Alternative way of describing hadronic processes within the parton model

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A set of evolution equations for correlators of densities of quarks and gluons is considered. Approximate solutions are derived in the framework of the gluon and quark dominance. A new approach to estimate the cross sections of the processes with the interaction of high energy hadrons within the parton model is proposed. This approach is demonstrated in a set of processes of hadron-hadron collisions with QCD subprocesses of $2 \to 2$ type. The process with the subprocess $gb \to tH^-$ was also considered.

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I. INTRODUCTION

The quark parton model of Feynman [1] provides a simple description of deep inelastic phenomena as well as the processes of the collision of hadrons with high energy. This model was theoretically justified in terms of asymptotically free gauge theories [2]. This approach is based on the factorization of contributions from small and large distances justified by the authors of Ref. [3]. The deviation from the naive Bjorken scaling of the structure functions of deep inelastic scattering (DIS) was recognized to be broken by the so-called "large logarithms"—the logarithms of the ratio of transferred momentum squared $Q^2 = -q^2$ (virtualities) of particles, which are far from the mass shell to their masses squared m^2, i.e., $L \equiv \ln(Q^2/m^2)$. The reasons for the appearance of these logarithms in QED were clarified with the use of methods of quasireal photons and electrons [4–7]. Nevertheless, even taking into account the contributions of higher orders of perturbation theory in the leading logarithmical approximation the description of the processes can be formulated at the parton language with the use of parton (quark, gluon) densities—the so-called structure functions, $q^i(x,t)$, $G(x,t)$. These structure functions depend on the scale Q^2 of consideration. This evolution of parton densities is described by the equations of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [8] (here we use the original notation of Ref. [8])

$$\frac{dq^i(x,L)}{dL} = \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[q^i(y,L)P_{qq}\left(\frac{x}{y}\right) + G(y,L)P_{qg}\left(\frac{x}{y}\right)\right],$$

$$L = \ln\left(\frac{Q^2}{m^2}\right),$$

$$\frac{dG(x,L)}{dL} = \frac{\alpha_s(l)}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_{i=1}^{2N_f} q^i(y,L)P_{Gq^i}\left(\frac{x}{y}\right) + G(y,L)P_{GG}\left(\frac{x}{y}\right)\right],$$

$$i = u,d,s,c,b,t, \tag{1}$$

where $q^i(x,L)$ and $G(x,L)$ are the quark and gluon densities of the proton, i.e., describe the densities of the parton with the energy fraction x of the total proton energy on a scale of Q^2 (where $Q^2 > 0$). Here m is the suitable normalization point $m \sim Q_0 \sim M_p \approx 1$ GeV. The quantity $\alpha_s(L)$ is the QCD coupling constant on the scale Q^2 and $P_{ij}(x)$ are the DGLAP equation kernels

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right),$$

$$P_{Gq}(z) = C_F\frac{1+(1-z)^2}{z},$$

$$P_{qG}(z) = \frac{1}{2}[z^2+(1-z)^2],$$

$$P_{GG}(z) = 2C_V\left(\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \frac{11}{12}\delta(z-1)\right),$$

with $C_F = \frac{N^2-1}{2N}$ and $C_V = N$ for the color group $SU(N)$. These quantities satisfy the following properties:

$$\int_0^1 dz P_{qq}(z) = 0, \quad \int_0^1 dz z P_{GG}(z) = 0. \tag{2}$$

It is useful to recall here the statistical interpretation of the DGLAP equations in terms of densities [9] by means of a set of correlation functions satisfying the system of statistical

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equations (renormalization group equation). It was a success of the numerous applications of the DGLAP set of equations, working with two densities q, G of quarks and gluons in a proton. However, the problems associated with processes with large multiplicity [10–12] require some generalization of traditional approach introducing the correlation functions [13].

This paper is organized in the following manner. In Sec. II, we define the parton density and the set of equations for them. In Sec. III, we apply the arguments of the quark dominance and derive the same equations within this approximation and write out the explicit analytical solutions. In Sec. IV, we show the general way how one can use parton densities obtained in the previous sections for the calculation of the cross sections of high energy proton collisions. In Sec. V, we demonstrate the application of this approach to two concrete processes. In Sec. VI, we discuss the results and give some note about logarithmical and double logarithmical regimes.

