Polarization observables in lepton-deuteron elastic scattering including the lepton mass

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Expressions for the unpolarized differential cross section and for various polarization observables in the lepton-deuteron elastic scattering, $\ell + D \to \ell + D$, $\ell = e, \mu, \tau$, have been obtained in the one-photon-exchange approximation, taking into account the lepton mass. Polarization effects have been investigated for the case of a polarized lepton beam and polarized deuteron target which can have vector or tensor polarization. Numerical estimations of the lepton mass effects have been done for the unpolarized differential cross section and for some polarization observables and applied to the case of low-energy muon deuteron elastic scattering.

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I. INTRODUCTION

The structure of hadrons and nuclei is traditionally studied through elastic and inelastic electron-hadron (nuclei) scattering as well as through elementary annihilation reactions, assuming a one-photon-exchange mechanism (for a recent review, see Ref. [1]). A review of the results obtained by the measurements of the unpolarized cross section and polarization observables in the elastic electron-nucleon scattering can be found in Ref. [2]. Nucleon form factors in the timelike region are reviewed in Ref. [3]. A review of the deuteron electromagnetic structure is given in Ref. [4].

Recently, results from the measurement of the proton charge radius were obtained in an experiment performed at the Paul Scherrer Institute (PSI, Switzerland) [5] from the Lamb shift in muonic hydrogen (CREMA collaboration). The value obtained is significantly different from earlier measurements based on electronic hydrogen spectroscopy and elastic electron-proton scattering and is smaller by 7σ than the 2010 CODATA official value [6]. Various explanations of this difference were proposed.

Some authors suggested the possible existence of new particles that interact with muons and hadrons but not with electrons. By adjusting the couplings of these particles one can, in principle, obtain an additional energy shift in the muonic hydrogen. This may lead to agreement between the measurement of the proton charge radius in the muonic and electronic experiments. Thus, for example, the existence of new particles with scalar and pseudoscalar (or vector and axial) couplings were proposed in Ref. [7]. The couplings are constrained by the existing data on the Lamb shift, muon magnetic moment, and kaon decay rate. New vector and scalar particles at the 100 MeV scale were proposed in Ref. [8]. The important consequence would be an enhancement by several orders of magnitude of the parity-violating asymmetries in the scattering of low-energy muons from nuclei.

On the other hand, the authors of the Ref. [9] have analyzed the recent electron-proton scattering data obtained at Mainz [10] (the cross sections were measured with statistical errors below 0.2%). By using a dispersive approach they obtained a small value for the proton charge radius which is consistent with the recent result obtained in the experiment with muonic hydrogen. In Ref. [11] it was shown that previous extractions of the proton charge radius from the electron-proton scattering data may have underestimated the errors.

In electron-proton elastic scattering experiments, the radius is related to the slope of the charge form factors as a function of the transferred momentum squared, Q^2, in the limit $Q^2 \to 0$:

$$\langle r_c^2 \rangle = -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}. \tag{1}$$

In Ref. [12] it was suggested that the error related to the extrapolation $Q^2 \to 0$ could be reduced by measuring this process in inverse kinematics.

Note that, about 40 years ago, there were tests of the muon-electron universality in the processes of the elastic and deep inelastic electron (muon) scattering. Measurements of the muon-proton elastic cross section in the range $0.15 \leqslant Q^2 \leqslant 0.85$ GeV^2 were compared with similar electron-proton data [13]. An apparent disagreement was found between muon and electron experiments which can possibly be accounted for by a combination of systematic normalization errors [13]. The data were obtained at rather high values of Q^2 in order to extract the proton charge radius. In Ref. [14], the muon-proton elastic scattering was measured in the range $0.6 \leqslant Q^2 \leqslant 3.2$ GeV^2. A possible difference from muon-electron universality was found, but the statistical accuracy of this observation was not compelling. The muon-proton deep inelastic scattering was measured in the range $0.4 \leqslant Q^2 \leqslant 3.6$ GeV^2 [15]. The data were consistent with muon-electron universality. Two-photon-exchange effects were investigated in the muon-proton elastic scattering [16]. The validity of the one-photon-exchange approximation was confirmed for Q^2 up to 0.85 GeV^2 and incident muon energies up to 17 GeV.

The fact that the proton charge radius was not measured in the process of the elastic muon-proton scattering led to the proposal of the MUon proton Scattering Experiment (MUSE) at the Paul Scherrer Institute (Zurich) [17]. This experiment

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plans a simultaneous measurement of the elastic $\mu^- p$ and $e^- p$ scattering as well as $\mu^+ p$ and $e^+ p$ and will establish the consistency or the difference of the muon-proton and electron-proton interaction with good precision in the considered kinematics. Three values of the muon beam momenta which are comparable with the muon mass: 115, 153, and 210 MeV, were chosen. However, in case of low energy and large lepton mass, the terms proportional to the lepton mass become important and the mass should be taken explicitly into account in the calculation of the kinematical variables and of the experimental observables. The expressions of the kinematical relations and of the polarized and unpolarized observables are different from those currently used. In Ref. [18] the effect of the lepton mass was discussed for muon-proton elastic scattering for the unpolarized cross section and the double spin asymmetry, where the lepton beam and the target are polarized.

The MUSE experiment at PSI will also determine the radii of light nuclei through muon elastic scattering. Of particular interest is a measurement on deuterium. The issue of taking into account finite-lepton-mass effects is also relevant for the case of the elastic muon-deuteron scattering.

In this paper we calculate the expressions for the unpolarized differential cross section and polarization observables, taking into account the lepton mass, for elastic lepton-deuteron scattering. We calculate the asymmetries due to the tensor polarization of the deuteron target and the spin correlation coefficients due to the lepton beam polarization and vector polarization of the deuteron target. Explicit formulas are given in two coordinate systems which are relevant for the experiment: in the first one the z axis is directed along the lepton beam momentum and in the second one the z axis is directed along the virtual photon momentum (or along the transferred momentum).

II. FORMALISM

Let us consider the reaction

$$\ell(k_1) + D(p_1) \to \ell(k_2) + D(p_2), \quad \ell = e,\mu,\tau, \tag{2}$$

where the momenta of the particles are written in parentheses. In the laboratory system, where we perform our analysis, the deuteron (lepton) four-momenta in the initial and final states are, respectively, p_1 and p_2 (k_1 and k_2) with components

$$p_1 = (M,0), \quad p_2 = (E_2,\vec{p}_2),$$
$$k_1 = (\varepsilon_1,\vec{k}_1), \quad k_2 = (\varepsilon_2,\vec{k}_2), \tag{3}$$

where M is the deuteron mass.

The matrix element of the reaction (2) can be written as follows in the one-photon-exchange approximation:

$$\mathcal{M} = \frac{e^2}{Q^2} j_\mu J^\mu, \quad j_\mu = \bar{u}(k_2)\gamma_\mu u(k_1). \tag{4}$$

By using the requirements of the Lorentz invariance, current conservation, parity and time-reversal invariance of the hadron electromagnetic interaction, the general form of the electromagnetic current for the spin-one deuteron is completely described by three form factors and can be written, following Ref. [19], as

$$J_\mu = (p_1+p_2)_\mu \bigg[-G_1(Q^2) U_1 \cdot U_2^* + \frac{G_3(Q^2)}{M^2} \times \bigg(U_1 \cdot q U_2^* \cdot q - \frac{q^2}{2} U_1 \cdot U_2^* \bigg) \bigg] + G_2(Q^2)(U_{1\mu} U_2^* \cdot q - U_{2\mu}^* U_1 \cdot q), \tag{5}$$

where $q = k_1 - k_2 = p_2 - p_1$, $Q^2 = -q^2 = -2M(M - E_d)$, E_d is the final deuteron energy, and $U_{1\mu}$ and $U_{2\mu}$ are the polarization four-vectors for the initial and final deuteron states. The functions $G_i(Q^2)$, $i = 1,2,3$, are the deuteron electromagnetic form factors, which are real functions in the region of the spacelike momentum transfer and depend only on the virtual photon four-momentum squared. These form factors are related to the standard deuteron form factors: G_C (the charge monopole), G_M (the magnetic dipole), and G_Q (the charge quadrupole) by

$$G_M = -G_2, \quad G_Q = G_1 + G_2 + 2G_3,$$
$$G_C = \tfrac{2}{3}\tau(G_2 - G_3) + \big(1 + \tfrac{2}{3}\tau\big) G_1, \tag{6}$$

with $\tau = Q^2/(4M^2)$. The standard form factors have the following normalization:

$$G_C(0) = 1, \quad G_M(0) = (M/m_n)\mu_d, \quad G_Q(0) = M^2 \mathcal{Q}_d,$$

where m_n is the nucleon mass, $\mu_d(\mathcal{Q}_d)$ is deuteron magnetic (quadrupole) moment and their values are $\mu_d = 0.857$ [20], $\mathcal{Q}_d = 0.2859\, fm^2$ [21].

