Breakup and finite-range effects on the ^{8}B$(d,n)^9$C reaction

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The astrophysical factor of ^{8}B$(p,\gamma)^9$C at zero energy, $S_{18}(0)$, is determined by a three-body coupled-channels analysis of the transfer reaction ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon. Effects of the breakup channels of d and ^{9}C are investigated with the continuum-discretized coupled-channels method. It is found that, in the initial and final channels, respectively, the transfer process through the breakup states of d and ^{9}C, its interference with that through their ground states in particular, gives a large increase in the transfer cross section. The finite-range effects with respect to the proton-neutron relative coordinate are found to be about 20%. As a result of the present analysis, $S_{18}(0) = 22 \pm 6$ eV b is obtained, which is smaller than the result of the previous distorted-wave Born approximation analysis by about 51%.

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I. INTRODUCTION

The explosive hydrogen burning called the hot pp chain [1] in low-metallicity supermassive stars plays an important role as a possible alternative path to the synthesis of the CNO elements. The proton capture reaction of ^{8}B, ^{8}B$(p,\gamma)^9$C, is expected to lead to this hot pp chain. Since it is very difficult to measure the cross section $\sigma_{p\gamma}$ for the ^{8}B$(p,\gamma)^9$C reaction at stellar energies, several experiments of alternative reactions such as the inclusive [2] and exclusive [3] ^{9}C breakup reactions and the proton transfer reaction ^{8}B$(d,n)^9$C [4] have been done to determine the astrophysical factor

$$S_{18}(\varepsilon_{p\mathrm{B}}) = \sigma_{p\gamma}\varepsilon_{p\mathrm{B}} \exp[2\pi\eta]. \quad (1)$$

Here, $\varepsilon_{p\mathrm{B}}$ is the relative energy of the p-^{8}B system in the center-of-mass (c.m.) frame and η is the Sommerfeld parameter. Because of the weak $\varepsilon_{p\mathrm{B}}$ dependence of $S_{18}(\varepsilon_{p\mathrm{B}})$, its value at zero energy, $S_{18}(0)$, is paid special attention as a reference value.

A problem with the results of the indirect measurements of $S_{18}(0)$ is that they are not consistent with each other, with values of 46 ± 6 eV b (from inclusive ^{9}C breakup [2]), 77 ± 15 eV b (from exclusive ^{9}C breakup [3]), and 45 ± 13 eV b (from transfer [4]). In Ref. [5], reanalysis of the two ^{9}C breakup reactions has been performed with a three-body coupled-channels reaction model, and $S_{18}(0) = 66 \pm 10$ eV b was obtained, resolving the discrepancy between the two results of ^{9}C breakup. There remains, however, about a 30% difference between the result of Ref. [5] and that of the transfer reaction. It was reported in Ref. [6] that, in the ^{7}Be$(d,n)^8$B reaction at 7.5 MeV, breakup channels of d played an essential role. One may expect a similar effect also in the ^{8}B$(d,n)^9$C reaction.

The purpose of the present study is to investigate the deuteron breakup effects on the cross section of ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon and $S_{18}(0)$, by means of the continuum-discretized coupled-channels method (CDCC) [7–9]. In the CDCC method, one nonperturbatively treats the channel couplings of the breakup (continuum) states of weakly bound nuclei, and the method has been highly successful in describing various real or virtual breakup reactions in a wide range of incident energies. The theoretical foundation of the CDCC method is given in Refs. [9–11]. As an advantage over the previous CDCC study on ^{7}Be$(d,n)^8$B [6], in this work the breakup channels of both the "projectile" d, the target nucleus in inverse kinematics, and the residual nucleus ^{9}C are taken into account. Furthermore, a finite-range (FR) calculation of the transition matrix (T matrix) of the transfer reaction is performed. We also propose a finite-range correction (FRC) to the zero-range (ZR) calculation, which is appropriate for three-body model calculation including breakup channels of both the projectile and the residual nucleus. Interpretation of the FR effects on $S_{18}(0)$ is given through this correction.

This paper is constructed as follows. In Sec. II, we give a formulation of the coupled-channels Born approximation (CCBA) for the ^{8}B$(d,n)^9$C reaction. In Sec. III, we extract $S_{18}(0)$ from the transfer cross section; the role of the breakup channels of d and ^{9}C are discussed. The formalism of the FRC for the three-body reaction model and discussion of the FR effects on the transfer cross section are also given. Finally, we summarize this study in Sec. IV.

II. COUPLED-CHANNELS BORN APPROXIMATION FORMALISM

In the present study we describe the transfer reaction ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon with the three-body ($p + n + ^8$B) model shown in Fig. 1. The transition matrix in the post form is given by

$$T_{\beta\alpha} = \langle \Psi_\beta^{(-)} | V_{pn} | \Psi_\alpha^{(+)} \rangle, \quad (2)$$

where $\Psi_\alpha^{(+)}$ and $\Psi_\beta^{(-)}$ are, respectively, three-body wave functions for the initial and final channels; their explicit definition is given below. The interaction between p and n, V_{pn}, is adopted as the transition interaction that causes the transfer process. The superscripts $(+)$ and $(-)$ represent the

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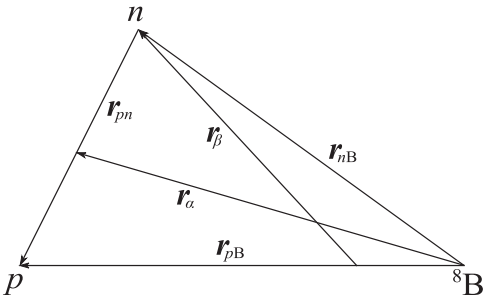

FIG. 1. Illustration of the three-body system.

outgoing and incoming boundary conditions for the scattering wave, respectively.

