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Neutron angular correlations in spontaneous and neutron-induced fission

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Background: Neutron emission is correlated in fission events because, on average, more than one neutron is emitted per fission. Measurements of these correlations, coupled with studies of more inclusive observables such as neutron multiplicity, provide sensitive information about the fission mechanism. Neutron-neutron angular correlations have been studied in both spontaneous fission of ^{252}Cf and neutron-induced fission of ^{235}U. These correlations, until recently incalculable in most available simulations of fission, can now be calculated in event-by-event simulations of fission.

Purpose: Phenomenological studies of fission are of interest both for basic science and for practical applications. Neutron-neutron angular correlations are characteristic of the fissioning isotope and could be used in material identification.

Method: We use our model of complete fission events, FREYA, to first study the sensitivity of two-neutron angular correlations to the model inputs and then compare to available data. We also compare our simulations to neutron-fragment angular correlations.

Results: We find that the correlations calculated with FREYA are fairly robust with respect to the input parameters. Any strong deviations in the correlations result in poor agreement with measured inclusive neutron observables such as neutron multiplicity as a function of fragment mass and the neutron multiplicity distribution. The agreement of FREYA with the present set of correlation data is found to be good.

Conclusions: FREYA can be used to reliably predict neutron-neutron angular correlations and could then be used to identify materials.

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I. INTRODUCTION

Neutron-neutron correlations as a function of the angle between the two emitted neutrons, θ_{nn}, are an observable that has been studied since early in the history of fission measurements. These correlations do not require simultaneous measurement of the fission fragments. Another early observable that does depend on detecting both a neutron and the fission fragment is the angular correlation between emitted neutrons and the light fission fragment, θ_{nL}. Taken together, these two observables are sensitive to the characteristics of neutron emission and are useful for testing models of neutron emission.

The first of the neutron-neutron angular correlation measurements dates as far back as 1948 by DeBenedetti *et al.* [1]. They bombarded a ^{235}U source with fast neutrons. The neutrons were detected by proportional counters placed at different angles around the source. The also took a calibration measurement of the ratio of neutron coincidences at 90° and 180°. They tried to account for cross correlations between detectors due to rescattering where a neutron producing a recoil in one counter is scattered into an adjacent detector. To do this, they used a Pb-Be source emitting single neutrons so that all observed neutron coincidences arise from only one neutron. They assumed that the same number of rescattering coincidences from the Pb-Be source also arose from the ^{235}U source. With this assumption, they found a flat correlation for $\theta_{nn} < 90°$ and an increase above 90°. They concluded that the neutrons are preferentially emitted in opposite directions by opposite fragments. Since most later experiments have observed a finite signal at $\theta_{nn} = 0°$, they may have overestimated this background for their source.

Neutrons are generally assumed to be emitted isotropically in the rest frame of the decaying fragment. When boosted to the laboratory frame, where observations are made, the neutrons thus move in the same direction as the fragments. If one neutron is emitted from each of the two fragments, the correlation is back to back, $\theta_{nn} = 180°$, because energy-momentum conservation requires the fragments to move in opposite directions after scission. If both neutrons come from the same fragment, then they appear at $\theta_{nn} = 0°$. These are the only neutron sources if one assumes neutrons are emitted only from the fully accelerated fragments and not before. However, there is also a possibility that neutrons are emitted from the nucleus before scission [2–5] or during acceleration of the fragments [6]. A number of the later experiments measuring neutron-neutron angular correlations were motivated by the search for these scission neutrons.

The measured correlations have been simulated by assuming the existence of these scission neutrons. These simulations include three neutron sources: the scissioning nucleus and the light and heavy fission fragments. Scission neutrons are emitted isotropically in the rest frame of the nucleus undergoing scission, equivalent to the laboratory frame in this case. There is then no boost and the correlation between two neutrons emitted at scission is independent of θ_{nn}. This is also the case if one of the neutrons comes from a fragment and the other is a scission neutron. This additional neutron source then leads to a flatter correlation than one in which no scission neutrons are emitted [7]. In previous simulations of these correlations a 0%–20% contribution to the neutron multiplicity from scission neutrons was assumed. In these simulations, in addition to the assumption of isotropic neutron emission in the emitter rest frame, it was also assumed that the neutron energy spectrum is the same for all neutrons and is independent of the number of neutrons emitted from a given fragment. The two fragments are assumed to have a common temperature for neutron emission and all emission is assumed to be independent and thus uncorrelated.

Another neutron-related angular correlation is that between a neutron and the light fragment. While the identity of the light fragment can be determined with fragment detectors, it is unknown whether the detected neutron comes from the light or the heavy fragment. Nonetheless, measurements show a strong peak at $\theta_{nL} = 0^\circ$. The first such measurements were by Bowman *et al.* [2] and by Skarsvag and Bergheim [3].

For the first time, with our model FREYA, these angular correlations are calculated with a complete event-by-event Monte Carlo simulation with a temperature that changes with each neutron emitted. The fission model FREYA (Fission Reaction Event Yield Algorithm) incorporates the relevant physics with a few key parameters determined by comparison to data [8–10]. It simulates the entire fission process and produces complete fission events with full kinematic information on the emerging fission products and the emitted neutrons and photons, incorporating sequential neutron and photon evaporation from the fission fragments. The event-by-event nature of FREYA makes it straightforward to extract the angular correlation between two evaporated neutrons [1,7,11–13] and between an evaporated neutron and the light fission fragment [2,3], neither of which can be addressed with standard fission models.