The differential cross section can be written in terms of the matrix element modulus squared as

$$d\sigma = \frac{(2\pi)^4}{4I} |\mathcal{M}|^2 \frac{d\vec{k}_2 d\vec{p}_2}{(2\pi)^6 4\varepsilon_2 E_2} \delta^{(4)}(k_1 + p_1 - k_2 - p_2), \tag{7}$$

where $I^2 = (k_1 \cdot p_1)^2 - m^2 M^2$ and m is the lepton mass.

Writing the matrix element in the form $\mathcal{M} = (e^2/Q^2)\overline{\mathcal{M}}$ one obtains the following expression for the differential cross section of the reaction (2) in the laboratory system for the case when the scattered lepton is detected in the final state

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4M} \frac{\vec{k}_2^2}{d|\vec{k}_1|} \frac{|\overline{\mathcal{M}}|^2}{Q^4}, \tag{8}$$

where $d = (M + \varepsilon_1)|\vec{k}_2| - \varepsilon_2 |\vec{k}_1| \cos\theta$, and θ is the lepton scattering angle (angle between the initial and final lepton momenta). The scattered lepton energy has the following form in terms of the lepton scattering angle:

$$\varepsilon_2 = \frac{(\varepsilon_1 + M)(M\varepsilon_1 + m^2) + \vec{k}_1^2 \cos\theta \sqrt{M^2 - m^2 \sin^2\theta}}{(\varepsilon_1 + M)^2 - \vec{k}_1^2 \cos^2\theta}. \tag{9}$$

In the limit of zero lepton mass this expression gives the well-known relation between the energy and angle of the scattered lepton:

$$\varepsilon_2 = \varepsilon_1 \left[1 + 2(\varepsilon_1/M) \sin^2 \frac{\theta}{2} \right]^{-1}.$$

The differential cross section for the case when the recoil deuteron is detected in the final state can be written as

$$\frac{d\sigma}{d\Omega_D} = \frac{\alpha^2}{4M}\frac{\vec{p}_2^{\,2}}{\bar{d}|\vec{k}_1|}\frac{|\overline{\mathcal{M}}|^2}{Q^4}, \quad (10)$$

where $\bar{d} = (M+\varepsilon_1)|\vec{p}_2| - E_2|\vec{k}_1|\cos\theta_D$, and θ_D is the angle between the momenta of the lepton beam and recoil deuteron. By using the relation

$$dQ^2 = |\vec{k}_1||\vec{p}_2|\frac{1}{\pi}\frac{E_2+M}{\varepsilon_1+M}d\Omega_D, \quad (11)$$

we obtain the following expression for the differential cross section over the Q^2 variable:

$$\frac{d\sigma}{dQ^2} = \frac{\pi\alpha^2}{4M}\frac{|\vec{p}_2|}{\bar{d}\vec{k}_1^2}\frac{\varepsilon_1+M}{E_2+M}\frac{|\overline{\mathcal{M}}|^2}{Q^4}. \quad (12)$$

The square of the reduced matrix element can be written as

$$|\overline{\mathcal{M}}|^2 = L_{\mu\nu}H^{\mu\nu}, \quad (13)$$

where the leptonic $L_{\mu\nu}$ and hadronic $H^{\mu\nu}$ tensors are defined as follows:

$$L_{\mu\nu} = j_\mu j_\nu^*, \quad H^{\mu\nu} = J^\mu J^{\nu *}. \quad (14)$$

If the initial and scattered leptons are unpolarized, then in this case the leptonic tensor is

$$L_{\mu\nu}(0) = 2q^2 g_{\mu\nu} + 4(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}). \quad (15)$$

In the case of polarized lepton beam the spin-dependent part of the leptonic tensor can be written as

$$L_{\mu\nu}(s) = 2im\langle\mu\nu q s_l\rangle, \quad (16)$$

where $\langle\mu\nu ab\rangle = \varepsilon_{\mu\nu\rho\sigma}a^\rho b^\sigma$ and $s_{l\mu}$ is the lepton polarization four-vector which satisfies the conditions $s_l^2 = -1$, $k_1\cdot s_l = 0$.

For an arbitrary polarization state of the initial and recoil deuterons, we may write the electromagnetic current in the following form:

$$J^\mu = J^{\mu\alpha\beta}U_{1\alpha}U_{2\beta}^*,$$

and the hadronic tensor $H^{\mu\nu}$ becomes

$$H^{\mu\nu} = J^{\mu\alpha\beta}J^{\nu\sigma\gamma *}\rho^i_{\alpha\sigma}\rho^f_{\gamma\beta}, \quad (17)$$

where $\rho^i_{\alpha\sigma}$ ($\rho^f_{\gamma\beta}$) is the spin-density matrix of the initial (final) deuteron.

Because we consider the case of a polarized deuteron target and unpolarized recoil deuteron, the hadronic tensor $H_{\mu\nu}$ can be expanded according to the polarization state of the initial deuteron:

$$H_{\mu\nu} = H_{\mu\nu}(0) + H_{\mu\nu}(V) + H_{\mu\nu}(T), \quad (18)$$

where the spin-independent tensor $H_{\mu\nu}(0)$ corresponds to an unpolarized initial deuteron and the spin-dependent tensor $H_{\mu\nu}(V)$ [$H_{\mu\nu}(T)$] describes the case where the deuteron target has a vector (tensor) polarization.

In the general case, the initial deuteron polarization state is described by the spin-density matrix. The general expression for the deuteron spin-density matrix in the coordinate representation is [22]

$$\rho^i_{\alpha\beta} = -\frac{1}{3}\left(g_{\alpha\beta} - \frac{p_{1\alpha}p_{1\beta}}{M^2}\right) + \frac{i}{2M}\langle\alpha\beta s p_1\rangle + Q_{\alpha\beta}, \quad (19)$$

where s_μ is the polarization four-vector describing the vector polarization of the deuteron target ($p_1\cdot s = 0$, $s^2 = -1$) and $Q_{\mu\nu}$ is the tensor which describes the quadrupole polarization of the initial deuteron and which satisfies the following conditions: $Q_{\mu\nu} = Q_{\nu\mu}$, $Q_{\mu\mu} = 0$, $p_{1\mu}Q_{\mu\nu} = 0$. In the laboratory system (the initial deuteron rest frame) all time components of the tensor $Q_{\mu\nu}$ are zero and the tensor polarization of the deuteron target is described by five independent space components:

$$Q_{ij} = Q_{ji}, \quad Q_{ii} = 0, \quad i,j = x,y,z.$$

If the polarization of the recoil deuteron is not measured, the deuteron spin-density matrix can be written as

$$\rho^f_{\alpha\beta} = -\left(g_{\alpha\beta} - \frac{p_{2\alpha}p_{2\beta}}{M^2}\right). \quad (20)$$

The relation between elements of the deuteron spin-density matrix in the helicity and spherical tensor representations as well as in the coordinate representation is given in the Appendix. The relations between the polarization parameters s_i and Q_{ij} and the population numbers n_+, n_-, and n_0 describing the polarized deuteron target, which is often used in spin experiments, are also given.

III. UNPOLARIZED DIFFERENTIAL CROSS SECTION

Let us consider the elastic scattering of unpolarized lepton beam by unpolarized deuteron target. The hadronic tensor $H_{\mu\nu}(0)$ can be written as

$$\begin{aligned} H_{\mu\nu}(0) &= H_1(Q^2)\tilde{g}_{\mu\nu} + H_2(Q^2)\tilde{p}_{1\mu}\tilde{p}_{1\nu}, \\ \tilde{g}_{\mu\nu} &= g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \\ \tilde{p}_{1\mu} &= p_{1\mu} - \frac{p_1\cdot q}{q^2}q_\mu. \end{aligned} \quad (21)$$

The real structure functions $H_{1,2}(Q^2)$ are expressed in terms of the deuteron electromagnetic form factors as

$$\begin{aligned} H_1(Q^2) &= \tfrac{2}{3}Q^2(1+\tau)G_M^2, \\ H_2(Q^2) &= 4M^2\left(G_C^2 + \tfrac{2}{3}\tau G_M^2 + \tfrac{8}{9}\tau^2 G_Q^2\right). \end{aligned} \quad (22)$$

The contraction of the spin-independent leptonic $L_{\mu\nu}(0)$ and hadronic $H^{\mu\nu}(0)$ tensors gives

$$\begin{aligned} S(0) = &-4(q^2+2m^2)H_1(Q^2) \\ &+ 2\left[(1+\tau)q^2 + \frac{4}{M^2}(k_1\cdot\tilde{p}_1)^2\right]H_2(Q^2), \end{aligned} \quad (23)$$

where the averaging over the spin of the initial deuteron is included in the structure functions $H_{1,2}(Q^2)$.

