Isospin-symmetry breaking and shape coexistence in $A \approx 70$ analogs

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The interplay between isospin-symmetry-breaking and shape-coexistence effects in $A \approx 70$ analogs is self-consistently treated within the beyond-mean-field *complex* excited VAMPIR variational model with symmetry projection before variation using an effective interaction obtained from a G matrix based on the charge-dependent Bonn CD potential. Results are presented on Coulomb energy differences, mirror energy differences, triplet energy differences, and triplet displacement energy in the $A = 70$ and $A = 74$ isovector triplets.

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I. INTRODUCTION

Proton-rich nuclei in the $A \approx 70$ mass region are proper candidates to get insight into fundamental symmetries and interactions. In nuclei the isospin-symmetry breaking occurs due to the Coulomb interaction between protons and in the strong interaction due to the differences in the proton-proton, neutron-neutron, and neutron-proton interaction strengths because of the mass difference between the up and down quarks and electromagnetic interactions among quarks. The charge-symmetry and charge-independence breaking could be investigated by studying different isospin-related phenomena such as Coulomb energy differences (CED), mirror energy differences (MED), triplet energy differences (TED), or triplet displacement energy (TDE) among the triplet $T = 1$ nuclei. Anomalies in the Coulomb energy differences have been identified in the $A \approx 70$ mass region for nuclei supposed to manifest shape mixing at low spins [1–4].

The investigation of the structure and dynamics of exotic nuclei around the $N = Z$ line in the $A \approx 70$ mass region is one of the most exciting challenges in low energy nuclear physics. These nuclei display some rather interesting nuclear structure effects generated by the interplay between shape coexistence and mixing, competing like-nucleon and neutron-proton $T = 1$ and $T = 0$ pairing correlations, and isospin-symmetry-breaking interactions.

In the present study I will examine the isospin-symmetry-breaking effects on Coulomb energy differences, mirror energy differences, triplet energy differences, and triplet displacement energy for the $A = 70$ and $A = 74$ isovector triplets using the beyond-mean-field *complex* excited VAMPIR variational approach. Recent experimental advances made possible the investigation of exotic nuclear structure phenomena in proton-rich medium mass nuclei, but the members of the $T = 1$ triplet with $Z - N = 2$ in the $A \approx 70$ mass region could be very difficult to populate. This is the case for the ^{70}Kr and ^{74}Sr nuclei, while properties of the other members of the corresponding isovector triplets, ^{70}Br, ^{70}Se and ^{74}Rb, ^{74}Kr have been extensively investigated [5–14]. Recently preliminary results on the spectroscopy of ^{74}Sr have been reported [15].

Effects of the isospin nonconserving forces on the structure of medium mass nuclei have been studied using different theoretical approaches and various effective interactions [2–4,16–19]. Recent studies on effects of the isospin nonconserving interactions in the $T = 1$ analog states in the $A \approx 70$ mass region by performing modern shell-model calculations indicated that the experimental trends in MED and TED can be reproducedby adding to the Coulomb interaction some phenomenological isospin nonconserving (INC) nuclear interactions, but the modern charge-dependent forces cannot account for the phenomenological strengths of the INC force [19].

Investigations based on the variational approaches of the VAMPIR model family have been successfully performed for the description of a variety of nuclear structure phenomena in the $A \approx 70$ mass region, not only in nuclei along the valley of β stability, but also in some exotic nuclei close to the proton drip line [20–24]. The *complex* excited VAMPIR approach allows for a unified description of low- and high-spin states including in the projected mean field neutron-proton correlations in both the $T = 1$ and $T = 0$ channels and general two-nucleon unnatural-parity correlations. The oblate-prolate coexistence and mixing, and the variation of the deformation with mass number, increasing spin, as well as excitation energy, have been compared with the available experimental information. Since the VAMPIR approaches enable the use of rather large model spaces and of general two-body interactions, *large-scale* nuclear structure studies going far beyond the abilities of the conventional shell-model configuration-mixing approach are possible. My previous investigations on microscopic aspects of shape coexistence in $N \simeq Z$ nuclei in this mass region indicated the presence of a strong competition between particular configurations based on large and small oblate and prolate quadrupole deformations in the intrinsic system. Furthermore, as expected, since in $N \simeq Z$ nuclei neutrons and protons fill the same single-particle orbits, the neutron-proton pairing correlations were found to play an important role [20]. On the other hand the theoretical results suggest that certain properties of these nuclei, like shape mixing, are extremely sensitive to small variations of particular parts of the effective Hamiltonian [21].

