Regularities of low-lying states with random interactions in the fermion dynamical symmetry model

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In this paper we study low-lying states under random interactions in the framework of the fermion dynamical symmetry model (FDSM), regardless of the ground state spin. Very strong correlations are found for R_6 versus R_4 (where $R_I \equiv E_{I_1^+}/E_{2_1^+}$) for *the entire ensemble*. We present arguments on the origin of these regular patterns in terms of the dynamical symmetries of the FDSM. The regular patterns of $B(E2;4_1^+ \longrightarrow 2_1^+)$ versus $B(E2;2_1^+ \longrightarrow 0_1^+)$ are found.

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An even-even nucleus always has spin-zero ground state (0 g.s.) without any exception. The origin of this phenomenon has long been recognized to be a reflection of the strong pairing interaction between like nucleons. Therefore the discovery of Johnson, Bertsch, and Dean in 1997 for the 0 g.s. dominance under two-body random ensemble (TBRE) [1] sparked off a sudden focus on its microscopic origin in nuclear structure theory [2–4].

In addition to studies of the 0 g.s. dominance, there have been many investigations of collective motions in atomic nuclei with random interactions, under the requirement of the spin-zero ground states. Among these studies, here we mention vibrational and rotational motions of sd bosons [5], collective motions of nucleons under random interactions with mixture of "reasonably" strong quadrupole-quadrupole interaction [6], the vibrational and rotational motions of SP(6) or SO(8) symmetric SD pairs [7] in the fermion dynamical symmetry model (FDSM) [8,9], an analysis of the Mallmann plot [10] [namely a plot of (R_I, R_4) [11], where $R_I \equiv E_{I_1^+}/E_{2_1^+}$], similar discussions (but goes one step further) for the sd bosons [12] and for the FDSM [13]. This work goes one more step further along the same line of Ref. [13] in the FDSM, *but without the requirement of the spin-zero ground states under random interactions*.

The FDSM [8,9] is an SD-pair truncated shell model, in which the building blocks (SD pairs) are constructed in the so-called k-i basis. The SD pairs and the Hamiltonian generators obey the SO(8) or the SP(6) symmetry. The FDSM Hamiltonian is as follows:

$$H = G_0 S^\dagger S + G_2 D^\dagger \cdot D + \sum_{r=1}^{2\text{ or }3} B_r P^{(r)} \cdot P^{(r)}, \qquad (1)$$

where $S^\dagger$ and $D^\dagger$ are creation operators of spin-0 and spin-2 pairs; $P^{(r)}$ is the multipole operator with r ranging 1–2 in the case of the SP(6) symmetry and 1–3 in the case of the SO(8) symmetry. G_0, G_2, and B_r are taken to be random values following the Gaussian distribution with average 0 and width 1.

In this paper we study random systems which follow the SO(8) symmetry or the SP(6) symmetry of the FDSM, with pair number $N = 3$–5. In each case we diagonalize 5000 sets of the random Hamiltonians and calculate their yrast state energies for both the cases with spin-zero ground states and those with spin-nonzero ground states. $E_{I_1^+}$ is defined to be the energy of the I_1^+ state (the subscript "1" of I means the yrast state with spin I) subtracted by that of the 0_1^+ state, therefore it is always positive because the ground state has spin 0 in previous studies [5–13]. Here for cases with nonzero g.s. spin, we take the same definition of $E_{I_1^+}$ although it might be negative.