By substituting this expression into Eq. (8) and averaging over the spin of the initial lepton, we obtain the expression for the unpolarized differential cross section of the reaction (2) in the laboratory system, taking into account the lepton mass, in the form

$$\frac{d\sigma_{un}}{d\Omega} = \sigma_0 D, \tag{24}$$

where σ_0 is the cross section for the scattering of lepton on the point spin-1 particle. It is a generalization of the Mott cross section (with a recoil factor) to the case when the lepton mass is not neglected:

$$\sigma_0 = 4\frac{\alpha^2}{q^4}\frac{M}{d}\frac{\vec{k}_2^2}{|\vec{k}_1|}\left[\varepsilon_1^2 - M(M+2\varepsilon_1)\tau\right]. \tag{25}$$

Note that, in the limit $m = 0$, this expression reduces to the Mott cross section

$$\sigma_0(m=0) = \sigma_{\rm Mott} = \frac{\alpha^2\cos^2\frac{\theta}{2}}{4\varepsilon_1^2\sin^4\frac{\theta}{2}}\left(1+2\frac{\varepsilon_1}{M}\sin^2\frac{\theta}{2}\right)^{-1}, \tag{26}$$

where θ is the lepton scattering angle (between the momenta of the initial and final leptons).

The quantity D, which contains the information about the structure of the deuteron, has a form

$$D = A(Q^2) + f(Q^2,\varepsilon_1,m)B(Q^2), \tag{27}$$

where the standard structure functions $A(Q^2)$ and $B(Q^2)$ which describe unpolarized differential cross section of the reaction (2) in the zero-lepton-mass approximation are explicitly singled out:

$$\begin{aligned} A(Q^2) &= G_C^2(Q^2) + \tfrac{8}{9}\tau^2 G_Q^2(Q^2) + \tfrac{2}{3}\tau G_M^2(Q^2),\\ B(Q^2) &= \tfrac{4}{3}\tau(1+\tau)G_M^2(Q^2). \end{aligned} \tag{28}$$

The function $f(Q^2,\varepsilon_1,m)$ has the form

$$f(Q^2,\varepsilon_1,m) = (Q^2-2m^2)\left[4\varepsilon_1^2 - Q^2\left(1+2\frac{\varepsilon_1}{M}\right)\right]^{-1}. \tag{29}$$

In the limit of zero lepton mass this function reduces to

$$f(Q^2,\varepsilon_1,m=0) = \tan^2\frac{\theta}{2}.$$

Thus, in this approximation, we obtain the standard expression for the unpolarized differential cross section of the reaction (2):

$$\frac{d\sigma_{un}}{d\Omega} = \sigma_{\rm Mott}\left\{A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta}{2}\right)\right\}. \tag{30}$$

In the standard approach (zero lepton mass) the measurement of the unpolarized differential cross section at various values of the lepton scattering angle and the same value of Q^2 allows us to determine the structure functions $A(Q^2)$ and $B(Q^2)$. Therefore, it is possible to determine the magnetic form factor $G_M(Q^2)$ and the following combination of the form factors $G_C^2(Q^2) + 8\tau^2 G_Q^2(Q^2)/9$.

The determination of this quantity in the nonzero-lepton-mass approximation requires the measurement of the unpolarized differential cross section at different values of lepton beam energy and at the same value of Q^2. The separation of the charge G_C and quadrupole G_Q form factors requires polarization measurements.

IV. VECTOR POLARIZED DEUTERON TARGET

The calculation of polarization observables requires us to choose a coordinate frame. Let us define the following coordinate frame in the lab system: the z axis is directed along the lepton beam momentum $\vec{k}_1$, the y axis is directed along the vector $\vec{k}_1 \times \vec{k}_2$, and the x axis is chosen in order to form a left-handed coordinate system. Therefore, the reaction plane is the xz plane.

In this section we consider the T-even polarization observables, which depend on the spin correlation $\vec{s}\cdot\vec{s}_l$ which determines the scattering of the polarized lepton beam (of spin s_l) by a vector polarized deuteron target (of spin s).

In the case considered, the spin-dependent tensor $H_{\mu\nu}(V)$, which describes the vector-polarized initial deuteron and unpolarized final deuteron, can be written as

$$\begin{aligned} H_{\mu\nu}(V) = {}& \frac{i}{M}S_1(Q^2)\langle\mu\nu sq\rangle\\ &+ \frac{i}{M^3}S_2(Q^2)[\tilde{p}_{1\mu}\langle\nu sqp_1\rangle - \tilde{p}_{1\nu}\langle\mu sqp_1\rangle]\\ &+ \frac{1}{M^3}S_3(Q^2)[\tilde{p}_{1\mu}\langle\nu sqp_1\rangle + \tilde{p}_{1\nu}\langle\mu sqp_1\rangle], \end{aligned} \tag{31}$$

where the three real structure functions $S_i(Q^2)$, $i = 1-3$, can be expressed in terms of the deuteron electromagnetic form factors as

$$\begin{aligned} S_1(Q^2) &= M^2(1+\tau)G_M^2,\\ S_2(Q^2) &= M^2\left[G_M^2 - 2\left(G_C + \frac{\tau}{3}G_Q\right)G_M\right],\\ S_3(Q^2) &= 0. \end{aligned} \tag{32}$$

The third structure function $S_3(Q^2)$ vanishes since the deuteron form factors are real functions for elastic scattering (in the spacelike region of momentum transfer). In the timelike region (for annihilation processes, for example, $e^- + e^+ \to D + \bar{D}$), where the form factors are complex functions, the structure function $S_3(Q^2)$ is not zero and it is determined by the imaginary part of the form factors; namely, $S_3(Q^2) = 2M^2{\rm Im}[G_C - \tau/(3G_Q)]G_M^*$.

The differential cross section of reaction (2) describing the scattering of polarized lepton beam on the vector-polarized deuteron target can be written as (referring only to the spin-dependent part of the cross section which is determined by the spin correlation coefficients)

$$\begin{aligned} \frac{d\sigma(s,s_l)}{d\Omega} = {}& \frac{d\sigma_{un}}{d\Omega}(1 + C_{xx}\xi_x\xi_{lx} + C_{yy}\xi_y\xi_{ly} + C_{zz}\xi_z\xi_{lz}\\ &+ C_{xz}\xi_x\xi_{lz} + C_{zx}\xi_z\xi_{lx}), \end{aligned} \tag{33}$$

where the vector $\vec{\xi}_l$ ($\vec{\xi}$) is the unit polarization vector in the rest frame of the lepton beam (deuteron target). The spin correlation coefficients C_{ij} have the following form in terms of the deuteron electromagnetic form factors:

$$\begin{aligned}
DC_{xx} &= \frac{m}{M}\frac{G_M}{z}\left[\tau\vec{k}_2^2\sin^2\theta G_M - 2(|\vec{k}_1| - |\vec{k}_2|\cos\theta)^2\left(G_C + \frac{\tau}{3}G_Q\right)\right],\\
DC_{yy} &= 2\frac{m}{M}\frac{q^2}{z}(1+\tau)\left(G_C + \frac{\tau}{3}G_Q\right)G_M,\\
DC_{zx} &= -\frac{m}{M}\frac{|\vec{k}_2|}{z}\sin\theta(|\vec{k}_1| - |\vec{k}_2|\cos\theta)\left(\tau G_M + 2G_C + \frac{2}{3}\tau G_Q\right)G_M,\\
DC_{xz} &= -\frac{|\vec{k}_2|}{zM}\sin\theta G_M\left[\varepsilon_1(|\vec{k}_1| - |\vec{k}_2|\cos\theta)\left(\tau G_M + 2G_C + \frac{2}{3}\tau G_Q\right) - 2M|\vec{k}_1|\tau(1+\tau)G_M\right],\\
DC_{zz} &= \frac{G_M}{zM}\left\{\tau G_M(|\vec{k}_1| - |\vec{k}_2|\cos\theta)[\varepsilon_1(|\vec{k}_1| - |\vec{k}_2|\cos\theta) - 2M|\vec{k}_1|(1+\tau)] - 2\varepsilon_1\vec{k}_2^2\sin^2\theta\left(G_C + \frac{\tau}{3}G_Q\right)\right\},
\end{aligned} \tag{34}$$

where $z = 4[\varepsilon_1^2 - \tau(M^2 + 2M\varepsilon_1)]$. Note that the spin correlation coefficients C_{xx}, C_{yy}, and C_{zx} correspond to the transverse (relative to the lepton beam momentum) components of the spin vector $\vec{\xi}_l$ and therefore they are proportional to the lepton mass. The spin correlation coefficients C_{xz} and C_{zz} describe the scattering of the longitudinally polarized lepton beam and they are not suppressed by the factor m/M.

