Phenomenological analysis of $B(E2)$ transition strengths in neutron-rich carbon isotopes

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Recent experimental results related to the quadrupole collectivity in neutron-rich carbon isotopes are analyzed in a phenomenological approach. $B(E2;2_1^+ \to 0_1^+)$ transitions rates derived from lifetime measurements are interpreted in terms of the mixing of basic neutron and proton 2^+ excitations. A seniority inspired scheme is used to describe the neutron component. The observed increase in collectivity can be explained with a corresponding increased role of proton excitations. This is likely due to the reduction of the proton $p_{3/2}$-$p_{1/2}$ spin-orbit splitting caused by the tensor and two-body spin-orbit components of the force between the protons and the added neutrons in the $(d_{5/2}+s_{1/2})$ shells.

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Introduction. The neutron-rich carbon isotopes, which are experimentally accessible up to the neutron drip line, provide a unique opportunity to study the evolution of nuclear structure and the coupling of neutron and proton degrees of freedom. In particular, the even-even carbon isotopes have received considerable attention since initial claims of a quenched $B(E2;2_1^+ \to 0_1^+)$ value[1] and large asymmetries in the proton and neutron quadrupole matrix elements were reported in ^{16}C [1] and interpreted as evidence of the decoupling of the valence neutrons from the core. Subsequent work [2–6] reported values that do not support the scenario of an anomalous decoupling of the valence neutrons from the core and can be understood without resorting to new or unexpected phenomena. The current experimental data are reviewed in Fig. 1. Contrary to a decrease in the quadrupole collectivity, which could signal a decoupling of neutrons and protons, the $B(E2)$s show an increase toward ^{20}C.

Here we report on the interpretation of these data within a simple framework which captures the main components contributing to the quadrupole collectivity in these nuclei. As we will show, the proton excitations seem to play a key role in understanding the rise in ^{20}C which, on the basis of neutrons only, should be at most comparable to ^{16}C.

The phenomenological framework. Let us start by considering the basic ingredients that may play a role in the low-lying 2^+ states in the carbon isotopes. Given the rather distinct energies of the 2^+ excitations in ^{18}O (neutrons, 1.98 MeV) and ^{14}C (protons, 7 MeV), and guided by a weak-coupling approach [8], one would expect that in ^{16}C the 2^+ will be also dominated by neutron excitations.

But what about for the whole chain, ^{16,18,20}C? Can we understand the overall behavior of the quadrupole collectivity by considering a seniority inspired scheme [9–11] to describe the neutron components? The rather constant energy of the 2^+ states (1.6–1.8 MeV) suggests that such a description may be adequate. Moreover, an inspection of Fig. 2, showing the effective neutron single-particle energies (ESPEs) derived from the WBT interaction [12], indicates that pairing ($\Delta \approx 3$ MeV) is expected to dominate over the single-particle spacing, $\Delta E = E_{s_{1/2}} - E_{d_{5/2}} \approx 1.5$ MeV. The strong mixing of these two levels, implied by the pairing force, is clearly observed in the ground state of ^{16}C, where spectroscopic factors measured in the ^{15}C(d,p) reaction determine a neutron wave function [13]. [See also Ref. [14] for results obtained in the analysis of the ^{14}C(t,p) reaction.]

$$|0^+;{}^{16}\mathrm{C}\rangle_\nu \approx 0.55|(s_{1/2})^2\rangle + 0.84|(d_{5/2})^2\rangle. \tag{1}$$

In addition to the neutrons, the proton excitations will contribute to the 2^+ state, and thus we can expect a wave function of the form

$$|2_1^+\rangle = \alpha|2^+\rangle_\nu + \beta|2^+\rangle_\pi. \tag{2}$$

In simple terms the proton structure of the ground state will be dominated by the configuration $|\pi(p_{3/2})^4; J=0\rangle$. In fact the pioneering calculations of Ref. [15] show that for the $A=14$ systems, the intermediate-coupling results are close to the jj scheme limit and predict, for example, $\langle n_\pi(p_{1/2})\rangle \approx 0.33$, in agreement with direct reaction measurements [16]. Furthermore, the dominant proton contribution to the 2^+ state, for which $\langle n_\pi(p_{3/2})\rangle \approx 2.9$, corresponds to a particle-hole excitation across the spin-orbit gap between the $p_{3/2}$ and $p_{1/2}$ levels, i.e., $|\pi(p_{3/2})^3(p_{1/2})^1; J=2\rangle$ [17].

