Stability of magnetized strange quark matter in the MIT bag model with a density dependent bag pressure

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The stability of magnetized strange quark matter (MSQM) is studied in the MIT bag model with the density dependent bag pressure. In the consistent thermodynamic description of MSQM, the quark chemical potentials, the total thermodynamic potential, and the anisotropic pressure acquire the corresponding additional term proportional to the density derivative of the bag pressure. The model parameter space is determined for which MSQM is absolutely stable, i.e., its energy per baryon is less than that of the most stable ^{56}Fe nucleus under zero external pressure and vanishing temperature. It is shown that there exists a magnetic field strength $H_{u\ \max}$ at which the upper bound B_∞^u on the asymptotic bag pressure $B_\infty \equiv B(\varrho_B \gg \varrho_0)$ (ϱ_0 being the nuclear saturation density) from the absolute stability window vanishes. The value of this field, $H_{u\ \max} \sim (1\text{–}3) \times 10^{18}$ G, represents the upper bound on the magnetic field strength, which can be reached in a strongly magnetized strange quark star. It is clarified how the absolute stability window and upper bound on the magnetic field strength are affected by varying the parameters in the Gaussian parametrization for the density dependence of the bag pressure.

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I. INTRODUCTION AND BASIC EQUATIONS

After the conjecture that strange quark matter (SQM), composed of deconfined u, d, and s quarks, can be the ground state of matter [1–3], it became the subject of intense research. In the astrophysical context, this would mean that the formation of strange quark stars, made up entirely of SQM and self-bound by strong interactions, is possible [4–6]. The birth of a strange quark star can proceed via conversion of a neutron star as a strong deflagration process during a few milliseconds [7], accompanied by a powerful neutrino signal [8]. If SQM is metastable at zero external pressure, it can be encountered in the cores of heavy neutron stars where the density of about several times nuclear saturation density can be sufficient for the deconfinement phase transition to occur [9]. Such stars, composed of the quark core and hadronic crust, are called hybrid stars. Modern astrophysical observations, including data on the masses and radii, spin-down rates, cooling history, glitches, and superbursts, do not disprove the existence of quark matter in compact stars.

The other important feature is that compact stars can be endowed with a strong magnetic field [10]. Near the surface of magnetars—strongly magnetized neutron stars—the field strength can reach values of about 10^{14}–10^{15} G [11,12]. Even stronger magnetic fields up to 10^{19}–10^{20} G can potentially occur in the cores of neutron stars [13]. The large pulsar kick velocities due to the asymmetric neutrino emission in direct Urca processes in the dense core of a magnetized neutron star could be the possible imprint of such ultrastrong magnetic fields [14–17]. The origin of magnetar's strong magnetic fields is yet under discussion, and, among other possibilities, it is not excluded that this can be due to spontaneous ordering of nucleon [18,19] or quark [20] spins in the dense interior of a neutron star.

Strong magnetic fields can have significant impact on thermodynamic properties of cold dense matter [21–29]. In particular, the pressure anisotropy, exhibited in the difference between the pressures along and perpendicular to the magnetic field, becomes relevant for strongly magnetized matter [30–35]. In this study, I consider strongly magnetized SQM (MSQM) taking into account the effects of the pressure anisotropy. I aim at finding the model parameter space for which MSQM is absolutely stable, i.e., its energy per baryon is less than that of the most stable nucleus ^{56}Fe under zero external pressure and vanishing temperature. For the parameters from this absolute stability window, the formation of a strongly magnetized strange quark star is possible.

Note that in order to describe the confinement property of quantum chromodynamics, in the conventional MIT bag model [36] this is achieved by introducing the density independent bag pressure by which quarks are confined in a finite region of space called a "bag." The standard thermodynamic equations can be used to study quarks confined to a bag. Another phenomenological way to describe the quark confinement is to consider the density dependent quark masses [37–40]. In this case, an important issue of thermodynamic consistency arises. Because of the density dependence of the quark masses, the quark chemical potentials acquire an additional density dependent term and become effective [41]. In fact, such a thermodynamically consistent approach was developed in recent work [42]. Note also that the quark confinement was modeled recently by the density and isospin dependent quark masses [43,44].

