Internal bremsstrahlung of β decay of atomic $^{35}_{16}$S

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We calculate the energy spectra and the branching ratio of the internal bremsstrahlung (IB) of the β^- decay of atomic $^{35}_{16}$S. We show that the theoretical spectrum of the IB, calculated within the standard model and QED, fits well the experimental spectra, measured by M. S. Powar and M. Singh [J. Phys. G: Nucl. Part. Phys. **2**, 43 (1976)] and by A. Singh and A. S. Dhaliwal [Appl. Radiat. Isot. **94**, 44 (2014)] for the photon energy regions $0.025 \leqslant \omega \leqslant 0.150$ MeV and $0.001 \leqslant \omega \leqslant 0.100$ MeV, respectively.

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I. INTRODUCTION

A theory of internal bremsstrahlung (IB), i.e., photon emission accompanying the nuclear β decays, has been formulated in the papers by Knipp and Uhlenbeck [1] and Bloch [2] and developed in Refs. [3–10]. In this paper we apply the theory of the IB to the calculation of the IB branching ratio and energy spectrum, produced in the β^- decay of atomic $^{35}_{16}$S [11]. The nucleus $^{35}_{16}$S of the atomic $^{35}_{16}$S in the ground state, characterized by the spin and parity $J^\pi = \frac{3}{2}^+$, is unstable under the β^--decay $^{35}_{16}\mathrm{S} \to {}^{35}_{17}\mathrm{Cl} + e^- + \bar{\nu}_e$ with the half-life $T_{1/2} = 87.51$ d [12]. Since the nucleus $^{35}_{17}$Cl of atomic $^{35}_{17}$Cl is characterized by the spin and parity $J^\pi = \frac{3}{2}^+$, the β^--decay $^{35}_{16}\mathrm{S} \to {}^{35}_{17}\mathrm{Cl} + e^- + \bar{\nu}_e$ is the pure Fermi transition [13]. The end-point energy of the electron spectrum and Q value of the β^- decay of $^{35}_{16}$S are equal to $E_0 = 0.67818(12)$ MeV and $Q_{\beta^-_c} = 0.16718(12)$ MeV [12], respectively. The IB of the β^- decay of $^{35}_{16}$S assumes the following process $^{35}_{16}\mathrm{S} \to {}^{35}_{17}\mathrm{Cl} + e^- + \bar{\nu}_e + \gamma$.

According to [1–10], photons in the reaction $^{35}_{16}\mathrm{S} \to {}^{35}_{17}\mathrm{Cl} + e^- + \bar{\nu}_e + \gamma$ should be emitted by the decay electron. In comparison with the theoretical analysis of the IB, developed in [3–10], we calculate in addition the recoil corrections of the daughter ion. For the calculation of the Coulomb corrections, caused by a distortion of the wave function of the decay electron in the Coulomb field of the daughter ion, we use the electron wave functions, obtained by Jackson, Treiman, and Wyld [14].

The experimental analysis of the IB from the β^- decay of ^{35}S has been carried out in Refs. [15–20]. We compare our theoretical results with the experimental data by Powar and Singh [16], obtained for the photon energy region $0.025 \leqslant \omega \leqslant 0.150$ MeV, since these are the most accurate data obtained in the last century, and with the most recent experimental data by Singh and Dhaliwal [20], investigated in the reaction $^{35}_{16}\mathrm{S} \to {}^{35}_{17}\mathrm{Cl} + e^- + \bar{\nu}_e + \gamma$ for the photon energy region $0.001 \leqslant \omega \leqslant 0.100$ MeV.

The paper is organized as follows. In Sec. II we analyze the IB in the neutron β^- decay. In Sec. III we give the photon-electron energy and photon-energy spectra and angular distributions of the IB, calculated within quantum electrodynamics (QED) and the standard model, for the β^- decay of atomic $^{35}_{16}$S. In the Sec. IV we discuss the obtained results and compare them with the experimental data.