I shall briefly describe the *complex* excited VAMPIR variational procedure and define the effective Hamiltonian in the next section. In Sec. III I shall then discuss the results on

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TABLE I. The amount of mixing for the lowest *complex* excited VAMPIR states in ^{70}Se.

I ($\hbar$)	Prolate content	Oblate content
0^+	41(4)(1)(1)%	51(1)%
2^+	56(2)%	39(2)%
4^+	52(2)%	43(2)%
6^+	76(3)(1)(1)%	17(1)%

TABLE III. The amount of mixing for the lowest *complex* excited VAMPIR states in ^{70}Kr.

I ($\hbar$)	Prolate content	Oblate content
0^+	69(3)%	24(3)%
2^+	70(3)%	24(1)%
4^+	75(3)%	19(2)%
6^+	86(3)(2)%	7(2)%

isospin-symmetry-breaking and shape-mixing effects on CED, MED, TED, and TDE in the $A = 70$ isovector triplet of nuclei, ^{70}Se, ^{70}Br, ^{70}Kr and the $A = 74$ triplet ^{74}Kr, ^{74}Rb, ^{74}Sr.

II. THEORETICAL FRAMEWORK

The *complex* excited VAMPIR (EXVAM) approach uses Hartree-Fock-Bogoliubov (HFB) vacua as basic building blocks, which are only restricted by time-reversal and axial symmetry. The underlying HFB transformations are essentially *complex* and do mix proton- with neutron-states as well as states of different parity and angular momentum. The broken symmetries of these vacua (nucleon numbers, parity, total angular momentum) are restored by projection techniques and the resulting symmetry-projected configurations are then used as test wave functions in chains of successive variational calculations to determine the underlying HFB transformations as well as the configuration mixing. The HFB vacua of the above type account for arbitrary two-nucleon correlations and thus simultaneously describe like-nucleon as well as isovector and isoscalar neutron-proton pairing correlations.

For nuclei in the $A \approx 70$ mass region I used a ^{40}Ca core and the $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$, $1d_{5/2}$, and $0g_{9/2}$ oscillator orbits for both protons and neutrons are introduced in the valence space. I start with an isospin symmetric basis and then introduce the Coulomb shifts for the proton single-particle levels resulting from the ^{40}Ca core by performing spherically symmetric Hartree-Fock calculations using the Gogny interaction D1S in a 21 major-shell basis [23].

The effective two-body interaction is constructed from a nuclear matter G matrix based on the charge-dependent Bonn one-boson-exchange potential Bonn CD. In order to enhance the pairing correlations this G matrix was modified by adding short-range (0.707 fm) Gaussians with strength of -35 MeV in the $T = 1$ proton-proton and neutron-neutron channnel, -20 MeV in the neutron-proton $T = 1$ channel, and -35 MeV in the neutron-proton $T = 0$ channel. In addition, the isoscalar interaction was modified by monopole shifts of -500 keV for all $T = 0$ matrix elements of the form $\langle 1p1d_{5/2}; IT = 0|\hat{G}|1p1d_{5/2}; IT = 0\rangle$, where $1p$ denotes either the $1p_{1/2}$ or the $1p_{3/2}$ orbit. For the matrix elements of the form $\langle 0g_{9/2}0f; IT = 0|\hat{G}|0g_{9/2}0f; IT = 0\rangle$, where $0f$ denotes either the $0f_{5/2}$ or the $0f_{7/2}$ orbitals, monopole shifts of -370 keV (for $A = 70$) and -275 keV (for $A = 74$) have been added. These monopole shifts have been introduced in the earlier EXVAM calculations in order to influence the onset of deformation. Previous results indicated that the oblate-prolate coexistence and mixing at low spins sensitively depend on the strengths of the neutron-proton $T = 0$ matrix elements involving nucleons occupying the $0f_{5/2}$ or $0f_{7/2}$ and $0g_{9/2}$ single-particle orbits. The Hamiltonian includes the two-body matrix elements of the Coulomb interaction between the valence protons.

