Short-range correlation effects on the nuclear matrix element of neutrinoless double-β decay

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(Received 26 May 2014; published 29 December 2014)

We report the results of a calculation of the nuclear matrix element of neutrinoless double-β decay of ^{48}Ca, carried out taking into account nucleon-nucleon correlations in both coordinate and spin space. Our numerical results, obtained using nuclear matter correlation functions, suggest that inclusion of correlations may lead to a ∼20% decrease of the matrix element, with respect to the shell-model prediction. This conclusion is supported by the results of an independent calculation, in which correlation effects are taken into account using the spectroscopic factors of ^{48}Ca, obtained from an *ab initio* many-body approach, to renormalize the shell-model states.

DOI: 10.1103/PhysRevC.90.065504 PACS number(s): 23.40.Hc, 24.10.Cn, 27.40.+z

I. INTRODUCTION

A fully quantitative approach to the calculation of the nuclear matrix element (NME) determining the neutrinoless double-β ($0\nu\beta\beta$) decay rate (see, e.g., Refs. [1,2]) requires the inclusion of correlation effects, not taken into account in the mean-field approximation underlying the nuclear shell model.

High-resolution electron-induced nucleon knock-out experiments have provided unambiguous evidence of the inadequacy of the independent particle model (IPM) to describe the full complexity of nuclear dynamics. While the peaks associated with knock out from shell-model orbits can be clearly identified in the measured missing energy spectra, the integrated strengths, yielding the corresponding spectroscopic factors, turn out to be significantly lower than the IPM predictions, independent of the nuclear mass number [3,4].

Long-range correlations are usually included within the framework of the quasiparticle random phase approximation (QRPA) and its extension, or carrying out large-scale shell-model calculations. The procedure routinely employed to take into account the effect of short-range correlations is based on the modification of the shell-model states entering the two-body transition matrix element through the action of a correlation function (see, e.g., Refs. [5–8]).

Many existing calculations have been performed using the somewhat oversimplified correlation function referred to as Miller-Spencer (MS), depending on the magnitude of the internucleon distance only. More advanced correlation functions, obtained using the Brueckner-Goldstone (BG) formalism [9] or the unitary correlation operator method (UCOM) [10] and projecting onto the two-nucleon channel of total spin and isospin $S = 0$ and $T = 1$, have been used in Refs. [5–7].

The results of Refs. [5–7] exhibit a strong dependence on the shape of the correlation function. The authors of Ref. [5], who carried out calculations for ^{48}Ca and ^{76}Ge, report a 30–40% reduction of the NME, with respect to the shell-model prediction, obtained using the MS model, to be compared to a 7–16% effect resulting from calculations carried out with the UCOM correlation function. In Ref. [7], different choices of the correlation function result in qualitatively different predictions for ^{48}Ca. The MS model yields a ∼20% suppression of the NME, while use of BG correlation functions leads to a ∼20% enhancement.

Nuclear matter studies carried out within *ab initio* many-body approaches, based on state-of-the-art models of the nuclear hamiltonian, clearly show that the correlation function features a complex operator structure, reflecting the strong spin and isospin dependence of the *NN* potential as well as its non-spherically-symmetric nature.

As a first step towards the development of a fully realistic and consistent implementation of short-range correlations in calculations of the $0\nu\beta\beta$ NME, we have studied the effect of spin- and isospin-dependent correlations on the NME of the $^{48}\text{Ca} \rightarrow {}^{48}\text{Ti}\, 0\nu\beta\beta$ decay using the results of accurate nuclear matter calculations, carried out within the correlated basis function (CBF) approach. The inclusion of isospin dependence is needed to take into account the differences between the correlation functions acting in the neutron-neutron and proton-neutron channels. Moreover, spin-dependent correlations affect the Fermi and Gamow-Teller character of the transition matrix elements, leading to a mixing of the corresponding contributions.

Being the simplest case from the point of view of nuclear structure, the $^{48}\text{Ca} \rightarrow {}^{48}\text{Ti}$ decay appears to be best suited for our exploratory analysis of correlation effects. In addition, searches of this decay are being carried out by the CANDLES [11] and CARVEL [12] experiments.

To gauge the robustness of our approach and assess the role of finite-size and shell effects—neglected when using

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nuclear matter correlation functions—we have also carried out an independent calculation, in which correlations have been taken into account through renormalization of the shell-model states of ^{48}Ca.