The Schrödinger equation for $\Psi_\alpha^{(+)}$ is given by

$$[H_\alpha - E]\,\Psi_\alpha^{(+)}(\boldsymbol{r}_{pn},\boldsymbol{r}_\alpha) = 0, \tag{3}$$

$$H_\alpha = K_{\boldsymbol{r}_\alpha} + h_{pn} + U_{p\mathrm{B}}^{(\alpha)}(r_{p\mathrm{B}}) + U_{n\mathrm{B}}^{(\alpha)}(r_{n\mathrm{B}}) + V_{\mathrm{C}}(r_\alpha), \tag{4}$$

where $K_{\boldsymbol{X}}$ is the kinetic energy operator with respect to the coordinate $\boldsymbol{X}$, h_{pn} is the internal Hamiltonian of d, and E is the total energy of the three-body system. The nuclear interaction between x ($= p$ or n) and ^{8}B is represented by $U_{x\mathrm{B}}$ with the superscript (α) specifying the initial channel. The Coulomb interaction between d and ^{8}B is denoted by V_{C}; we disregard the Coulomb breakup in this study. We describe $\Psi_\alpha^{(+)}$ with CDCC as

$$\Psi_\alpha^{(+)}(\boldsymbol{r}_{pn},\boldsymbol{r}_\alpha) \approx \sum_i \psi_{pn}^i(\boldsymbol{r}_{pn})\chi_\alpha^{ii_0(+)}(\boldsymbol{r}_\alpha), \tag{5}$$

where ψ_{pn}^i is the internal wave function of d with i its energy index; $i = i_0$ corresponds to the ground state of d and $i \neq i_0$ to the discretized continuum states of the p-n system. ψ_{pn}^i satisfies

$$\left(h_{pn} - \varepsilon_{pn}^i\right)\psi_{pn}^i(\boldsymbol{r}_{pn}) = 0, \tag{6}$$

where ε_{pn}^i is the energy eigenvalue of the p-n system. One may obtain the d-^{8}B distorted wave $\chi_\alpha^{ii_0(+)}$ by solving the CDCC equations under the standard boundary condition [7–9]. Note that, in the present study, we ignore the intrinsic spin of each particle for simplicity. Details of the description of $\Psi_\alpha^{(+)}$ with CDCC are given in Ref. [12].

In the exact form of Eq. (2), $\Psi_\alpha^{(+)}$ includes not only the deuteron components, consisting of the elastic and breakup ones, but also rearrangement components. The latter are not explicitly taken into account in the present CCBA calculation, which has been justified in Refs. [10,11].

The three-body wave function $\Psi_\beta^{(+)}$ in the final channel, which is the time reversal of $\Psi_\beta^{(-)}$, satisfies the following Schrödinger equation:

$$[H_\beta - E]\Psi_\beta^{(+)}(\boldsymbol{r}_{p\mathrm{B}},\boldsymbol{r}_\beta) = 0, \tag{7}$$

$$H_\beta = K_{\boldsymbol{r}_\beta} + h_{p\mathrm{B}} + U_{n\mathrm{B}}^{(\beta)}(r_{n\mathrm{B}}), \tag{8}$$

where $h_{p\mathrm{B}}$ is the p-^{8}B internal Hamiltonian given by

$$h_{p\mathrm{B}} = K_{\boldsymbol{r}_{p\mathrm{B}}} + U_{p\mathrm{B}}^{(\beta)}(\boldsymbol{r}_{p\mathrm{B}}) + V_{\mathrm{C}}(r_{p\mathrm{B}}). \tag{9}$$

The superscript (β) represents the final channel. Note that H_β does not contain the term V_{pn} that has been used as a transition interaction in Eq. (2). In the CDCC framework $\Psi_\beta^{(+)}$ is expressed by

$$\Psi_\beta^{(+)}(\boldsymbol{r}_{p\mathrm{B}},\boldsymbol{r}_\beta) \approx \sum_j \psi_{p\mathrm{B}}^j(\boldsymbol{r}_{p\mathrm{B}})\chi_\beta^{jj_0(+)}(\boldsymbol{r}_\beta), \tag{10}$$

where

$$\left(h_{p\mathrm{B}} - \varepsilon_{p\mathrm{B}}^j\right)\psi_{p\mathrm{B}}^j(\boldsymbol{r}_{p\mathrm{B}}) = 0 \tag{11}$$

with $\psi_{p\mathrm{B}}^j$ the overlap functions of the ground and discretized continuum states of ^{9}C with the p-^{8}B(g.s.) configuration; here the ground state is denoted by $j = j_0$ and $\varepsilon_{p\mathrm{B}}^j$ is the eigenenergy of ^{9}C in the jth state. The n-^{9}C distorted wave $\chi_\beta^{jj_0(+)}$ can be calculated with the same procedure as for $\chi_\alpha^{ii_0(+)}$. Since the ground state of ^{9}C includes the component that cannot be described by the p-^{8}B(g.s.) configuration, $\psi_{p\mathrm{B}}^{j_0}$ has to be normalized by the square root of the spectroscopic factor $\mathcal{S}$. The breakup components $\psi_{p\mathrm{B}}^j$ ($j \neq j_0$) also have to be normalized by the same factor $\sqrt{\mathcal{S}}$, because

$$\begin{aligned}\Psi_\beta^{(+)}(\boldsymbol{r}_{p\mathrm{B}},\boldsymbol{r}_\beta) &= \lim_{\epsilon\to+0}\frac{i\epsilon}{E - H_\beta + i\epsilon}e^{i\boldsymbol{k}_\beta\cdot\boldsymbol{r}_\beta}\sqrt{\mathcal{S}}\psi_{p\mathrm{B}}^{j_0}(\boldsymbol{r}_{p\mathrm{B}}) \\ &= \sqrt{\mathcal{S}}\lim_{\epsilon\to+0}\frac{i\epsilon}{E - H_\beta + i\epsilon}e^{i\boldsymbol{k}_\beta\cdot\boldsymbol{r}_\beta}\psi_{p\mathrm{B}}^{j_0}(\boldsymbol{r}_{p\mathrm{B}});\end{aligned} \tag{12}$$

note that the $\psi_{p\mathrm{B}}^j$ ($j \neq j_0$) are generated by the Møller wave operator $i\epsilon/(E - H_\beta + i\epsilon)$. Here, $\mathcal{S}$ has only one quantum number, i.e., $\ell = 1$ for the orbital angular momentum between p and ^{8}B(g.s.) in the ground state of ^{9}C. This is due to the neglect of the intrinsic spin of each particle in the present study. Thus $\mathcal{S}$ is understood as an averaged value of the $\mathcal{S}$'s, each with a different value of the total angular momentum of the p-^{8}B(g.s.) system.

III. RESULTS AND DISCUSSION

A. Model setting

We adopt the one-range Gaussian interaction [13] for V_{pn}. The pseudostate method with the real-range Gaussian basis functions [14] is used for obtaining the discretized-continuum states of d; we include the s and d states and neglect the intrinsic spin of d. The number of basis functions taken is 20, and the minimum (maximum) range parameter of the Gaussian is 1.0 (30.0) fm. We include in the CDCC pseudostates with $\varepsilon_{pn}^i < 65$ MeV and $\varepsilon_{pn}^i < 80$ MeV for the s and d states, respectively. To obtain $\Psi_\alpha^{(+)}$, ψ_{pn}^i is calculated up to $r_{pn} = 100.0$ fm.