III. RESULTS AND DISCUSSION

In the isovector triplets the Coulomb energy differences are defined by $\mathrm{CED}(J) = E_x(J,T = 1,T_z = 0) - E_x(J,T = 1,T_z = 1)$, the mirror energy differences by $\mathrm{MED}(J) = E_x(J,T = 1,T_z = -1) - E_x(J,T = 1,T_z = 1)$, the triplet energy differences by $\mathrm{TED}(J) = E_x(J,T = 1,T_z = -1) + E_x(J,T = 1,T_z = 1) - 2E_x(J,T = 1,T_z = 0)$, and the triplet displacement energy by $\mathrm{TDE}(A,T) = \mathrm{BE}(T,T_z = -1) + \mathrm{BE}(T,T_z = 1) - 2\mathrm{BE}(T,T_z = 0)$, where E_x represents the excitation energy, BE represents the binding energy, and $T_z = -1$ for the proton-proton pair. In order to investigate the CED, MED, TED, and TDE in the $A = 70$ isovector triplet up to 25 many-nucleon excited VAMPIR configurations for the spins 0^+, 2^+, 4^+, and 6^+ in ^{70}Se, ^{70}Br, and ^{70}Kr have been calculated using the above defined Hamiltonian. In the $A = 74$ triplet are calculated the lowest 16 0^+, 2^+, 4^+, and 6^+ excited VAMPIR projected configurations in ^{74}Kr, ^{74}Rb, and ^{74}Sr. First the VAMPIR solutions, representing the optimal mean-field description of the yrast states by single symmetry-projected HFB determinants, were obtained. Then the excited VAMPIR approach was used to construct additional excited states by

TABLE II. The amount of mixing for the lowest *complex* excited VAMPIR states in ^{70}Br.

I ($\hbar$)	Prolate content	Oblate content
0^+	68(1)%	26(2)(1)%
2^+	66(2)%	29(1)%
4^+	68(2)(1)%	26(1)%
6^+	81(4)(2)(1)(1)%	10(1)%

TABLE IV. The amount of mixing for the lowest *complex* excited VAMPIR states in ^{74}Kr.

I ($\hbar$)	Prolate content	Oblate content
0^+	82(1)(1)%	14(1)(1)%
2^+	92(1)(1)%	6%
4^+	95(1)(1)%	3%
6^+	97(1)%	1(1)%

TABLE V. The amount of mixing for the lowest *complex* excited VAMPIR states in ^{74}Rb.

I ($\hbar$)	Prolate content	Oblate content
0^+	85(1)%	12(1)%
2^+	94(1)%	4%
4^+	96(1)%	2%
6^+	97(1)%	1%

independent variational calculations. The final solutions for each spin have been obtained by diagonalizing the residual interaction between the successively constructed orthogonal many-nucleon excited VAMPIR configurations. The variational procedure which is used involves projection before variation on particle number, angular momentum, and parity.

