106 27	0.167 72	− 0.001 70
			− 0.780 12	0.063 97	0.146 81	− 0.000 50
^{35}Ar	0.023 3631	− 0.122 6819	0.196 58	0.116 36	0.184 53	− 0.001 83
			− 0.536 03	0.070 47	0.195 15	− 0.000 55
^{37}K	0.022 9710	− 0.125 1564	0.183 69	0.124 91	0.193 42	− 0.001 80
			0.892 86	0.090 10	0.411 92	− 0.000 84
^{39}Ca	0.022 4606	− 0.112 3165	0.217 79	0.132 59	0.206 53	− 0.001 85
			0.582 46	0.097 57	0.373 16	− 0.000 87
^{41}Sc	0.022 0044	− 0.104 6436	0.209 89	0.140 01	0.221 65	− 0.001 93
			3.945 90	0.126 35	0.915 61	− 0.001 24
^{43}Ti	0.021 6749	− 0.113 4605	0.158 01	0.156 50	0.236 44	− 0.002 00
			3.546 35	0.132 77	0.826 30	− 0.001 35
^{45}V	0.021 1420	− 0.091 0683	0.314 18	0.162 34	0.257 79	− 0.002 18
			4.544 43	0.144 48	0.992 22	− 0.001 34

A. Superallowed $0^+ \rightarrow 0^+$ Fermi transitions

For Fermi (vector) transitions,

$$\begin{aligned} B_0^F &= -\tfrac{1}{5}(W_0R)^2 + \tfrac{1}{15}R^2 - \tfrac{6}{35}(\alpha Z)(W_0R) + \tfrac{61}{630}(\alpha Z)^2, \\ B_1^F &= \tfrac{4}{15}(W_0R)R - \tfrac{48}{35}(\alpha Z)R, \\ B_2^F &= \tfrac{2}{15}(W_0R)R - \tfrac{18}{35}(\alpha Z)R, \\ B_3^F &= -\tfrac{4}{15}R^2, \end{aligned} \tag{10}$$

where R is the radius of the nuclear charge distribution expressed in electron Compton wavelength units. We derived these equations from the work of Behrens and Bühring [8] who give algebraic expressions for the shape-correction function as expansions in the small quantities R and (αZ). Our Eqs. (10) and (14) below are correct to second order in these quantities, namely to order R^2, $(\alpha Z)^2$, and $(\alpha Z)R$. Inserting Eqs. (9) and (10) into Eq. (8) we obtain

$$\delta_S \simeq B_0 + B_1\langle W\rangle + B_2\langle 1/W\rangle + B_3\langle W^2\rangle - \frac{3W_0}{2M_A}, \tag{11}$$

where $\langle W^n\rangle$ is the value of W^n averaged over the electron spectrum. Estimates of these quantities are:$\langle W\rangle = W_0/2$, $\langle W^{-1}\rangle = 5W_0^{-1}/2$ and $\langle W^2\rangle = 2W_0^2/7$.

This leads to our final choice of parametrization for the correction δ_S:

$$\delta_S = b_0 + b_1W_0 + b_2/W_0 + b_3W_0^2, \tag{12}$$

where approximate values of the coefficients are

$$b_0^F \simeq \tfrac{2}{5}R^2 + \tfrac{61}{630}(\alpha Z)^2, \quad b_1^F \simeq -\tfrac{6}{7}(\alpha Z)R - \tfrac{3}{2M_A},$$
$$b_2^F \simeq -\tfrac{9}{7}(\alpha Z)R, \quad b_3^F \simeq -\tfrac{1}{7}R^2. \tag{13}$$

We fitted the expression in Eq. (12) to the exactly computed value of δ_S from Eq. (5) to obtain the parameters b_0, b_1, b_2, and b_3. Again, it was found that all four parameters could not be uniquely determined with precision, so the coefficients b_2 and b_3 were fixed at the values given in Eq. (13) for b_2^F and b_3^F, and the fitting process was used to determine b_0 and b_1. Table I gives the values of the parameters b_0, b_1, b_2, and b_3 for the superallowed $0^+ \to 0^+$ Fermi transitions.