We describe the inputs to FREYA that could affect the shape of these correlations in Sec. II. We then discuss the sensitivity of the neutron-neutron angular correlation result to these inputs in Sec. III. In Sec. IV we make a comparison to available data on neutron-neutron angular correlations (Sec. IV A) and neutron-light fragment angular correlations (Sec. IV B). We summarize our findings in Sec. V.

II. INPUTS

FREYA relies on data-related inputs of the fission fragment yields, $Y(A)$, as a function of energy (for neutron-induced fission) and total fragment kinetic energy, TKE. Additionally, the Gaussian widths of the fragment charge distributions depend on previous measurements [10]. There are some universal inputs including ground-state masses, taken from data [14] and supplemented by theory [15] when required; fission barrier heights; and pairing energies and shell corrections. FREYA also has several input parameters that can depend on the identity of the fissile nucleus. These include dTKE, the shift of the measured TKE required to match the average neutron multiplicity; e_0, the asymptotic level density parameter; x, the advantage in excitation energy given to the light fragment; c, the relative thermal fluctuations in the fragment temperature distribution; $Q_{\rm min}$, the energy above the neutron separation energy where photon emission begins to dominate over neutron emission; and c_S, the ratio of the "spin temperature" to the scission temperature.

In the remainder of this section, we introduce these inputs more fully and describe the consequences of varying these inputs on neutron observables.

We use a parametrization of the level density parameter based on the back-shifted Fermi gas (BSFG) model [16],

$$\tilde{a}_i(E_i^*) = \frac{A_i}{e_0}\left[1 + \frac{\delta W_i}{U_i}(1 - e^{-\gamma U_i})\right], \tag{1}$$

where $U_i = E_i^* - \Delta_i$ and $\gamma = 0.05$ [17]. The pairing energy of the fragment, Δ_i, and its shell correction, δW_i, are tabulated in Ref. [16] based on the mass formula of Koura *et al.* [18]. We take e_0 as a model parameter. We note that, if the shell corrections are negligible, $\delta W \approx 0$, or the available energy, U, is large, then this renormalization is immaterial and the BSFG level-density parameter is proportional to the mass, $\tilde{a}_i \sim A_i/e_0$. In Ref. [10], we found $e_0 \sim 10$ MeV, which we use in these studies for all fissile actinides. To test the effect of the choice of e_0 on the neutron angular correlations, we will vary e_0 by 20%, between 8 MeV and 12 MeV.

If the two fragments are in mutual thermal equilibrium, $T_L = T_H$, the total excitation energy will, on average, be partitioned in proportion to the respective heat capacities, which in turn are proportional to the level density parameters, i.e., $\overline{E}_i^* \sim \tilde{a}_i$. FREYA therefore first assigns average excitation energies based on such an equipartition,

$$\acute{E}_i^* = \frac{\tilde{a}_i(\tilde{E}_i^*)}{\tilde{a}_L(\tilde{E}_L^*) + \tilde{a}_H(\tilde{E}_H^*)}\overline{\rm TXE}, \tag{2}$$

where $\tilde{E}_i^* = (A_i/A_0)\overline{\rm TXE}$. Subsequently, because the observed neutron multiplicities suggest that the light fragments tend to be disproportionately excited, the average values are adjusted in favor of the light fragment:

$$\overline{E}_L^* = x\acute{E}_L^*,\ \overline{E}_H^* = \overline{\rm TKE} - \overline{E}_L^*, \tag{3}$$

where x is an adjustable model parameter, expected to be larger than unity. We find $x = 1.3$ agrees well with $\nu(A)$ for ^{252}Cf(sf) while $x = 1.2$ is used for ^{235}U(n,f) [19]. To test the effect of the choice of x on the correlation observables, we will vary x for ^{252}Cf by $\sim$30%, between 1 and 1.6. We also test the effect of taking $x < 1$, using $x = 0.75$. Since this parameter has a strong effect on the calculated $\nu(A)$, we also show how this observable changes with x.

After the mean excitation energies have been assigned, FREYA considers the effect of thermal fluctuations. In Weisskopf's statistical model of the nucleus, in which the excited nucleus is treated as a degenerate Fermi gas, the mean excitation of a fragment is related to its temperature T_i by $\overline{E}_i^* = \tilde{a}_i T_i^2$ [20–22] and the associated variance in the excitation is $\sigma_{E_i}^2 = -\partial^2 \ln \rho_i(E_i)/\partial E_i^2 = 2\overline{E}_i^* T_i$. Therefore, for each of the two fragments, we sample a thermal energy fluctuation δE_i^* from a Gaussian distribution of variance $2c\overline{E}_i^* T_i$ and modify the fragment excitations accordingly, arriving at

$$E_i^* = \overline{E}_i^* + \delta E_i^*, \quad i = L,H. \tag{4}$$

Due to energy conservation, there is a compensating opposite fluctuation in the total kinetic energy, so that

$$\mathrm{TKE} \;=\; \overline{\mathrm{TKE}} - \delta E_L^* - \delta E_H^*. \qquad (5)$$

The factor c multiplying the variance can, in principle, be tuned to the neutron multiplicity distribution $P(\nu)$. As a default value, used in our previous work, we take $c = 1.0$. We vary c by 20%, between 0.8 and 1.2. Since $P(\nu)$ is also sensitive to this quantity, we will show the effect of changing c on $P(\nu)$ for ^{252}Cf(sf).