These phenomenological QCD models were applied to study MSQM in Refs. [21,22,24,27,29,35,45,46]. In particular, the effects of the pressure anisotropy were disregarded in Refs. [21,22,24,27], in Refs. [45,46] only the matter

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contribution to the pressure anisotropy was considered, and both the matter and field contributions were accounted for in Refs. [35,44]. In this study, I consider the absolute stability of MSQM in the MIT bag model with the density dependent bag pressure $B(\varrho_B)$. The advisability to extend the conventional MIT bag model came from the necessity to reconcile the different constraints on the bag pressure at low and high baryon densities obtained from heavy-ion experiments at the CERN Super Proton Synchrotron (SPS) and astrophysical observations of neutron stars with masses well above the mass of a canonical neutron star $M \sim 1.4 M_\odot$ ($M_\odot$ being the solar mass) [47]. In other model frameworks, the density dependent bag pressure was used in Refs. [48,49]. Note that in the extended MIT bag model, the quark chemical potentials acquire a term proportional to the density derivative $\frac{\partial B}{\partial \varrho_B}$, and the issue of the consistent thermodynamic description of MSQM becomes relevant. To that aim, I explore the formalism of the work [42], developed initially to describe the quark confinement by the density (and/or temperature) dependent quark masses, and, after proper modification, apply it to the case of the extended MIT bag model.

Here MSQM will be considered as an uniform matter permeated by an external uniform magnetic field. In the bag model, the matter part of the total energy density (excluding the magnetic field energy contribution) reads

$$E_m = \Omega_m^0 + \sum_i \bar{\mu}_i \varrho_i, \tag{1}$$

where

$$\Omega_m^0 = \sum_i \Omega_i^0 + B(\varrho_B), \tag{2}$$

and Ω_i^0 is the thermodynamic potential for free relativistic fermions of ith species ($i = u,d,s,e$) in the external magnetic field, which is given by the same expression as in Refs. [21,27,35] with the only difference that, according to the approach of Ref. [42], the real (nonrenormalized) chemical potentials μ_i should be substituted there by the effective (renormalized) chemical potentials $\bar{\mu}_i$.

At the given H, the differential form of Eq. (1) is

$$(dE_m)_H = \left(d\Omega_m^0\right)_H + \sum_i \bar{\mu}_i\, d\varrho_i + \sum_i \varrho_i\, d\bar{\mu}_i, \tag{3}$$

where

$$\left(d\Omega_m^0\right)_H = \sum_i \frac{\partial \Omega_i^0}{\partial \bar{\mu}_i}\, d\bar{\mu}_i + \sum_i \frac{\partial B}{\partial \varrho_i}\, d\varrho_i.$$

With account of equation

$$\varrho_i = -\left(\frac{\partial \Omega_i^0}{\partial \bar{\mu}_i}\right)_H, \tag{4}$$

Eq. (3) acquires the form

$$(dE_m)_H = \sum_i \left(\bar{\mu}_i + \frac{\partial B}{\partial \varrho_i}\right) d\varrho_i. \tag{5}$$

On the other hand, the fundamental thermodynamic relation at zero temperature reads [50]

$$(dE_m)_H = \sum_i \mu_i d\varrho_i. \tag{6}$$

By comparing Eqs. (5) and (6), and taking into account expression for the baryon number density $\varrho_B = \frac{1}{3}(\varrho_u + \varrho_d + \varrho_s)$, one gets the relationship between the real and effective chemical potentials

$$\mu_e = \bar{\mu}_e, \quad \mu_f = \bar{\mu}_f + \frac{1}{3}\frac{\partial B}{\partial \varrho_B}, \quad f = u,d,s. \tag{7}$$