II. GENERAL FORMALISM

Let us introduce three distributions $D^a(x,L)$ (where $a = q,\bar{q},g$), which are the densities of partons of type a with the energy fraction x inside the parent quark q on the scale Q^2 (see Fig. 1). Similarly, we must introduce three quantities $\bar{D}^a$ that are the densities of a parton of type a inside the parent antiquark. We also need three distributions G^a that are similar parton densities inside the parent gluon. The evolution of these densities with the scale Q^2 is described by the evolution equations similar to DGLAP.[1] That is, for quark densities we have

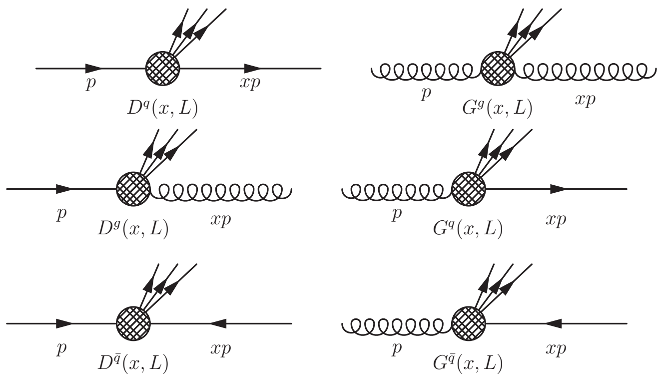

FIG. 1. Definition of correlator densities (see Sec. II).

$$
\begin{aligned}
\frac{d}{dL}D^q(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[D^q(y,L)P_{qq}\left(\frac{x}{y}\right) + D^g(y,L)P_{qg}\left(\frac{x}{y}\right)\right];\\
\frac{d}{dL}D^{\bar{q}}(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[D^{\bar{q}}(y,L)P_{qq}\left(\frac{x}{y}\right) + D^g(y,L)P_{qg}\left(\frac{x}{y}\right)\right];\\
\frac{d}{dL}D^g(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[D^g(y,L)P_{gg}\left(\frac{x}{y}\right) + D^{\bar{q}}(y,L)P_{gq}\left(\frac{x}{y}\right) + D^q(y,L)P_{gq}\left(\frac{x}{y}\right)\right].
\end{aligned} \tag{3}
$$

Here we notice that in the lowest order of perturbation theory the processes of the transition of a quark to an antiquark and an antiquark to a quark are absent, i.e., $P_{q\bar{q}} = P_{\bar{q}q} = 0$. A similar set of equations for parton densities inside the antiquark stems from Eq. (3) by the replacement $D^a \to \bar{D}^a$. For gluon densities we have

$$
\begin{aligned}
\frac{d}{dL}G^q(x,L) = \frac{d}{dL}G^{\bar{q}}(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[G^q(y,L)P_{qq}\left(\frac{x}{y}\right) + G^g(y,L)P_{qg}\left(\frac{x}{y}\right)\right];\\
\frac{d}{dL}G^g(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[G^g(y,L)P_{gg}\left(\frac{x}{y}\right) + G^{\bar{q}}(y,L)P_{gq}\left(\frac{x}{y}\right) + G^q(y,L)P_{gq}\left(\frac{x}{y}\right)\right].
\end{aligned} \tag{4}
$$

It follows from Eqs. (3) and (4) that

$$
\begin{aligned}
\frac{d}{dL}(D^g(x,L) + G^g(x,L)) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[(D^g(y,L) + G^g(y,L))P_{gg}\left(\frac{x}{y}\right) + (D^{\bar{q}}(y,L) + G^{\bar{q}}(y,L))P_{gq}\left(\frac{x}{y}\right)\right.\\
&\quad\left. + (D^q(y,L) + G^q(y,L))P_{gq}\left(\frac{x}{y}\right)\right];\\
\frac{d}{dL}(D^q(x,L) + G^q(x,L)) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}\left[(D^q(y,L) + G^q(y,L))P_{qq}\left(\frac{x}{y}\right) + (D^g(y,L) + G^g(y,L))P_{qg}\left(\frac{x}{y}\right)\right].
\end{aligned}
$$

We also note that omitting the densities of antiquarks $D^{\bar{q}}$ and $G^{\bar{q}}$ inside the quark and the gluon and identifying

$$
\begin{aligned}
D^q(x,L) + G^q(x,L) &= q(x,L),\\
D^g(x,L) + G^g(x,L) &= G(x,L),
\end{aligned}
$$

we reproduce the DGLAP equations (1).

[1] A similar consideration was used in the framework of QED in Ref. [14].

Our statement about the numerical smallness of the contribution of the intermediate quark (antiquark) states in the evolution of gluon density follows from the iteration procedure in solving the first equation of a gluon set. So this can be taken into account by including a relevant contribution to the K factor. In addition, only light quarks must be considered by describing the experiments without quark jet production.