B. Mirror $T = 1/2$ transitions

For pure Gamow-Teller (axial-vector) transitions, coefficients in the expression for the shape-correction function in Eq. (9) are

$$\begin{aligned}
B_0^{GT} \simeq & -\tfrac{1}{5}(W_0R)^2 + \tfrac{11}{45}R^2\left(1 - \tfrac{2}{11}x\right) \\
& + \tfrac{2}{35}(\alpha Z)(W_0R)(1-x) + \tfrac{1}{3}(W_0R)[\mp 2\overline{b} + \overline{d}] \\
& + \tfrac{1}{3}\beta(\alpha Z)[\pm 2\overline{b} + \overline{d}] + \tfrac{61}{630}(\alpha Z)^2, \\
B_1^{GT} \simeq & \tfrac{4}{9}(W_0R)R\left(1 - \tfrac{1}{10}x\right) - \tfrac{8}{5}(\alpha Z)R\left(1 - \tfrac{1}{28}x\right) \pm \tfrac{4}{3}R\overline{b}, \\
B_2^{GT} \simeq & -\tfrac{2}{45}(W_0R)R(1-x) - \tfrac{18}{35}(\alpha Z)R - \tfrac{1}{3}R[\pm 2\overline{b} + \overline{d}], \\
B_3^{GT} \simeq & -\tfrac{4}{9}R^2\left(1 - \tfrac{1}{10}x\right),
\end{aligned} \tag{14}$$

where

$$x = -\sqrt{10}\mathcal{M}_{1y}/\mathcal{M}_{\sigma r^2}, \tag{15}$$

$$\overline{b} = \frac{1}{MR}\left[\frac{g_M}{g_A} + \frac{\mathcal{M}_L}{\mathcal{M}_{GT}}\right], \tag{16}$$

$$\overline{d} = \frac{1}{MR}\frac{\mathcal{M}_{\sigma L}}{\mathcal{M}_{GT}}, \tag{17}$$

and also $\beta \simeq 6/5$, $g_M = 4.706$, and M is the nucleon mass in electron rest-mass units. Where there is a $\pm$ symbol, the upper sign is used for electron emission beta decays, the lower sign for positron emitters. All the transitions discussed in this work are positron emitters, so the lower sign is consistently used. The nuclear matrix elements are defined in Eq. (68) of Ref. [5]. Schematically, they are written $\mathcal{M}_{GT} = \langle\sigma\rangle$, $\mathcal{M}_{\sigma r^2} = \langle r^2\sigma\rangle$, $\mathcal{M}_{1y} = (16\pi/5)^{1/2}\langle r^2[Y_2 \times \sigma]\rangle$, $\mathcal{M}_L = \langle L\rangle$, and $\mathcal{M}_{\sigma L} = \langle\sigma \times L\rangle$. Note that the matrix element $\mathcal{M}_{\sigma L}$, and hence $\overline{d}$, vanishes in diagonal matrix elements, as would occur in a mirror transition between isobaric analog states.

The correction δ_S is again parametrized as in Eq. (12) with approximate expressions for the coefficients derived from Eq. (14). For pure Gamow-Teller transitions they yield

$$\begin{aligned}
b_0^{GT} & \simeq \tfrac{2}{15}R^2 + \tfrac{1}{15}R^2x + \tfrac{1}{3}\beta(\alpha Z)[\pm 2\overline{b} + \overline{d}] + \tfrac{61}{630}(\alpha Z)^2, \\
b_1^{GT} & \simeq -\tfrac{26}{35}(\alpha Z)R - \tfrac{1}{35}(\alpha Z)Rx + \tfrac{1}{3}R\overline{d} - \tfrac{3}{2M_A}, \\
b_2^{GT} & \simeq -\tfrac{9}{7}(\alpha Z)R - \tfrac{5}{6}R[\pm 2\overline{b} + \overline{d}], \\
b_3^{GT} & \simeq -\tfrac{11}{105}R^2\left(1 + \tfrac{1}{11}x\right).
\end{aligned} \tag{18}$$

Again in fitting the exact values of δ_S from Eq. (5) with the expression in Eq. (12) we held the parameters b_2 and b_3 fixed at the values given for b_2^{GT} and b_3^{GT} in Eq. (18) and then obtained the parameters b_0 and b_1 from the fit.

The $T = 1/2$ mirror transitions are mixed transitions, with both Fermi and Gamow-Teller components. The fitted b coefficients for both the Fermi and Gamow-Teller components are given in Table II along with the a_0 and a_1 coefficients. In such mixed transitions the inverse of the partial lifetime is proportional to

$$t^{-1} \propto f_V\left[|\mathcal{M}_F|^2 + \frac{f_A}{f_V}|g_A\mathcal{M}_{GT}|^2\right], \tag{19}$$

where

$$f_V = f_0\left(1 + \delta_S^F\right), \quad f_A = f_0\left(1 + \delta_S^{GT}\right). \tag{20}$$

The statistical rate functions f_V and f_A are easily obtained from the parameters listed in Table II.