The results concerning the structure of the 0^+, 2^+, 4^+, and 6^+ states in ^{70}Se, ^{70}Br, and ^{70}Kr indicate a strong mixing of different prolate and oblate deformed configurations in the intrinsic system in the final wave functions. The amounts of mixing for the lowest calculated 0^+, 2^+, 4^+, and 6^+ states are presented in Tables I–III for the $A = 70$ triplet of nuclei. In these tables is indicated the contribution of different prolate (p) and oblate (o) projected configurations in the intrinsic system in the structure of the final *complex* excited VAMPIR wave functions as percentage of the total amplitude. The contributions larger than 1% are indicated in decreasing order. Comparing the wave functions of the ground states in ^{70}Se and ^{70}Br, one can observe that in ^{70}Se oblate and prolate components bring almost equal contributions (the oblate content is larger), while in ^{70}Br the mixing is strong, but the prolate content dominates. For the higher spin analog states the prolate components dominate, but in ^{70}Se the oblate components make a much larger contribution than in ^{70}Br. The analysis of the structure of the investigated states in ^{70}Kr reveals similarity with the shape mixing found in ^{70}Br: strong prolate-oblate mixing decreasing with increasing spin, the prolate components representing altogether 91% of the total amplitude at spin 6^+.

The results concerning the structure of the 0^+, 2^+, 4^+, and 6^+ states in ^{74}Kr, ^{74}Rb, and ^{74}Sr indicate a variable, sometimes strong mixing of prolate and oblate deformed configurations in the final wave functions for the above defined Hamiltonian. The amount of mixing for the lowest calculated 0^+, 2^+, 4^+, and 6^+ analog states is presented in Tables IV–VI. In the $A = 74$ triplet of nuclei prolate components dominate the structure of the wave functions of the analog states and the prolate-oblate mixing is weaker than in the $A = 70$ case.

TABLE VI. The amount of mixing for the lowest *complex* excited VAMPIR states in ^{74}Sr.

I ($\hbar$)	Prolate content	Oblate content
0^+	77(2)%	19(1)%
2^+	87(1)%	11%
4^+	90(1)%	8%
6^+	92(1)%	5(1)%

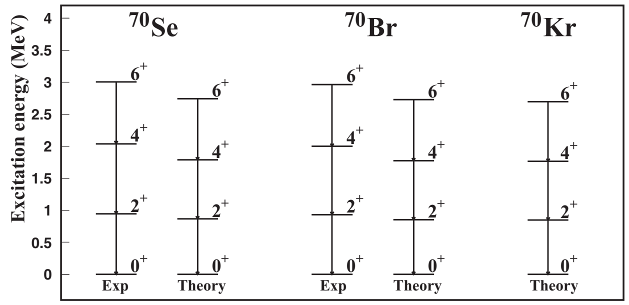

FIG. 1. The *complex* excited VAMPIR spectra for the analog states in ^{70}Se, ^{70}Br, and ^{70}Kr compared with the available data.

Figures 1 and 2 present the *complex* excited VAMPIR spectra for the $A = 70$ and $A = 74$ analogs, respectively, compared with the available data.

In both $A = 70$ and $A = 74$ triplets the structure of the lowest 2^+, 4^+, and 6^+ states is dominated by prolate deformed configurations in the intrinsic system, while the first excited states manifest oblate-dominated content as illustrated by the spectroscopic quadrupole moments presented in Tables VII and VIII. As one may expect in ^{70}Se due to the very strong oblate-prolate mixing the spectroscopic quadrupole moments for the lowest two 2^+ and 4^+ states is very small, changing sign from the lowest to the first excited state. In the $A = 74$ triplet where the prolate-oblate mixing is small at spin 6^+ the spectroscopic quadrupole moments reveal the fact that the quadrupole deformation of the main prolate component of the wave functions is larger ($\beta_2 \sim 0.36$) than the deformation of the main oblate component for the first excited states ($\beta_2 \sim -0.33$). The *complex* excited VAMPIR results are in rather good agreement with the available data on spectroscopic quadrupole moments in ^{74}Kr, which indicate for the yrast 2^+ state -53 $(+24/-23)$ $e\,\mathrm{fm}^2$, but for the first excited 2^+ state $+24$ $(+21/-17)$ $e\,\mathrm{fm}^2$ [13] and the corresponding EXVAM values are $-54\ e\,\mathrm{fm}^2$ and $+49\ e\,\mathrm{fm}^2$. For the yrast 4^+ and 6^+ states the experimental values are -80 $(+40/-20)$ $e\,\mathrm{fm}^2$ and -130 $(+30/-50)$ $e\,\mathrm{fm}^2$, respectively, while the corresponding EXVAM results are $-74\ e\,\mathrm{fm}^2$ and $-85\ e\,\mathrm{fm}^2$. For a comparison with the available data, $B(E2)$ transition strengths have been evaluated using as effective charges $e_p = 1.3$ and $e_n = 0.3$. In ^{70}Se the calculated $B(E2)$ values for the 2^+, 4^+, 6^+ states amount to 623, 928,