Further, I study charge neutral states of MSQM and assume that the chemical equilibrium with respect to weak processes is established among the fermion species with the corresponding conditions on the real chemical potentials [5,42]. Note that, in view of Eq. (7), the effective chemical potentials $\bar{\mu}_i$ satisfy the same equations as the real ones μ_i:

$$\bar{\mu}_d = \bar{\mu}_u + \mu_{e^-}, \quad \bar{\mu}_d = \bar{\mu}_s. \tag{8}$$

The Hugenholtz–van Hove theorem establishes the thermodynamic relation between the pressure and energy density at zero temperature for nonmagnetized fermion matter [51]. For magnetized fermion matter, the total pressure is the anisotropic function of the magnetic field strength [30–35]. In particular, the longitudinal p^l and transverse p^t pressures are different. By comparing expressions for the longitudinal pressure p^l and energy density [31,34,35], one can get the Hugenholtz–van Hove theorem for magnetized matter in the form

$$p_m^l = -E_m + \sum_i \mu_i \varrho_i, \tag{9}$$

where p_m^l is the matter part of the longitudinal pressure. I will preserve this equation also for MSQM in the extended MIT bag model. With account of Eq. (1), Eq. (9) takes the form

$$p_m^l = -\Omega_m^0 + \sum_i (\mu_i - \bar{\mu}_i)\varrho_i = -\Omega_m^0 + \varrho_B \frac{\partial B}{\partial \varrho_B}. \tag{10}$$

At zero temperature, the matter part of the thermodynamic potential Ω_m, determined according to the standard thermodynamic equation

$$\Omega_m = E_m - \sum_i \mu_i \varrho_i, \tag{11}$$

with account of Eq. (1) becomes

$$\Omega_m = \Omega_m^0 - \sum_i (\mu_i - \bar{\mu}_i)\varrho_i = \Omega_m^0 - \varrho_B \frac{\partial B}{\partial \varrho_B}. \tag{12}$$

By comparing Eqs. (10) and (12), one arrives at the thermodynamic relationship $p_m^l = -\Omega_m$. The matter parts of the longitudinal and transverse pressures are related by the equation [34,44,52]

$$p_m^l - p_m^t = HM, \tag{13}$$

where $M = -\frac{\partial \Omega_m}{\partial H}$ is the system magnetization. After summarizing Eqs. (1), (2), (10), and (13) and accounting for the pure magnetic field contribution, the total energy density E, and the longitudinal p^l and transverse p^t pressures for MSQM with the density dependent bag pressure can be written in the form

$$E = \sum_i \left(\Omega_i^0 + \bar{\mu}_i \varrho_i\right) + \frac{H^2}{8\pi} + B, \tag{14}$$

$$p^l = -\sum_i \Omega_i^0 - \frac{H^2}{8\pi} - B + \varrho_B \frac{\partial B}{\partial \varrho_B}, \tag{15}$$

$$p^t = -\sum_i \Omega_i^0 - HM + \frac{H^2}{8\pi} - B + \varrho_B \frac{\partial B}{\partial \varrho_B}. \tag{16}$$

In the case of the density independent bag pressure, Eqs. (14)–(16) go over to the corresponding equations of Ref. [35]. Because of the breaking of the rotational symmetry by the magnetic field, the longitudinal p^l and transverse p^t pressures are not the same. There are two different contributions to the pressure anisotropy: the matter contribution proportional to the magnetization M, and the magnetic field contribution given by the Maxwell term $\frac{H^2}{8\pi}$. Note that it was argued recently [53] (and discussed before in Refs. [54,55]) that the magnetization contribution to the energy-momentum tensor is canceled by the Lorentz force associated with the magnetization current, and, taking that into account, the pressure anisotropy does not occur. In the given case, the direct answer to this question is that I consider a spatially uniform distribution of the magnetic field and matter density, and, hence, the magnetization current density $\mathbf{j}_b = \nabla \times \mathbf{M}$ (in units with $c = 1$) is exactly zero. Therefore, the associated Lorentz force density $\mathbf{j}_b \times \mathbf{H} = 0$, and cannot compensate the term with the magnetization in the transverse pressure. The other point is that the main, most principal source of the pressure anisotropy is provided by the pure magnetic field contribution $H^2/8\pi$, and there is no compensating effect for it. Just this term, as will be shown later, plays the main role in establishing the upper bound for the magnetic field strength in a magnetized medium.