II. RADIATIVE β^- DECAY OF THE NEUTRON

According to classical electrodynamics [21], the number of photons $dN(\omega,E_e)$ with energies from the interval $(\omega + d\omega,\omega)$, emitted by the decay electrons instantly accelerated from velocity zero to velocity β and energy E_e in the neutron β^- decay, is equal to

$$dN(\omega,E_e) = \frac{\alpha}{\pi}\left[\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right]\frac{d\omega}{\omega}, \qquad (1)$$

where $\alpha = 1/137.036$ is the fine-structure constant [22].

The rate of the neutron β^- decay with the IB, calculated within the standard model (SM), can be taken in the following form [23]:

$$\lambda_{\beta^-_c\gamma}(\omega_{\max},\omega_{\min}) = (1+3\lambda^2)\frac{G_F^2|V_{ud}|^2}{2\pi^3}\int_{\omega_{\min}}^{\omega_{\max}}\int_{m_e}^{E_0-\omega} dN(\omega,E_e)\,dE_e \times F(E_e,Z=1)\,(E_0-E_e-\omega)^2\sqrt{E_e^2-m_e^2}\,E_e, \qquad (2)$$

where $dN(\omega,E_e)$ is the number of photons, emitted into the energy interval $\omega_{\min} \leqslant \omega \leqslant \omega_{\max}$, $G_F = 1.1664 \times 10^{-11}\,\mathrm{MeV}^{-2}$ is the Fermi coupling constant, $V_{ud} = 0.97427(15)$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element [22], $\lambda = -1.2750(9)$ is the axial coupling constant [24] (see also [23]), and $F(E_e,Z)$ is the relativistic Fermi function [13,14] (see also [23])

$$F(E_e,Z) = \left(1+\frac{1}{2}\gamma\right)\frac{4(2r_pm_e\beta)^{2\gamma}}{\Gamma^2(3+2\gamma)}\frac{e^{\pi\alpha Z/\beta}}{(1-\beta^2)^\gamma}\left|\Gamma\left(1+\gamma+i\,\frac{\alpha Z}{\beta}\right)\right|^2, \qquad (3)$$

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where $\gamma = \sqrt{1-\alpha^2 Z^2} - 1$, $r_p = 0.841$ fm is the decay proton electric radius [25] and $\Gamma(x)$ is the Euler Γ-function. The Fermi function describes the final-state electron-proton interaction. Then, $E_0 = (m_n^2 - m_p^2 + m_e^2)/2m_n = 1.2927$ MeV is the end-point energy of the electron spectrum of the β^- decay of the neutron [23].

A consistent quantum field theoretic calculation (the QED calculation) of the number of photons, emitted into the energy interval $(\omega + d\omega, \omega)$, gives [23]

$$dN(\omega, E_e) = \frac{\alpha}{\pi}\left\{\left(1 + \frac{\omega}{E_e} + \frac{1}{2}\frac{\omega^2}{E_e^2}\right) \times \left[\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right] + \frac{\omega^2}{E_e^2}\right\}\frac{d\omega}{\omega}. \quad (4)$$

The term in the curly brackets independent of the photon energy ω can be explained by the classical model of the IB [see Eq. (1)], whereas the terms, proportional to ω/E_e and ω^2/E_e^2, are the QED corrections. The IB energy spectrum of the radiative β^- decay of the neutron is

$$\begin{aligned}\frac{d\mathrm{BR}_{\beta_c^-\gamma}(\omega)}{d\omega} &= \frac{C_n}{m_e^5}\int_{m_e}^{E_0-\omega} m_e \frac{dN(\omega, E_e)}{d\omega}\, dE_e\, F(E_e, Z=1) \\ &\quad \times (E_0 - E_e - \omega)^2\sqrt{E_e^2 - m_e^2}\, E_e \\ &= \frac{C_n}{m_e^5}\int_{m_e}^{E_0-\omega} dE_e\, F(E_e, Z=1) \\ &\quad \times (E_0 - E_e - \omega)^2\sqrt{E_e^2 - m_e^2}\, E_e \\ &\quad \times \frac{\alpha}{\pi}\frac{1}{\omega}\left\{\left(1 + \frac{\omega}{E_e} + \frac{1}{2}\frac{\omega^2}{E_e^2}\right)\right. \\ &\quad \left.\times \left[\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right] + \frac{\omega^2}{E_e^2}\right\}, \end{aligned} \quad (5)$$