In the calculation of $\psi_{p\mathrm{B}}^j$ in the final channel, we adopt a Woods-Saxon central potential as $U_{p\mathrm{B}}^{(\beta)}$ with radial parameter $R_0 = 1.25 \times 8^{1/3}$ fm and diffuseness parameter $a_0 = 0.65$ fm. Its depth is determined to reproduce the proton separation energy of 1.30 MeV in the p state. The interaction between a point charge and a uniformly charged sphere with the charge radius 2.5 fm is used as V_{C}, which is used also in

the CDCC calculation in the initial channel. The pseudostate method is also used for the final channel. For the expansion of $\psi^{j}_{p\mathrm{B}}$ we take 20 Gaussian basis functions with the minimum (maximum) range parameter of 1.0 (20.0) fm. We take into account the s, p, d, f, and g waves of $\psi^{j}_{p\mathrm{B}}$ with maximum values of $\varepsilon^{j}_{p\mathrm{B}}$ of 70, 75, 85, 90, and 70 MeV, respectively. $\psi^{j}_{p\mathrm{B}}$ is calculated up to $r_{p\mathrm{B}} = 100.0$ fm.

For $U^{(\alpha)}_{p\mathrm{B}}$, $U^{(\alpha)}_{n\mathrm{B}}$, and $U^{(\beta)}_{n\mathrm{B}}$, we adopt the nucleon global optical potential for p-shell nuclei by Watson *et al.* [15] (WA). The nonlocal correction proposed by Timofeyuk and Johnson [16–18] (TJ) to the nucleon distorting potentials of the initial channel is used. The calculated energy shift [16–18] with the above-mentioned p-n model is 17.8 MeV in the c.m. frame. We thus evaluate $U^{(\alpha)}_{p\mathrm{B}}$ and $U^{(\alpha)}_{n\mathrm{B}}$ at 33.0 MeV in the laboratory frame, which is shifted from the incident energy of 14.4 MeV/nucleon. The nonlocal correction to $U^{(\beta)}_{n\mathrm{B}}$ is made following Perey and Buck [19] with the nonlocal parameter $\beta = 0.85$ fm.

For describing the transfer reaction, Eq. (2) is integrated over $\boldsymbol{r}_\alpha$ and $\boldsymbol{r}_\beta$ up to 25.0 and 20.0 fm, respectively. The number of partial waves for $\chi^{ii_0(+)}_{\alpha}$ and $\chi^{jj_0(-)}_{\beta}$ is 25. As mentioned above, we include only the s states of ψ^{i}_{pn}, consisting of the ground and discretized-continuum states, in the calculation of the T matrix of the transfer process. It should be noted that the coupling between the s and d states of ψ^{i}_{pn} is taken into account in the calculation of $\Psi^{(+)}_{\alpha}$ with the CDCC method. It is found that D^{i}_{pn} defined below by Eq. (22) is negligibly small for the d states of the deuteron, which justifies their neglect in the transfer process.

B. Asymptotic normalization coefficient and astrophysical factor $S_{18}(0)$

We show in Fig. 2 the cross section of the transfer reaction ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon as a function of the neutron emission angle in the c.m. frame. The solid line shows the CCBA result. We have normalized the result to reproduce the experimental data [4] multiplied by $\mathcal{S} = 0.361$. Note that, from the present transfer reaction, $\mathcal{S}$ cannot be determined because the reaction is peripheral, as will be confirmed below. Instead, the asymptotic normalization coefficient (ANC) [5,12,20] $C^{^9\mathrm{C}}_{p^8\mathrm{B}}$ for the overlap of the ^{9}C wave function with the p-^{8}B(g.s.) configuration is well determined. From $\mathcal{S}$ and the so-called single-particle ANC of $\psi^{j_0}_{p\mathrm{B}}$, one can obtain the ANC: $(C^{^9\mathrm{C}}_{p^8\mathrm{B}})^2 = 0.59$ fm^{-1}.

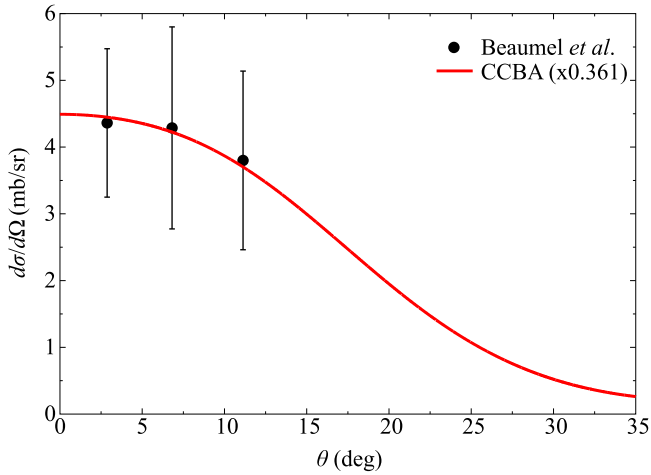

FIG. 2. (Color online) Cross section of the transfer reaction ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon as a function of the neutron emission angle in the c.m. frame. The result of the CCBA (solid line) is normalized to the experimental data [4].

The accuracy of the value of the ANC depends on how the transfer reaction ^{8}B$(d,n)^9$C is peripheral with respect to $r_{p\mathrm{B}}$. This can be examined by estimating the dependence of $C^{^9\mathrm{C}}_{p^8\mathrm{B}}$ on the parameters of $U^{(\beta)}_{p\mathrm{B}}$; each of R_0 and a_0 is changed by 20%. As mentioned above, we put a constraint on the depth of the potential so that the proton separation energy is reproduced. It is found that, by this change of R_0 and a_0, $(C^{^9\mathrm{C}}_{p^8\mathrm{B}})^2$ varies by only 2%, which indicates the peripherality of the transfer reaction and guarantees the reliability of $C^{^9\mathrm{C}}_{p^8\mathrm{B}}$.

Uncertainty due to the distorting potential is estimated by using another nucleon global potential set for p-shell nuclei. We adopt the parameter set by Dave and Gould [21] (DG). Since the incident energy corrected with the TJ prescription for nonlocality, 33.0 MeV, is out of the range of the DG parametrization, we see the difference between the values of ANC calculated with WA and DG potentials, both without the nonlocal correction. As a result, the uncertainty of the ANC coming from the optical potential is found to be 3%.