For a spatially nonuniform case, the argument, raised in [53], is based on the consideration of the balance of the *volume* forces in the stationary state: $\partial_l T^{kl} = 0$, T^{kl} being the spatial components of the quantum-statistical average of the energy-momentum tensor of the system. Under writing the condition $\partial_l T^{kl} = 0$ with account of the Maxwell equation for the electromagnetic field tensor, the Lorentz force density associated with the magnetization current density cancels the term with the magnetization, and the corresponding first integral does not contain the magnetization as well. While this step is correct, it is then wrongly concluded in Ref. [53] that the system's equilibrium (stationary state) is determined by the thermodynamic pressure and any anisotropy in the matter pressure does not appear. Note that the pressures in the system are determined as the spatial diagonal elements T^{kk} of the energy-momentum tensor in the system's rest frame. In the stable stationary state, they should be positive, $T^{kk} > 0$. The conditions $T^{kk} > 0$ and $\partial_l T^{kl} = 0$ are, obviously, different and *both* should be satisfied in the stable stationary state.

In fact, the consistent derivation shows that the above mentioned cancelation occurs, with account of the Maxwell equation, only in the derivative $\partial_l T^{kl}$, but not in the energy-momentum tensor itself. An example of the spatially uniform case, discussed above, clearly confirms that. The introduction by hand of the Lorentz force contribution associated with the magnetization current to the energy-momentum tensor is the artificial step, not confirmed by the consistent derivation. Just the opposite, the consistent microscopic derivations [31,52] show that a such contribution is missing in the energy-momentum tensor, and, hence, the anisotropy in the matter part of the total pressure is present as well as the anisotropy caused by the contribution of the magnetic field.

II. NUMERICAL RESULTS AND DISCUSSION

Now I will determine the absolute stability window of MSQM, subject to charge neutrality and chemical equilibrium conditions, at zero temperature. The equilibrium conditions for MSQM require vanishing the longitudinal p^l and transverse p^t pressures

$$p^l = -\Omega = 0, \quad p^t = -\Omega + H\frac{\partial \Omega}{\partial H} = 0, \tag{17}$$

where $\Omega = \Omega_m + \frac{H^2}{8\pi}$ is the total thermodynamic potential of the system.

In order to be absolutely stable, the energy per baryon of MSQM should be less than that of the most stable ^{56}Fe nucleus under the equilibrium conditions (17). On the other hand, at $H = 0$, the experimental observation proves that two-flavor quark matter, consisting of u and d quarks, is less stable compared to the ^{56}Fe nucleus at zero external pressure and vanishing temperature [3]. I will also retain this constraint for strong magnetic fields $H \gtrsim 10^{17}$ G, although, strictly speaking, it is unknown from the experimental point of view whether two-flavor quark matter is less stable than the ^{56}Fe nucleus under the equilibrium conditions (17) in such strong fields. Thus, for determining the absolute stability window of MSQM, I will use the following constraints:

$$\left.\frac{E_m}{\varrho_B}\right|_{uds} \leq \epsilon_H(^{56}\mathrm{Fe}) \leq \left.\frac{E_m}{\varrho_B}\right|_{ud}. \tag{18}$$

Regarding, for the rough estimate, the ^{56}Fe nucleus as a system of noninteracting nucleons, magnetic fields $H > 10^{20}$ G are necessary in order to significantly alter its energy per nucleon $\epsilon_H(^{56}\mathrm{Fe})$ [23]. Since I will consider magnetic fields $H < 5 \times 10^{18}$ G, further I use the approximation $\epsilon_H(^{56}\mathrm{Fe}) \approx \epsilon_0(^{56}\mathrm{Fe}) = 930$ MeV.