III. APPROXIMATE EVOLUTION EQUATIONS

The quark dominance consists of the suggestion $D^q \gg \bar{D}^q, G^q$. The gluon dominance implies $G^g \gg D^g, \bar{D}^g$ and in addition $D^g \gg D^{\bar{q}}$. The set of equations in these approximations reads as

$$\begin{aligned}
\frac{d}{dL}D^q(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}D^q(y,L)P_{qq}\left(\frac{x}{y}\right);\\
\frac{d}{dL}D^{\bar{q}}(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}D^g(y,L)P_{qg}\left(\frac{x}{y}\right);\\
\frac{d}{dL}D^g(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}D^q(y,L)P_{gq}\left(\frac{x}{y}\right);\\
\frac{d}{dL}G^q(x,L) &= \frac{d}{dL}G^{\bar{q}}(x,L)\\
&= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}G^g(y,L)P_{qg}\left(\frac{x}{y}\right);\\
\frac{d}{dL}G^g(x,L) &= \frac{\alpha_s(L)}{2\pi}\int_x^1 \frac{dy}{y}G^g(y,L)P_{gg}\left(\frac{x}{y}\right). \qquad (5)
\end{aligned}$$

Note that the equation for D^q coincides with the equation for the nonsinglet quark density $q_{NS} = q - \bar{q}$.

Solving the equation for G^g by the iteration method we see that the regeneration of the gluon density in the channel $G^g \to D^q \to G^g$ is associated with the small factor

$$K_0 = \left(\frac{C_F}{2C_V}\right)^2 = \left(\frac{4/3}{6}\right)^2 \approx 0.05. \qquad (6)$$

The terms of such a magnitude can be neglected thus determining the accuracy of the approximation. Alternatively, it can be included as some contribution to the K factor. Thus, we have the following equations:

$$\begin{aligned}
D^q(x,L) &= \delta(x-1) + \int_{m^2}^{Q^2}\frac{\alpha_s(L')}{2\pi}\frac{dQ'^2}{Q'^2}\\
&\quad\times\int_x^1\frac{dy}{y}P_{qq}\left(\frac{x}{y}\right)D^q(y,L'),\\
L' &= \ln\left(\frac{Q'^2}{m^2}\right),
\end{aligned} \qquad (7)$$

$$\begin{aligned}
xG^g(x,L) &= \delta(x-1) + \int_{m^2}^{Q^2}\frac{\alpha_s(L')}{2\pi}\frac{dQ'^2}{Q'^2}\\
&\quad\times\int_x^1\frac{dy}{y}P_{GG}\left(\frac{x}{y}\right)yD_g(y,L'). \qquad (8)
\end{aligned}$$

Using the solutions of the homogeneous equations for quark in the nonsinglet density D^q and G^g (we use the method similar to one developed in the framework of QED in Ref. [15] (see Eqs. (11) and (20) in Ref. [15]):

$$\begin{aligned}
D^q(x,L) &= 2\beta_q(1-x)^{2\beta_q}\left[\frac{1}{1-x}\left(1+\frac{3}{2}\beta_q\right) - \frac{1}{2}(1+x)\right]\\
&\quad + O\left(\beta_q^2\right),\\
xG^g(x,L) &= 2\beta_g(1-x)^{2\beta_g}\left[\frac{1}{1-x}\left(1+\frac{11}{6}\beta_g\right)\right.\\
&\quad\left. - 2x + x^2(1-x)\right] + O\left(\beta_g^2\right), \qquad (9)
\end{aligned}$$

where

$$\beta_q = C_F\frac{\alpha_s(L)}{2\pi}(L-1); \quad \beta_g = 2C_V\frac{\alpha_s(L)}{2\pi}(L-1). \qquad (10)$$

These solutions satisfy the properties (2), i.e.,

$$\int_0^1 dx D^q(x,L) = 1, \quad \int_0^1 dx x G^g(x,L) = 1. \qquad (11)$$

Similar solutions for other functions have the form

$$\begin{aligned}
D^g(x,L) &= G^q(x,L) = \frac{1}{2}(1+(1-x)^2)\beta_q^2 + O\left(\beta_q^3\right);\\
D^{\bar{q}}(x,L) &= D^{q'}(x,L) = D^{\bar{q}'}(x,L) = \frac{1}{2}\phi(x)\beta_q^2 + O\left(\beta_q^3\right),\\
\phi(x) &= \frac{1}{3x}(1-x)(4+7x+4x^2) + 2(1+x)\ln x. \qquad (12)
\end{aligned}$$