We generally assume that neutron emission continues until no further neutron emission is energetically possible, i.e., when $E_d^* < S_n + Q_{\min}$, where S_n is the neutron separation energy in the prospective daughter nucleus, $S_n = M(^{A_d}Z_d) - M(^{A_d-1}Z_d) - m_n$. We have chosen $Q_{\min} = 0.01$ MeV so that neutrons are emitted even if the energy is very close to the neutron separation energy. We will take this as the default and raise it to 1 MeV to see how the correlations are affected.

In Ref. [23], we introduced the possibility for the fissile nucleus to have some initial angular momentum. In addition to the rigid rotation of the dinuclear configuration prior to scission, assumed to be inherited by the fragments, the fragments also acquire fluctuations around the rigid rotation axis, including wriggling and bending modes, with rotation in the same or opposite sense around an axis perpendicular to the dinuclear axis. We assume that these fluctuations are statistically excited during scission. Thus, in each event, the values of $s_\pm$, the spin of the normal modes [where the plus refers to wriggling modes (with parallel rotations) while the minus refers to bending modes (with opposite rotations)] are being sampled from distributions of the form

$$P_\pm(\boldsymbol{s}_\pm = (s_\pm^x, s_\pm^y, 0))\, ds_\pm^x ds_\pm^y \sim \mathrm{e}^{-s_\pm^2/2\mathcal{I}_\pm T_S} ds_\pm^x ds_\pm^y, \qquad (6)$$

where the "spin temperature" T_S is regarded as a global but somewhat adjustable parameter. We take $T_S = c_S T_{\rm sc}$, where $T_{\rm sc}$ is the scission temperature. As the default value, we use $c_S = 1$, which corresponds to assuming that the spin degrees of freedom are fully equilibrated at scission. This value of c_S yields $\overline{S}_L \sim 6.2\hbar$, $\overline{S}_H \sim 7.6\hbar$, values that are in rather good agreement with the average energy of photons emitted in fission. As an alternative, we have also employed $c_S = 0.1$ to dial down the photon multiplicity; this yields $\overline{S}_L \sim 1.8\hbar$, $\overline{S}_H \sim 2.2\hbar$, eliminating most of the collective yrast photons. See Ref. [23] for details.

The moments of inertia, $\mathcal{I}_\pm$, depend on the moments of inertia of the light and heavy fragments, $\mathcal{I}_L$ and $\mathcal{I}_H$, as well as the moment of inertia of the relative fragment motion, $\mathcal{I}_R$. We use the rigid-rotator moment of inertia, $\mathcal{I} = c_I(2/5)MR^2$, where M and R are the mass and radius of the fragment, $R = r_0 A^{1/3}$. We use the commonly accepted value of $c_I = 0.5$ [23,24] and leave it fixed since it only affects the photon observables.

III. SENSITIVITY OF NEUTRON-NEUTRON ANGULAR CORRELATIONS TO INPUTS

We first study the robustness of the correlation observable by changing the input parameters one at a time from their default values of $x = 1.3$, $e_0 = 10$ MeV, $c_S = 1$, $Q_{\min} = 0.01$ MeV, and $c = 1$ for ^{252}Cf(sf). We do not change dTKE from its value of 0.5 MeV with $c_S = 1$ because most of the changes we make do not strongly affect the calculated $\overline{\nu}$. We also do not change c_I because changing c_I only has an effect on the photon observables; the effect on the neutron observables is negligible. The results are shown in Fig. 1 for neutrons emitted during spontaneous fission of ^{252}Cf. We choose ^{252}Cf(sf) because these correlations have been studied most for this system. We employ a minimum neutron energy of $E_n = 0.5$ MeV for both emitted neutrons. Increasing the minimum neutron energy tends to enhance the correlation at $\theta_{nn} = 0^\circ$ while giving only a negligible change at $\theta_{nn} = 180^\circ$ (see Ref. [19]).