By compiling the uncertainties due to peripherality (2%) and the optical potential (3%) as well as the experimental error of 22% [4], we obtain $(C^{^9\mathrm{C}}_{p^8\mathrm{B}})^2 = 0.59 \pm 0.02(\mathrm{theor.}) \pm 0.13(\mathrm{exp.})$ fm^{-1}, where (theor.) and (exp.), respectively, stand for the theoretical and experimental uncertainties. Using the proportionality of $(C^{^9\mathrm{C}}_{p^8\mathrm{B}})^2$ to $S_{18}(0)$, we have

$$S_{18}(0) = 22 \pm 1(\mathrm{theor.}) \pm 5(\mathrm{exp.})\ \mathrm{eV\,b}. \tag{13}$$

C. Breakup effects of d and ^{9}C on transfer cross section

The result for $S_{18}(0)$ in the present study, 22 ± 6 eV b, is somewhat smaller than the result from the previous analysis (45 ± 13 eV b) extracted from the same experimental data [4] with the distorted-wave Born approximation (DWBA), which does not explicitly take into account the breakup states of nuclei. In this section we discuss this difference in view of the breakup effects of d and ^{9}C in the transfer reaction. In Fig. 3, we show by the thick (thin) solid line the cross section of ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon calculated with (without) the breakup states of both d and ^{9}C. Inclusion of the breakup channels gives a large increase of about 58% in the cross section at 0°.

To see this in more detail, we decompose the T matrix into

$$T_{\beta\alpha} = T_{\beta(\mathrm{el}),\alpha(\mathrm{el})} + T_{\beta(\mathrm{el}),\alpha(\mathrm{br})} + T_{\beta(\mathrm{br}),\alpha(\mathrm{el})} + T_{\beta(\mathrm{br}),\alpha(\mathrm{br})}, \tag{14}$$

$$T_{\beta(\mathrm{el}),\alpha(\mathrm{el})} \equiv \left\langle \psi^{j_0}_{p\mathrm{B}} \chi^{j_0 j_0(-)}_{\beta} \middle| V_{pn} \middle| \psi^{i_0}_{pn} \chi^{i_0 i_0(+)}_{\alpha} \right\rangle, \tag{15}$$

$$T_{\beta(\mathrm{el}),\alpha(\mathrm{br})} \equiv \left\langle \psi^{j_0}_{p\mathrm{B}}\chi^{j_0 j_0(-)}_{\beta}\middle|V_{pn}\middle|\sum_{i\neq i_0}\psi^{i}_{pn}\chi^{i i_0(+)}_{\alpha}\right\rangle, \quad (16)$$

$$T_{\beta(\mathrm{br}),\alpha(\mathrm{el})} \equiv \left\langle \sum_{j\neq j_0}\psi^{j}_{p\mathrm{B}}\chi^{j j_0(-)}_{\beta}\middle|V_{pn}\middle|\psi^{i_0}_{pn}\chi^{i_0 i_0(+)}_{\alpha}\right\rangle, \quad (17)$$

$$T_{\beta(\mathrm{br}),\alpha(\mathrm{br})} \equiv \left\langle \sum_{j\neq j_0}\psi^{j}_{p\mathrm{B}}\chi^{j j_0(-)}_{\beta}\middle|V_{pn}\middle|\sum_{i\neq i_0}\psi^{i}_{pn}\chi^{i i_0(+)}_{\alpha}\right\rangle. \quad (18)$$

The T matrix with the subscript $\gamma(\mathrm{el})$ and $\gamma(\mathrm{br})$ corresponds to the elastic transfer (ET) and the breakup transfer (BT) in the γ channel, respectively. The dash-dotted line in Fig. 3 shows the cross section due to the ET described by $T_{\beta(\mathrm{el}),\alpha(\mathrm{el})}$. Note that $T_{\beta(\mathrm{el}),\alpha(\mathrm{el})}$ includes the breakup effects as the back-coupling between the elastic channel and the breakup channels for both d and ^{9}C. However, the small difference between the thin solid line and the dash-dotted line indicates that those back-coupling effects are not significant in the present case. The dashed line shows the result including the breakup states of only d, which is about 23% larger than that shown by the thin solid line at 0°. It is also found that the transfer cross section through the breakup states of d is less than 1% of that shown by the dashed line. We thus conclude that the increase in the cross section caused by the breakup states of d is due to the interference between $T_{\beta(\mathrm{el}),\alpha(\mathrm{el})}$ and $T_{\beta(\mathrm{el}),\alpha(\mathrm{br})}$. This conclusion holds also for the role of the breakup states of ^{9}C; large interference between $T_{\beta(\mathrm{el}),\alpha(\mathrm{el})}$ and $T_{\beta(\mathrm{br}),\alpha(\mathrm{el})}$ increases the cross section by about 38% at 0°, as shown by the dotted line. Furthermore, it is found numerically that the contribution of $T_{\beta(\mathrm{br}),\alpha(\mathrm{br})}$ to the cross section is negligibly small.

These properties of the numerical result can be understood as follows. If we make the adiabatic approximation [22–24] to

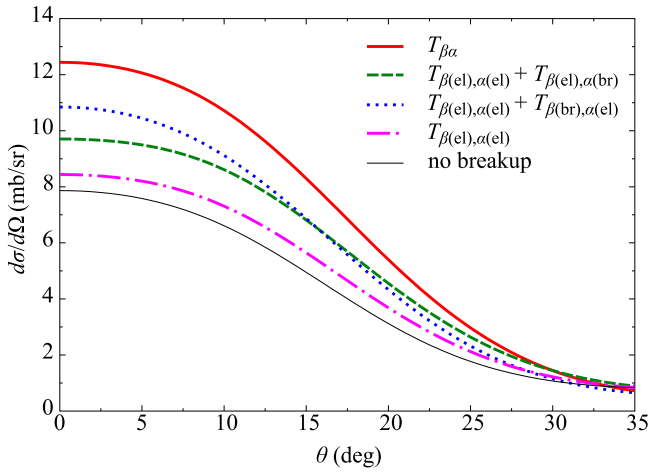

FIG. 3. (Color online) Breakup effects of d and ^{9}C on the cross section of ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon. The thick solid and thin solid lines show, respectively, the results with and without the breakup states of both d and ^{9}C. The dashed (dotted) line represents the result with the breakup states of d (^{9}C) in the transition matrix $T_{\beta\alpha}$ being neglected. The cross section corresponding to the ET is shown by the dash-dotted line. See the text for detail.