In Sec. II, after recollecting the expressions of the Fermi and Gamow-Teller contributions to $0\nu\beta\beta$ decay within the closure approximation, we discuss the shell-model structure of the two-nucleon matrix elements (Sec. II A) and the modifications arising from the inclusion of nucleon-nucleon correlations (Sec. II B). The results of our calculations, including those obtained from the alternative approach based on spectroscopic factors, are reported in Sec. III, while in Sec. IV we summarize our findings and state the conclusions.

II. $0\nu\beta\beta$ DECAY

The half life associated with the $0\nu\beta\beta$ decay of a nucleus of mass A and charge Z

$$(\mathrm{A,Z}) \rightarrow (\mathrm{A,Z}-2) + 2e^-, \tag{1}$$

τ, can be written in the form (see, e.g., Ref. [2])

$$\frac{1}{\tau} = G|M|^2\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2, \tag{2}$$

where G is a phase-space factor, m_e is the electron mass and the so called effective neutrino mass is defined in terms of neutrino mass eigenvalues and elements of the mixing matrix according to

$$\langle m_{\beta\beta}\rangle = \left|\sum_k U_{ek}^2 m_k\right|^2. \tag{3}$$

The NME can be cast in the form

$$M = M_{\mathrm{GT}} - \left(\frac{g_V}{g_A}\right)^2 M_{\mathrm{F}}, \tag{4}$$

where g_V and g_A are the vector and axial-vector coupling constant, respectively, while M_{F} and M_{GT} denote the Fermi (F) and Gamow-Teller (GT) transition matrix elements.

Within the closure approximation (see, e.g., Ref. [1]) M_{F} and M_{GT} can be written in the general form

$$M_\alpha = \left\langle \Psi_f, \mathcal{J}_f^\pi \right| \sum_{jk} \tau_j^+ \tau_k^+ O_{jk}^\alpha(r) \left| \Psi_i, \mathcal{J}_i^\pi \right\rangle, \tag{5}$$

where $\alpha = \mathrm{F, GT}$, τ_i^+ is the charge-raising operator acting in the isospin space of the ith nucleon and Ψ_i and Ψ_f are the initial and final nuclear states, the total angular momentum and parity of which are labeled $\mathcal{J}_i^\pi$ and $\mathcal{J}_f^\pi$.

The transition operators $O_{jk}^\alpha(r)$ are defined as

$$O_{jk}^F(r) = \mathbb{1}\, H(r_{jk}), \quad O_{jk}^{GT}(r) = (\boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k)\, H(r_{jk}), \tag{6}$$

where $H(r_{jk})$ is the so-called neutrino potential, given by

$$H(r_{jk}) = R_A \frac{2}{\pi} \int_0^{+\infty} \frac{j_0(q r_{jk})}{q + \langle E\rangle} q dq, \tag{7}$$

with $j_0(x) = \sin x/x$. In the above equations, $r_{jk} = |\mathbf{r}_j - \mathbf{r}_k|$ is the magnitude of the distance between the two nucleons involved in the decay process, R_A is the nuclear radius and $\langle E\rangle$ is the average energy of the virtual intermediate states employed in the closure approximation.

Note that the above equations do not take into account the effect of nucleon form factors, which—based on the results available in the literature—is expected to be small. For example, the authors of Ref. [6] report a change of less that 2% in the NME of $^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$ $0\nu\beta\beta$ decay.

A. Two-body matrix elements

We assume that two neutrons of the initial-state nucleus decay, while the other nucleons act as spectators. Owing to the two-body nature of the transition operators, the matrix element in Eq. (5) can be reduced to a sum of products of two-body transition densities (TBTD) and antisymmetrized two-body matrix elements [7]

$$M_\alpha = \sum_{j_1,j_2,j_1',j_2',J^\pi} TBTD\,(j_1,j_2,j_1',j_2';J^\pi) \times \langle j_1' j_2';\ J^\pi\ T|\,\tau_1^+\tau_2^+ O_{12}^\alpha(r)\,|j_1 j_2;\ J^\pi\ T\rangle_a. \tag{8}$$

Here, the indices 1 and 2 label the quantum numbers of the two decaying neutrons, while 1′ and 2′ refer to the final-state protons. The angular momentum of a nucleon participating in the decay is denoted j_i or j_i' ($i = 1,2$), while J^π and T specify the total angular momentum, parity and isospin of the nucleon pair, respectively. Finally, the notation $|\cdots\rangle_a$ refers to antisymmetrized two-particle states.