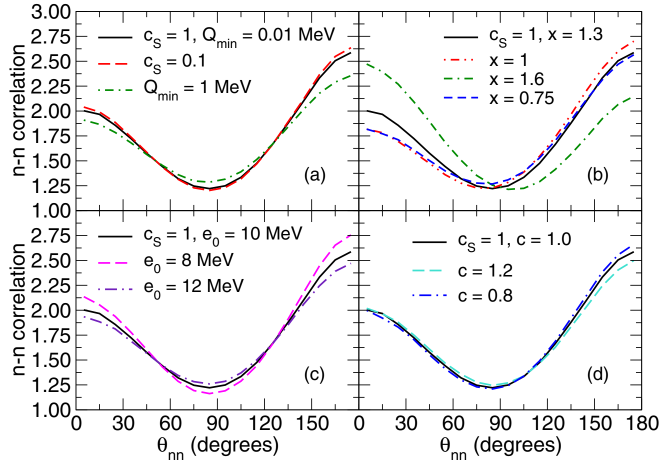

FIG. 1. (Color online) The angular correlation between two neutrons emitted from ^{252}Cf(sf) as a function of the opening angle between the two neutrons, θ_{nn}. The FREYA results are shown for neutron kinetic energies $E_n > 0.5$ MeV. The results of different parameter choices in FREYA are compared to the default results: $c_S = 1$, $Q_{\min} = 0.01$ MeV, $x = 1.3$, $e_0 = 10$ MeV, and $c = 1$. (a) Parameters affecting photon emission are varied. The dashed red curve is with $c_S = 0.1$, $Q_{\min} = 0.01$ MeV while the dot-dashed green curve is with $c_S = 1$, $Q_{\min} = 1$ MeV. (b) The parameter affecting the relative excitation energy of the light fragment, x, is varied. The dot-dot-dashed red curve is the result for $x = 1$, equal partition between the light and heavy fragments. The dot-dashed-dashed green curve shows the result when giving the light fragment even more energy, $x = 1.6$, while the dashed blue curve shows a result with $x = 0.75$, with more excitation given to the heavy than to the light fragment. (c) The parameter governing the level density is varied. The dashed magenta curve is with $e_0 = 8$ MeV while the dot-dashed maroon curve is with $e_0 = 12$ MeV. (d) The parameter governing thermal fluctuations is varied with $c = 1.2$ (dashed turquoise curve) increasing the width of the fluctuation while $c = 0.8$ (dot-dashed blue) decreases it.

Figure 1(a) shows the sensitivity of the correlation to the parameters most closely related to the photon observables, c_S and $Q_{\min}$. Changing c_S from 1 to 0.1 reduces the initial spin from $\sim 7\hbar$ to $\sim 2\hbar$. The higher value, $c_S = 1$, is most compatible with previous extractions of fragment spins at scission [23]. The two calculations effectively coincide; thus the correlation is insensitive to this parameter. Changing the minimum energy for neutron emission relative to the separation energy, $S_n + Q_{\min}$, from $Q_{\min} = 0.01$ MeV, effectively allowing neutron emission to dominate down to $\sim S_n$, to 1 MeV has a small effect on the correlation, reducing the value at $\theta_{nn} = 180^\circ$ somewhat and making the correlation slightly more symmetric around $\theta_{nn} = 90^\circ$. The higher value of $Q_{\min}$ is more compatible with the energy spectra of photon emission (see Ref. [24]). Therefore, while these parameters do not have a strong effect on the neutron-neutron angular correlation, they do have an important effect on photon observables.

The most striking effect on the shape of the neutron-neutron angular correlation is the partition of the excitation energy between the light and heavy fragments. Changing x while keeping $x \geqslant 1$ has a somewhat larger effect at $\theta_{nn} = 0^\circ$ than changing $Q_{\min}$, but the difference is not large. The small-angle enhancement for neutrons coming from the same fragment is larger for those emitted by the light fragment because of its higher velocity from Coulomb repulsion at scission. Furthermore, the relative magnitude of the small-angle and large-angle enhancements evolves as the energy sharing is changed, due to the change in the origin of the emitted neutrons. Thus, for $x = 1.6$ when the light fragment has a large excess energy, the peak at $\theta_{nn} = 0^\circ$ is higher than the peak at $\theta_{nn} = 180^\circ$, and the angular correlation function tilts steadily in favor of the 180° peak as x is decreased. Such a large dependence on x should appear also in other observables and may provide a bound on how much x can be varied and still agree with data on other observables.

In Fig. 2, we show $\nu(A)$ for the same values of x as in Fig. 1(b), $x = 0.75$, 1, 1.3, and 1.6. For the default value of $x = 1.3$, we also show the variance in $\nu(A)$ over the range of possible Z values for each A for FREYA. We also show several sets of recent data which agree well with each other. The agreement of our default calculations with these data is quite good for $105 < A < 145$, covering the symmetric region and the A range where the yields are highest. To improve the overall agreement, we would have to introduce an A-dependent temperature distribution, as has been done in some other calculations [28,29], or point-by-point yields for each fragment pair [30]. Increasing x to 1.6 increases $\nu(A)$ for the light fragment to well above the data while underestimating the neutron multiplicity for the heavy fragment. It enhances the difference in neutron emission between $A = 120$ and 132. Taking $x = 1$ decreases the variation of $\nu(A)$ considerably. While it actually improves agreement with the data for $A < 100$, the edge of the sawtooth is not sufficiently sharp. Finally, $x = 0.75$ actually inverts the sawtooth shape, significantly underestimating the yield below symmetry while overestimating the neutron multiplicity for the heavy fragment. Thus, while we can see some dependence of the correlation function on x, these variations can be ruled out by the data on $\nu(A)$.

Next, we show the dependence of the neutron-neutron angular correlation on the asymptotic value of the level density parameter, e_0, in Fig. 1(c). Changing e_0 by ± 2 MeV modifies the correlation somewhat but generally by less than changing $Q_{\min}$ in Fig. 1(a). While the chosen range of e_0 is within the range of acceptable values, it is constrained by the shape of the prompt fission neutron spectrum (see, e.g., Ref. [10]).

Finally, we check the sensitivity of our results to the variance of the thermal fluctuations in Fig. 1(d). These fluctuations can modify the intrinsic excitation energy and thus the neutron multiplicity distribution, $P(\nu)$ (see Fig. 3). While changing c has a negligible effect on the correlation function, it has a stronger effect on $P(\nu)$. Increasing c to 1.2 broadens $P(\nu)$ relative to the data while decreasing it to 0.8 narrows the multiplicity distribution relative to the data. Thus significant changes of c can be ruled out.