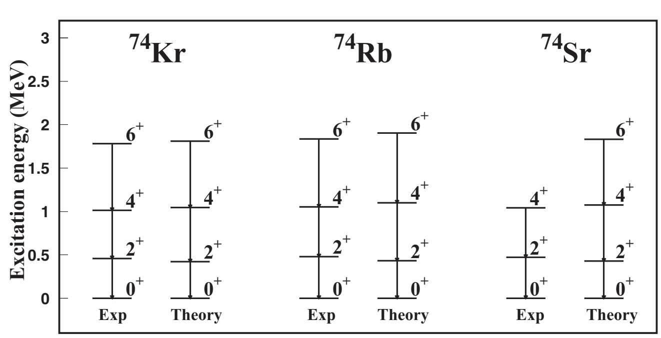

FIG. 2. The *complex* excited VAMPIR spectra for the analog states in ^{74}Kr, ^{74}Rb, and ^{74}Sr compared with the available data.

TABLE VII. Spectroscopic quadrupole moments (in $e\,\mathrm{fm}^2$) for the lowest EXVAM states of ^{70}Se, ^{70}Br, and ^{70}Kr. As effective charges $e_p = 1.3$ and $e_n = 0.3$ are used.

I ($\hbar$)	^{70}Se	^{70}Br	^{70}Kr
2_1^+	−6.8	−18.3	−24.7
2_2^+	4.1	16.2	18.0
4_1^+	−7.2	−30.1	−42.0
4_2^+	0.1	24.9	33.3
6_1^+	−48.7	−59.2	−65.7
6_2^+	38.4	51.4	53.7

971 $e^2\mathrm{fm}^4$, respectively, while the corresponding data are 342(19), 370(24), 530(96) $e^2\mathrm{fm}^4$ [8]. The results of the configuration mixing of constrained HFB wave functions from the generator coordinate calculations with Gaussian overlap approximation using the Gogny D1S interaction indicate 549, 955, 1404 $e^2\mathrm{fm}^4$ for the corresponding strengths [8]. Future experiments scheduled at HIE-ISOLDE and RIKEN for the investigation of the electromagnetic properties of ^{70}Se and ^{70}Kr could help the beyond-mean-field approaches to get better insights into the shape coexistence and mixing specific for the proton-rich $A \approx 70$ nuclei. In ^{70}Br the experimental strength for the deexcitation of the 2^+ state is 291(43) $e^2\mathrm{fm}^4$ [12] and the EXVAM result is 686 $e^2\mathrm{fm}^4$. In ^{74}Kr the latest experimental $B(E2)$ values for the 2^+, 4^+, and 6^+ states indicate 1290(90), 2560(260), 3020(300) $e^2\mathrm{fm}^4$ [13,14], and the corresponding calculated EXVAM values are 878, 1306, 1432 $e^2\mathrm{fm}^4$, respectively.

Figures 3 and 4 present the proton and neutron occupation of the valence spherical orbitals $1p_{3/2}$, $0f_{5/2}$, and $0g_{9/2}$ for the analog states in the $A = 70$ and $A = 74$ triplets, respectively. The trend manifested by the prolate-oblate mixing with increasing spin in the structure of the wave functions for the analog states is corroborated by the evolution of the spherical occupations. In the $A = 74$ triplet both the proton and the neutron occupations manifest a much smoother evolution up to spin 6^+ than the analog states in the $A = 70$ nuclei presenting variable and strong prolate-oblate mixing for 0^+, 2^+, and 4^+ states. Since in the $A = 74$ nuclei for the spin 6^+ the prolate-deformed configurations represent more than 94% of the total amplitude of the wave functions of the analog states, while for the first excited states the oblate components make this kind of contribution, it is worthwhile to compare the

TABLE VIII. Spectroscopic quadrupole moments (in $e\,\mathrm{fm}^2$) for the lowest EXVAM states of ^{74}Kr, ^{74}Rb, and ^{74}Sr. As effective charges $e_p = 1.3$ and $e_n = 0.3$ are used.