While the detailed components of the ^{14}C wave function are required to explain for example its long β-decay lifetime [18], for the current analysis the assumption of a closed-shell $Z=6$, $N=8$ core appears justified. Considering the basic excitations schematically shown in Fig. 3, the wave functions of the 0^+ and 2_1^+ states of ${}^A_6\mathrm{C}_{8+n}$ with n neutrons in the $(d_{5/2}+s_{1/2})$ combined shell are

$$\begin{aligned}|0^+;{}^A\mathrm{C}\rangle &= |\nu(sd)^n; J=0\rangle \otimes |\pi(p_{3/2})^4; J=0\rangle;\\ |2_1^+;{}^A\mathrm{C}\rangle &= \alpha|\nu(sd)^n; J=2\rangle \otimes |\pi(p_{3/2})^4; J=0\rangle\\ &\quad + \beta|\nu(sd)^n; J=0\rangle \otimes |\pi(p_{3/2})^3(p_{1/2})^1; J=2\rangle.\end{aligned} \tag{3}$$

The proton amplitude β in Eqs. (2) and (3) can be studied by measuring spectroscopic factors from one-proton removal reactions ${}^{A+1}_{7}\mathrm{N}_N$ to ${}^A_6\mathrm{C}_N$, since for the specific case of carbons,

[1]In what follows, we will use $B(E2)$ for short.

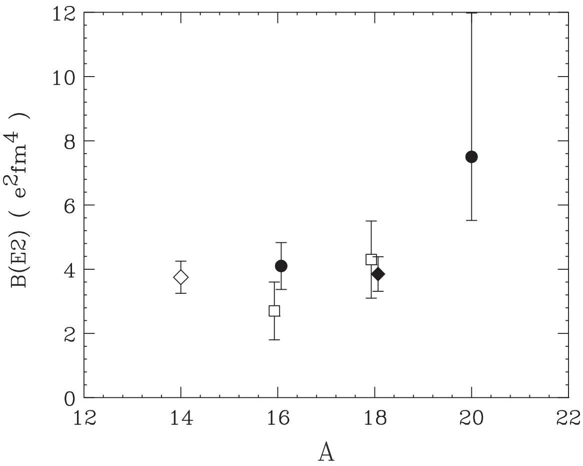

FIG. 1. Summary of $B(E2;2_1^+ \to 0_1^+)$ values in the carbon isotopes. Data are from Refs. [7] (open diamond), [4,5] (circles), [3] (open squares), and [6] (diamond).

population of the 2^+ is expected to proceed only through the proton component, since the $\pi p_{1/2}$ cannot contribute to the $1/2^-$ ground state in the nitrogen isotopes by coupling to the 2^+ in the carbons. The relative cross sections to populate the ground state and the 2^+ state can be expressed as (see discussion in Ref. [5])

$$\sigma(2_1^+)/\sigma(0_1^+) \approx \beta^2 \times 5/2.$$

This ratio has been measured experimentally in ^{16}C in the one-proton knockout reaction from ^{17}N. The resulting proton amplitude of 11(1)% confirms the dominant neutron character of the state [5].

Turning our attention to the $B(E2)$s, these can be easily calculated with the wave functions in Eqs. (2) and (3)

$$B(E2) = (\alpha\langle 2^+|E2|0^+\rangle_\nu + \beta\langle 2^+|E2|0^+\rangle_\pi)^2. \quad (4)$$

Realizing that $\Delta/\Delta E > 1$, and following from the arguments above, we further consider the neutron configuration $(sd)^n$ as

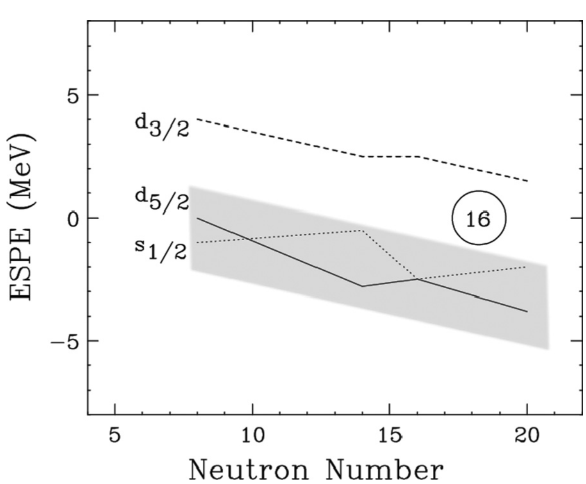

FIG. 2. ESPEs for the carbon isotopes obtained from the WBT interaction. The shaded area indicates the size of the pairing gap (Δ). Adapted from [12].