$\Psi^{(+)}_{\alpha}$, we have

$$\Psi^{(+)}_{\alpha}(\boldsymbol{r}_{pn},\boldsymbol{r}_{\alpha}) \approx \psi^{i_0}_{pn}(\boldsymbol{r}_{pn})\chi^{\mathrm{AD}(+)}_{\alpha}(\boldsymbol{r}_{pn},\boldsymbol{r}_{\alpha}). \quad (19)$$

The adiabatic wave function $\chi^{\mathrm{AD}(+)}_{\alpha}$ satisfies

$$\left[K_{\boldsymbol{r}_{\alpha}} + U^{(\alpha)}_{p\mathrm{B}}(r_{p\mathrm{B}}) + U^{(\alpha)}_{n\mathrm{B}}(r_{n\mathrm{B}}) - E_{\alpha}\right]\chi^{\mathrm{AD}(+)}_{\alpha}(\boldsymbol{r}_{pn},\boldsymbol{r}_{\alpha}) = 0, \quad (20)$$

where $E_{\alpha} = E + \varepsilon^{i_0}_{pn}$. The $\boldsymbol{r}_{pn}$ dependence of $U^{(\alpha)}_{N\mathrm{B}}$ ($N = p$ or n) gives that of $\chi^{\mathrm{AD}(+)}_{\alpha}$. Consequently, $\Psi^{(+)}_{\alpha}$ contains not only the elastic-channel but also the breakup-channel components:

$$\chi^{i i_0\mathrm{AD}(+)}_{\alpha}(\boldsymbol{r}_{\alpha}) \equiv \left\langle \psi^{i}_{pn}(\boldsymbol{r}_{pn})\middle|\psi^{i_0}_{pn}(\boldsymbol{r}_{pn})\chi^{\mathrm{AD}(+)}_{\alpha}(\boldsymbol{r}_{pn},\boldsymbol{r}_{\alpha})\right\rangle. \quad (21)$$

The $\boldsymbol{r}_{pn}$ dependence of $U^{(\alpha)}_{N\mathrm{B}}$ is, however, quite weak within the range of V_{pn}. Then one can expect that, for $\chi^{i i_0\mathrm{AD}(+)}_{\alpha}$ with $i \neq i_0$, the amplitude would be much smaller than that of $\chi^{i_0 i_0\mathrm{AD}(+)}_{\alpha}$ and the phase would be very similar to that of $\chi^{i_0 i_0\mathrm{AD}(+)}_{\alpha}$. The former is the reason for the very small contribution of the BT and the latter is that for the constructive interference between the ET and BT amplitudes. These properties have been confirmed numerically. This interpretation of the breakup effects can also be applied to $\Psi^{(-)}_{\beta}$ in the final channel. It should be noted that the adiabatic approximation [22–24] itself is found to work well; it makes $C^{^9\mathrm{C}}_{p^8\mathrm{B}}$ smaller by about 6% (12%) when applied to $\Psi^{(+)}_{\alpha}$($\Psi^{(-)}_{\beta}$).

As mentioned above, the back-coupling effects are found to be small in the present case. In fact, if we evaluate $C^{^9\mathrm{C}}_{p^8\mathrm{B}}$ and $S_{18}(0)$ from the thin solid line, we obtain $(C^{^9\mathrm{C}}_{p^8\mathrm{B}})^2 = 0.95$ fm^{-1} and $S_{18}(0) = 36$ eV b. This value is, within only about 2% difference, consistent with the result corresponding to the D1-N1 set for the distorting potentials, $(C^{^9\mathrm{C}}_{p^8\mathrm{B}})^2 = 0.97$ fm^{-1}, shown in Table 1 of Ref. [4]; N1 corresponds to the WA potential. We have confirmed by our DWBA calculation that the result with the D1-N1 set agrees well with the thin solid line in Fig. 3. From these findings we conclude that inclusion of the breakup states of both d and ^{9}C is necessary to accurately describe the transfer reaction, which gives quite a large increase in the cross section, that is, decrease in $S_{18}(0)$.

The non-negligible BT component in each channel is opposite to what was found in the analysis [12] of ^{13}C(^{6}Li,d)^{17}O below the Coulomb barrier energy, in which breakup effects of ^{6}Li ($= \alpha + d$) were investigated. Below we discuss the difference between the breakup properties of d and ^{6}Li in the two reactions. The origin of the difference can be understood from the behavior of D^{i}_{pn} defined by

$$D^{i}_{pn}(r_{pn}) = V_{pn}(r_{pn})\phi^{i}_{pn}(r_{pn}), \quad (22)$$

where ϕ^{i}_{pn} is the radial part of ψ^{i}_{pn}. We show in Fig. 4(a) D^{i}_{pn} for some s-wave eigenstates of d; the eigenvalue ε^{i}_{pn} is given in the legend. Similarly, we plot in Fig. 4(b) $D^{i}_{\alpha d}(r_{\alpha d}) = V_{\alpha d}(r_{\alpha d})\phi^{i}_{\alpha d}(r_{\alpha d})$ for the α-d system; the two-range Gaussian interaction $V_{\alpha d}$ given in Ref. [25] is adopted to generate the radial part $\phi^{i}_{\alpha d}$ of the s-wave eigenstate $\psi^{i}_{\alpha d}$.

In Figs. 4(a) and 4(b), respectively, D^{i}_{pn} and $D^{i}_{\alpha d}$ for some eigenstates are plotted. One sees that the amplitude of D^{i}_{pn} for breakup states (the dashed and dotted lines) are comparable

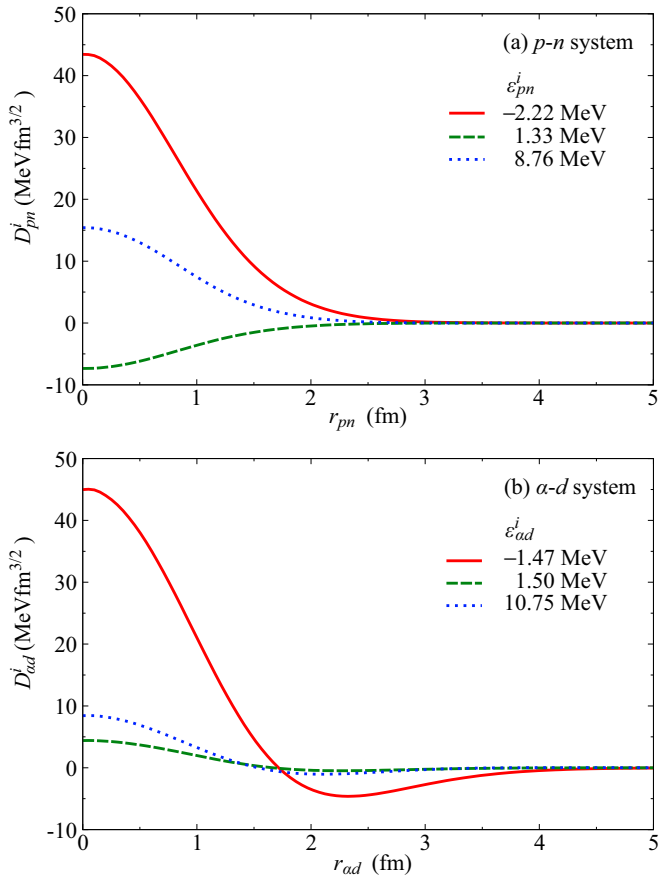

FIG. 4. (Color online) (a) D^i_{pn} for several ith states with the eigenenergy ε^i_{pn}. (b) Same as in panel (a) but for the α-d system.

to that of $D^{i_0}_{pn}$ (solid line). On the other hand, $D^i_{\alpha d}$ for the breakup states are much smaller than $D^{i_0}_{\alpha d}$, which is found to be due to the Coulomb interaction between α and d. Thus, the difference in the BT components between the ^{8}B$(d,n)^9$C and ^{13}C$(^6$Li$,d)^{17}$O reactions can be understood. It should be noted that a large value of D^i for a breakup state does not necessarily give a large BT cross section, because even in this case $\chi^{ii_0}_\alpha$ can be small as a result of the channel couplings. Furthermore, the importance of the back-coupling effect depends on the reaction system in a nontrivial manner.