where $\mathrm{BR}_{\beta_c^-\gamma}(\omega)$ is the branching ratio. Then, $C_n = \tau_n(1 + 3\lambda^2)G_F^2|V_{ud}|^2 m_e^5/2\pi^3 = 0.56985$ is the dimensionless coefficient, calculated for $\tau_n = 879.6$ s and $\lambda = -1.2750$ [23]. One may show that in the photon energy region $\omega_{\min} = 0.001 \leqslant \omega \leqslant \omega_{\max} = 0.015$ MeV, the contribution of the QED corrections makes up only 0.6 % of the contribution, caused by the classical mechanism of photon emission.

III. INTERNAL BREMSSTRAHLUNG IN β^- DECAY OF $^{35}_{16}$S

Now let us discuss the energy spectra of the IB of the β^- decay of $^{35}_{16}$S. Following the procedure, proposed in [26–28] for the investigation of weak decays of heavy ions, we obtain the rate of the β^- decay of $^{35}_{16}$S:

$$\lambda_{\beta^-}\left(^{35}_{16}\mathrm{S}\right) = \frac{|\mathcal{M}_F|^2}{2\pi^3}\int_{m_e}^{E_0} dE_e\, F(E_e, Z=17) \times (E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}\, E_e. \quad (6)$$

The nuclear matrix element $\mathcal{M}_F$ of the transition $^{35}_{16}\mathrm{S} \to {}^{35}_{17}\mathrm{Cl}$ is determined by [26–28]

$$\mathcal{M}_F = -\frac{G_F}{\sqrt{2}} V_{ud}\int d^3x\, \Psi_d^*(\vec{r}\,)\Psi_m(\vec{r}\,), \quad (7)$$

where $\Psi_m(\vec{r}\,)$ and $\Psi_d(\vec{r}\,)$ are the wave functions of the mother $^{35}_{16}$S and daughter $^{35}_{17}$Cl nuclei, respectively. Since in the β^- decay of $^{35}_{16}$S the decay electron is sufficiently fast (e.g., the end-point energy velocity is of about $\beta \sim 0.7$, which is commensurable with the end-point energy velocity of the decay electron in the neutron β^--decay $\beta \sim 0.9$), we will use the relativistic Fermi function $F(E_e, Z=17)$, given by Eq. (3) with the *root mean square* radius of $^{35}_{17}$Cl equal to $R_d = 3.439(5)$ fm [29].

Following [23] for the rate of the IB of the β^- decay of $^{35}_{16}$S with photons, emitted in the energy interval $\omega_{\min} \leqslant \omega \leqslant \omega_{\max}$, we obtain the expression

$$\lambda_{\beta^-\gamma}\left(^{35}_{16}\mathrm{S}\right)^{\omega_{\max}}_{\omega_{\min}} = \frac{|\mathcal{M}_F|^2}{2\pi^3}\int_{\omega_{\min}}^{\omega_{\max}}\int_{m_e}^{E_0-\omega} dN^{(\mathrm{QED})}(\omega, E_e)\, dE_e \times F(E_e, Z=17)(E_0 - E_e - \omega)^2 \times \sqrt{E_e^2 - m_e^2}\, E_e. \quad (8)$$