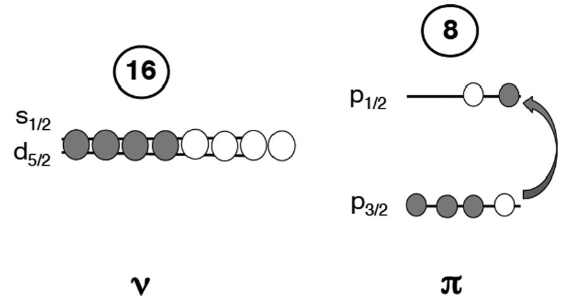

FIG. 3. Basic excitations considered in our description of the carbon isotopes.

arising from an *effective* j^n shell (with $j = 7/2$), where the $B(E2)$ for n neutrons follows the well-known behavior of the seniority scheme, i.e.,

$$B(E2,n)_\nu = \frac{n(8-n)}{12} B(E2,n=2)_\nu. \quad (5)$$

We have now all the ingredients to confront Eq. (4) with the experimental data. Our strategy starts by fixing the proton $E2$ matrix element from ^{14}C [7], taking into account the fact that the 2^+ in ^{14}C also has a component of neutrons excited into the sd shell. Actually, its wave function can be written in the form of Eq. (3), with $\alpha \approx \beta \approx 0.7$ [17,19,20]. In this particular case, however, the neutrons do not contribute to the $B(E2)$ since their configuration is a 2p2h excitation.

At this point, one would be tempted to determine $B(E2,n=2)_\nu$ from ^{18}O but an important component of the quadrupole strength here comes from the presence of deformed states. Rather, we take the proton amplitude in ^{16}C to be 11%, as measured experimentally, and then adjust the value of $B(E2,n=2)_\nu$.

In Fig. 4, the data are compared to the results of the seniority approach. The individual contributions required to reproduce

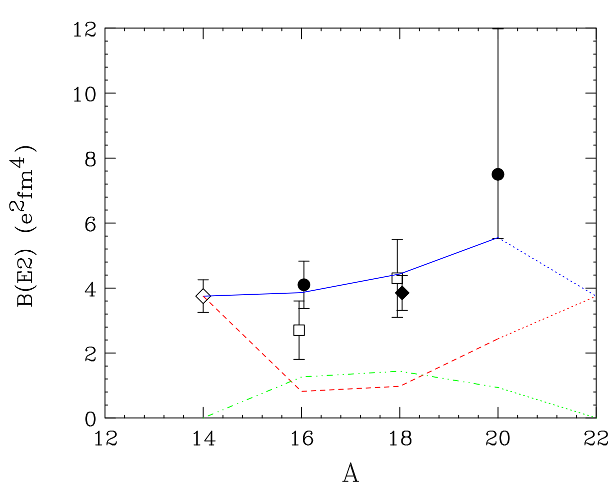

FIG. 4. (Color online) Summary of $B(E2)$ values in carbon isotopes from Fig. 1, and the results of the phenomenological treatment (blue solid line). Contributions from neutrons [green (dash-dot)] and protons [red (dash)] are shown. Possible extrapolations to ^{22}C are indicated by dotted lines.

TABLE I. Proton amplitudes of the 2_1^+ states in ^{14}C and those in ^{16,18,20}C derived from our analysis of $B(E2)$s.

^{14}C	^{16}C	^{18}C	^{20}C
≈50%	11%	13%	$\gtrsim$30%[a]

[a]This value corresponds to the upper limit of the lifetime of the state, firmly established in Ref. [4]. (See also Fig. 4.)

the data are shown for neutrons (green dash-dotted line) and protons (red dashed line). Due to the well-known (parabolic) shape of the $B(E2,n)_\nu$ in the seniority scheme, it is clear that an increase in the proton contributions is needed to reproduce the trend of the experimental points. The adjusted proton amplitudes are summarized in Table I. The rise in the $B(E2)$ cannot be understood in a neutron-only scenario. Such a case will require an anomalous behavior in the effective charges and not even a possible closure at $N = 20$, instead of the well-established $N = 16$ [21], is enough to reach the experimental limits.

We are therefore confident that our scenario explaining the increase in $B(E2)$s due to the increased proton component is rather robust. Of course, it will be important to measure these amplitudes in the heavier carbon isotopes ^{18,20}C in order to confirm the increase in the proton contribution to the 2_1^+ states, and one-proton knockout reaction experiments are possible today [22]. Obviously, the increase in the proton component can also be also tested by g-factor measurements in the isotopic chain. Again, with the wave functions above, it is easy to show that the g factor of the 2^+ state is given by

$$g_{2^+} = \alpha^2 g_\nu + \beta^2 g_\pi, \quad (6)$$

from which a simple estimate can be obtained. To do so, we determine $g_\nu \approx -0.69$ from ^{15}C, and $g_\pi \approx 1.45$ from ^{13}B and ^{15}N using the data in Ref. [23]. As anticipated, and confirmed in Fig. 5, the evolution of the g factors clearly signals the increase of the proton amplitude. Unfortunately, these experiments are currently very challenging given the available beam intensities, in particular for ^{20}C.