In the limit of zero lepton mass we have the following expressions for the spin correlation coefficients:

$$\begin{aligned}
\bar{D}C_{xz}^{(0)} &= \frac{1}{2}\frac{\tau}{\varepsilon_1}\tan\frac{\theta}{2}\left[(\varepsilon_1 + \varepsilon_2)G_M - 4(M+\varepsilon_1)\left(G_C + \frac{\tau}{3}G_Q\right)\right]G_M,\\
\bar{D}C_{zz}^{(0)} &= -2\tau G_M\frac{M}{\varepsilon_1}\left[G_C + \frac{\tau}{3}G_Q + \frac{\varepsilon_2}{2M^2}(M+\varepsilon_1)\left(1 + \frac{\varepsilon_1}{M}\sin^2\frac{\theta}{2}\right)\tan^2\frac{\theta}{2}G_M\right],\\
C_{xx}^{(0)} &= C_{yy}^{(0)} = C_{zx}^{(0)} = 0, \quad \bar{D} = A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}.
\end{aligned} \tag{35}$$

The expressions of these coefficients coincide with the results obtained in Ref. [19].

Another coordinate system is also used in the description of the elastic lepton-deuteron scattering: the Z axis is directed along the virtual photon momentum (transferred momentum) $\vec{q}$, the Y axis is directed along the vector $\vec{k}_1 \times \vec{k}_2$ and coincides with the y axis, and the X axis is chosen in order to form a left-handed coordinate system. These coordinate systems are connected by a rotation in the reaction scattering plane, which it is determined by the angle ψ between the direction of the lepton beam and the virtual photon momentum:

$$\cos\psi = \frac{M+\varepsilon_1}{|\vec{k}_1|}\sqrt{\frac{\tau}{1+\tau}}, \quad \sin\psi = -\frac{1}{|\vec{k}_1|}\frac{1}{\sqrt{1+\tau}}\sqrt{\varepsilon_1\varepsilon_2 - \tau M^2 - m^2(1+\tau)}. \tag{36}$$

Thus, the spin correlation coefficients in the new coordinate system can be related to the ones in the previously considered coordinate system by the following relations:

$$\begin{aligned}
C_{Zz} &= \cos\psi C_{zz} + \sin\psi C_{xz}, \quad C_{Xz} = -\sin\psi C_{zz} + \cos\psi C_{xz},\\
C_{Zx} &= \cos\psi C_{zx} + \sin\psi C_{xx}, \quad C_{Xx} = -\sin\psi C_{zx} + \cos\psi C_{xx}.
\end{aligned} \tag{37}$$

The spin correlation coefficient C_{yy} is the same in both coordinate systems. We do not transform the spin components of the lepton beam in order not to mix the transverse and longitudinal components (relative to the lepton beam momentum) since only the last one leads to the spin correlation coefficients which are not proportional to the lepton mass. So, in this coordinate system the z component of the spin of the lepton beam corresponds to the longitudinal polarization and the x component corresponds to the transverse polarization which belongs to the reaction scattering plane.

After performing the rotation we obtain the following expressions for the spin correlation coefficients in the new coordinate system:

$$\begin{aligned}
DC_{Zz} = \frac{1}{Mz|\vec{k}_1|}\frac{G_M}{\sqrt{\tau(1+\tau)}}\Big\{&2M|\vec{k}_1|\tau(1+\tau)[\tau(M+\varepsilon_1)(|\vec{k}_2|\cos\theta - |\vec{k}_1|) - r|\vec{k}_2|\sin\theta]G_M + \varepsilon_1(|\vec{k}_2|\cos\theta - |\vec{k}_1|)\\
&\times[\tau^2(M+\varepsilon_1)(|\vec{k}_2|\cos\theta - |\vec{k}_1|) - r|\vec{k}_2|\sin\theta]G_M - 2\varepsilon_1|\vec{k}_2|\sin\theta[\tau(M+\varepsilon_1)|\vec{k}_2|\sin\theta + r(|\vec{k}_2|\cos\theta - |\vec{k}_1|)]\\
&\times\left(G_C + \frac{\tau}{3}G_Q\right)\Big\},
\end{aligned}$$

$$
\begin{aligned}
DC_{Xz} &= \frac{1}{Mz|\vec{k}_1|}\frac{G_M}{\sqrt{\tau(1+\tau)}}\Big\{2\varepsilon_1|\vec{k}_2|\sin\theta[\tau(M+\varepsilon_1)(|\vec{k}_2|\cos\theta-|\vec{k}_1|)-r|\vec{k}_2|\sin\theta]\Big(G_C+\frac{\tau}{3}G_Q)\Big)\\
&\quad+\tau[\tau(M+\varepsilon_1)|\vec{k}_2|\sin\theta+r(|\vec{k}_2|\cos\theta-|\vec{k}_1|)][\varepsilon_1(|\vec{k}_2|\cos\theta-|\vec{k}_1|)+2M|\vec{k}_1|(1+\tau)]G_M\Big\},\\
DC_{Zx} &= -\frac{m}{M}\frac{1}{z|\vec{k}_1|}\frac{G_M}{\sqrt{\tau(1+\tau)}}\Big\{\tau|\vec{k}_2|\sin\theta[r|\vec{k}_2|\sin\theta-\tau(M+\varepsilon_1)(|\vec{k}_2|\cos\theta-|\vec{k}_1|)]\\
&\quad\times G_M+2(|\vec{k}_1|-|\vec{k}_2|\cos\theta)[\tau(M+\varepsilon_1)|\vec{k}_2|\sin\theta-r(|\vec{k}_1|-|\vec{k}_2|\cos\theta)]\Big(G_C+\frac{\tau}{3}G_Q\Big)\Big\},\\
DC_{Xx} &= \frac{m}{M}\frac{1}{z|\vec{k}_1|}\frac{G_M}{\sqrt{\tau(1+\tau)}}\Big\{\tau|\vec{k}_2|\sin\theta[\tau|\vec{k}_2|(M+\varepsilon_1)\sin\theta+r(|\vec{k}_2|\cos\theta-|\vec{k}_1|)]G_M-2(|\vec{k}_1|-|\vec{k}_2|\cos\theta)\\
&\quad\times[\tau(M+\varepsilon_1)(|\vec{k}_1|-|\vec{k}_2|\cos\theta)+r|\vec{k}_2|\sin\theta]\Big(G_C+\frac{\tau}{3}G_Q\Big)\Big\},\\
r &= \sqrt{\tau[\varepsilon_1\varepsilon_2-\tau M^2-m^2(1+\tau)]}.
\end{aligned}
\tag{38}
$$

In the limit of zero lepton mass there are only two nonzero spin correlation coefficients corresponding to the longitudinal polarization of the lepton beam, and they have following form:

$$
\begin{aligned}
\bar{D}C_{Zz}^{(0)} &= -\tau\sqrt{(1+\tau)\Big(1+\tau\sin^2\frac{\theta}{2}\Big)}\tan\frac{\theta}{2}\sec\frac{\theta}{2}G_M^2,\\
\bar{D}C_{Xz}^{(0)} &= -2\sqrt{\tau(1+\tau)}\tan\frac{\theta}{2}G_M\Big(G_C+\frac{\tau}{3}G_Q\Big).
\end{aligned}
\tag{39}
$$

Along with the transformation of the spin correlation coefficients it is necessary to transform the vector which describes the vector polarization of the deuteron target. The new components of this vector are related to s_z and s_x, Eq. (19), as follows:

$$
s_I = V_{Ii}(\psi)s_i, \quad V(\psi)=\begin{pmatrix}\cos\psi & \sin\psi\\ -\sin\psi & \cos\psi\end{pmatrix},
\tag{40}
$$

where $I = Z,X$ and $i = z,x$.

V. TENSOR-POLARIZED DEUTERON TARGET

In the case of tensor-polarized deuteron target, the general structure of the spin-dependent tensor $H_{\mu\nu}(T)$ can be written in terms of five structure functions as follows:

$$
\begin{aligned}
H_{\mu\nu}(T) &= V_1(Q^2)\bar{Q}\tilde{g}_{\mu\nu}+V_2(Q^2)\frac{\bar{Q}}{M^2}\tilde{p}_{1\mu}\tilde{p}_{1\nu}+V_3(Q^2)(\tilde{p}_{1\mu}\widetilde{Q}_\nu+\tilde{p}_{1\nu}\widetilde{Q}_\mu)\\
&\quad+V_4(Q^2)\widetilde{Q}_{\mu\nu}+iV_5(Q^2)(\tilde{p}_{1\mu}\widetilde{Q}_\nu-\tilde{p}_{1\nu}\widetilde{Q}_\mu),
\end{aligned}
$$

where we introduce the following notations:

$$
\begin{aligned}
\widetilde{Q}_\mu &= Q_{\mu\nu}q_\nu-\frac{q_\mu}{q^2}\bar{Q}, \quad \widetilde{Q}_\mu q_\mu=0,\\
\widetilde{Q}_{\mu\nu} &= Q_{\mu\nu}+\frac{q_\mu q_\nu}{q^4}\bar{Q}-\frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha}-\frac{q_\mu q_\alpha}{q^2}Q_{\nu\alpha},\\
\widetilde{Q}_{\mu\nu}q_\nu &= 0, \quad \bar{Q}=Q_{\alpha\beta}q_\alpha q_\beta.
\end{aligned}
\tag{41}
$$

The structure functions $V_i(Q^2)$ ($i = 1-5$), which describe the part of the hadronic tensor due to the tensor polarization of the deuteron target, have the following form in terms of the deuteron form factors:

$$
\begin{aligned}
V_1(Q^2) &= -G_M^2, \quad V_5(Q^2)=0,\\
V_2(Q^2) &= G_M^2+\frac{4}{1+\tau}\Big(G_C+\frac{\tau}{3}G_Q+\tau G_M\Big)G_Q,\\
V_3(Q^2) &= -2\tau\big[G_M^2+2G_QG_M\big], \quad V_4(Q^2)=4M^2\tau(1+\tau)G_M^2.
\end{aligned}
\tag{42}
$$

The fifth structure function $V_5(Q^2)$ is zero since deuteron form factors are real functions in the kinematical region considered. In the timelike region of momentum transfers this structure function is not zero and is given by $V_5(Q^2) = -4\tau ImG_Q G_M^*$.