The branching ratio of the IB of the β^- decay of ^{35}S is given by

$$\mathrm{BR}^{(\mathrm{QED})}_{\beta^-\gamma}\left(^{35}_{16}\mathrm{S}\right)^{\omega_{\max}}_{\omega_{\min}} = \frac{\int_{\omega_{\min}}^{\omega_{\max}}\frac{d\omega}{\omega}\int_{m_e}^{E_0-\omega} dE_e\, dN^{(\mathrm{QED})}(\omega, E_e)\, F(E_e, Z=17)(E_0 - E_e - \omega)^2\sqrt{E_e^2 - m_e^2}\, E_e^2}{\int_{m_e}^{E_0} dE_e\, F(E_e, Z=17)(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}\, E_e m_e}. \quad (9)$$

For the photon-electron $X^{(\mathrm{QED})}(\omega, E_e, \theta_{e\gamma})$ and photon energy $Y^{(\mathrm{QED})}(\omega, \theta_{e\gamma})$ spectra and angular distributions of the IB in the β^- decay of $^{35}_{16}$S, we obtain the following expressions:

$$X^{(\mathrm{QED})}(\omega, E_e; \theta_{e\gamma}) = 4\pi\frac{d^3\mathrm{BR}^{(\mathrm{QED})}_{\beta^-\gamma}\left(^{35}_{16}\mathrm{S}\right)}{d\omega dE_e d\Omega_{e\gamma}} = 4\pi\frac{d^2N^{(\mathrm{QED})}(\omega, E_e, \theta_{e\gamma})}{d\omega d\Omega_{e\gamma}}\frac{F(E_e, Z=17)(E_0 - E_e - \omega)^2\sqrt{E_e^2 - m_e^2}\, E_e}{\int_{m_e}^{E_0} dE_e\, F(E_e, Z=17)(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}\, E_e} \quad (10)$$

and

$$Y^{(\mathrm{QED})}(\omega; \theta_{e\gamma}) = 4\pi\frac{d^2\mathrm{BR}^{(\mathrm{QED})}_{\beta^-\gamma}\left(^{35}_{16}\mathrm{S}\right)}{d\omega d\Omega_{e\gamma}} = \int_{m_e}^{E_0-\omega} dE_e\, 4\pi\frac{d^2N^{(\mathrm{QED})}(\omega, E_e, \theta_{e\gamma})}{d\omega d\Omega_{e\gamma}}\frac{F(E_e, Z=17)(E_0 - E_e - \omega)^2\sqrt{E_e^2 - m_e^2}\, E_e}{\int_{m_e}^{E_0} dE_e\, F(E_e, Z=17)(E_0 - E_e)^2\sqrt{E_e^2 - m_e^2}\, E_e}, \quad (11)$$