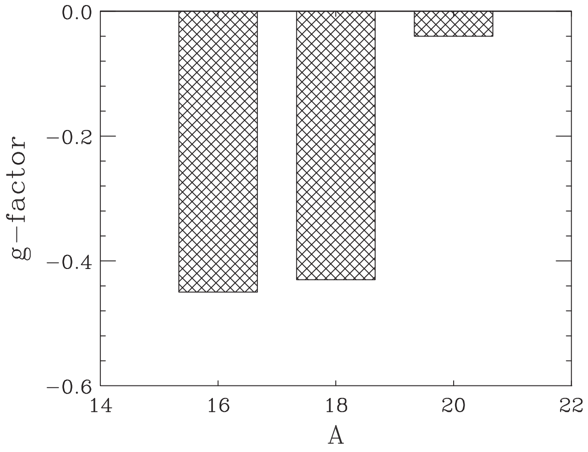

FIG. 5. Predicted g factors for the 2^+ states in ^{16,18,20}C.

Discussion. Do we understand the increased role of the proton component? The answer is yes, and it can be attributed to a reduction in the $p_{3/2}$-$p_{1/2}$ spin-orbit splitting, as neutrons are added in the sd shell. A similar effect has been discussed in the case of yttrium isotopes in [24].

To lowest order the proton amplitude is given by $\beta \sim V_{\pi\nu}/(E_{2_\pi^+} - E_{2_\nu^+})$, with $V_{\pi\nu}$ the matrix element mixing the unperturbed 2_π^+ and 2_ν^+ states. Because the energy denominator $E_{2_\pi^+} - E_{2_\nu^+}$ is dominated by $E_{p1/2} - E_{p3/2}$, it is clear that a reduced splitting will help promote proton excitations.

The change in the splitting of these two levels with the mass number A can estimated by

$$\Delta E_{p3/2-p1/2} = (V_{p_{3/2},d_{5/2}} - V_{p_{1/2},d_{5/2}})n_{d_{5/2}}(A) + (V_{p_{3/2},s_{1/2}} - V_{p_{1/2},s_{1/2}})n_{s_{1/2}}(A), \quad (7)$$

where V_{j_1,j_2} are monopole averages [24]. In the limit of strong neutron pairing discussed earlier, $n_{d_{5/2}}(A) \approx \frac{3}{4}(A-14)$ and $n_{s_{1/2}}(A) \approx \frac{1}{4}(A-14)$. Because of the similar radial forms of the $p_{3/2}$ and $p_{1/2}$ orbits the effect due to the central potential is small. The major contribution to the difference in the monopole terms arises from the tensor part of the effective interaction for the $d_{5/2}$ shell and from the two-body spin orbit for the $s_{1/2}$ shell. The tensor force appears to play a general role in driving the shell evolution across the nuclear chart [25].

Using, for example, the Schiffer-True interaction [26], as done in [24], we estimate a reduction of approximately 3 MeV between ^{14}C and ^{20}C. Proton ESPEs, presented in [27], are in agreement with our findings. We end by briefly mentioning that the coupling of the unperturbed 2_π^+ and 2_ν^+ states also gives rise to a second 2^+ of *mixed symmetry* character [28],

$$|2^+\rangle_{\rm MS} = -\beta|2^+\rangle_\nu + \alpha|2^+\rangle_\pi,$$

expected to be strongly populated in the knockout reactions mentioned earlier. Its observation will undoubtedly add weight to our picture but, unfortunately, it should lie at an excitation energy $\gtrsim$7 MeV, above the neutron separation energy, and thus likely decay by neutron emission.

Summary. We have presented a phenomenological analysis of recent data on the $B(E2)$ transition strengths in neutron-rich carbon isotopes. We argued that a seniority inspired scheme could be applicable for these cases, which allows us to write simple wave functions and formulas for electromagnetic properties and spectroscopic factors.

When contrasted with the experimental data, and due to the expected behavior of the neutron component in the seniority scheme, the most plausible explanation for the increase in the $B(E2)$s is by an increase in the proton amplitude. We suggested proton removal reactions and g-factor measurements to probe this component. This effect can be traced back to the quenching of the spin-orbit splitting between the proton $p_{3/2}$ and $p_{1/2}$ levels caused by the tensor and two-body spin-orbit components of the force between the protons and the added neutrons in the $(d_{5/2} + s_{1/2})$ shells.

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