The differential cross section of the reaction (2) describing the scattering of unpolarized lepton beam on the tensor-polarized deuteron target can be written as

$$\frac{d\sigma(Q)}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}[1 + A_{xx}(Q_{xx} - Q_{yy}) + A_{xz}Q_{xz} + A_{zz}Q_{zz}], \tag{43}$$

where A_{ij} are the asymmetries caused by the tensor polarization of the deuteron target. Here we used the conditions that the tensor Q_{ij} is symmetrical and traceless: $Q_{xx} + Q_{yy} + Q_{zz} = 0$. The asymmetries have the following form in terms of the deuteron electromagnetic form factors:

$$\begin{aligned}
DA_{xx} &= \frac{1}{2}\frac{\vec{k}_2^2}{M^2}\frac{\sin^2\theta}{z}\bigg\{\left(\varepsilon_1^2 + \tau M^2 - m^2\right)G_M^2 + 4(1+\tau)^{-1}G_Q\bigg[\tau(M+\varepsilon_1)(\varepsilon_1 - \tau M)G_M \\
&\quad + \left(\varepsilon_1^2 - \tau M^2 - 2\tau M\varepsilon_1\right)\left(G_C + \frac{\tau}{3}G_Q\right)\bigg]\bigg\}, \\
DA_{xz} &= -2\frac{\varepsilon_1|\vec{k}_2|}{M^2 z}\sin\theta\Bigg(\bigg[\varepsilon_1(|\vec{k}_1| - |\vec{k}_2|\cos\theta)\left(1 + \tau\frac{M^2}{\varepsilon_1^2} - \frac{m^2}{\varepsilon_1^2}\right) - 2\tau M|\vec{k}_1|\left(1 + \frac{M}{\varepsilon_1}\right)\bigg]G_M^2 + 4\frac{\varepsilon_1 G_Q}{1+\tau}\bigg\{(|\vec{k}_1| - |\vec{k}_2|\cos\theta) \\
&\quad \times\left(1 - \tau\frac{M^2}{\varepsilon_1^2} - 2\tau\frac{M}{\varepsilon_1}\right)\left(G_C + \frac{\tau}{3}G_Q\right) + \tau\left(1 - \tau\frac{M}{\varepsilon_1}\right)G_M\bigg[|\vec{k}_1|\left(1 - \tau\frac{M}{\varepsilon_1}\right) - |\vec{k}_2|\left(1 + \frac{M}{\varepsilon_1}\right)\cos\theta\bigg]\bigg\}\Bigg), \\
DA_{zz} &= \frac{1}{zM^2}\Bigg(\bigg[(|\vec{k}_1| - |\vec{k}_2|\cos\theta)^2 - \frac{1}{2}\vec{k}_2^2\sin^2\theta\bigg]\bigg\{\left(\varepsilon_1^2 + \tau M^2 - m^2\right)G_M^2 + 4(1+\tau)^{-1}G_Q\bigg[\tau(M+\varepsilon_1)(\varepsilon_1 - \tau M)G_M \\
&\quad + \left(\varepsilon_1^2 - \tau M^2 - 2\tau M\varepsilon_1\right)\left(G_C + \frac{\tau}{3}G_Q\right)\bigg]\bigg\} \\
&\quad + 4\tau M|\vec{k}_1|G_M\{M|\vec{k}_1|(1+\tau)G_M - (|\vec{k}_1| - |\vec{k}_2|\cos\theta)[(M+\varepsilon_1)G_M + 2(\varepsilon_1 - \tau M)G_Q]\}\Bigg).
\end{aligned} \tag{44}$$

In the limit of zero lepton mass we have the following expressions for the asymmetries due to the tensor polarization of the deuteron target:

$$\begin{aligned}
\bar{D}A_{xx}^{(0)} &= \frac{\tau}{2}\bigg\{\left(1 + \tau\frac{M^2}{\varepsilon_1^2}\right)G_M^2 + 4(1+\tau)^{-1}G_Q\bigg[\tau\left(1 + \frac{M}{\varepsilon_1}\right)\left(1 - \tau\frac{M}{\varepsilon_1}\right)G_M + \left(1 - \tau\frac{M^2}{\varepsilon_1^2} - 2\tau\frac{M}{\varepsilon_1}\right)\left(G_C + \frac{\tau}{3}G_Q\right)\bigg]\bigg\}, \\
\bar{D}A_{xz}^{(0)} &= -\frac{\varepsilon_2}{M}\frac{\tau}{1+\tau}\sin\theta\bigg\{4\left(1 + \frac{M}{\varepsilon_1}\right)G_Q\left(G_C + \frac{\tau}{3}G_Q\right) + (1+\tau)\left(1 + \frac{M}{\varepsilon_1}\right)\tan^2\frac{\theta}{2}G_M^2 \\
&\quad + 2\left(1 - \tau\frac{M}{\varepsilon_1}\right)\bigg[-1 - \tau + 2\sin^2\frac{\theta}{2}\left(1 + \frac{\varepsilon_1}{M} + \frac{\varepsilon_1^2}{M^2} - \tau\frac{\varepsilon_1}{M}\right)\bigg]\left(1 + \tan^2\frac{\theta}{2}\right)G_M G_Q\bigg\}, \\
\bar{D}A_{zz}^{(0)} &= -\frac{\tau}{2}\bigg\{\bigg[6\frac{\tau}{1+\tau}\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1}\left(1 + \frac{M}{\varepsilon_1}\right)G_Q - G_M\bigg]G_M + \tan^2\frac{\theta}{2}\bigg[1 - 2\tau - 6\tau\frac{M}{\varepsilon_1}\left(1 + \frac{M}{2\varepsilon_1}\right)\bigg] \\
&\quad \times\bigg[G_M^2 + \frac{4}{1+\tau}\cot^2\frac{\theta}{2}G_Q\left(G_C + \frac{\tau}{3}G_Q\right)\bigg]\bigg\}.
\end{aligned} \tag{45}$$

In the coordinate system where the Z axis is directed along the virtual photon momentum, the asymmetries due to the tensor polarization of the deuteron target have the following form:

$$A_\alpha = T_{\alpha\beta}(\psi)A^\beta, \tag{46}$$

where the indices of the rotation matrix have following meaning: $\alpha = ZZ, XX, XZ$ and $\beta = zz, xx, xz$. The rotation matrix can be written as

$$T(\psi) = \begin{pmatrix} \frac{1}{4}(1 + 3\cos 2\psi) & \frac{3}{4}(1 - \cos 2\psi) & \frac{3}{4}\sin 2\psi \\ \frac{1}{4}(1 - \cos 2\psi) & \frac{1}{4}(3 + \cos 2\psi) & \frac{-1}{4}\sin 2\psi \\ -\sin 2\psi & \sin 2\psi & \cos 2\psi \end{pmatrix}. \tag{47}$$

Along with the transformation of the asymmetries it is necessary to transform the tensor of the quadrupole polarization which describes the tensor polarization of the deuteron target. The new tensor polarization parameters are related to Q_{zz}, $(Q_{xx} - Q_{yy})$, and Q_{xz}, Eq. (19), as follows:

$$Q_{ZZ} = \tfrac{1}{4}(1+3\cos 2\psi)Q_{zz} + \tfrac{1}{4}(1-\cos 2\psi)(Q_{xx}-Q_{yy}) + \sin 2\psi\, Q_{xz},$$
$$Q_{XX}-Q_{YY} = \tfrac{3}{4}(1-\cos 2\psi)Q_{zz} + \tfrac{1}{4}(3+\cos 2\psi)(Q_{xx}-Q_{yy}) + \sin 2\psi\, Q_{xz}, \quad (48)$$
$$Q_{XZ} = -\tfrac{3}{4}\sin 2\psi\, Q_{zz} + \tfrac{1}{4}\sin 2\psi(Q_{xx}-Q_{yy}) + \cos 2\psi\, Q_{xz}.$$

VI. NUMERICAL ESTIMATES

In this section we give numerical estimates of the effect of the mass on the kinematical variables and on some of the experimental observables. Because two variables completely define the kinematics for a binary process, the results are preferentially illustrated as bidimensional plots as function of the muon beam energy and the muon scattering angle.