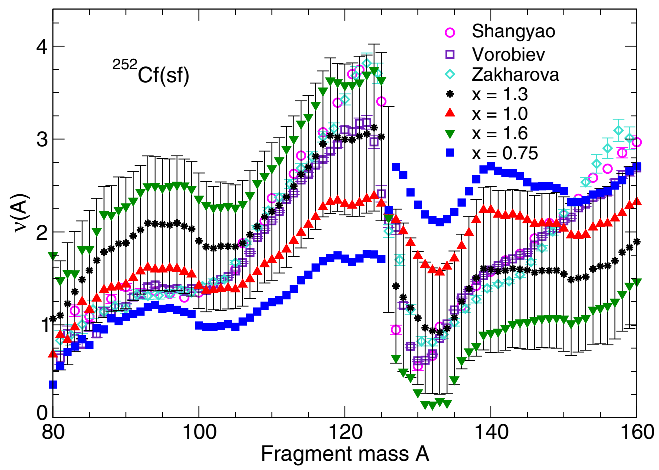

FIG. 2. (Color online) The variation of $\nu(A)$ with the parameter governing the partition of the excitation energy between the two fragments, x, compared to data [25–27]. The default result, black stars, including bars representing the variance in A for fragment Z, is shown by the magenta stars. The results with $x = 1$ (solid red, upward-facing triangles), $x = 1.6$ (solid green, downward-facing triangles), and $x = 0.75$ (blue stars) do not show the variances.

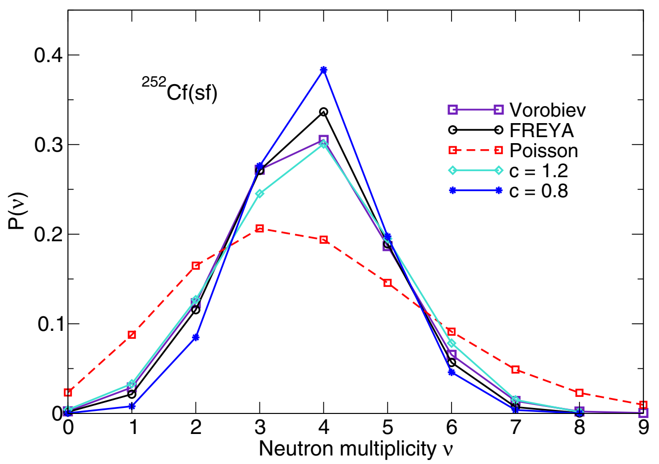

FIG. 3. (Color online) The variation in the neutron multiplicity distribution obtained from varying the thermal fluctuations, c. The data [31] (violet squares) are compared to the default (labeled FREYA) result as well as that with $c = 1.2$ (turquoise diamonds) and that with $c = 0.8$ (blue stars). The corresponding result for a Poisson distribution with the same average multiplicity is shown by the red dashed curve.

IV. COMPARISON TO DATA

Here we take our default calculation for FREYA, shown as the solid black curve in Fig. 1 for neutron-neutron angular correlations in ^{252}Cf(sf), and compare these results to available data. We first compare our calculations to existing neutron-neutron angular correlation data in Sec. IV A. We focus on ^{252}Cf(sf) data from Pringle and Brooks [12] and then from Skarsvag and Bergheim [3]. Our calculations have also been compared to the recent ^{252}Cf(sf) data of Pozzi *et al.* and appear in Ref. [13]. Therefore we do not reproduce them again here but rather refer the reader to their work. We also compare to the ^{235}U($n_{\rm th}$,f) data of Franklyn *et al.* [11]. In Sec. IV B, we compare our calculations of neutron–light fragment angular correlations to ^{252}Cf(sf) data from Bowman *et al.* [2] and Gagarski *et al.* [7]. Some of these early data have also been compared to previous Monte Carlo studies, albeit not with complete events [32].

A. Neutron-neutron angular correlation data

As we have already discussed, prompt neutrons from fission tend to be either forward or backward correlated. We have considered three separate neutron sources: both neutrons from the light fragment, both from the heavy fragment, and one neutron emitted from each fragment. We do not include scission neutrons as a source. In Ref. [33], we analyzed ^{239}Pu($n_{\rm th}$,f) for $\nu = 2$ and found a significant correlation at $\theta_{nn} = 0^\circ$ when both neutrons are emitted from the same fragment, with a higher peak when both neutrons are emitted from the light fragment, due to its higher velocity. On the other hand, when one neutron is emitted from each fragment, there is a peak at $\theta_{nn} = 180^\circ$. The overall result is a stronger backward correlation because emission from both fragments is most likely. Indeed, the backward correlation is strongest when the overall neutron multiplicity is low since large multiplicities reduce the angular correlation [19]. This is because larger overall neutron multiplicities make it likely that more than one neutron is emitted by each fragment.

As we will see, the agreement of our calculations with the data is quite good without requiring a contribution from scission neutrons.

1. ^{252}*Cf(sf)*

An early neutron-neutron angular correlation measurement was performed by Pringle and Brooks in 1975 [12]. They used two liquid scintillator neutron detectors and employed pulse-shape discrimination to reject photon events. One detector was fixed in the horizontal plane with the source while the second detector was rotated around the vertical axis. At $\theta_{nn} < 45^\circ$ the distance between the detector and the source was increased and shielding was inserted to reduce neutron rescattering. The minimum detected neutron energy was 0.7 MeV [12].