For determining the absolute stability window from constraints (17) and (18), I utilize the Gaussian parametrization for the bag pressure as a function of the baryon density [47]:

$$B(\varrho_B) = B_\infty + (B_0 - B_\infty)\, e^{-\beta(\frac{\varrho_B}{\varrho_0})^2}, \tag{19}$$

where $B_0 = B(\varrho_B = 0)$, $B_\infty = B(\varrho_B \gg \varrho_0)$ ($\varrho_0 \simeq 0.17$ fm^{-3}), and the parameter β controls the rate of decrease of the bag pressure from B_0 to B_∞. In numerical analysis, I use $\beta = 0.01$ and $\beta = 0.17$. Note that the equilibrium conditions (17) contain the derivatives $\frac{\partial B}{\partial H}$, $\frac{\partial^2 B}{\partial \varrho_B \partial H}$. Further, I will assume that the magnetic field affects the bag pressures B_0 and B_∞ in the same way, and, hence, the difference $\Delta B_0 \equiv B_0 - B_\infty$ is independent of H and will

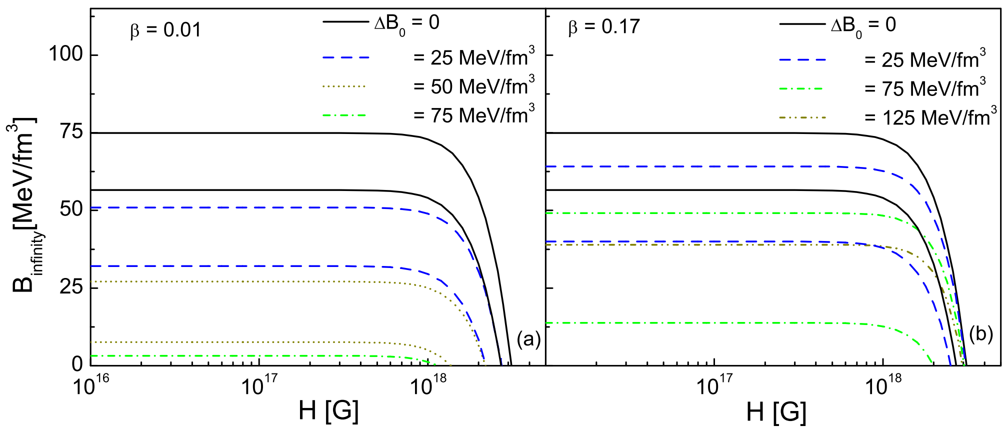

FIG. 1. (Color online) The absolute stability window in the plane (H,B_∞) for MSQM at zero temperature with $B(\varrho_B)$ given by Eq. (19) at (a) $\beta=0.01$ and (b) $\beta=0.17$, and with a variable parameter ΔB_0. The upper B^u_∞ and lower B^l_∞ bounds are shown as the upper and lower curves, respectively, in the pairs of the similar curves (see also comments in the text).

be considered as a parameter. Then the actual dependence of the bag pressure B on H is contained in the asymptotic bag pressure B_∞: $\frac{\partial B}{\partial H}=\frac{\partial B_\infty}{\partial H}$ while $\frac{\partial^2 B}{\partial\varrho_B\partial H}=0$. The absolute stability window will be built in the plane (H,B_∞) under various fixed values of ΔB_0 and β. The upper (lower) bound B^u_∞ (B^l_∞) on the asymptotic bag pressure B_∞ can be found from the first of the equilibrium conditions (17):