A. Kinematics

The effect of the lepton mass on the kinematical variables is illustrated for the scattered lepton energy and for the momentum transfer squared.

The relative difference between the scattered lepton energy taking and not taking into account the lepton mass is shown in Fig. 1(a) as a bi-dimensional plot as a function of the muon incident energy and scattering angle.

The effect of the mass on the momentum transfer squared is shown in Fig. 1(b).

From these figures it appears that the effect of the lepton mass on the energy and angle of the scattered muon is sizable in the considered kinematical range, in particular at low beam energies. The scattered muon energy is largely modified at small Q^2 and large scattering angles and a huge (relative) effect appears for the momentum transfer squared. These effects are quantified in Table I.

B. Parametrization of deuteron form factors

For the calculation of unpolarized and polarized observables, knowledge of the deuteron form factors is needed. We used the parametrization from Ref. [23] which is based on a

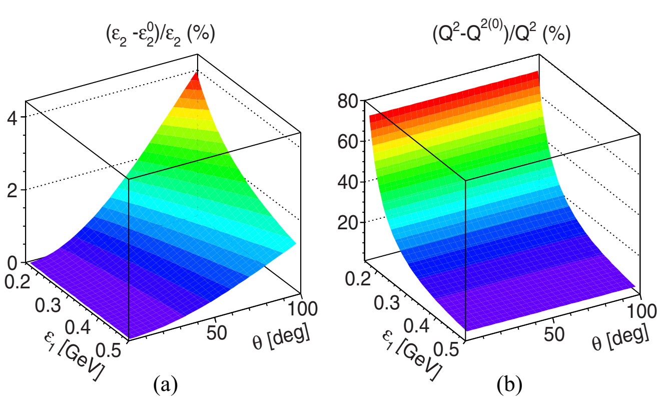

FIG. 1. (Color online) Effect of the lepton mass on the kinematics of the scattered lepton: (a) the relative difference in percent between the scattered lepton energy and (b) the momentum transfer squared taking and not taking into account, respectively, the lepton mass as a function of the scattering muon angle and of the incident energy.

"two-component model" of the deuteron, inspired from vector meson dominance [24] where the pn core is surrounded by an (isoscalar) meson cloud. This parametrization has a simple analytical form and reproduces best the existing experimental data. We recall here the formulas and the parameter set that we used.

The three deuteron form factors are parametrized as

$$G_i(Q^2) = N_i g_i(Q^2)F_i(Q^2), \quad i = C,Q,M, \quad (49)$$

where N_i is the normalization of the ith form factor at $Q^2 = 0$:

$$N_C = G_C(0) = 1,$$
$$N_Q = G_Q(0) = M^2 Q_d = 25.83,$$
$$N_M = G_M(0) = \frac{M}{m_p}\mu_d = 1.714.$$

where m_p is the proton mass.

The expression for the meson cloud is

$$F_i(Q^2) = 1 - \alpha_i - \beta_i + \alpha_i \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_i \frac{m_\phi^2}{m_\phi^2 + Q^2},$$

where $m_\omega = 0.784$ GeV ($m_\phi = 1.019$ GeV) is the mass of the ω (ϕ) meson. These expressions are built in such a form that $F_i(0) = 1$ for any values of the free parameters α_i and β_i, which are real numbers.

The intrinsic core is parametrized as

$$g_i(Q^2) = 1/[1 + \gamma_i Q^2]^{\delta_i}. \quad (50)$$

The terms $g_i(Q^2)$ are functions of two parameters, also real. We took common values for all form factors: $\gamma_i = 12.1$ and $\delta_i = 1.05$.

TABLE I. Effect of including the lepton mass on the relevant kinematical quantities, Q^2, ε_2 and ratio of cross sections for three values of the beam energy and two scattering angles. The index $^{(0)}$ indicates the value when the lepton mass is set to zero.

ε_1 [GeV]	θ [deg]	Q^2 [GeV^2]	$Q^{2(0)}$ [GeV^2]	ε_2 [GeV]	ε_2^0 [GeV]	σ/σ^0
0.1	20	0.00232	0.00120	0.10004	0.09968	0.23
0.1	80	0.00222	0.01582	0.10049	0.09578	45.41
0.15	20	0.00136	0.00270	0.14964	0.14928	3.96
0.15	80	0.01758	0.03488	0.14531	0.14070	3.94
0.2	20	0.00346	0.00479	0.19908	0.19872	1.95
0.2	80	0.04381	0.06075	0.18832	0.18380	1.22

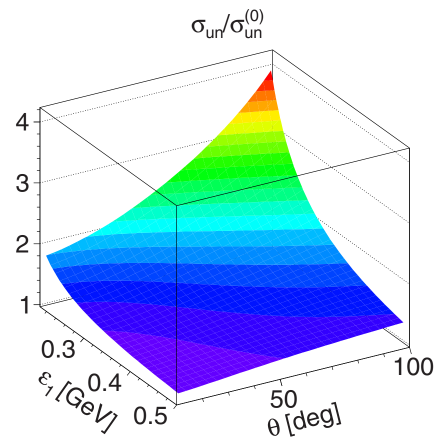

FIG. 2. (Color online) Bidimensional plot of the ratio between the cross section taking and not taking into account the lepton mass as function of ε_1 and θ.

The parameters α_C and α_M have been fixed by the value of the experimental node which appears at $Q^2 = 1.9$ GeV^2 and $\simeq$0.7 GeV^2 for G_M and G_C, respectively. The other parameters are $\beta_i(G_C) = -5.11$, $\alpha_Q = 4.21$, $\beta_Q = -3.41$, and $\beta_M = -2.86$.

C. Experimental observables

Using Eqs. (23)–(29) with the parametrization of form factors above described, the unpolarized cross section is calculated in the relevant kinematical range. The correction to the Born cross section due to the finite lepton mass is illustrated in Fig. 2, where the ratio between the cross section taking and not taking into account the lepton mass is shown as a function of ε_1 and θ. One can see that this ratio increases essentially at small energies and large angles. Besides the change in the kinematical variables, this is due to the presence of $\vec{k}_1$ in the denominator of Eq. (25) and to the additional terms accompanying the structure functions [see Eqs. (27) and (29)].

Including the lepton mass changes essentially the kinematics and the dynamics, particularly at low beam energy. For the evaluation of the counting rates in preparing the experiments, the complete formula of the cross section [Eqs. (24) and (25)] has to be taken into account, because the effect of including the lepton mass is proportionally the same, as illustrated for the cross-section ratio in Fig. 2, at the corresponding kinematical conditions.

The extraction of the magnetic form factor from the cross section μd at backward angles, or the relevant combination of G_C and G_Q at forward angles has also to take into account the complete formula and the additional mass-dependent terms.

The extrapolation of form factors to $Q^2 \to 0$ is therefore affected in both coordinates: the calculated Q^2 and the extracted form factors. The effect is much larger than the percent precision of the expected data and should be taken into account.

The polarization observables C_{xx}, C_{yy}, C_{zx}, C_{xz}, induced by a polarized lepton beam on a vector-polarized deuteron target are illustrated in Fig. 3, in bidimensional plots and as

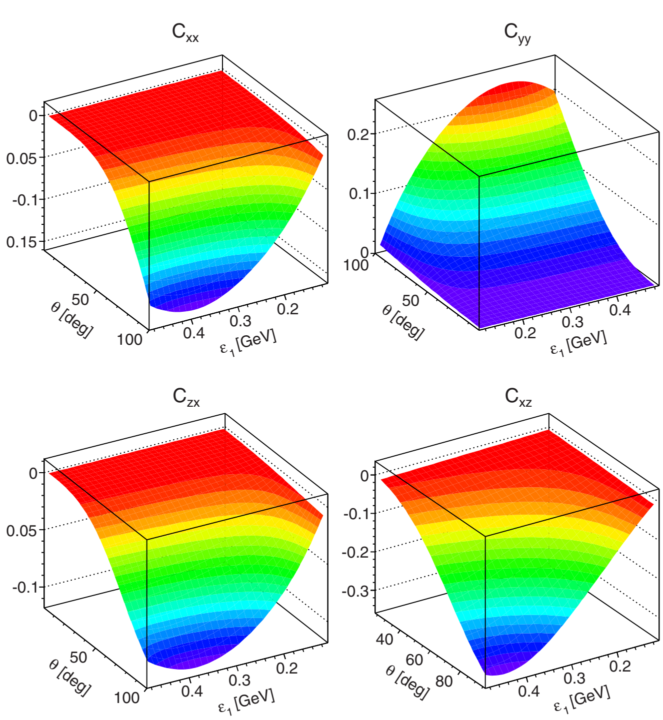

FIG. 3. (Color online) Correlation coefficients C_{xx} (top left), C_{yy} (top right), C_{zx} (bottom left), and C_{xz} (bottom right) as functions of ε_1 and θ.