The coefficients $TBTD\,(j_1,j_2,j_1',j_2';J^\pi)$ describe how the spectator nucleons rearrange themselves as a result of the decay process. They are computed in a model space using an effective nucleon-nucleon interaction.

In order to carry out the calculation, the two-body matrix element in Eq. (8) must be decomposed into products of reduced matrix elements of operators acting in spin and coordinate space. In addition, the coordinate-space two-nucleon state is rewritten in terms of relative and center of mass coordinates, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R}_{12} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, according to

$$\langle \mathbf{r}_1|k_1 l_1\rangle\langle \mathbf{r}_2|k_2 l_2\rangle = \sum_{k,l,K,L} \langle kl,\, KL|k_1 l_1,\, k_2 l_2\rangle_\Lambda \times \langle \mathbf{R}_{12}|KL\rangle\langle \mathbf{r}_{12}|kl\rangle, \tag{9}$$

where k_i and l_i are the principal and angular momentum quantum numbers, respectively, while $\langle\ldots\rangle_\Lambda$, Λ being the angular momentum of the proton pair in the final nucleus, are the coefficients of the Talmi-Moshinski transformation of the harmonic oscillator basis [13,14].

B. Correlated wave functions

Within CBF, the correlated nuclear states, $|\Psi_n\rangle$, are obtained from the shell-model eigenstates, $|\Phi_n\rangle$, through the transformation

$$|\Psi_n\rangle = F|\Phi_n\rangle, \tag{10}$$

where the operator F, embodying the correlation structure induced by the NN interaction, is written in the form

$$F = \mathcal{S}\prod_{ij} f_{ij}. \tag{11}$$

Note that, in general, $[f_{ij}, f_{ik}] \neq 0$. As a consequence, the product in the right-hand side of Eq. (11) has to be symmetrized through the action of the operator $\mathcal{S}$.

The two-body correlation functions f_{ij}, the operator structure of which reflects the structure of the NN potential, can be cast in the form

$$f_{ij} = \sum_{m=1}^{6} f^{(m)}(r_{ij}) O_{ij}^{(m)}, \quad (12)$$

with

$$O_{ij}^{(m)} = [1,(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}] \otimes [1,(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)] \quad (13)$$

where $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ are Pauli matrices acting in spin and isospin space, respectively, and

$$S_{ij} = \frac{3}{r_{ij}^2}(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{ij}) - (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j). \quad (14)$$

The scheme routinely employed to include correlations in the nuclear matrix elements of Eq. (5)—which can be loosely related to the lowest-order (i.e., two-body) approximation of the cluster expansion formalism (see, e.g., Ref. [15])—amounts to modifying the state describing the relative motion of the nucleon pair involved in the decay process, appearing in Eq. (9), according to

$$|kl\rangle \to f_{12}|kl\rangle. \quad (15)$$

Note that the above prescription can be seen just as well as a replacement of the Fermi and Gamow-Teller transition operators with the effective operators $\widetilde{O}_{12}^{\alpha}$, defined as

$$\widetilde{O}_{12}^{\alpha} = f_{12} O_{12}^{\alpha} f_{12}. \quad (16)$$

Equation (16) implies that using the correlation function defined by Eqs. (12) and (13) affects the operator structure of the transition operators. To see this, consider, for example, the somewhat simplified case of a correlation function including contributions with $m \leqslant 4$ only. Because for nucleons participating in double-β decay $(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) = 1$, the resulting correlation functions can be rewritten in the form [see Eqs. (12) and (13)]

$$f_{12} = f(r_{12}) + g(r_{12})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \quad (17)$$

with

$$f(r_{12}) = f^{(1)}(r_{12}) + f^{(2)}(r_{12}), \quad (18)$$

$$g(r_{12}) = f^{(3)}(r_{12}) + f^{(4)}(r_{12}). \quad (19)$$

From the above definitions and the relation $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)^2 = 3 - 2(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$, it follows that inclusion of correlations in the two-body matrix elements leads to the appearance of a Gamow-Teller contribution to the matrix element of O_{12}^F, along with a Fermi contribution to the matrix element of O_{12}^{GT}.