D. Formalism of finite-range correction for CCBA transition amplitude and finite-range effect on transfer cross section

In this section we describe a procedure for an FRC to the ZR CCBA transition matrix. The essence of this correction is similar to that given in Ref. [26], except that the present method is based on a three-body reaction model including continuum states of both the projectile and the residual nucleus. The integral expression of Eq. (2), with Eq. (10), is given by

$$T_{\beta\alpha} = \sum_j \int d\boldsymbol{r}_{pn} d\boldsymbol{r}_\alpha \chi^{jj_0(-)*}_\beta(\boldsymbol{r}_\beta) \psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_{p\mathrm{B}}) V_{pn}(r_{pn}) \times \Psi^{(+)}_\alpha(\boldsymbol{r}_{pn},\boldsymbol{r}_\alpha). \quad (23)$$

By using

$$\psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_{p\mathrm{B}}) = \psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_\alpha + \sigma\boldsymbol{r}_{pn}) = e^{\sigma\nabla_{\boldsymbol{r}_{p\mathrm{B}}}\cdot\boldsymbol{r}_{pn}}\psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_\alpha),$$
$$\chi^{jj_0(-)*}_\beta(\boldsymbol{r}_\beta) = \chi^{jj_0(-)*}_\beta(\tau^{-1}\boldsymbol{r}_\alpha + \xi\boldsymbol{r}_{pn}) = e^{\tau\xi\nabla_{\boldsymbol{r}_\beta}\cdot\boldsymbol{r}_{pn}}\chi^{jj_0(-)*}_\beta(\tau^{-1}\boldsymbol{r}_\alpha) \quad (24)$$

with $\sigma = 1/2$, $\tau = 9/8$, and $\xi = \sigma/\tau - 1$, Eq. (23) can be rewritten as

$$T_{\beta\alpha} = \sum_j \int d\boldsymbol{r}_{pn} d\boldsymbol{r}_\alpha e^{(\sigma\nabla_{\boldsymbol{r}_{p\mathrm{B}}} + \tau\xi\nabla_{\boldsymbol{r}_\beta})\cdot\boldsymbol{r}_{pn}} \chi^{jj_0(-)*}_\beta(\tau^{-1}\boldsymbol{r}_\alpha) \times \psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_\alpha) V_{pn}(r_{pn}) \Psi^{(+)}_\alpha(\boldsymbol{r}_{pn},\boldsymbol{r}_\alpha). \quad (25)$$

It should be noted that $\nabla_{\boldsymbol{r}_{p\mathrm{B}}}$ and $\nabla_{\boldsymbol{r}_\beta}$ operate on only $\psi^{j*}_{p\mathrm{B}}$ and $\chi^{jj_0(-)*}_\beta$, respectively.

As in Ref. [26], we use

$$e^{(\sigma\nabla_{\boldsymbol{r}_{p\mathrm{B}}} + \tau\xi\nabla_{\boldsymbol{r}_\beta})\cdot\boldsymbol{r}_{pn}} \approx 1 + \tfrac{1}{6}\left(\sigma\nabla_{\boldsymbol{r}_{p\mathrm{B}}} + \tau\xi\nabla_{\boldsymbol{r}_\beta}\right)^2 r^2_{pn}. \quad (26)$$

Here, we assume that only the s-wave component of the deuteron wave function contributes to the T matrix, which has eliminated the first-order term of the expansion series in Eq. (26); justification of this assumption is given in Sec. III A. With the local energy approximation [26], one may find

$$T_{\beta\alpha} \approx \sum_j \int d\boldsymbol{r}_{pn} d\boldsymbol{r}_\alpha \chi^{jj_0(-)*}_\beta(\tau^{-1}\boldsymbol{r}_\alpha) \psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_\alpha) V_{pn}(r_{pn}) \times \hat{F}_{\mathrm{LEA}} \Psi^{(+)}_\alpha(\boldsymbol{r}_{pn},\boldsymbol{r}_\alpha) \quad (27)$$

with

$$\hat{F}_{\mathrm{LEA}} \equiv 1 + \frac{1}{6} r^2_{pn} \frac{2\mu_{pn}}{\hbar^2}\left[U^{(\beta)}_{p\mathrm{B}}(r_{p\mathrm{B}}) + U^{(\beta)}_{n\mathrm{B}}(r_{n\mathrm{B}}) + \Delta V_{\mathrm{C}} - U^{(\alpha)}_{p\mathrm{B}}(r_{p\mathrm{B}}) - U^{(\alpha)}_{n\mathrm{B}}(r_{n\mathrm{B}}) - h_{pn}\right] \quad (28)$$

and

$$\Delta V_{\mathrm{C}} \equiv V_{\mathrm{C}}(r_{p\mathrm{B}}) - V_{\mathrm{C}}(r_\alpha), \quad (29)$$

where μ_{pn} is the reduced mass of the p-n system. Here we assume $\Delta V_{\mathrm{C}} \sim 0$. Note that, if we include the Coulomb breakup in the initial channel, $V_{\mathrm{C}}(r_\alpha)$ is replaced with $V_{\mathrm{C}}(r_{p\mathrm{B}})$, which results in $\Delta V_{\mathrm{C}} = 0$. Using $\boldsymbol{r}_{p\mathrm{B}} = \boldsymbol{r}_\alpha + \sigma\boldsymbol{r}_{pn}$ and $\boldsymbol{r}_{n\mathrm{B}} = \boldsymbol{r}_\alpha - \sigma\boldsymbol{r}_{pn}$, we make the following expansion:

$$U^{(\gamma)}_{p\mathrm{B}}(r_{p\mathrm{B}}) \approx U^{(\gamma)}_{p\mathrm{B}}(r_\alpha) + \left[\nabla_{\boldsymbol{r}_\alpha} U^{(\gamma)}_{p\mathrm{B}}(r_\alpha)\right]\cdot\sigma\boldsymbol{r}_{pn}, \quad (30)$$

$$U^{(\gamma)}_{n\mathrm{B}}(r_{n\mathrm{B}}) \approx U^{(\gamma)}_{n\mathrm{B}}(r_\alpha) - \left[\nabla_{\boldsymbol{r}_\alpha} U^{(\gamma)}_{n\mathrm{B}}(r_\alpha)\right]\cdot\sigma\boldsymbol{r}_{pn}. \quad (31)$$