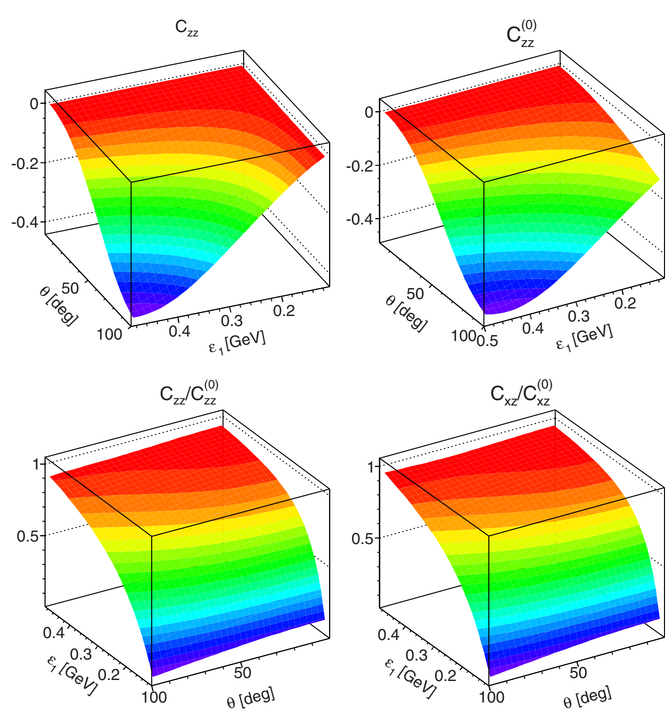

FIG. 4. (Color online) Correlation coefficients C_{zz} (top left) and $C_{zz}^{(0)}$ (top right) as functions of ε_1 and θ. The ratio of the observables taking and not taking into account the lepton mass is shown for C_{zz} (bottom left) and for C_{xz} (bottom right), respectively.

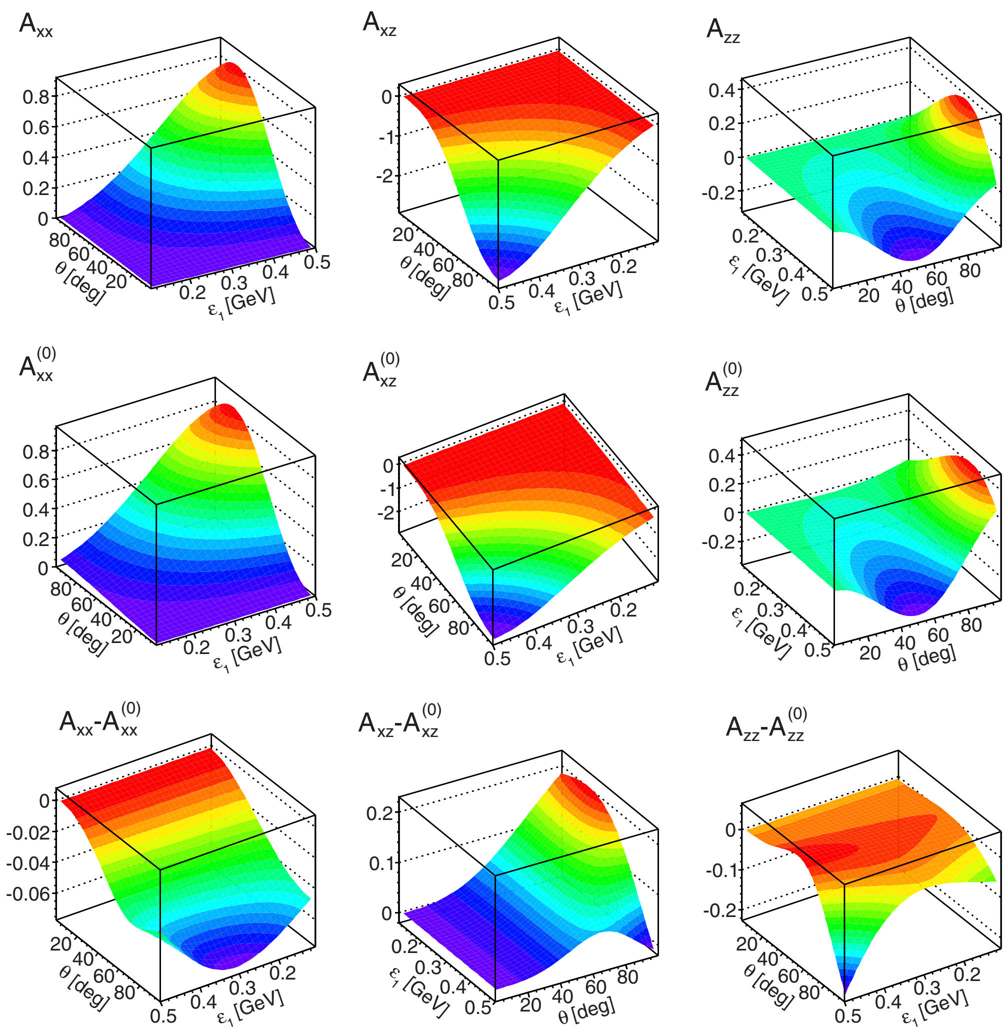

FIG. 5. (Color online) Tensor asymmetries from left to right A_{xx}, A_{xz}, A_{zz}. From top to bottom, the first (second) row represents the observables taking (not taking) into account the lepton mass, respectively. The third row shows their difference.

functions of ε_1 and θ. The correlation coefficients vanish for $\theta = 0$ and at small energies. C_{xx}, C_{zx}, and C_{xz} become sizable and negative as the angle and energy increase, whereas C_{yy} becomes positive.

In Fig. 4, the polarization coefficient C_{zz} is shown (top left). The same coefficient, setting the mass of the lepton to zero, is shown (top right). The ratio of the observables taking and not taking into account the lepton mass is shown for C_{zz} (bottom left) and for C_{xz} (bottom right), respectively. Again, the relative effect of the mass is very large at low energies and large angles.

The tensor asymmetries induced by a tensor-polarized deuteron target and unpolarized lepton beam are illustrated in Fig. 5. From left to right, A_{xx}, A_{xz}, and A_{zz} are shown. From top to bottom, the first (second) row represents the observables taking (not taking) into account the lepton mass. The third row shows their difference. We do not plot the ratio because it diverges for the observable A_{zz}, which changes of sign in the considered range. The relative effect of the mass is of about 10% on A_{xx} and A_{xz} and can reach 50% on A_{zz}.

VII. CONCLUSION

We calculated the polarized and unpolarized cross section for lepton deuteron elastic scattering, taking into account the lepton mass and showed an application to the case of muon scattering.

Besides the unpolarized cross section, different observables have been calculated, according to the possible polarizations of the lepton beam and the deuteron target. The spin correlation coefficients due to the lepton beam polarization and to the vector polarization of the deuteron target, as well as the asymmetries due to the tensor polarization of the deuteron target have been explicitly derived. The calculations have been done for two coordinate systems: in the first one the z axis is directed along the lepton beam momentum and, in the second one, along the virtual photon momentum.

The numerical application needs a parametrization of the deuteron form factors. We chose the parametrization from Ref. [23] which is based on a two component model of the deuteron, where the pn core is surrounded by an (isoscalar) meson cloud. This model reproduces very well the existing experimental data.

It is shown that the effect of the finite lepton mass is sizable in particular at low incident energies and large scattering angles.

These results are particularly important in relation to planned measurements of low energy muon-deuteron scattering, which aim for a precise determination of the charge radius. The determination of the radius requires experimentally the extrapolation of the extracted form factor to $Q^2 = 0$; Eq. (1). Neglecting the muon mass brings an error in the determination of Q^2 which is reflected directly in the slope. Note that the experimental systematic precision sought is 1%.

The issue addressed here has a wider interest. Since the advent of high-energy accelerators, lepton-hadron scattering concerns mostly high-energy electron scattering, and the commonly used formulas neglect the electron mass, which is acceptable. But the lepton mass cannot always be neglected: besides the present case, where the energy is of the order of the muon mass, the terms related to the lepton mass have a large effect at high energies in case of inverse kinematics as proton-electron scattering, which is object of Ref. [25], as well as in the crossed channel, the antiproton annihilation into a lepton pair [26]) of interest for the antiproton annihilation experiment at the GSI Facility for Antiproton and Ion Research (PANDA at FAIR). In this respect, the subject of the present paper is relevant to other elementary processes. It is also an issue not only for fixed-target experiments, but also for electron-ion colliders (this case was discussed in Ref. [27]).

Finally we would like to stress that the formulas derived in the paper are valid at any energy (outside the very low-energy region where capture takes place instead of scattering). The relations in terms of form factors are model independent: they are fully relativistic expressions based on the most general symmetry properties of electromagnetic interactions and on the spin-1 nature of the exchanged photon.