Substituting Eq. (17) into Eq. (16) one finds

$$\widetilde{O}_{12}^{F} = [f^2(r_{12}) + 3g^2(r_{12})]O_{12}^F + 2g(r_{12})[f(r_{12}) - g(r_{12})]O_{12}^{GT}, \quad (20)$$

and

$$\widetilde{O}_{12}^{GT} = [f^2(r_{12}) - 4f(r_{12})g(r_{12}) + 7g^2(r_{12})]O_{12}^{GT} + 6g(r_{12})[f(r_{12}) - g(r_{12})]O_{12}^{F}. \quad (21)$$

III. RESULTS

As stated in Sec. I, our analysis is aimed at studying the effects of nucleon-nucleon correlations. Therefore, we have kept the complications associated with the shell-model description of the nuclear states to a minimum.

We focused on the reaction

$$^{48}_{20}Ca \to ^{48}_{22}Ti + 2e^-, \quad (22)$$

in which the initial and the final nucleus are both in their ground states, having $\mathcal{J}^{\pi} = 0^+$. Note that ^{48}Ca is the lightest nucleus that can undergo double-β decay, and its shell structure is quite simple, $Z = 20$ and $(A - Z) = 28$ being both magic numbers, corresponding to closed shells.

We consider the case in which the neutrons and protons involved in the decay process occupy the $1f_{7/2}$ shell. As a consequence, in the matrix element of Eqs. (8) and (9) $j_1 = j_2 = j_1' = j_2' = 7/2$, $k_1 = k_2 = k_1' = k_2' = 0$, and $l_1 = l_2 = l_1' = l_2' = 3$. Numerical calculations have been carried out using the TBTD reported in Ref. [16] and harmonic oscillator wave functions corresponding to $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$MeV. The vector and axial-vector coupling constant and the average energy of Eq. (7) have been set to the values reported in Ref. [7]: $g_V = 1$, $g_A = 1.25$ and $\langle E \rangle = 7.72$MeV. Note that the dependence of the NME on the average energy is quite weak. Changing the value of $\langle E \rangle$ from 2.5 MeV to 12.5 MeV results in a variation of the NME of less than 5% [7].

The correlation operator employed in this work includes the components with $m \leqslant 4$ of Eq. (12), needed to take into account spin and isospin dependence. The radial dependence of the functions $f^{(m)}(r_{12})$ has been obtained from a realistic nuclear hamiltonian including the Argonne v_6' NN potential, solving the set of Euler-Lagrange equations derived from the minimization of the ground-state energy of isospin-symmetric nuclear matter at equilibrium density [17].

In Fig. 1 the correlation functions $f(r_{12})$ and $g(r_{12})$ (the latter multiplied by a factor 5) of Eq. (17) are compared to those employed in the study of the ^{48}Ca $\to$ ^{48}Ti $0\nu\beta\beta$ decay

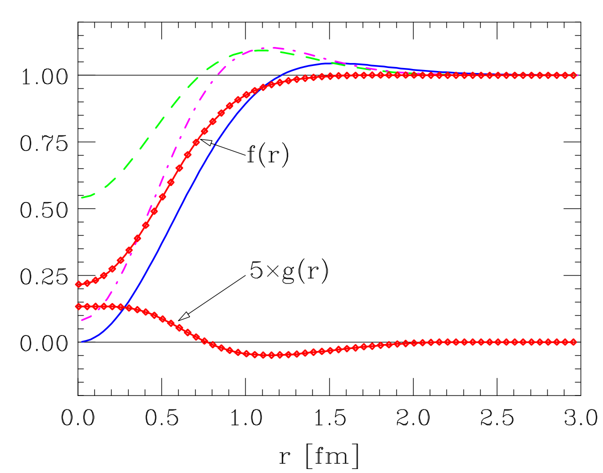

FIG. 1. (Color online) Radial behavior of the correlation functions of Eq. (17). The MS (solid line), AV 18 (dot-dash line), and CD Bonn (dashed line) correlation functions employed in Ref. [7] are also shown, for comparison.

TABLE I. Ratio between the $0\nu\beta\beta$ NME of Eq. (4), computed including central and central plus spin-dependent correlations and the corresponding quantity obtained setting $f(r_{12}) = 1$ and $g(r_{12}) = 0$.