I ($\hbar$)	^{74}Kr	^{74}Rb	^{74}Sr
2_1^+	−53.5	−57.0	−50.4
2_2^+	49.1	53.0	47.5
4_1^+	−74.0	−76.8	−70.4
4_2^+	68.0	71.6	67.1
6_1^+	−84.8	−86.4	−80.7
6_2^+	78.2	80.7	79.8

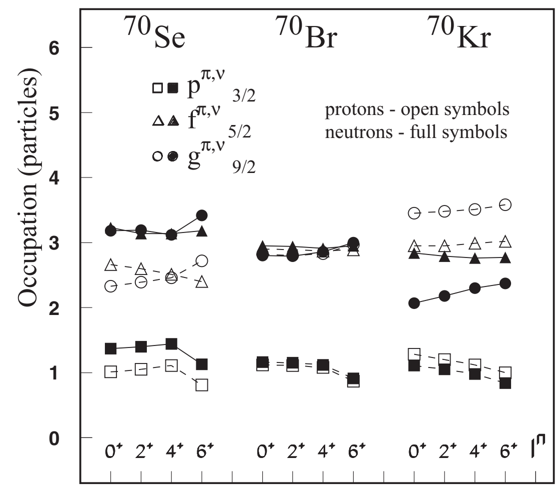

FIG. 3. Occupation of valence spherical orbitals for the analog states in ^{70}Se, ^{70}Br, and ^{70}Kr.

spherical occupations for the corresponding states. Thus for the $1p_{3/2}$ orbital the neutron occupation (in particles) changes from 1.84 for the lowest 6^+ state to 2.18 for the first excited 6^+ state in ^{74}Kr, from 1.74 to 2.14 in ^{74}Rb, and from 1.76 to 2.20 in ^{74}Sr. The corresponding proton occupation changes from 1.41 to 2.06, from 1.70 to 2.12, and from 1.86 to 2.20 in ^{74}Kr, ^{74}Rb, and ^{74}Sr, respectively. The change in the neutron occupation of the $0f_{5/2}$ orbital is from 3.65 to 3.32, from 3.41 to 3.32, and from 3.34 to 3.14 in Kr, Rb, and Sr, respectively. The $0f_{5/2}$ proton occupation changes from 2.93 to 3.30, from 3.39 to 3.29, and from 3.46 to 3.11 in Kr, Rb, and Sr, respectively. The corresponding $0g_{9/2}$ neutron occupation changes from 3.80 to 3.99, from 3.24 to 3.10, and from 2.38 to 2.19, while the proton occupation changes from 2.98 to 2.26, from 3.07 to 3.10, and from 3.65 to 3.26 in Kr, Rb, and Sr, respectively.

Figure 5 presents the *complex* excited VAMPIR results on Coulomb energy differences for the ^{70}Br-^{70}Se and ^{74}Rb-^{74}Kr analogs obtained using the above defined Hamiltonians based on Bonn CD potential compared with available data [5–11].