IV. GENERAL FORM OF THE CROSS SECTION OF HADRON COLLISION

Let us consider the collision of protons

$$p + p \to \text{jet}_1 + \text{jet}_2 + F, \qquad (13)$$

with the hard subprocess

$$a + b \to F, \qquad (14)$$

where a and b are partons that are taken from initial protons and F is some final state produced by them. Thus, the scale of hard subprocess is of order of $Q^2 = M_F^2$, where M_F is the mass of produced system F. In our approach we write the cross section of the process (13) in the center-of-mass system in the following factorized form:

$$\begin{aligned}
d\sigma_{pp\to F+X} &= \int_0^1 dx_1 \sum_{a_1} W_{a_1}(x_1)\int_0^1 dy_1 \sum_{b_1} D_{a_1}^{b_1}(y_1,L)K_{a_1}\\
&\quad\times\int_0^1 dx_2 \sum_{a_2} W_{a_2}(x_2)\int_0^1 dy_2 \sum_{b_2} D_{a_2}^{b_2}(y_2,L)\\
&\quad\times K_{a_2} d\hat{\sigma}^{b_1 b_2\to F}(\hat{s},\hat{t},\hat{u})\Theta(z - z_{\text{th}}),\\
z &= x_1 y_1 x_2 y_2, \qquad (15)
\end{aligned}$$

where the functions $W^a(x)$ describe the probability to find a parton a inside a proton. This quantity is taken on the scale of the order of $Q^2 \sim 1\,\text{GeV}^2$ and was defined by the authors of Ref. [16] as a result of self-consistent analysis of many subprocesses and it was shown that it satisfies the momentum conservation law and the parton number normalization. We present these parametrizations in the Appendix. The next piece of Eq. (15) is the convolution of these distributions with the quantities $D_a^b(x,L) = \{D^b(x,L), \bar{D}^b(x,L), G^b(x,L)\}$, which are the densities of the parton of sort b inside the parton of sort a. This convolution in our approach is the explicit realization of QCD evolution from the initial scale $Q^2 \sim 1\,\mathrm{GeV}^2$ to the scale of hard subprocess $Q^2 = M_F^2$. The summation over $\{a_1,a_2\}$ is performed over all possible partons inside the proton, i.e., $\{a\} = \{u,d,s,\bar{u},\bar{d},\bar{s},g\}$. The summation over $\{b_1,b_2\}$ is performed over all possible partons that can be found inside the parton of sort $\{a_1,a_2\}$, i.e., in principle all possible partons also, i.e., $\{b\} = \{u,d,s,\bar{u},\bar{d},\bar{s},g\}$.

The quantity

$$d\hat{\sigma}^{b_1 b_2 \to F}(\hat{s},\hat{t},\hat{u}) \tag{16}$$

in Eq. (15) is the cross section of a hard subprocess of two parton b_1 and b_2 fusion that actually produces the final system F, which is of experimental interest. This cross section should be taken in the system of the center of mass of this subprocess $b_1 + b_2$, i.e., these invariants $\{\hat{s},\hat{t},\hat{u}\}$ are in this reference frame of the subprocess and have a z-shifted form

$$\hat{s} = 4E^2 z, \quad \hat{t} = -2E^2 z(1-\cos\hat{\theta}), \quad \hat{u} = -\hat{s} - \hat{t}, \tag{17}$$

where $\hat{\theta}$ is the angle between three vectors of the initial parton b_1 momentum and the momentum of one of the particles from the created state F in the center-of-mass reference frame of the subprocess, which can be expressed in terms of the angle θ between the directions of the initial beam and the momentum of the same particles from the created state F

$$\cos\hat{\theta} = \frac{x_1 y_1 + x_2 y_2 \cos\theta}{x_2 y_2 + x_1 y_1 \cos\theta}. \tag{18}$$

The quantities K_a in Eq. (15) are the so-called K factors, which take into account the nonleading contribution of evolution. The K factor associated with the quark density has the form [17]

$$K_q = 1 + \frac{\alpha_s}{2\pi} k_q, \quad k_q = \frac{1}{2} C_V \left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5 n_f}{18} \approx 1.5$$

and K_g is the K factor associated with the gluon density and has the form

$$K_g = 1 + \tfrac{1}{2} K_0, \tag{19}$$

where K_0 is given in Eq. (6).

The Θ function in Eq. (15) assures that an experimental setup allows registration of the jets of produced particles with some finite threshold invariant mass s_{th} only, i.e., the jets with the invariant mass $s_j \geqslant s_{\mathrm{th}}$. The quantity z_{th} that characterizes this threshold has the form

$$z_{\mathrm{th}} = \frac{s_{\mathrm{th}}}{s}. \tag{20}$$

In these setups, where the product of the subprocess is detected at large angles with invariant mass square exceeding some threshold value $s_{\mathrm{th}} = s\, z_{\mathrm{th}}$, the role of "sea" partons in the proton can be neglected.

V. APPLICATION TO SOME DEFINITE SUBPROCESS

Below we consider two types of subprocesses. First, for the sake of demonstration we will consider the process of associative production of the top quark and the charged Higgs boson H^- since the application of our approach is simpler in this case. Then we will use our approach to describe the experimental data from the Tevatron [18].