$$B^{u\,(l)}_\infty(H) = -\sum_{\substack{i=u,d,s,e\\(i=u,d,e)}}\Omega^0_i - \frac{H^2}{8\pi} - \Delta B_0\, e^{-\beta(\frac{\varrho_B}{\varrho_0})^2}\left(1+\frac{2\beta\varrho_B^2}{\varrho_0^2}\right) \tag{20}$$

after finding the effective chemical potentials $\bar\mu_i$ from the first constraint to the left (right) in (18), taken with the equality sign, and charge neutrality and chemical equilibrium conditions (8). In Eq. (20), the baryon density ϱ_B should be determined from Eq. (4) after finding the chemical potentials $\bar\mu_i$ at the given H.

Figure 1 shows the dependences $B^u_\infty(H)$ and $B^l_\infty(H)$ for the current quark masses $m_u=m_d=5$ MeV and $m_s=150$ MeV. The upper bound B^u_∞ stays, at first, practically constant and then, beginning from the magnetic field strength H somewhat smaller than 10^{18} G, decreases. For example, the maximum value of B^u_∞, corresponding to $H=0$ (which is practically indistinguishable from the value of B^u_∞ at $H=10^{16}$ G) is $B^u_{\infty,\,\max}\approx 74.9$ MeV/fm^3 for $\Delta B_0=0$, independently of the value of β; for $\Delta B_0=75$ MeV/fm^3, $B^u_{\infty,\,\max}\approx 3.2$ MeV/fm^3 at $\beta=0.01$, and $B^u_{\infty,\,\max}\approx 49.1$ MeV/fm^3 at $\beta=0.17$. The upper bound B^u_∞ vanishes at $H_{u\,\max}\approx 3.1\times10^{18}$ G for $\Delta B_0=0$ at any β; for $\Delta B_0=75$ MeV/fm^3, B^u_∞ vanishes at $H_{u\,\max}\approx 1.1\times10^{18}$ G for $\beta=0.01$, and at $H_{u\,\max}\approx 3\times10^{18}$ G for $\beta=0.17$. In stronger magnetic fields $H>H_{u\,\max}$, in order to satisfy the equilibrium conditions, the upper bound B^u_∞ on the asymptotic bag pressure B_∞ had to become negative, contrary to the constraint $B_\infty>0$. This means that, under the equilibrium conditions and in magnetic fields $H>H_{u\,\max}$, MSQM cannot be absolutely stable.

The behavior of the lower bound $B^l_\infty(H)$ is similar to that of $B^u_\infty(H)$. At $H=0$, the maximum value of B^l_∞ (which almost coincides with the value of B^l_∞ at $H=10^{16}$ G) is $B^l_{\infty,\,\max}\approx 56.5$ MeV/fm^3 for $\Delta B_0=0$, independently of the value of β; for $\Delta B_0=25$ MeV/fm^3, $B^l_{\infty,\,\max}\approx 32.0$ MeV/fm^3 at $\beta=0.01$, and $B^l_{\infty,\,\max}\approx 40.0$ MeV/fm^3 at $\beta=0.17$. The lower bound B^l_∞ stays practically constant till magnetic fields somewhat smaller than 10^{18} G, beyond which B^l_∞ decreases. The lower bound B^l_∞ vanishes at $H_{l\,0}\approx 2.7\times10^{18}$ G for $\Delta B_0=0$ at any β; for $\Delta B_0=25$ MeV/fm^3, B^l_∞ vanishes at $H_{l\,0}\approx 2.2\times10^{18}$ G for $\beta=0.01$, and at $H_{l\,0}\approx 2.5\times10^{18}$ G for $\beta=0.17$. Under the equilibrium conditions and in the fields $H>H_{l\,0}$, the lower bound B^l_∞ would be negative. Because $B_\infty>0$, the inequality $B_\infty>B^l_\infty$ would be fulfilled always in the fields $H>H_{l\,0}$. Thus, in order for MSQM to be absolutely stable, the magnetic field strength should satisfy the constraint $H<H_{u\,\max}$. In fact, the value $H_{u\,\max}$ represents the upper bound on the magnetic field strength which can be reached in a magnetized strange quark star. Note that the upper bound B^u_∞ decreases with increasing the parameter ΔB_0 at the given β. Hence, B^u_∞ vanishes at a smaller magnetic field for a larger value of ΔB_0, i.e., the upper bound on the magnetic field strength $H_{u\,\max}$ decreases with increasing ΔB_0 at the given β. On the other hand, the upper bound B^u_∞ increases with increasing the parameter β at the given ΔB_0. Hence, the larger β is, the larger the upper bound $H_{u\,\max}$ is at the given ΔB_0.