	$f(r_{12})$	$f(r_{12}) + g(r_{12})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$
M/M_{SM}	0.77	0.79

described in Ref. [7]. The solid, dot-dash, and dashed line correspond to the correlation functions referred to as MS, AV 18, and CD Bonn, respectively [7].

The numerical values of the ratio M/M_{SM}, where M_{SM} is the NME computed without including correlations—which amounts to setting $f(r_{ij}) = 1$ and $g(r_{ij}) = 0$—are listed in Table I. It appears that inclusion of central correlations leads to a $\gtrsim$20% decrease of the NME, while the effect of spin-dependent correlations is small, and goes in the opposite direction.

Our result turns out to be close to that obtained in Ref. [7] using the MS correlation function, while the the authors of Ref. [5] find an even larger effect. On the other hand, Ref. [7] also reports a ∼10% and ∼20% enhancement of the ratio M/M_{SM}, resulting from calculations carried out with the CD-Bonn and AV 18 correlation functions, respectively. Comparison between the shapes of the correlation functions, displayed in Fig. 1 suggests that the qualitative differences in the calculated M/M_{SM} ratios reflect the differences in shape of the correlation functions. The enhancement of the NME, yielding $M/M_{SM} > 1$, appears to be associated with the use of correlation functions that sizeably overshoot unity at intermediate distance, while exhibiting a less pronounced correlation hole at short distance. The authors of Ref. [5] also attribute the small reduction of the NME obtained using the UCOM approach to the reduced correlation hole exhibited by the corresponding correlation function.

Valuable insight on the behavior of nucleon-nucleon correlations can be obtained from theoretical studies of infinite nuclear matter. The simplifications arising from translation invariance allow one to carry out accurate calculations of the two-nucleon distribution functions—yielding the probability distribution of finding two nucleons at separation distance r—in both the neutron-neutron (nn) [or, equivalently, proton-proton (pp)] and proton-neutron (pn) channels. They are defined as

$$g^{nn}(r) = \frac{1}{4\pi r^2}\left\langle \sum_{j>i} \delta(r - r_{ij})\frac{1}{2}\big(1 - \tau_i^3\big)\frac{1}{2}\big(1 - \tau_j^3\big)\right\rangle, \quad (23)$$

$$g^{pn}(r) = \frac{1}{4\pi r^2}\left\langle \sum_{j>i} \delta(r - r_{ij})\frac{1}{2}\big(1 + \tau_i^3\big)\frac{1}{2}\big(1 - \tau_j^3\big)\right\rangle, \quad (24)$$

where τ_i^3 is the matrix describing the third component of the isospin of particle i, while $\langle \ldots \rangle$ denotes the ground-state expectation value.

Figure 2 shows the radial dependence of the distribution functions $g^{nn}(r)$ (solid line) and $g^{pn}(r)$ (dashed line), at nuclear matter equilibrium density, computed using the Fermi hypernetted chain (FHNC) summation scheme and the Argonne v_6' NN potential [17]. The dot-dash line corresponds to the results obtained at lowest order of the cluster expansion with the correlation function of Eqs. (17)–(19).

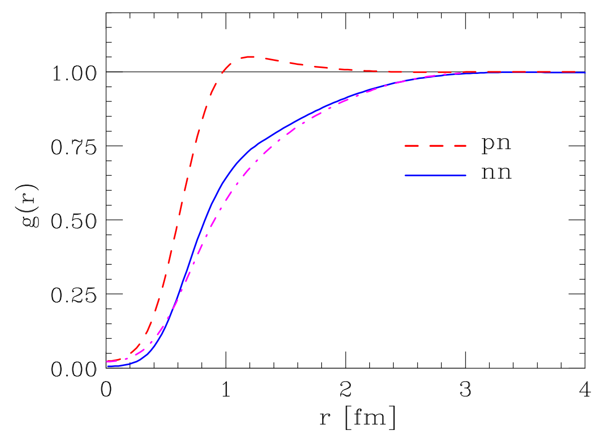

FIG. 2. (Color online) Radial dependence of the neutron-neutron (solid line) and proton-neutron (dashed line) distribution functions of Eqs. (23) and (24), computed within the FHNC approach using the Argonne v_6' NN potential [17]. The dot-dash line corresponds to the results obtained at two-body cluster level using a correlation function defined as in Eqs. (17)–(19).