A. Process $p + p \to t + H^- + jjjj$

Let us consider now the important application of our approach to the process

$$p + p \to t + H^- + jjjj, \tag{21}$$

where j denotes the jet. In this case, the dominant channel of the charge Higgs production is through the subprocess

$$b + g \to t + H^-. \tag{22}$$

The cross section of this subprocess has the form (see Eq. (6.22) in Ref. [19] or Eq. (2.1) in Ref. [20])

$$\begin{aligned} \frac{d\hat{\sigma}^{bg\to tH^-}}{d\cos\hat{\theta}} = \frac{\sigma_0}{\hat{s}} \Bigg\{ & \frac{\hat{s} + \hat{t} - M_{H^-}^2}{2\hat{s}} \\ & - \frac{m_t^2(\hat{u} - M_{H^-}^2) + M_{H^-}^2(\hat{t} - m_t^2) + \hat{s}(\hat{u} - m_t^2)}{\hat{s}(\hat{u} - m_t^2)} \\ & - \frac{m_t^2(\hat{u} - M_{H^-}^2 - \hat{s}/2) + \hat{s}\hat{u}/2}{(\hat{u} - m_t^2)^2} \Bigg\}, \end{aligned} \tag{23}$$

where the subprocess invariants $\hat{s}$, $\hat{t}$, $\hat{u}$ are defined as

$$\hat{s} = (p_b + p_g)^2, \quad \hat{t} = (p_b + p_t)^2, \quad \hat{u} = (p_b + p_{H^-})^2, \tag{24}$$

and the angle $\hat{\theta}$ from Eq. (17) is the angle between the momenta of the initial b quark and produced t quark in the reference frame of the center of mass of the subprocess (i.e., $\mathbf{p_b} = -\mathbf{p_g}$ where $\mathbf{p_b}$ and $\mathbf{p_g}$ are the three-momenta of the initial b quark and the gluon, respectively). The quantity σ_0 is the following constant:

$$\sigma_0 = \frac{\pi\alpha\alpha_s(m_b^2 \tan^2\beta + m_t^2 \cot^2\beta)}{6\, M_W^2 \sin^2\theta_W}, \tag{25}$$

where θ_W is the Weinberg angle and β is the parameter of the minimal supersymmetric standard model (MSSM). For $\tan\beta = 40$ and $\alpha_s = 0.1$ (which is the approximate value of α_s at the scale of t-quark mass) we have $\sigma_0 \approx 0.06$.

The application of our master formula (15) to the process (21) gives the cross section in the following form:

$$\begin{aligned} & \frac{d\sigma_{pp\to tH^- + jjjj}}{d\cos\theta} \\ & = \int_0^1 dx_1 \int_0^1 dy_1 \int_0^1 dx_2 \int_0^1 dy_2\, \Theta(z - z_{\mathrm{th}}) \frac{d\hat{\sigma}^{bg\to tH^-}}{d\cos\theta} \\ & \times (W_u(x_1) D^{q'}(y_1,L) K_q + W_d(x_1) D^{q'}(y_1,L) K_q \\ & + W_g(x_1) G^q(y_1,L) K_g)(W_u(x_2) D^g(y_2,L) K_q \\ & + W_d(x_2) D^g(y_2,L) K_q + W_g(x_2) G^g(y_2,L) K_g), \end{aligned} \tag{26}$$

which is graphically illustrated in Fig. 2(a). Let us note that in Eq. (26) there are many terms that appear from the braces. Figure 2(a) corresponds to the term that comes from the multiplication of the first term in the first braces with the last term in the second braces. At this stage, we need to notice that the cross section of the subprocess in Eq. (26) depends on the angle θ between the direction of the momentum of the produced t quark and the beam in

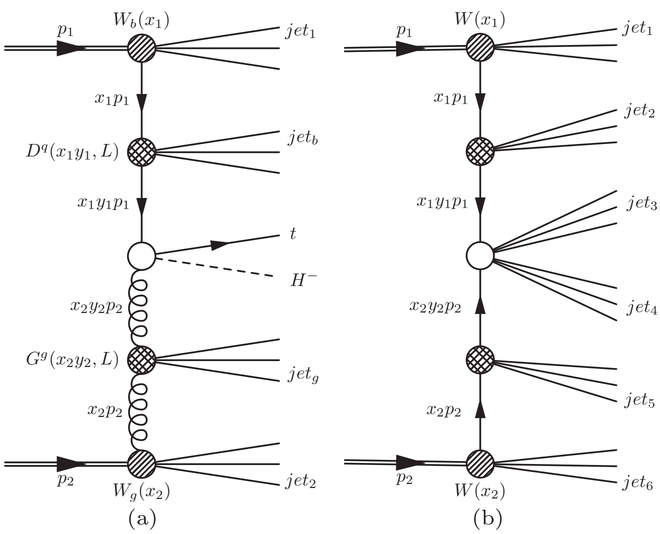

FIG. 2. Process factorization schemes.