The second terms of Eqs. (30) and (31) vanish after being integrated over $\boldsymbol{r}_{pn}$, because we consider only the s-wave states of ψ^i_{pn}, as mentioned above. By using Eqs. (5) and (6), we then obtain

$$T_{\beta\alpha} \approx \sum_{ij} \int d\boldsymbol{r}_\alpha \chi^{jj_0(-)*}_\beta(\tau^{-1}\boldsymbol{r}_\alpha) \psi^{j*}_{p\mathrm{B}}(\boldsymbol{r}_\alpha) \times D^i_0 F^i_{\mathrm{LEA}}(r_\alpha) \chi^{ii_0(+)}_\alpha(\boldsymbol{r}_\alpha) \quad (32)$$

with

$$F^i_{\rm LEA}(r_\alpha) \equiv 1 + \frac{\rho_i^2}{6}\frac{2\mu_{pn}}{\hbar^2}\big[U^{(\beta)}_{p\rm B}(r_\alpha) + U^{(\beta)}_{n\rm B}(r_\alpha) - U^{(\alpha)}_{p\rm B}(r_\alpha) - U^{(\alpha)}_{n\rm B}(r_\alpha) - \varepsilon^i_{pn}\big]. \quad (33)$$

In Eqs. (32) and (33) D^i_0 and ρ_i^2 are defined by

$$D^i_0 = \sqrt{4\pi}\int dr_{pn} r^2_{pn} D^i_{pn}(r_{pn}), \quad (34)$$

$$\rho_i^2 = \frac{\int dr_{pn} r^4_{pn} D^i_{pn}(r_{pn})}{\int dr_{pn} r^2_{pn} D^i_{pn}(r_{pn})}. \quad (35)$$

Thus, the integration over $\boldsymbol{r}_{pn}$ is factored out in the evaluation of the T matrix. It should be noted that the FRC function $F^i_{\rm LEA}$ does not depend on j.

If we take only the first term on the right-hand-side (r.h.s) of Eq. (33), we obtain a T matrix with the ZR approximation to D^i_{pn}:

$$D^i_{pn}(r_{pn}) = \frac{D^i_0}{\sqrt{4\pi}}\delta(r_{pn}). \quad (36)$$

Therefore, the second term on the r.h.s. of Eq. (33) is regarded as the FRC to the ZR calculation. Equations (32) and (33) give a natural extension of the FRC proposed in Ref. [26] that can be used in the CCBA formalism.

When the breakup states in the final channel are neglected as in the previous study [6], Eq. (33) reduces to

$$F^i_{\rm LEA}(r_\alpha) = 1 + \frac{\rho_i^2}{6}\frac{2\mu_{pn}}{\hbar^2}\big[U^{(\beta)}_{p\rm B}(r_\alpha) + U^{(\beta)}(\tau^{-1}r_\alpha) - U^{(\alpha)}_{p\rm B}(r_\alpha) - U^{(\alpha)}_{n\rm B}(r_\alpha) - \varepsilon^i_{pn}\big], \quad (37)$$

where $U^{(\beta)}$ is the distorting potential for the n-^{9}C scattering wave function. This expression is useful when we adopt the CDCC wave function in only the initial channel.

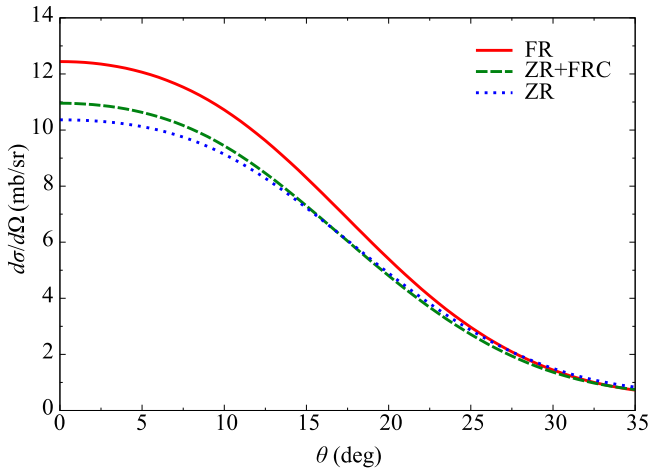

FIG. 5. (Color online) CCBA results of the FR calculation (solid line), the ZR calculation (dotted line), and the ZR calculation with the FRC (dashed line).

Further simplification of Eq. (33) can be done if $U^{(\beta)}_{p\rm B} \approx U^{(\alpha)}_{p\rm B}$ and $U^{(\beta)}_{n\rm B} \approx U^{(\alpha)}_{n\rm B}$, that is,

$$F^i_{\rm LEA}(r_\alpha) \approx 1 - \frac{\rho_i^2}{6}\frac{2\mu_{pn}}{\hbar^2}\varepsilon^i_{pn}. \quad (38)$$

By definition, ε^i_{pn} is negative for the ground state ($i = i_0$) and positive for the breakup states ($i \neq i_0$). Thus, we can see from Eq. (38) that for the transfer process through the deuteron ground state, the ET, the FRC increases the T-matrix element. On the other hand, for the transfer process through the breakup states of d, the BT, the correction gives a decrease in the T-matrix element. This behavior is useful to interpret the difference between the results of the ZR and FR calculations, as shown below. It should be noted that ρ_i^2 can be negative when ε^i_{pn} is very large. However, the contribution of such state to the T matrix is found to be negligibly small. Note also that in the actual calculation we use Eq. (33); Eq. (38) is used just for interpretation of the numerical result.