where we have denoted

$$4\pi\frac{d^2N^{(\rm QED)}(\omega,E_e,\theta_{e\gamma})}{d\omega d\Omega_{e\gamma}}=\frac{\alpha}{\pi}\frac{1}{\omega}\frac{1}{(1-\vec{n}\cdot\vec{\beta}\,)^2}\Big\{\Big[\Big(\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2\Big)\Big(1+\frac{\omega}{E_e}\Big)+(1-\vec{n}\cdot\vec{\beta}\,)\frac{\omega^2}{E_e^2}\Big]+\alpha^2Z^2\frac{m_e^2}{E_e^2}\Big(1+\frac{\omega}{E_e}\Big)$$
$$\times\Big[1+\frac{(\vec{n}\cdot\vec{\beta}\,)}{\beta^2\sqrt{1+\frac{2}{\beta^2}\frac{\omega}{E_e}(\vec{n}\cdot\vec{\beta}\,)+\frac{1}{\beta^2}\frac{\omega^2}{E_e^2}}}\Big(\vec{n}\cdot\vec{\beta}+\frac{\omega}{E_e}\Big)\Big]+3\frac{E_e}{M_d}\Big\{1-(\vec{n}\cdot\vec{\beta}\,)\frac{\omega}{E}\Big[\frac{2}{3}\theta(E-\omega)$$
$$+\frac{E}{\omega}\Big(1-\frac{1}{3}\frac{E^2}{\omega^2}\Big)\theta(\omega-E)\Big]\Big\}\Big[\Big(\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2\Big)\Big(1+\frac{\omega}{E_e}\Big)+(1-\vec{n}\cdot\vec{\beta}\,)\frac{\omega^2}{E_e^2}\Big]$$
$$-\frac{E_e}{M_d}\Big\{\Big[\Big(1-\frac{1}{5}\frac{\omega^2}{E^2}\Big)\theta(E-\omega)+\frac{E}{\omega}\Big(1-\frac{1}{5}\frac{E^2}{\omega^2}\Big)\theta(\omega-E)\Big]\Big[\Big(\beta^2+(\vec{n}\cdot\vec{\beta}\,)\frac{\omega}{E_e}\Big)(\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2)$$
$$+(\vec{n}\cdot\vec{\beta}\,)(1-\vec{n}\cdot\vec{\beta}\,)\frac{\omega}{E_e}-(\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2)(1-\vec{n}\cdot\vec{\beta}\,)\frac{\omega^2}{E_e^2}\Big]-\frac{2}{5}\frac{\omega^2}{E^2}\Big[\theta(E-\omega)+\frac{5}{2}\frac{E}{\omega}\Big(1-\frac{3}{5}\frac{E^4}{\omega^4}\Big)$$
$$\times\theta(\omega-E)\Big]\Big[(\vec{n}\cdot\vec{\beta}\,)\Big(\vec{n}\cdot\vec{\beta}+\frac{\omega}{E_e}\Big)(\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2)+(\vec{n}\cdot\vec{\beta}\,)(1-\vec{n}\cdot\vec{\beta}\,)\frac{\omega}{E_e}\Big]\Big\}\Big\}. \quad (12)$$

Here $\vec{n}$ is a unit vector in the direction of the photon momentum, $\vec{n}\cdot\vec{\beta}=\beta\cos\theta_{e\gamma}$, and β is the decay electron velocity, $d\Omega_{e\gamma}=2\pi\sin\theta_{e\gamma}d\theta_{e\gamma}$ is the infinitesimal element of the solid angle, and $\theta(E-\omega)$ and $\theta(\omega-E)$ are the Heaviside functions, where $E=E_0-E_e-\omega$ is the electron antineutrino energy. The mass M_d of the daughter ion $^{35}_{17}$Cl is equal to $M_d=32573.276$ MeV [12].

For the calculation of $d^2N^{(\rm QED)}(\omega,E_e,\theta_{e\gamma})/d\omega d\Omega_{e\gamma}$, we have followed Felsner [8]. The electron bispinor wave function $u_e(\vec{k}_e,\sigma_e)$ is taken in the form accounting for the Coulomb distortion caused by the Coulomb interaction of the decay electron with the daughter nucleus [14] (see also [13] and [7]). It reads

$$u_e(\vec{k}_e,\sigma_e)=\sqrt{E_e+m_e(1+\gamma)}\begin{pmatrix}1\\ \big(1+i\,\frac{\alpha Zm_e}{k_e}\big)\frac{\vec{\sigma}\cdot\vec{k}_e}{E_e+m_e(1+\gamma)}\end{pmatrix}\otimes\varphi_{\sigma_e}, \quad (13)$$

where $\vec{k}_e$, $\sigma_e=\pm1$ and φ_{σ_e} are the 3-momentum, the polarization and the Pauli spinor of the decay electron, respectively. The electron wave function Eq. (13) satisfies the Dirac equation

$$\Big(\hat{k}_e-m_e(1+\gamma)+i\frac{\alpha Zm_e}{k_e}\gamma^0\vec{\gamma}\cdot\vec{k}_e\Big)u_e(\vec{k}_e,\sigma_e)=0, \quad (14)$$

and it is normalized by $\bar{u}_e(\vec{k}_e,\sigma'_e)u_e(\vec{k}_e,\sigma_e)=2m_e(1+\gamma)\,\delta_{\sigma'_e\sigma_e}$. In addition we have taken into account the contributions of the daughter nucleus recoil, that was not calculated in the earlier theoretical works on the internal bremsstrahlung of the nuclear β decays [1–9].

