Reexamining shell-model predictions for the mass of ^{17}Na(g.s.)

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A new shell-model calculation for ^{17}C provides spectroscopic factors for use in a computation of mirror energy differences between ^{17}C and ^{17}Na. Results are compared with previous predictions.

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Three recent papers [1–3] have presented predictions for the ground-state (g.s.) mass of ^{17}Na and for energies of some excited states. Reference [1] used the two-center microscopic cluster model (MCM) and an interaction V2, whose parameters had been tailored to fit the properties of the low-lying states of ^{17}C, the mirror of ^{17}Na. Reference [2] used the experimental ^{17}C energies as input to a calculation of Coulomb energies in ^{17}Na in a potential model. The nuclear potential was of Woods-Saxon shape with $r_0, a = 1.25, 0.65$ fm. The Coulomb potential was that of a uniform sphere. Both papers considered s and d nucleons coupled to various core states of ^{16}C and ^{16}Ne. As only the 0^+ g.s. and the 2^+ first-excited state were known in ^{16}Ne, the energies of the other ^{16}Ne states were calculated in Ref. [2] from the mirrors in ^{16}C and by using published wave functions [4]. Reference [2] took spectroscopic factors for ^{17}C from Ref. [1] for both V2 and shell-model (sm) interactions. Amos *et al.* [3] presented calculations of energies and widths for ^{17}Na by using the multichannel algebraic scattering (MCAS) theory. Reference [3] treated ^{17}C as a neutron coupled to collective 0^+, 2^+, and 4^+ states of ^{16}C and treated ^{17}Na as a proton coupled to their mirrors in ^{16}Ne.

Results in Ref. [2] were similar for the V2 and sm models for the two excited states. The g.s. mass prediction differed by 340 keV in the two calculations, but proton resonance energies for both $3/2^+$ and $5/2^+$ differed by only 3 and 18 keV, respectively. Perhaps more importantly, V2 results in Refs. [1,2] differed by 310, 810, and 770 keV for $1/2^+$, $3/2^+$, and $5/2^+$, respectively. All calculations agreed that the g.s. of ^{17}Na will be $1/2^+$. In addition to an overall energy shift, it seems that the MCM results do not fully account for the lowering of the $s_{1/2}$ proton energy (the so-called Thomas-Ehrman effect). Predicted energies of Amos *et al.* were quite different from those in the other two papers. After I pointed out some problems [5] with the Pauli principle in their approach, Amos *et al.* restated their belief in the validity of their method [6]. I do not intend to reopen those discussions here. Rather, I present results of a new shell-model calculation to investigate the robustness of the shell-model predictions. This calculation is smaller than the previous one [1,2] in scope, but it is complete within the $d_{5/2}s_{1/2}$ space as is the coupling of ^{17}C states to ^{16}C core states.

The differences mentioned above prompted me to repeat the calculations with spectroscopic factors from a completely different sm interaction. Relevant states are the ^{17}Na mirrors of the first three states in ^{17}C. The present sm is in the spirit of Lawson [7], who explicitly computed levels of ^{19}O in a basis of $(sd)^3$ where the sd orbitals were restricted to $s_{1/2}$ and $d_{5/2}$ only. I have used this procedure to compute wave functions in ^{17}C as ^{14}C(g.s.) $\times$ $(sd)^3$. As the first-excited state of ^{14}C is above 6 MeV, this is expected to be a reasonable assumption for low-lying levels of ^{17}C. I took single-particle energies from the $1/2^+$ and $5/2^+$ states of ^{15}C. The two-body residual interaction matrix elements were taken from Ref. [8] where they first arose in a consideration of the properties of ^{18}O. This interaction and space restriction is the same as in Ref. [4].

TABLE I. Spectroscopic factors for $^{16}\mathrm{C} + n = {}^{17}\mathrm{C}$ (V2 and SM are from Ref. [1]).

		$3/2^+$			$1/2^+$			$5/2^+$		
Core	sp	V2	SM	Present	V2	SM	Present	V2	SM	Present
0_1^+	s				0.828	0.644	0.465			
	d	0.010	0.035	∼0				0.558	0.701	0.821
0_2^+	s				Na[a]	Na[a]	0.534	Na[a]	Na[a]	
	d	Na	Na	∼0	Na[a]	Na[a]		Na[a]	Na[a]	0.0008
2_1^+	s	0.328	0.163	0.174				0.037	0.096	0.197
	d	1.260	1.445	1.647	0.034	0.415	0.507	0.520	0.226	0.166
2_2^+	s	0.030	0.225	0.270				0.050	0.014	0.163
	d	0.127	0.090	0.137	0.366	0.372	0.327	0	0.631	0.279
4^+	d	0.372	0.381	0.477				0.969	0.916	0.802
3^+	s							0	0.301	0.543
	d	0.026	0.285	0.371	0.091	1.027	1.167	0.060	0.003	0.0005

[a]Na stands for not available.

The present model differs from Ref. [2] in two other minor features: (1) The current geometrical parameters of the Woods-Saxon well are r_0, $a = 1.26, 0.60$ fm, and the Coulomb potential has $r_0 = 1.40$ fm; (2) I have included the second 0^+ core state, which was omitted in both Refs. [1,2]. As the energy of this state is not known in ^{16}Ne, it was necessary to compute its energy. I did this by using the wave functions [4] referred to earlier. The resulting excitation energy in ^{16}Ne is 2.735 MeV. With this inclusion, the current set of six core states for $A = 16$, $T = 2$ are all that exist in our sd space, so there is no missing strength. In Refs. [1,2], the different interactions resulted in different amounts of the total strength that reside in the chosen set of core states.

Present spectroscopic factors are listed in Table I where they are compared with two sets from Ref. [1] that were used in Ref. [2]. Numerical results are not drastically different, apart from the discrepancies already noted in Ref. [2], but some differences do exist. One difference is that the present S's sum to 3.0 for each of the three final states. In Ref. [1], this sum ranged from 1.32 to 2.15 for V2 and from 2.46 to 2.89 for sm.

TABLE II. Calculated proton energies (MeV) of the three lowest resonances in $^{17}\text{Na} = {}^{16}\text{Ne} + p$.

J^π	Reference [1]	Reference [2]		Present	MCAS
	V2	V2	SM	Brief Report	(Ref. [3])
$1/2^+$	2.40	2.71	3.05	3.02	1.03
$3/2^+$	2.57	3.38	3.39	3.22	2.26
$5/2^+$	2.97	3.74	3.72	3.47	1.13

Final energies are listed in Table II where they are compared with previous values [1–3]. Despite the strong differences in the input of the two sm calculations, differences in the final energies are smaller than the differences between V2 and either shell model. It would appear that the sm differences for the excited states can be traced to the smaller $s_{1/2}$ parentage in the sm of Refs. [1,2]. (For the g.s., the s parentage was already large there.) The MCAS results are seen to bear little resemblance to the other predictions. The differences between the various calculations are large enough to be easily tested if a suitable reaction can be found to produce ^{17}Na. I still await an experimental test of these predictions.

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