We show in Fig. 5 the results obtained by the FR calculation (solid line), the ZR calculation (dotted line), and the ZR calculation with the FRC described by Eqs. (32) and (33) (dashed line). One finds that the FR effect gives about a 20% increase in the cross section at $\theta = 0^\circ$. The FRC works well

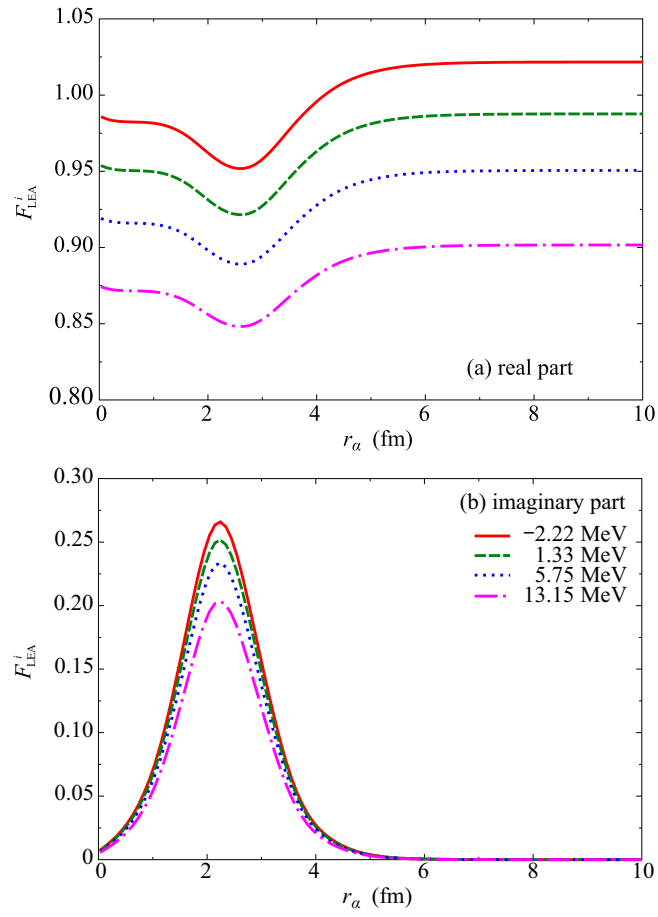

FIG. 6. (Color online) (a) Real and (b) imaginary parts of the correction function $F^i_{\rm LEA}$ defined by Eq. (33). Each line corresponds to the result with ε^i_{pn} specified in the legends.

qualitatively but is not sufficient to get good agreement with the solid line. This suggests that the FR effect found in ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon contains a higher-order component that cannot be included in the present procedure.

The correction function $F^i_{\rm LEA}$ of Eq. (33) is plotted in Fig. 6; panels (a) and (b) correspond to the real and imaginary parts of $F^i_{\rm LEA}$, respectively. It is found that $F^i_{\rm LEA}$ has a nontrivial behavior in the interior region, say, $r_\alpha \lesssim 6$ fm. As clarified in Sec. III B, however, the ^{8}B$(d,n)^9$C reaction at 14.4 MeV/nucleon is peripheral with respect to $r_{p\rm B}$, which is the same as r_α in the ZR limit. Thus, the contribution of $F^i_{\rm LEA}$ in the interior region to the T matrix is expected to be very small. In this case, a simple estimation of the FR effect based on Eq. (38) works well. At higher incident energies, where we have less peripherality, the FR effect can change significantly.

IV. SUMMARY

We have analyzed the transfer reaction ^{8}B$(d,n)^9$C at 14.4 MeV/nucleon by means of the $p+n+^8$B three-body coupled-channels framework. The ANC of ^{9}C in the p-^{8}B(g.s.) configuration, $C^{^9{\rm C}}_{p^8{\rm B}}$, and the astrophysical factor at zero energy, $S_{18}(0)$, for the ^{8}B$(p,\gamma)^9$C reaction have been determined. Our results are $(C^{^9{\rm C}}_{p^8{\rm B}})^2 = 0.59 \pm 0.15$ fm^{-1} and $S_{18}(0) = 22 \pm 6$ eV b. It is found that the breakup states of both d and ^{9}C increase the transfer cross section through the interference between the ET and BT amplitudes. As a result, the present result is smaller than the previous value [4] extracted from the same experimental data by about 51%. The back-coupling effects on the elastic channel are found to be small.

We proposed a new prescription of the FRC to the ZR calculation of the T matrix, which can be used in the CCBA formalism. For the ^{8}B$(d,n)^9$C reaction at 14.4 MeV/nucleon, the FRC is not sufficient to reproduce the result of the FR calculation, indicating the importance of higher-order correction terms. The FR effect on the transfer reaction considered turns out to be about 20%.

In Fig. 7 we compare the present result for $S_{18}(0)$ with previous results extracted from indirect measurements. As mentioned, we obtained a smaller $S_{18}(0)$ than that of Ref. [4] because of the contribution of d and ^{9}C breakup states. The present result is not consistent with the result of a three-body model analysis [5] of the inclusive [2] and exclusive [3] ^{9}C breakup reactions within 2σ. Further investigation is necessary

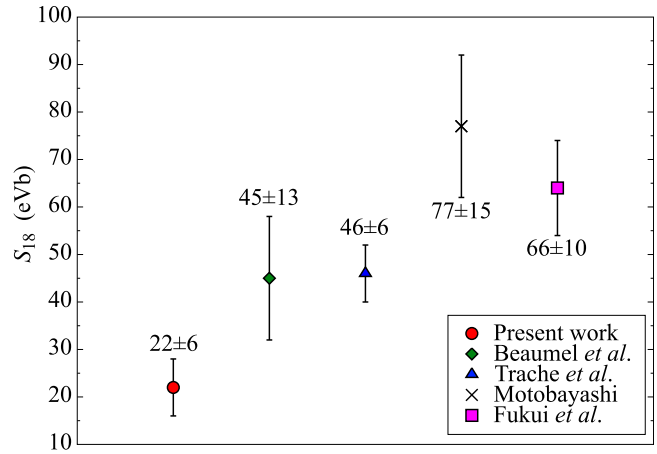

FIG. 7. (Color online) $S_{18}(0)$ in the present work (circle) compared with the results evaluated from the ^{8}B$(d,n)^9$C reaction (diamond) [4] and values extracted from ^{9}C breakup reactions (triangle [2], cross [3], and square [5]).

to understand the reason for this discrepancy. Extension of the present framework to include breakup channels of ^{8}B as well as the three-body model description of ^{9}C will be importan future work. Another possible reason for the discrepancy in $S_{18}(0)$ is the Pauli blocking effect on the transfer reaction [27,28]. Antisymmetrization between a nucleon in d and that in ^{8}B in calculation of the d-^{8}B three-body wave function will be an important subject.

In Ref. [29], $S_{18}(0) = 44 \pm 11$ eV b was extracted from ^{8}Li$(d,p)^9$Li, which is the mirror reaction to ^{8}B$(d,n)^9$C, by means of the DWBA. It will be interesting to estimate breakup effects of d in this mirror reaction. Furthermore, a compilation of the ANCs for the p-shell nuclei has been made recently [30], in which $C^{^9{\rm C}}_{p^8{\rm B}} = 1.080$ fm^{-1} was reported. It will be important to elucidate the difference between this value and the present result.

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