Note also that the lower bound B^l_∞ decreases with increasing the parameter ΔB_0. Above certain value of ΔB_0, which depends on β, the lower bound B^l_∞ would become negative, and, hence, for $B_\infty>0$ the inequality $B_\infty>B^l_\infty$ would be always fulfilled at any H. For such ΔB_0 and β, the absolute stability window corresponds to $0<B_\infty<B^u_\infty$. This is just the case for $\Delta B_0=75$ MeV/fm^3 at $\beta=0.01$, and for $\Delta B_0=125$ MeV/fm^3 at $\beta=0.17$, shown in Figs. 1(a) and 1(b), respectively.

Figure 2 shows the dependences $\varrho_b(H)$ for MSQM and magnetized two-flavor quark matter, determined under the respective equilibrium conditions at $E_m/\varrho_b=930$ MeV, for magnetic fields $H<H_{u\,\max}$. In fact, the corresponding lines $\varrho^u_B(H)$ and $\varrho^l_B(H)$ represent the upper and lower bounds on the

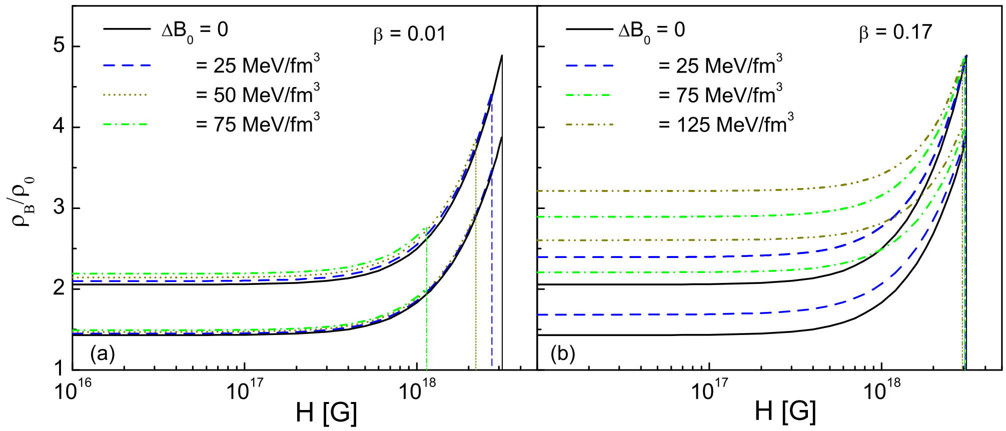

FIG. 2. (Color online) The dependences $\varrho_B(H)$ for MSQM and magnetized two-flavor quark matter (the upper and lower curves, respectively, in the pairs of the similar curves), determined under the equilibrium conditions at $E_m/\varrho_b = 930$ MeV for the same values of the parameters ΔB_0 and β as in Fig. 1. The bounding vertical lines on the right correspond to $H = H_{u\ \max}$.