the system of the center of mass of the initial proton-proton beams, while expression (23) depends on the scattering angle $\hat{\theta}$ in the center-of-mass reference frame of the subprocess. Since these angles correspond to each other according to relation (18), i.e., we obtain

$$d\cos\hat{\theta} = \frac{x_2^2 y_2^2 - x_1^2 y_1^2}{(x_2 y_2 + x_1 y_1 \cos\theta)^2}\, d\cos\theta, \tag{27}$$

and hence the following final form of the cross section (26):

$$\begin{aligned}
&\frac{d\sigma_{pp\to tH^- + jjjj}}{d\cos\theta}\\
&= \int_0^1 dx_1 \int_0^1 dy_1 \int_0^1 dx_2 \int_0^1 dy_2\ \Theta(z - z_{\rm th})\\
&\quad \times \frac{d\hat{\sigma}^{bg\to tH^-}}{d\cos\hat{\theta}} \frac{x_2^2 y_2^2 - x_1^2 y_1^2}{(x_2 y_2 + x_1 y_1 \cos\theta)^2} (W_u(x_1) D^{q'}(y_1, L)\\
&\quad \times K_q + W_d(x_1) D^{q'}(y_1, L) K_q + W_g(x_1) G^q(y_1, L) K_g)\\
&\quad \times (W_u(x_2) D^g(y_2, L) K_q + W_d(x_2) D^g(y_2, L) K_q\\
&\quad + W_g(x_2) G^g(y_2, L) K_g).
\end{aligned} \tag{28}$$

The dependence of this cross section on the scattering angle θ is presented in Fig. 3 via the quantity

$$F_H(\theta) = \frac{s}{4\,\sigma_0} \frac{d\sigma_{pp\to tH^- + jjjj}}{d\cos\theta}, \tag{29}$$

which is built for a few values of masses of Higgs boson M_H. The total cross section of this process is proportional to the quantity T_H

$$T_H(s) = \int_0^\pi d\theta\ F_H(\theta), \tag{30}$$

which is presented in Fig. 4.

B. Six-jets production at Tevatron

In Ref. [18] the data for the six-jet production

$$p + p \to 6\,\text{jets} \tag{31}$$

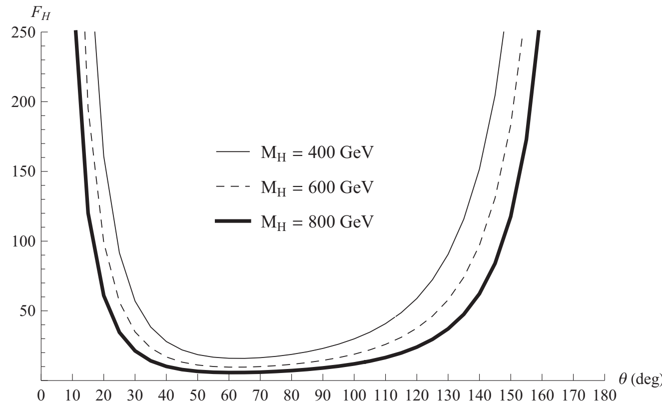

FIG. 3. The angular dependence of the quantity F_H defined in Eq. (29), for $\sqrt{s} = 14$ TeV.

for the total energy of the proton-proton in the center-of-mass system equal to $\sqrt{s} = 1.8\,$TeV are present. The application of the master formula (15) to the process (31) gives a more complicated result since we need to take into account a few subprocesses [21]

$$\begin{aligned}
\frac{d\sigma(q\bar{q}\to q\bar{q})}{d\cos\hat{\theta}} &= \frac{\alpha_s^2}{9\hat{s}}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} + \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - \frac{2\hat{u}^2}{3\hat{s}\hat{t}}\right],\\
\frac{d\sigma(q\bar{q}'\to q\bar{q}')}{d\cos\hat{\theta}} &= \frac{\alpha_s^2}{9\hat{s}} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2},\\
\frac{d\sigma(gg\to q\bar{q})}{d\cos\hat{\theta}} &= \frac{\alpha_s^2}{24\hat{s}}(\hat{t}^2 + \hat{u}^2)\left(\frac{1}{\hat{t}\hat{u}} - \frac{9}{4\hat{s}^2}\right),
\end{aligned} \tag{32}$$

where $\hat{\theta}$ is the angle between the direction of motion of the initial parton and the momentum of the final quark, and invariants $\hat{s}, \hat{t}, \hat{u}$ are defined in the same manner as in Eq. (24). The factorization scheme for this process is shown in Fig. 2(b).