In comparing Figs. 1 and 2, it has to be kept in mind that they show different quantities. The leading term in the cluster expansion of the distribution functions, defined by Eqs. (23) and (24), is quadratic in the correlation functions displayed in Fig. 1. Moreover, owing to the effect of Pauli's exclusion principle, the distribution functions have longer range.

It clearly appears that inclusion of higher-order cluster contributions does not appreciably affect the shape of the neutron-neutron correlation function in uniform matter. Most notably, it does not lead to either a reduction of the correlation hole or to the appearance of a region in which $f(r) > 1$. However, surface and shell effects are expected to play a role, and their importance needs to be estimated.

A different procedure to include correlation effects in the NME of the $0\nu\beta\beta$ decay, based on concept of quasiparticle in interacting many-body systems, exploits the renormalization of the shell-model states. Within this scheme the single nucleon state of quantum numbers $k_i l_i j_i$ is modified according to [compare to Eq. (15)]

$$|k_i l_i j_i\rangle \to \sqrt{Z_{k_i l_i j_i}}\, |k_i l_i j_i\rangle. \quad (25)$$

The spectroscopic factor $Z_{k_i l_i j_i}$ is the residue of the Green's function at the single-particle pole, not to be confused with the occupation probability [18]. It is defined as [19]

$$Z^{\alpha}_{k_i l_i j_i} = \int d^3x \left|\phi^{\alpha}_{k_i l_i j_i}(x)\right|^2, \quad (26)$$

where the superscript $\alpha = p, n$ specifies the third component of the isospin, while the quasihole wave function $\phi^{\alpha}_{k_i l_i j_i}$ is given by

$$\phi^{\alpha}_{k_i l_i j_i}(x_1) = \frac{\sqrt{A}}{N^{\alpha}_{k_i l_i j_i}}\left\langle \Psi^{\alpha}_{k_i l_i j_i}(x_2,\ldots,x_A)\middle|\Psi_0(x_1,\ldots,x_A)\right\rangle. \quad (27)$$

In the above equation, $|\Psi_0\rangle$ and $|\Psi^\alpha_{k_i l_i j_i}\rangle$ denote the nuclear ground state and the $(A-1)$-nucleon state obtained removing a nucleon with quantum numbers $k_i l_i j_i$ and isospin projection α, respectively. The normalization factor is

$$N^\alpha_{k_i l_i j_i} = \langle \Psi^\alpha_{k_i l_i j_i} | \Psi^\alpha_{k_i l_i j_i} \rangle^{1/2} \langle \Psi_0 | \Psi_0 \rangle^{1/2}. \qquad (28)$$

It is important to realize that using renormalized single-particle states is conceptually equivalent to using correlated states. In the absence of correlations, $Z_{k_i l_i j_j} = 1$ for all occupied shell-model states, and $Z_{k_i l_i j_i} = 0$ otherwise.

The authors of Ref. [19] have carried out an *ab initio* calculation of the spectroscopic factors of ^{48}Ca within the FHNC approach, using a nuclear hamiltonian including the Argonne v_8' *NN* potential supplemented with the *UIX* three-nucleon potential. It has to be pointed out that their results take into account both finite-size and shell effects and the contributions of many-nucleon correlations.

We have employed the results of Ref. [19] to describe correlation effects in the NME of $0\nu\beta\beta$ decay through the replacement

$$M_\alpha \to \widetilde{M}_\alpha = Z^p_{1f_{7/2}}(^{48}\text{Ti}) Z^n_{1f_{7/2}}(^{48}\text{Ca}) M_\alpha, \qquad (29)$$

which, under the additional assumption

$$Z^p_{1f_{7/2}}(^{48}\text{Ti}) \approx Z^n_{1f_{7/2}}(^{48}\text{Ca}), \qquad (30)$$

yields

$$\widetilde{M}_\alpha = \left[Z^n_{1f_{7/2}}(^{48}\text{Ca})\right]^2 M_\alpha. \qquad (31)$$

The correspondence between the above result and the expression of the NME involving the correlation functions can be easily grasped substituting correlated states in Eq. (27), and using the two-body cluster approximation to evaluate the overlap.

Substitution of the numerical value reported in Ref. [19], $Z^n_{1f_{7/2}}(^{48}\text{Ca}) = 0.91$, in the NME of Eq. (31) yields $M/M_{SM} = 0.83$, in fair agreement with the results listed in Table I.