baryon density to ensure the absolute stability of MSQM. It is seen that the upper ϱ_B^u and lower ϱ_B^l bounds stay practically constant till the magnetic field strength being somewhat smaller than 10^{18} G, and then increase till it reaches the corresponding maximum value. The increase of the parameter ΔB_0 at the given β, and the increase of the parameter β at the given ΔB_0, lead to the increase of both the upper ϱ_B^u and lower ϱ_B^l bounds. For example, at $H = 0$ the minimum value of ϱ_B^u (being almost the same as the value of ϱ_B^u at $H = 10^{16}$ G) is $\varrho^u_{B\ \min} \approx 2.1\varrho_0$ for $\Delta B_0 = 0$, independently of β; for $\Delta B_0 = 75\ \text{MeV/fm}^3$, $\varrho^u_{B\ \min} \approx 2.2\varrho_0$ at $\beta = 0.01$, and $\varrho^u_{B\ \min} \approx 2.9\varrho_0$ at $\beta = 0.17$. Similarly, at $H = 0$ the minimum value of ϱ_B^l is $\varrho^l_{B\ \min} \approx 1.4\varrho_0$ for $\Delta B_0 = 0$, independently of β; for $\Delta B_0 = 75\ \text{MeV/fm}^3$, $\varrho^l_{B\ \min} \approx 1.5\varrho_0$ at $\beta = 0.01$, and $\varrho^l_{B\ \min} \approx 2.2\varrho_0$ at $\beta = 0.17$. Note that magnetic fields $H \gtrsim 10^{18}$ G strongly affect the upper ϱ_B^u and lower ϱ_B^l bounds from the absolute stability window.

In conclusion, I have considered MSQM under charge neutrality and chemical equilibrium conditions in the MIT bag model with the density dependent bag pressure $B(\varrho_B)$. I aimed to determine the range for the magnetic field strength H, asymptotic bag pressure $B_\infty \equiv B(\varrho_B \gg \varrho_0)$, and baryon density ϱ_B, for which MSQM is absolutely stable, i.e., its energy per baryon is less than that of the most stable ^{56}Fe nucleus under zero-pressure conditions (17) and vanishing temperature. In fact, this requirement sets the upper bound on the parameters from the absolute stability window. The lower bound is determined from the constraint that magnetized two-flavor quark matter under equilibrium conditions (17) and zero temperature should be less stable than the most stable ^{56}Fe nucleus. This constraint is extended from the weak terrestrial magnetic fields, where it has direct experimental confirmation, to possible strong magnetar interior magnetic fields $H \gtrsim 10^{17}$ G, where such confirmation is wanting. It has been shown that there exists a magnetic field strength $H_{u\ \max}$ at which the upper bound B^u_∞ on the asymptotic bag pressure B_∞ from the absolute stability window vanishes. In fact, the value of this field, $H_{u\ \max} \sim (1\text{–}3) \times 10^{18}$ G, represents the upper bound on the magnetic field strength, which can be reached in a strongly magnetized strange quark star. I have studied the effect of the parameters in the Gaussian parametrization for the bag pressure on the absolute stability window and upper bound $H_{u\ \max}$.

It is interesting to note that the obtained estimate for $H_{u\ \max}$ in strange quark stars is similar to the estimate $H \sim (1\text{–}3) \times 10^{18}$ G for the maximum average magnetic fields in stable neutron stars, composed of strange baryonic matter [56]. The found estimate of the upper bound $H_{u\ \max}$ in strange quark stars may be further improved by including within the MIT bag model the effects of the perturbative quark interactions [57]. It would be of interest also to extend this research to the case of the spatially nonuniform magnetic field, whose realistic profile should be determined from the solution of the coupled Einstein and Maxwell equations [58]. The other interesting problem is to take into account the vacuum corrections due to the magnetic field, which are, however, beyond the scope of the MIT bag model. To that aim, one can utilize the quark chiral models [59–61]. Nevertheless, as was shown in Ref. [62], these corrections become noticeable only in strong magnetic fields $H \gtrsim 3 \times 10^{19}$ G, and, therefore, one can expect that their effect on the maximum magnetic field in strange quark stars will be of less importance.

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