The photon energy spectrum of the QED IB in the β^- decay of $^{35}_{16}$S takes the form

$$S^{(\rm QED)}(\omega)=\frac{d{\rm BR}^{(\rm QED)}_{\beta^-\gamma}(^{35}_{16}{\rm S})}{d\omega}=\int^{E_0-\omega}_{m_e}dE_e\,\frac{dN^{(\rm QED)}(\omega,E_e)}{d\omega}\frac{F(E_e,Z=17)(E_0-E_e-\omega)^2\sqrt{E_e^2-m_e^2}\,E_e}{\int^{E_0}_{m_e}dE_e\,F(E_e,Z=17)(E_0-E_e)^2\sqrt{E_e^2-m_e^2}\,E_e}, \quad (15)$$

where $dN^{(\rm QED)}(\omega,E_e)/d\omega$ is given by

$$\frac{dN^{(\rm QED)}(\omega,E_e)}{d\omega}=\frac{\alpha}{\pi}\frac{1}{\omega}\Big\{\Big[\Big(1+\frac{\omega}{E_e}+\frac{1}{2}\frac{\omega^2}{E_e^2}\Big)\Big[\frac{1}{\beta}\ln\Big(\frac{1+\beta}{1-\beta}\Big)-2\Big]+\frac{\omega^2}{E_e^2}\Big]+\alpha^2Z^2\frac{m_e^2}{E_e^2}\Big(1+\frac{\omega}{E_e}\Big)\int\frac{d\Omega_{e\gamma}}{4\pi}\frac{1}{(1-\vec{n}\cdot\vec{\beta}\,)^2}$$
$$\times\Big[1+\frac{(\vec{n}\cdot\vec{\beta}\,)}{\beta^2\sqrt{1+\frac{2}{\beta^2}\frac{\omega}{E_e}(\vec{n}\cdot\vec{\beta}\,)+\frac{1}{\beta^2}\frac{\omega^2}{E_e^2}}}\Big(\vec{n}\cdot\vec{\beta}+\frac{\omega}{E_e}\Big)\Big]+3\frac{E_e}{M_d}\int\frac{d\Omega_{e\gamma}}{4\pi}\Big\{1-(\vec{n}\cdot\vec{\beta}\,)\frac{\omega}{E}\Big[\frac{2}{3}\theta(E-\omega)$$
$$+\frac{E}{\omega}\Big(1-\frac{1}{3}\frac{E^2}{\omega^2}\Big)\theta(\omega-E)\Big]\Big\}\Big[\frac{\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2}{(1-\vec{n}\cdot\vec{\beta}\,)^2}\Big(1+\frac{\omega}{E_e}\Big)+\frac{1}{1-\vec{n}\cdot\vec{\beta}}\frac{\omega^2}{E_e^2}\Big]$$
$$-\frac{E_e}{M_d}\int\frac{d\Omega_{e\gamma}}{4\pi}\Big\{\Big[\Big(1-\frac{1}{5}\frac{\omega^2}{E^2}\Big)\theta(E-\omega)+\frac{E}{\omega}\Big(1-\frac{1}{5}\frac{E^2}{\omega^2}\Big)\theta(\omega-E)\Big]$$
$$\times\Big[\Big(\beta^2+(\vec{n}\cdot\vec{\beta}\,)\frac{\omega}{E_e}\Big)\frac{\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2}{(1-\vec{n}\cdot\vec{\beta}\,)^2}+\frac{\vec{n}\cdot\vec{\beta}}{1-\vec{n}\cdot\vec{\beta}}\frac{\omega}{E_e}-\frac{\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2}{1-\vec{n}\cdot\vec{\beta}}\frac{\omega^2}{E_e^2}\Big]-\frac{2}{5}\frac{\omega^2}{E^2}\Big[\theta(E-\omega)+\frac{5}{2}\frac{E}{\omega}$$
$$\times\Big(1-\frac{3}{5}\frac{E^4}{\omega^4}\Big)\theta(\omega-E)\Big]\Big[(\vec{n}\cdot\vec{\beta}\,)\Big(\vec{n}\cdot\vec{\beta}+\frac{\omega}{E_e}\Big)\frac{\beta^2-(\vec{n}\cdot\vec{\beta}\,)^2}{(1-\vec{n}\cdot\vec{\beta}\,)^2}+\frac{\vec{n}\cdot\vec{\beta}}{1-\vec{n}\cdot\vec{\beta}}\frac{\omega}{E_e}\Big]\Big\}\Big\}. \quad (16)$$