Note that, although the validity of the approximation of Eq. (30) should be carefully investigated, nuclear matter results clearly support its accuracy [20].

IV. CONCLUSIONS

We have carried out a study aimed at analyzing the effects of short-range *NN* correlations on the NME of the $0\nu\beta\beta$ decay of ^{48}Ca.

The results of calculations performed using spin- and isospin-dependent correlation functions, obtained from the minimization of the ground-state energy of isospin symmetric nuclear matter at equilibrium density, indicate that inclusion of correlations leads to a $\sim 20\%$ decrease of the NME, with respect to the shell-model prediction. A similar or larger reduction has been obtained by the authors of Ref. [5,7] using the MS correlation function.

Comparison between our results and those of Ref. [5–7] suggests that the radial behavior of the correlation function plays a critical role. Using correlation functions that sizeably overshoot unity and feature a reduced correlation hole leads to predict a small decrease [5,6], or even an enhancement [7] of the NME.

The approach employed to obtain the correlation functions used in our work provides a realistic description of the short-range structure of two-nucleon states in nuclear matter, properly taking into account the differences between *nn* and *pn* pairs. Moreover, Fig. 2 shows that in uniform nuclear matter the lowest-order result is close to that obtained taking into account many-body contributions within the FHNC summation scheme.

The accuracy of the two-body cluster approximation has been recently questioned on the basis of isospin considerations [8]. The authors of Ref. [8] have shown that, owing to the sizable overshoot, the correlation function of Ref. [6] preserves isospin symmetry better than the nuclear matter correlation function of Akmal and Pandharipande, which turns out to be very similar to the one employed in our work. However, as correctly pointed out in the concluding section of Ref. [8], this feature does not arise from many-body cluster contributions, which are not taken into account in the Brueckner-Goldstone approach. In this context, it is worth noting that the summation of ladder diagrams carried out to obtain the correlation function of Ref. [6] has long been shown to be equivalent to the use of a correlated two-nucleon state, defined as in Eq. (15) [21].

The differences between the correlation function employed in this work and those referred to as CD-Bonn and AV 18 can be probably traced back to the fact that, while the former has been obtained from a nuclear matter calculation, the derivation of the latter explicitly takes into account finite-size and shell effects [6].

In order to gauge the robustness of our result against inclusion of these effects, we have estimated the $0\nu\beta\beta$ decay NME using the spectroscopic factors of ^{48}Ca computed in Ref. [19] within the framework of the FHNC approach. The $\sim 20\%$ suppression obtained from the alternative approach turns out to be remarkably close to that obtained from modifying the two-nucleon matrix elements through the action of the nuclear matter correlation function.

In view of the correspondence of the two approaches, pointed out in Sec. III, the agreement between the results obtained using the spectroscopic factors and those obtained from modifying the two-nucleon states entering the transition matrix element is likely to reflect the fact that the correlation functions of Ref. [19], derived within a framework in which finite-size and shell effects—as well as many-body cluster contributions—are taken into account, turn out to be similar to the nuclear matter correlation functions displayed in Fig. 1.

A fully consistent implementation of the formalism based on spectroscopic factor would require that the renormalization procedure be extended to all the shell-model states, not just those entering the two-nucleon transition matrix element. However, it is important to realize that the formalism founded on the modification of the two-nucleon states suffers from the same lack of consistency, as it ignores correlations between the spectator particles, not involved in the decay process. Using spectroscopic factors or correlation functions in the two-body matrix element is in fact conceptually equivalent.

The inconvenient truth is that, whatever the theoretical approach taken, the consistent inclusion of short-range correlations in the shell-model picture is a longstanding and still very elusive issue.

While the results reported in this paper appear to be encouraging, further studies, aimed at firmly establishing the relation between the two schemes employed in our work, are certainly called for. These studies will have to involve the analysis of higher-order contributions to the cluster expansion of the quasihole wave function, discussed in Ref. [19], as well as the use of a correlation function obtained from the minimization of the ground-state energy of ^{48}Ca, and the inclusion of the full operator structure of Eqs. (12)–(13).

ACKNOWLEDGMENTS

The authors are grateful to Alessandro Lovato for providing tables of the nuclear matter distribution functions shown in Fig. 2, as well as for many illuminating discussions. This work was supported by INFN under Grant No. MB31.

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