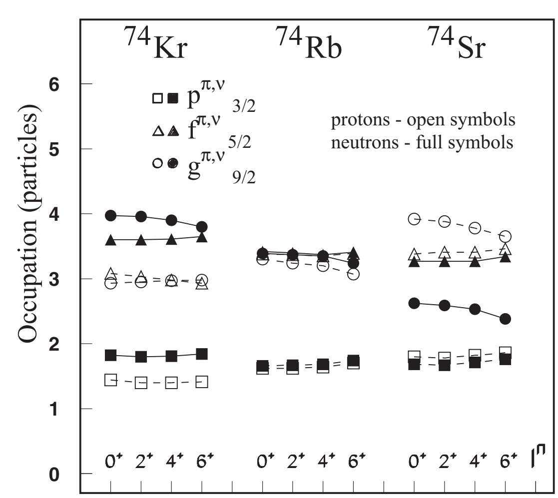

FIG. 4. Occupation of valence spherical orbitals for the analog states in ^{74}Kr, ^{74}Rb, and ^{74}Sr.

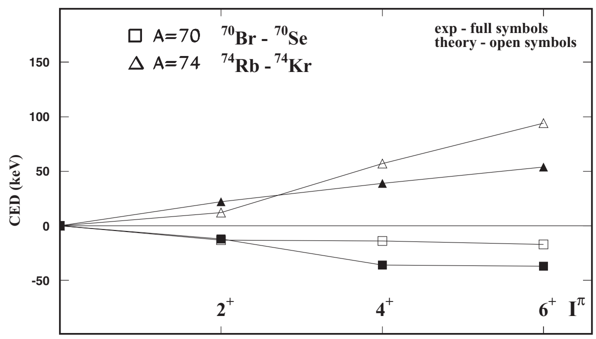

FIG. 5. Comparison of the *complex* excited VAMPIR results for CED to the experimental data [5–11].

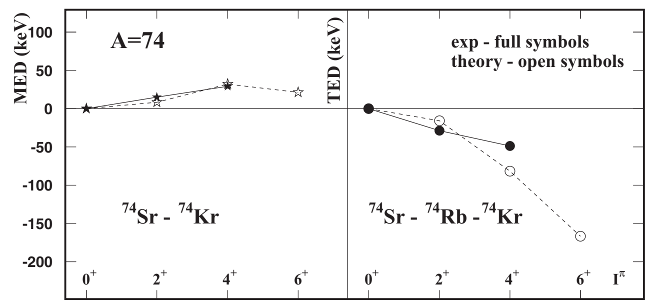

FIG. 7. The *complex* excited VAMPIR results for MED and TED in the A=74 isovector triplet compared to data [9–11,15].

The trend manifested in the data is reproduced by the EXVAM results for the $A = 74$ pair of nuclei as well as the anomalous behavior revealed for the ^{70}Br-^{70}Se case. The previous results for $A = 70$ nuclei obtained with isospin mixing induced only by Coulomb interaction (the G matrix obtained from the Bonn A potential was used) reproduced also this anomaly [2]. It is worthwhile to mention that in the present calculations I have used the same monopole shifts as in [2], but different strengths for the Gaussians in the $T = 1$ and $T = 0$ neutron-proton channel. In both calculations the shape mixing is very strong, changing drastically with increasing spin, but manifesting different behavior in ^{70}Se with respect to ^{70}Br. Consequently, one obtains an anomalous behavior of CED for $A = 70$, but a normal one for the $A = 74$ analogs.

Figure 6 illustrates the *complex* excited VAMPIR predictions on mirror energy differences and triplet energy differences for the $A = 70$ isovector triplet. MED manifest a negative trend, while TED indicate small positive values up to spin 4^+ and a small negative value for the spin 6^+. Of course, the trend manifested in TED is influenced by the evolution of shape mixing with increasing spin, which is significantly different in ^{70}Se with respect to ^{70}Br and ^{70}Kr.

Figure 7 illustrates the *complex* excited VAMPIR predictions on mirror energy differences and triplet energy differences for the $A = 74$ isovector triplet. MED manifest a positive trend, while TED indicate a negative trend, in agreement with the recent experimental available results [15].

The intensively hunted but still unknown member of the $A = 70$ triplet, the ^{70}Kr nucleus, could bring support to the EXVAM theoretical predictions and interpretation of the particular behavior revealed by TED connected with the anomaly identified in CED behavior. Within the *complex* excited VAMPIR model using an effective interaction including charge dependence in the strong force, the interplay between the effects of the isospin-nonconserving interaction and the variable strong shape mixing specific for each nucleus belonging to the $A = 70$ triplet is responsible for the discussed anomalous behavior.