The more recent measurement by Gagarski *et al.* [7] was published in 2008. They used a similar setup of two neutron detectors with varying angular difference around the source. The detectors were stilbene crystals with photomultiplier tubes surrounded by shielding. They used time of flight to separate neutrons from photons and showed that they could achieve neutron and photon separation with the photomultiplier tubes down to the detector threshold. By changing the event selection boundaries, several different neutron detection thresholds could be employed: 0.425, 0.55, 0.75, 0.8, 1.2, and 1.6 MeV [7].

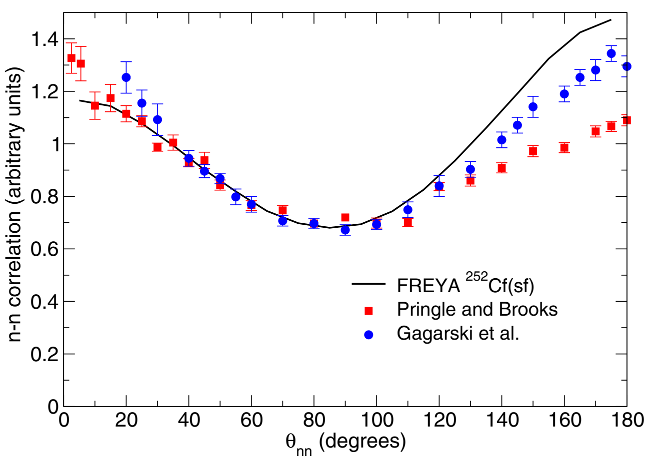

FIG. 4. (Color online) The default FREYA calculations compared to ^{252}Cf(sf) two-neutron angular correlation data from [12] (red squares) and [7] (blue circles) for neutron kinetic energies greater than 0.7 MeV.

The Gagarski measurement for the 0.75 MeV neutron threshold was compared to the Pringle and Brooks measurement at 0.7 MeV in Ref. [7]. Relatively good agreement was found between the two measurements at $\theta_{nn} < 90^\circ$ but the Gagarski result shows a stronger back-to-back correlation than that of Pringle and Brooks (see Fig. 4). The difference between the two measurements was noted in Ref. [7], but no reason for the discrepancy was proposed. The 0.05 MeV difference in energy thresholds is too small to account for it.

Figure 4 also shows the FREYA result. The calculation agrees well with both data sets at $\theta_{nn} < 90^\circ$ but overestimates the back-to-back correlation at larger angles. Our calculation is relatively close to the Gagarski result, although it is slightly above. Given that increasing $Q_{\rm min}$ was seen to decrease the calculated correlation at $\theta_{nn} \to 180^\circ$, taking a higher $Q_{\rm min}$ would improve our agreement with these data.

Our results are compared to the Gagarski *et al.* data [7] with their other energy thresholds in Fig. 5. We again see that the agreement between the calculation and the data is very good for $\theta_{nn} < 90^\circ$ while the calculation overestimates the back-to-back peak for neutrons with kinetic energies less than 1 MeV. The improvement of the agreement between the calculations and the data at higher neutron energy thresholds suggests that the dependence of the correlation on $Q_{\rm min}$ may diminish with neutron energy.

2. ^{235}*U*($n_{\rm th}$, *f*)

In 1978 Franklyn, Hofmeyer, and Mingay [11] studied neutron-neutron angular correlations in ^{235}U($n_{\rm th}$,f). They used two stilbene neutron detectors and employed pulse-shape discrimination to reject photon events. The used boron-loaded shadow shields to suppress neutron rescattering effects at low θ_{nn}. They obtained correlation results for minimum neutron

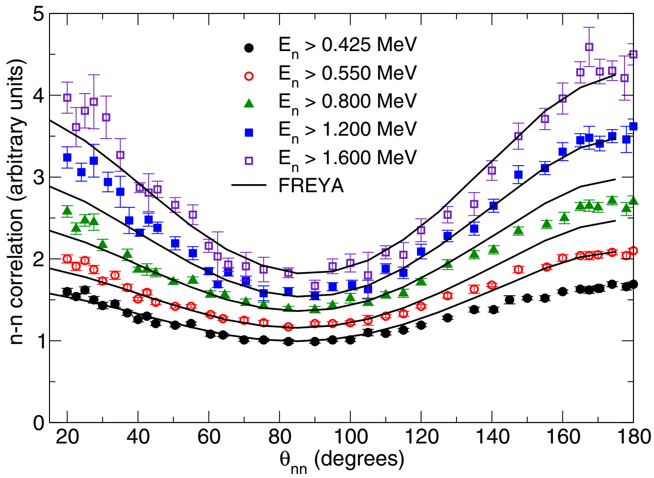

FIG. 5. (Color online) The default FREYA calculations compared to ^{252}Cf(sf) two-neutron angular correlation data from [7] for neutron kinetic energies greater than 0.425, 0.55, 0.8, 1.2, and 1.6 MeV.

energies of 1.0, 1.75, and 2.5 MeV. The lower limit on the neutron kinetic energy was imposed by a pulse-shape discriminator [11].