The comparison of the angular distribution of jet momenta in the proton-proton scattering from Eq. (32)

$$F_{qq}(\theta) = \frac{1}{\sigma_{\rm tot}} \frac{d\sigma_{pp\to 6j}}{d\cos\theta} \tag{33}$$

(where $\sigma_{\rm tot}$ is the total cross section of six-jet production) with the experimental results (see Fig. 6 in Ref. [18]) is shown in Fig. 5. To compare to the experimental results we need to

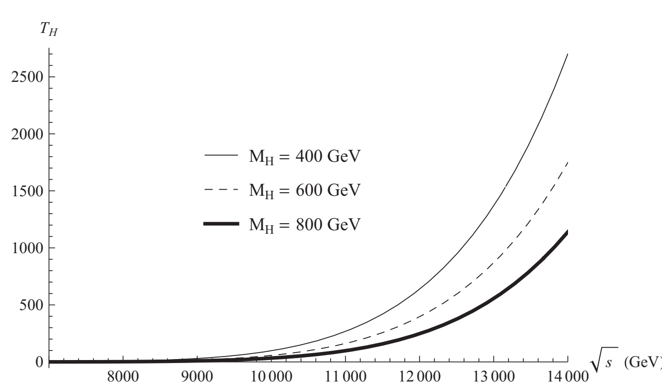

FIG. 4. The quantity T_H defined in Eq. (30) as a function of the total invariant mass s.

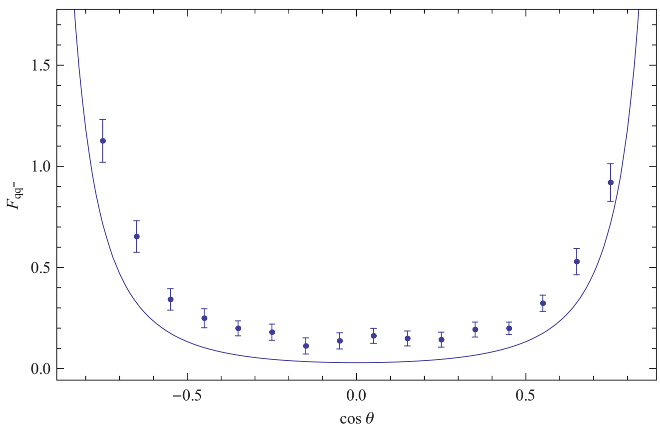

FIG. 5. (Color online) The angular dependence of the quantity F_{qq} defined in Eq. (33) is compared to the experimental data [18] for the six-jet production in the proton-proton collision with the total center-of-mass energy $\sqrt{s} = 1.8$ TeV.

take into account the same kinematical cuts that were used in the experimental analysis. In our rough estimation we use only one cut on the total invariant mass of the produced six jets, i.e., we select the threshold invariant mass of the order of $\sqrt{s_{\text{th}}} \sim 1$ TeV.

VI. CONCLUSION

In this paper, we discuss some modification of the method of taking into account the QCD leading logarithm radiative corrections based on the structure function approach. The modification consists in the construction of a set of evolution equations for the density of the parton of sort a in the initial quark $D^a(x,L)$ and the density of the parton of sort b to be in the initial gluon $G^b(x,L)$. This set of equations is solved analytically in the quark and gluon dominance approximation $D^q \gg D^a$, $a \neq q$ and $G^g \gg G^a$, $a \neq g$. This approximation can be improved for the accuracy level, which is required, by using the iteration procedure. This assumption is known as the gluon dominance which is used in the description of multiplicity of π mesons in the hadron collisions [10–12].

We present the approximate solution for D^a, G^a and demonstrate the application of this function to the problem of calculation of QCD radiative correction calculation in some particular processes.

In the literature, some efforts were made to calculate the subprocess cross section in the next-to-leading approximation. The main attention in Refs. [22,23] was paid to the two-loops level contributions. As a result, the terms of the order $(\alpha_s L^2)$, $(\alpha_s L^2)^2$ were taken into account. However, the emission of real (soft and hard) gluons with the one-loop radiative corrections was not considered. The role of radiative (virtual and real) corrections leads to the change of the $(\alpha L^2)^n$ regime to a single-logarithmical regime [i.e., only terms $\sim(\alpha L)^n$ remain] in the inclusive experimental approach. The single-logarithmical approach is determined by the renormalization group evolution equations, and thus allows us to use the structure function approach to obtain the cross section in the leading [i.e., $(\alpha L)^n$] and next-to-leading [i.e., $\alpha(\alpha L)^n$] approximation. Further improvement can be made by using the DGLAP equation kernels $P_{ij}(z)$ calculated in the next order of perturbation theory. This would also modify the solutions of Eqs. (7) and (8), which are also present in Ref. [15]. Thus the main advantage of our approach is that one can easily take into account a specific order of perturbation theory of QCD evolution from the soft to the hard scale.