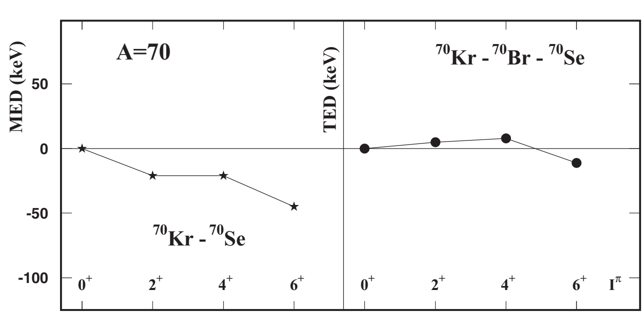

FIG. 6. The *complex* excited VAMPIR results for MED and TED in the $A = 70$ isovector triplet.

Recently new experimental results on $T = 1$ states in ^{66}As [25] confirmed the EXVAM predictions on CED values. Furthermore, the published data on excited states identified in ^{66}Se [26] give support to my *complex* excited VAMPIR results, which reveal the experimental negative trend manifested in the MED and TED evolution with spin in the $A = 66$ isovector triplet [27].

The EXVAM prediction for the triplet displacement energy indicates TDE($A = 70, T = 1$) = 111 keV and TDE($A = 74, T = 1$) = 107 keV. These values for TDE are smaller than the results of the calculations presented in [18] using different model spaces and effective interactions. Precise experimental data on mass measurements are necessary for the refining of the effective interaction.

This paper represents the first beyond-mean-field treatment—based on an effective two-body interaction constructed from the nuclear matter G matrix starting from the charge-dependent Bonn CD potential—able to describe self-consistently the isospin-symmetry-breaking effects in a region dominated by shape coexistence and mixing. Furthermore, I used a model space adequate for the description of proton-rich nuclei in the $A \approx 70$ mass regions which is not yet numerically feasible for the large-scale shell-model calculations. However, the investigated observables CED, MED, TED, TDE are rather small quantities created in this mass region by the interplay of shape mixing and isospin-symmetry-breaking forces. Consequently, it is difficult to disentangle between the two effects at least based on the available data. To further refine the renormalization of the two-body interaction adequate for the involved model space in the $A \approx 70$ mass region and to improve the estimation of the isospin-mixing effects on the structure of the analog states one needs more data on electromagnetic properties. Precise experimental spectroscopic quadrupole moments for the analog 2^+ states in each triplet could test the EXVAM predictions concerning the shape mixing.

IV. CONCLUSIONS

In this paper I present the first results on the effect of isospin mixing on CED, MED, TED, and TDE in the $A = 70$ and $A = 74$ isovector triplets calculating the 0^+, 2^+, 4^+, and 6^+ states in these nuclei in the frame of the *complex* excited VAMPIR model, using an effective interaction obtained for the $A \approx 70$ mass region starting from the charge-dependent Bonn CD potential. For the first time I estimated the isospin-symmetry-breaking effects, taking into account both the Coulomb interaction and the isospin-symmetry violation in the strong force as it is considered by the Bonn CD potential. In order to improve the estimation of the isospin-symmetry-breaking effects one could increase the dimension of the many-nucleon bases used to describe the investigated analog states in all members of the isovector triplets. Furthermore, the refining of the effective interaction appropriate for the mass region under consideration requires more experimental data sensitive to particular parts of the Hamiltonian which could be relevant for the shape mixing and consequently for isospin-mixing effects. Of course, precise experimental data are needed on the analog states in ^{70}Kr and ^{74}Sr to confirm the EXVAM predictions on mirror energy differences, triplet energy differences, and triplet displacement energy. Extended experimental information on characteristic properties of $A \approx 70$ isovector triplets testing the interplay between shape coexistence and isospin-nonconserving forces effects could test the theoretical predictions.

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