These data are compared to our default FREYA calculations for ^{235}U($n_{\rm th}$,f) in Fig. 6. The agreement is generally rather good for $\theta_{nn} < 140^\circ$ with $E_n \geqslant 1.0$ and 1.75 MeV, where our results again overestimate the back-to-back correlation somewhat. For $E_n \geqslant 2.5$ MeV, the agreement is good over all θ_{nn}, again suggesting that we should employ a larger value of $Q_{\rm min}$.

B. Neutron–light fragment angular correlations

In 1962 Bowman *et al.* made the first measurement of correlations between neutrons and light fragments [2]. Their setup consisted of two neutron detectors and two fission fragment detectors, both plastic scintillators of different

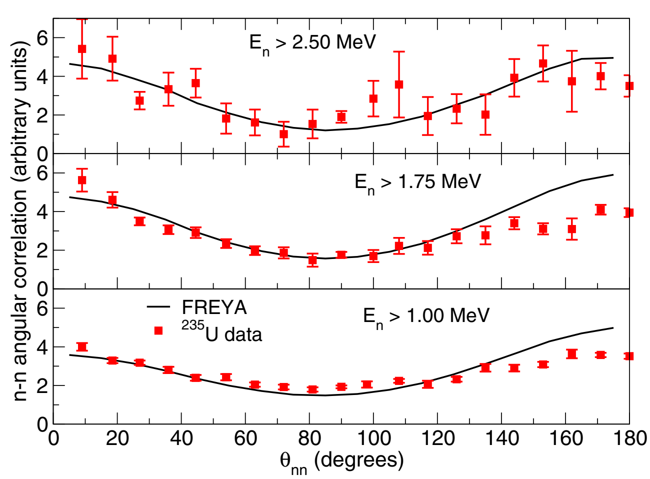

FIG. 6. (Color online) The default FREYA calculations compared to ^{235}U($n_{\rm th}$,f) two-neutron angular correlation data from [11] for neutron kinetic energies greater than 1.0 MeV (bottom), 1.75 MeV (center), and 2.5 MeV (top).

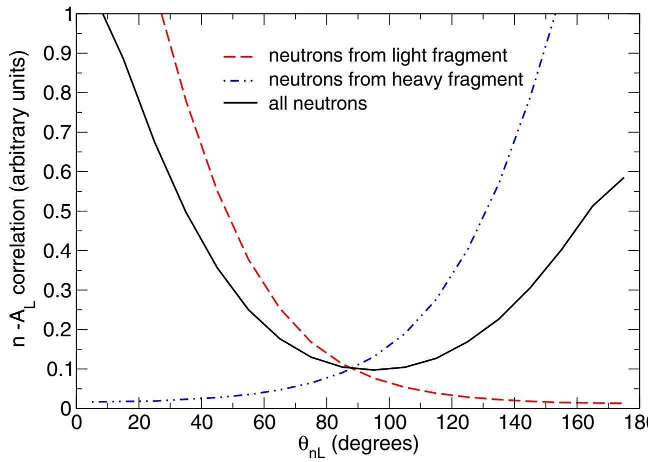

FIG. 7. (Color online) The default FREYA calculations shown for ^{252}Cf(sf) neutrons correlated with the light fragment, A_L. The neutrons emitted from the light fragment (dashed red) and those emitted from the heavy fragment (dot-dot-dashed blue) are compared to the correlation of all neutrons with the light fragment. All neutrons have a minimum kinetic energy of 0.5 MeV.

thickness, mounted around a steel drum of 2 m in diameter. A ^{252}Cf(sf) source was placed at the center of the drum and put under vacuum. The fragment detectors were mounted on opposite sides of the drum, at 180° from each other. One neutron detector was held fixed at $\theta_{nn} = 11.25^\circ$ while the other was moved through angles 22.5° to 90° with respect to one fragment detector (111.5° to 180° relative to the other fragment detector). Time of flight was used to detect one neutron in coincidence with two fission fragments as well as to separate the light and heavy fragments from each other. They presented the angular correlation between all measured neutrons and the identified light fragment. While the correlation is made with the light fragment, it was not possible to determine which fragment emitted the neutron [2].

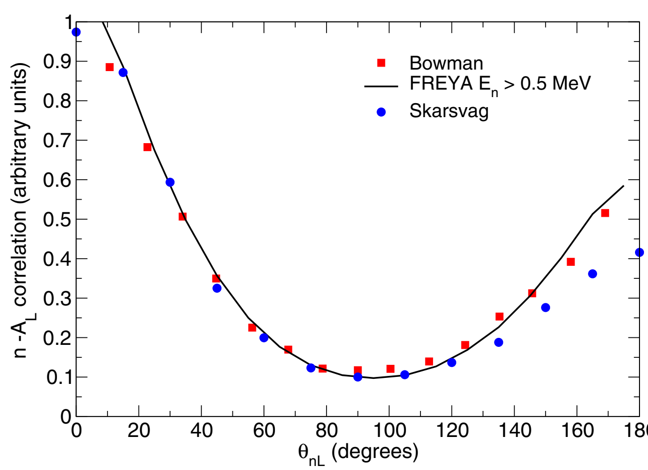

FIG. 8. (Color online) The default FREYA calculations compared to ^{252}Cf(sf) neutron–light fragment (A_L) angular correlation data from [2] (red squares) and [3] (blue circles). The minimum kinetic energy of the neutrons is 0.5 MeV.