The dynamics and the structure of the deuteron are fully contained in form factors. We used the parametrization of Ref. [23] which reproduces best the existing data: G_C up to $Q^2 = 2$ GeV^2, G_M up to $Q^2 = 2.8$ GeV^2, and G_Q up to $Q^2 = 1.7$ GeV^2. Moreover, in the low-Q^2 region of interest here, form factors are constrained by the static values (electric charge and magnetic and quadrupole moments) which are very precisely known. Due to these reasons, we can consider the present results to be “model independent.”

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APPENDIX

We give here the expressions which relate the description of the polarization state of the deuteron target for different approaches. For the case of arbitrary polarization of the target, the deuteron polarization is described by the spin-density matrix which is defined, in the general case, by eight parameters.

1. Coordinate representation

The deuteron spin-density matrix in the coordinate representation has the form

$$\rho_{\mu\nu} = -\frac{1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}\right) + \frac{i}{2M}\varepsilon_{\mu\nu\lambda\rho} s_\lambda p_\rho + Q_{\mu\nu},$$
$$Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_\mu Q_{\mu\nu} = 0, \tag{A1}$$

where p_μ is the deuteron four-momentum, s_μ and $Q_{\mu\nu}$ are, respectively, the deuteron polarization four-vector describing the vector polarization and the deuteron quadrupole-polarization tensor describing the tensor polarization.

In the deuteron rest frame Eq. (A1) becomes

$$\rho_{ij} = \frac{1}{3}\delta_{ij} - \frac{i}{2}\varepsilon_{ijk} s_k + Q_{ij}, \quad ij = x,y,z. \tag{A2}$$

2. Helicity representation

The spin-density matrix can be written in the helicity representation by using the following relation:

$$\rho_{\lambda\lambda'} = \rho_{ij} e_i^{(\lambda)*} e_j^{(\lambda')},$$
$$\rho_{\lambda\lambda'} = (\rho_{\lambda'\lambda})^*, \tag{A3}$$
$$\lambda,\lambda' = +,-,0,$$

where $e_i^{(\lambda)}$ are the deuteron spin functions which have the deuteron spin projection λ onto the quantization axis (z axis). They are

$$e^{(\pm)} = \mp\frac{1}{\sqrt{2}}(1, \pm i, 0), \quad e^{(0)} = (0,0,1). \tag{A4}$$

The elements of the spin-density matrix in the helicity representation are related to those in the coordinate representation as follows:

$$\rho_{++} = \frac{1}{3} + \frac{1}{2}s_z - \frac{1}{2}Q_{zz},$$
$$\rho_{--} = \frac{1}{3} - \frac{1}{2}s_z - \frac{1}{2}Q_{zz},$$
$$\rho_{00} = \frac{1}{3} + Q_{zz},$$
$$\rho_{+-} = -\frac{1}{2}(Q_{xx} - Q_{yy}) + iQ_{xy},$$
$$\rho_{+0} = \frac{1}{2\sqrt{2}}(s_x - is_y) - \frac{1}{\sqrt{2}}(Q_{xz} - iQ_{yz}),$$
$$\rho_{-0} = \frac{1}{2\sqrt{2}}(s_x + is_y) - \frac{1}{\sqrt{2}}(Q_{xz} + iQ_{yz}). \tag{A5}$$

To obtain these relations we used the condition $Q_{xx} + Q_{yy} + Q_{zz} = 0$.

3. Representation in terms of population numbers

The description of the polarized deuteron target in terms of the population numbers n_+, n_-, and n_0 is often used in the formulation of spin experiments (see, for example, Ref. [28]). Here n_+, n_-, and n_0 are the fractions of the atoms in the polarized target with the nuclear-spin projection onto the quantization axis $m = +1$, $m = -1$, and $m = 0$, respectively. If the spin-density matrix is normalized to 1, i.e., $\mathrm{Tr}\rho = 1$, then we have $n_+ + n_- + n_0 = 1$. Thus, the polarization state of the deuteron target is defined in this case by two parameters: the so-called V (vector) and T (tensor) polarizations:

$$V = n_+ - n_-, \quad T = 1 - 3n_0. \tag{A6}$$

By using the definitions for the quantities $n_{\pm,0}$:

$$n_\pm = \rho_{ij} e_i^{(\pm)*} e_j^{(\pm)}, \quad n_0 = \rho_{ij} e_i^{(0)*} e_j^{(0)}, \tag{A7}$$

we have the following relation between the V and T parameters and parameters of the spin-density matrix in the coordinate representation (in the case when the quantization axis is directed along the z axis)

$$n_0 = \tfrac{1}{3} + Q_{zz}, \quad n_\pm = \tfrac{1}{3} \pm \tfrac{1}{2}s_z - \tfrac{1}{2}Q_{zz}, \tag{A8}$$

or

$$T = -3Q_{zz}, \quad V = s_z. \tag{A9}$$

4. Representation of spherical tensors

Let us relate now the parameters of the spin-density matrix in the coordinate representation to the parameters of the matrix in the representation of the spherical tensors.

According to the Madison Convention [29], the spin-density matrix of a spin-1 particle is given by the expression

$$\rho = \frac{1}{3}\sum_{kq} t_{kq}^* \tau_{kq}, \tag{A10}$$

where t_{kq} are the polarization parameters of the deuteron spin-density matrix and τ_{kq} are the spherical tensors. The spherical tensors are expressed as

$$\tau_{00} = 1, \quad \tau_{10} = \sqrt{\frac{3}{2}}S_z, \quad \tau_{1\pm1} = \mp\frac{\sqrt{3}}{2}(S_x \pm iS_y),$$
$$\tau_{20} = \frac{3}{\sqrt{2}}\left(S_z^2 - \frac{2}{3}\right),$$
$$\tau_{2\pm2} = \frac{\sqrt{3}}{2}(S_x \pm iS_y)^2,$$
$$\tau_{2\pm1} = \mp\frac{\sqrt{3}}{2}[(S_x \pm iS_y)S_z + S_z(S_x \pm iS_y)], \tag{A11}$$

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \tag{A12}$$
$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

From Eq. (A11) and the Hermiticity of the spin operator it is straightforward to get

$$\tau_{kq}^+ = (-1)^q \tau_{k-q}. \tag{A13}$$

The Hermiticity condition for the density matrix yields for t_{kq}

$$t_{kq}^* = (-1)^q t_{k-q}. \tag{A14}$$

From this equation one can see that

$$t_{10}^* = t_{10}, \quad t_{11}^* = -t_{1-1},$$
$$t_{20}^* = t_{20}, \quad t_{22}^* = t_{2-2}, \quad t_{21}^* = -t_{2-1}, \tag{A15}$$

i.e., the parameters t_{10} and t_{20} are real, and the parameters t_{11}, t_{21} and t_{22} are complex. So, in total there are eight independent real parameters as required for spin-1 massive particles.

The explicit expression of the deuteron density matrix is

$$\rho = \frac{1}{3}\begin{pmatrix} 1 + \sqrt{\frac{3}{2}}t_{10} + \frac{1}{\sqrt{2}}t_{20} & \sqrt{\frac{3}{2}}(t_{1-1} + t_{2-1}) & \sqrt{3}t_{2-2} \\ -\sqrt{\frac{3}{2}}(t_{11} + t_{21}) & 1 - \sqrt{2}t_{20} & \sqrt{\frac{3}{2}}(t_{1-1} - t_{2-1}) \\ \sqrt{3}t_{22} & -\sqrt{\frac{3}{2}}(t_{11} - t_{21}) & 1 - \sqrt{\frac{3}{2}}t_{10} + \frac{1}{\sqrt{2}}t_{20} \end{pmatrix}. \tag{A16}$$

The density matrix is normalized to 1, i.e., $\mathrm{Tr}\rho = 1$. By using the expression for the density matrix in the helicity representation, Eq. (A5), we get the following relations between the parameters of the density matrix in the coordinate representation and in the spherical tensor representation:

$$t_{10} = \sqrt{\frac{3}{2}}s_z, \quad \mathrm{Re}t_{11} = -\mathrm{Re}t_{1-1} = -\frac{\sqrt{3}}{2}s_x,$$
$$\mathrm{Im}t_{11} = \mathrm{Im}t_{1-1} = -\frac{\sqrt{3}}{2}s_y, \quad t_{20} = -\frac{3}{\sqrt{2}}Q_{zz},$$
$$\mathrm{Re}t_{21} = -\mathrm{Re}t_{2-1} = \sqrt{3}Q_{xz}, \quad \mathrm{Im}t_{21} = \mathrm{Im}t_{2-1} = \sqrt{3}Q_{yz},$$
$$\mathrm{Re}t_{22} = \mathrm{Re}t_{2-2} = -\frac{\sqrt{3}}{2}(Q_{xx} - Q_{yy}), \quad \mathrm{Im}t_{22} = -\mathrm{Im}t_{2-2} = -\sqrt{3}Q_{xy}. \tag{A17}$$

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