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APPENDIX: PARTON DENSITIES IN THE PROTON

Keeping in mind the problem of the description of inelastic processes in the high energy collision of protons, it seems natural to consider protons as an objects with definite contents from quarks and gluons. This implies the presence of the preliminary evolution from the mass shell to virtuality of the order 1 GeV^2 of all the constituents of the proton.

Note that due to the condition $x_1 x_2 > z_{\text{th}}$ only valence quarks and gluons inside the proton take part in the process. We will choose the density of the valence quarks and gluons approximately as found in Ref. [16]

$$\begin{aligned} xW_u(x) &= A_u x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u x); \\ xW_d(x) &= A_d x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d x); \\ xW_g(x) &= A_g x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) \\ &\quad + A_{g'} x^{\delta_{g'}}(1-x)\eta_{g'}, \end{aligned} \tag{A1}$$

where some numerical fitting parameters are

$$\begin{aligned} &A_u = 0.2; \quad \eta_1 = -0.73; \quad \eta_2 = 3.3; \\ &A_d = 18; \quad \eta_3 = 0{,}1; \quad \eta_4 = 6; \\ &A_g = 0.0012216. \end{aligned}$$

Numerical constants were chosen so as to satisfy the constrains from the number sum rules

$$\int_0^1 dx W_u(x) = 2; \quad \int_0^1 dx W_d(x) = 1, \tag{A2}$$

and also the momentum sum rule

$$\int_0^1 dx x[W_u(x) + W_d(x) + W_g(x) + S(x)] = 1, \tag{A3}$$

with $S(x)$ being the sea contribution, where $\gamma_u = 8.9924$, $\gamma_d = 7.4730$, $\eta_g = 2.3882$, $\eta_{g'} = 0$, $A_{g'} = 0$, $\delta_g = -0.83657$, $\delta_{g'} = 0$, $\gamma_g = 1445.5$, $\epsilon_u = -2.3737$, $\epsilon_d = -4.3654$, and $\epsilon_g = -38.997$. More complicated expressions for the densities, which were extracted from the description of the fixed target HERA and Tevatron experiments, are presented in Ref. [16].

[1] R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).

[2] H. D. Politzer, Phys. Rep. **14**, 129 (1974).

[3] J. C. Collins, D. E. Soper, and G. F. Sterman, Adv. Ser. Direct. High Energy Phys. **5**, 1 (1988).

[4] C. F. von Weizsäcker, Z. Phys. **88**, 612 (1934).

[5] E. J. Williams, Phys. Rev. **45**, 729 (1934).

[6] P. Kessler, Nuovo Cimento X. Ser. **17**, 809 (1960).

[7] V. N. Baier, V. S. Fadin, and V. A. Khoze, Nucl. Phys. B **65**, 381 (1973).

[8] G. Altarelli and G. Parisi, Nucl. Phys. B **126**, 298 (1977).

[9] L. N. Lipatov, Yad. Fiz. **20**, 181 (1974) [Sov. J. Nucl. Phys. **20**, 94 (1975)].

[10] E. Kokoulina, Acta Phys. Polon. B **35**, 295 (2004).

[11] E. S. Kokoulina and V. A. Nikitin, arXiv:hep-ph/0502224.

[12] P. F. Ermolov *et al.*, arXiv:hep-ph/0503254.

[13] A. Bukhvostov, G. Frolov, L. Lipatov, and E. Kuraev, Nucl. Phys. B **258**, 601 (1985).

[14] A. B. Arbuzov, V. V. Bytev, E. A. Kuraev, E. Tomasi-Gustafsson, and Y. M. Bystritskiy, Phys. Part. Nucl. **41**, 394 (2010).

[15] E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. **41**, 466 (1985).

[16] A. Martin, W. Stirling, R. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).

[17] J. Kodaira and L. Trentadue, Phys. Lett. B **112**, 66 (1982).

[18] F. Abe *et al.* (CDF Collaboration), Phys. Rev. D **56**, 2532 (1997).

[19] R. M. Barnett, H. E. Haber, and D. E. Soper, Nucl. Phys. B **306**, 697 (1988).

[20] N. Kidonakis, J. High Energy Phys. 05 (2005) 011.

[21] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[22] N. Kidonakis, Phys. Rev. D **64**, 014009 (2001).

[23] N. Kidonakis and R. Vogt, Phys. Rev. D **68**, 114014 (2003).