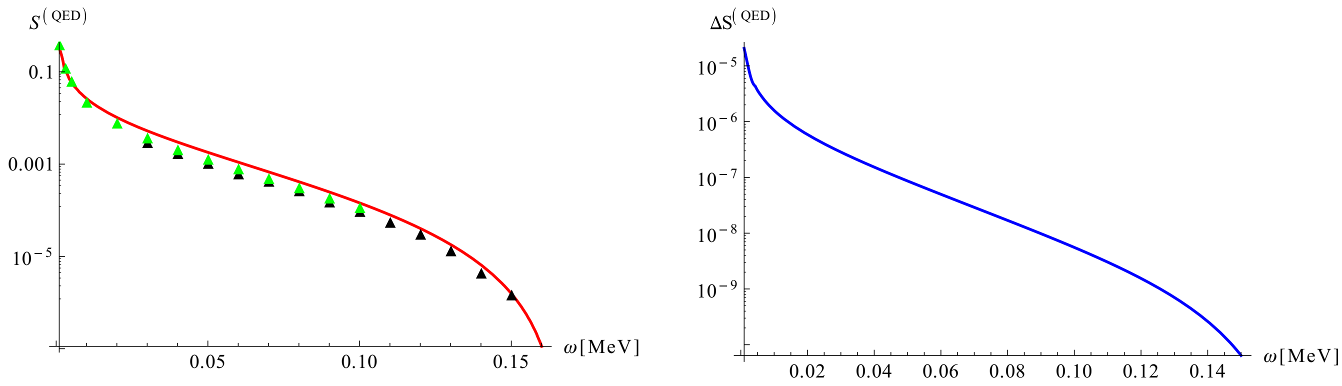

FIG. 1. (Color online) The photon energy spectrum of the QED internal bremsstrahlung for the photon energy region $\omega_{\min} = 0.001 \leqslant \omega \leqslant \omega_{\max} = E_0 - m_e = 0.16718\,\text{MeV}$ with the account for the daughter ion recoil (left). The difference ΔS between the photon energy spectra, calculated with and without the daughter ion recoil (right). The theoretical curve (left) is compared with the experimental data by Powar and Singh [16], who measured the photon energy spectrum for the photon energies $\omega_{\min} = 0.025 \leqslant \omega \leqslant \omega_{\max} = 0.150\,\text{MeV}$ [black (dark) triangles], and by Singh and Dhaliwal [20], who measured the photon energy spectrum for the photon energies $\omega_{\min} = 0.001 \leqslant \omega \leqslant \omega_{\max} = 0.100\,\text{MeV}$ [green (light) triangles].

The expression in the first brackets coincides with the number of photons, emitted by the decay electron in the neutron β^- decay, where the term independent of the photon energy is determined by the classical model of the IB [21]. Then, the terms, proportional to $\alpha^2 Z^2$, are caused by the quantum effect of the distortion of the wave function of the decay electron by the Coulomb field of the daughter nucleus, and the contributions, proportional to $1/M_d$, are induced by the recoil of the daughter nucleus.