III. CONCLUSIONS

We have provided simple parametrizations of the statistical rate functions, f, for nuclear β transitions of current interest in determining V_{ud} and testing CKM unitarity. In most but not all cases, the transition Q_{EC} values have already been measured with $\sim$1-keV precision or better. In a few cases they are much less well known. In all cases, the Q_{EC} values will undoubtedly be remeasured, leading possibly to different values and certainly to reduced uncertainties. When this happens, experimenters will need f values of equivalent precision, and the parametrizations presented here will satisfy that need without complicated computing.

It is important to note that our parametrization is only valid for the transitions identified and only for a limited range of energies ($\pm$60 keV for all cases except for the decay of ^{70}Br which covers $\pm$600 keV) around the currently accepted Q_{EC} values for those transitions [2,3]. The coefficients of our parametrization should not be applied outside the range of energies specified or to any other transitions.

ACKNOWLEDGMENTS

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Grant No. DE-FG03-93ER40773, and by the Welch Foundation under Grant No. A-1397.

APPENDIX: KINEMATIC RECOIL CORRECTIONS

Let M_A be the average mass of the initial and final nuclei. Then the kinematic recoil corrections are of order W_0/M_A and, in all but the most precise work, they can generally be ignored. The recoil correction enters the calculation in two places: firstly, the end-point energy is slightly modified, a correction we denote Δf^a; and secondly, additional terms are added to the shape-correction function $f_1(W)$, providing a correction we call Δf^b.

For the first correction, if W_0 is the end-point energy without consideration of recoil and $W_0^{\rm corr}$ is the corrected value, then from Eq. (3) of Holstein [5] we get

$$W_0^{\rm corr} = W_0\left(1+\frac{1}{2W_0M_A}\right)\left(1+\frac{W_0}{2M_A}\right)^{-1} \simeq W_0\left(1-\frac{W_0}{2M_A}+\frac{1}{2W_0M_A}\right). \tag{A1}$$

So, since the statistical rate function is approximately proportional to W_0^5, the correction to f must be of order

$$\frac{\Delta f^a}{f} \simeq 1-\frac{5}{2}\frac{W_0}{M_A}+\frac{5}{2}\frac{1}{W_0M_A}. \tag{A2}$$

Unlike Δf^a, the recoil correction to the shape-correction function, Δf^b, is different for Fermi and Gamow-Teller transitions. The modifications are

$$f_1^{F,\rm corr}(W) = f_1^F(W)\left(1+2\frac{W}{M_A}\right),$$
$$f_1^{GT,\rm corr}(W) = f_1^{GT}(W)\left(1-\frac{2}{3}\frac{W_0}{M_A}+\frac{10}{3}\frac{W}{M_A}-\frac{2}{3}\frac{1}{M_AW}\right). \tag{A3}$$

If these corrections are integrated over the electron spectrum, they yield corrections to the statistical rate function of

$$\frac{\Delta f^{b,F}}{f} \simeq 1+\frac{W_0}{M_A},$$
$$\frac{\Delta f^{b,GT}}{f} \simeq 1-\frac{2}{3}\frac{W_0}{M_A}+\frac{5}{3}\frac{W_0}{M_A}-\frac{5}{3}\frac{1}{M_AW_0}. \tag{A4}$$

Finally, combining corrections Δf^a and Δf^b, we obtain the final recoil correction to the statistical rate function

$$\frac{\Delta f^F}{f} = 1-\frac{3}{2}\frac{W_0}{M_A}+\frac{5}{2}\frac{1}{W_0M_A} \simeq 1-\frac{3}{2}\frac{W_0}{M_A},$$
$$\frac{\Delta f^{GT}}{f} = 1-\frac{3}{2}\frac{W_0}{M_A}+\frac{5}{6}\frac{1}{W_0M_A} \simeq 1-\frac{3}{2}\frac{W_0}{M_A}. \tag{A5}$$

Thus, Fermi and Gamow-Teller transitions are subject to essentially the same correction and it is this correction that we have recorded in Eq. (2) and used in our fitting algorithms. Of course, the exactly computed f values, to which our parametrizations are fitted, include the complete kinematic recoil treatment.

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