In Fig. 7, we show how the neutron–light fragment angular correlation is built up in FREYA. The dashed curve shows the result if all neutrons come from the light fragment. There is a strong peak at $\theta_{nL} = 0$ with essentially no signal in the opposite direction. If all detected neutrons arise from the heavy fragment, the correlation is effectively reflected around $\theta_{nL} = 90^\circ$. The shape of the correlation from all emitted neutrons retains the largest peak at zero degrees while, in the backward direction, the signal is reduced. This is because more neutrons are emitted by the light fragment because it gets more intrinsic excitation energy.

We compare the measurements of Bowman *et al.* [2] and also Skarsvag and Bergheim [3] (reproduced in Gagarski *et al.* [7]) to our FREYA results in Fig. 8. The agreement of our correlation with the shape of the data is very good.

V. SUMMARY

We have shown that event-by-event models of fission, such as FREYA, provide a powerful tool for studying fission neutron angular correlations. The calculations are robust, being relatively insensitive to the input parameters, which can be constrained by other data. The agreement of our calculations with the available data is good and does not lend strong support for the requirement of scission neutrons to explain the correlations. However, further data on these correlations based on fission of other isotopes and, for neutron-induced fission, at higher incident neutron energies would be welcome to help verify these results.

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[1] S. DeBenedetti, J. E. Francis Jr., W. M. Preston, and T. W. Bonner, Phys. Rev. **74**, 1645 (1948).
[2] H. R. Bowman, S. G. Thompson, J. C. D. Milton, and W. J. Swiatecki, Phys. Rev. **126**, 2120 (1962); H. R. Bowman, J. C. D. Milton, S. G. Thompson, and W. J. Swiatecki, Phys. Rev. **129**, 2133 (1963).
[3] K. Skarsvag and K. Bergheim, Nucl. Phys. **45**, 72 (1963).
[4] S. S. Kapoor, R. Ramanna, and P. N. Rama Rao, Phys. Rev. **131**, 283 (1983).
[5] M. V. Bilnov, N. M. Kazarinov, and I. T. Krisyuk, Yad. Fiz. **16**, 1155 (1973) [Sov. J. Nucl. Phys. **16**, 634 (1973)].
[6] K. Skarsvag, Phys. Scr. **7**, 160 (1973).
[7] A. M. Gagarski *et al.*, Bull. Russ. Acad. Sci.: Phys. **72**, 773 (2008).
[8] R. Vogt, J. Randrup, J. Pruet, and W. Younes, Phys. Rev. C **80**, 044611 (2009).
[9] J. Randrup and R. Vogt, Phys. Rev. C **80**, 024601 (2009).
[10] R. Vogt, J. Randrup, D. A. Brown, M. A. Descalle, and W. E. Ormand, Phys. Rev. C **85**, 024608 (2012).
[11] C. B. Franklyn, C. Hofmeyer, and D. W. Mingay, Phys. Lett. B **78**, 564 (1978).
[12] J. S. Pringle and F. D. Brooks, Phys. Rev. Lett. **35**, 1563 (1975).
[13] S. A. Pozzi *et al.*, Nucl. Sci. Eng. **178**, 250 (2014).
[14] G. Audi and A. H. Wapstra, Nucl. Phys. A **595**, 409 (1995).
[15] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[16] T. Kawano (private communication).
[17] S. Lemaire, P. Talou, T. Kawano, M. B. Chadwick, and D. G. Madland, Phys. Rev. C **72**, 024601 (2005).
[18] H. Koura, M. Uno, T. Tachibana, and M. Yamada, Nucl. Phys. A **674**, 47 (2000).
[19] R. Vogt and J. Randrup, Phys. Rev. C **84**, 044621 (2011).
[20] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).
[21] J. Terrell, Phys. Rev. **113**, 527 (1959).
[22] D. G. Madland and J. R. Nix, Nucl. Sci. Eng. **81**, 213 (1982).
[23] J. Randrup and R. Vogt, Phys. Rev. C **89**, 044601 (2014).
[24] R. Vogt and J. Randrup, Phys. Rev. C **87**, 044602 (2013).
[25] S. Shengyao, X. Jincheng, L. Zuhua, Z. Qixin, L. Shaoming and Z. Huanqiao, Chin. Phys. **5**, 678 (1984).
[26] V. N. Dushin *et al.*, Nucl. Instrum. Methods A **516**, 539 (2004).
[27] V. P. Zakharova, D. K. Ryazanov, B. G. Basova, A. D. Rabinovich, and V. A. Korostylev, Yad. Fiz. **30**, 36 (1979) [Sov. J. Nucl. Phys. **30**, 19 (1979)].
[28] P. Talou, B. Becker, T. Kawano, M. B. Chadwick, and Y. Danon, Phys. Rev. C **83**, 064612 (2011).
[29] O. Litaize and O. Serot, Phys. Rev. C **82**, 054616 (2010).
[30] G. Vladuca and A. Tudora, Ann. Nucl. Energy **28**, 419 (2001); A. Tudora, *ibid.* **36**, 72 (2009).
[31] A. S. Vorobiev *et al.*, http://www-nds.iaea.org/exfor/servlet/X4sGetSubent?subID=41425004
[32] J. P. Lestone, Los Alamos Report LA-UR-99-5444, 1999.
[33] J. Randrup and R. Vogt, JRC Sci. and Tech. Rep. **64789**, 115 (2011).