IV. CONCLUSIVE DISCUSSION

We have proposed the theoretical analysis of the internal bremsstrahlung (IB) in nuclear β decays. For the calculation of the energy spectra, angular distributions, decay rates, and the branching ratios, we have extended the classical electrodynamical analysis of the nuclear β decays, given by Jackson [21], to the quantum electrodynamical one. According to Jackson [21], one may calculate the number of photons $dN(E_e,\omega)$, emitted in the photon energy region $(\omega,\omega + d\omega)$ by instantly accelerated decay electrons from zero velocity to the velocity β, and insert $dN(E_e,\omega)$ into the electron energy spectrum of the nuclear β decay. By example of the IB of the neutron β^- decay we have calculated the number of photons $dN(E_e,\omega)$, emitted by the decay electrons, and shown that such a number of photons, inserted into the electron energy spectrum of the neutron β^- decay, describes fully the photon-electron energy spectrum of the radiative β^- decay of the neutron.

For the quantum field theoretic calculation (the QED calculation) of the number of photons $dN(E_e,\omega)$, emitted by the decay electrons in the nuclear β decays, we have accepted the approach, based on the use of the Coulomb wave functions of the decay electrons calculated by Jackson, Treiman, and Wyld [14] and used, for the first time, by Felsner [8] for the calculation of the IB in the nuclear β decays. In addition to the leading order contributions in the large daughter nucleus mass M_d expansion we have calculated the next-to-leading terms of order $1/M_d$, caused by the daughter nucleus recoil.

For the illustration of such a theoretical approach we have chosen the atomic $^{35}_{16}\text{S}$ [11], which is unstable under the allowed β^- decay. Such a choice can be also justified by the existence of the well-measured photon energy spectrum, the measurement of which was carried out by Powar and Singh [16] for the photon energies $\omega_{\min} = 0.025 \leqslant \omega \leqslant \omega_{\max} = 0.150\,\text{MeV}$ and recently by Singh and Dhaliwal [20] for the photon energies $\omega_{\min} = 0.001 \leqslant \omega \leqslant \omega_{\max} = 0.100\,\text{MeV}$. For the subsequent experimental analyzes of the IB of the β^- decay of atomic $^{35}_{16}\text{S}$ we have calculated the photon-electron energy and the photon energy spectra and angular distributions.

For the verification of the consistency of our theoretical results for the QED IB of the β^- decay of $^{35}_{16}\text{S}$, we have compared the QED IB spectrum, calculated for the photon energies $\omega_{\min} = 0.001 \leqslant \omega \leqslant \omega_{\max} = 0.16718\,\text{MeV}$, with the experimental spectra (see Fig. 1), measured by Powar and Singh [16] and Singh and Dhaliwal [20]. In Fig. 1 (left) we plot the theoretical curve, defined by Eq. (15), and the experimental data by Powar and Singh [16] [black (dark) triangles] and by Singh and Dhaliwal [20] [green (light) triangles]. One may see that the agreement of the theoretical spectrum with the experimental data is sufficiently good. However, one may see that in the photon energy region $0.030 \leqslant \omega \leqslant 0.140\,\text{MeV}$ the theoretical spectrum is a little bit large compared with the experimental data. In Fig. 1 (right) we plot the difference between the theoretical spectra, calculated with and without the daughter nucleus recoil. The smallness of such a difference is expected because of the large value of M_d and the relatively small energies of the decay electron. Of course, as for the observed small deviation of the theoretical spectrum from the experimental ones and the possible contributions of the external bremsstrahlung, i.e., an additional emission of photons accompanying the nuclear β decays, caused by an acceleration of the decay electrons and the electron atomic shell, we are planning to investigate them in our forthcoming publication. We would like to mention that the theory of the external bremsstrahlung has been developed by Bethe and Heitler [30,31] and Sauter [32].

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