We present the Mallmann plot (R_6 versus R_4) for the random ensemble with spin-zero ground states in Figs. 1(a)–1(f) and that for the random ensemble with spin-nonzero ground states in Figs. 1(a′)–1(f′), under the SO(8) symmetry or the SP(6) symmetry, with pair number $N = 3,4,5$. The results of panels (a)–(f) have already been obtained in Ref. [13] and here we include them in order to explicitly compare with results of the random ensemble with spin-nonzero ground states. Let us first look at the linear correlations, denoted by α, β, γ, and δ in Fig. 1 of Ref. [13], or Figs. 1(a)–1(f) here. Analytical formulas of correlations α, β, γ, and δ are

$$\alpha: \ R_6 = 3R_4 - 3, \quad \beta: R_6 = \frac{21}{10}R_4,$$
$$\gamma: \ R_6 = \frac{9}{5}R_4 + 1, \quad \delta: R_6 = \frac{18}{7}R_4 - \frac{11}{7},$$

respectively. We shall derive these relations from dynamical symmetry limits of the FDSM later in this paper.

In Figs. 1(a′)–1(f′), one sees that random samples with spin-nonzero ground states may give $R_4 < 0$ and/or $R_6 < 0$. The statistical peaks are found at $R_6 \approx 0$. The correlations α, β, γ, and δ for the cases with spin-zero ground states are the *same* as those for the cases with spin-nonzero ground states, although the statistics of the samplings following the correlations α and γ is much lower. The correlation β in Fig. 1(c′) is more pronounced than that in Fig. 1(c), and the correlation δ in Figs. 1(b′), 1(c′), 1(e′), and 1(f′) is more pronounced than that in Figs. 1(b), 1(c), 1(e), and 1(f). In Figs. 1(a′) and 1(c′) one sees a new correlation (we denote it by ξ), $R_6 = \frac{7}{5}R_4 + 1$, which was not found in the cases with spin-zero ground states [13].

The correlations α, β, γ, δ, and ξ can be obtained by dynamical symmetry (vibrational and rotational) limits in the FDSM. In order to study the connection between them, the

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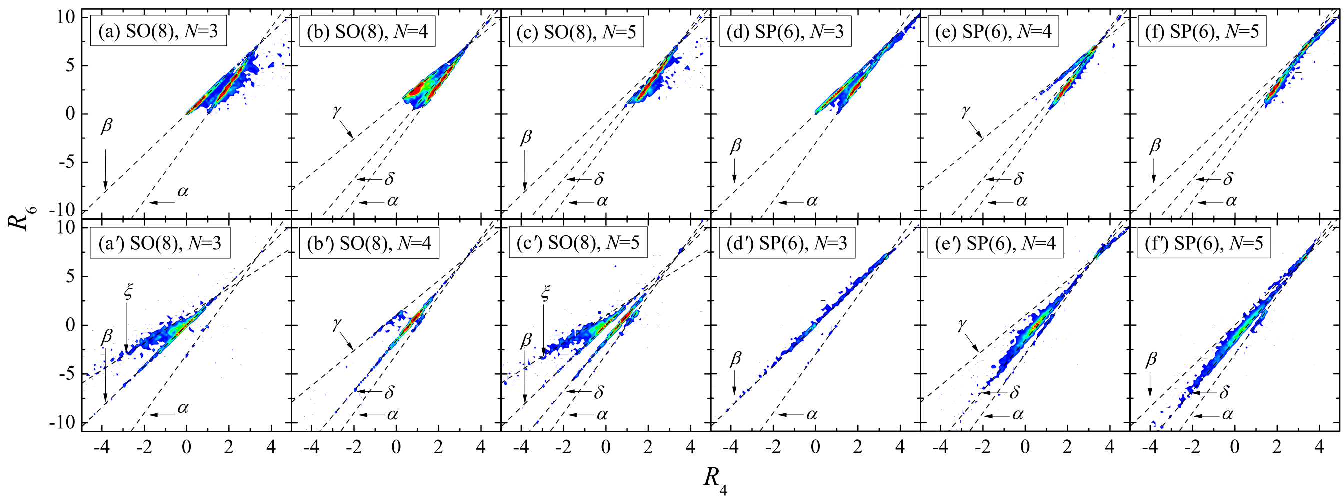

FIG. 1. (Color online) Distribution of (R_6, R_4) of the random Hamiltonian in the FDSM. Panels (a)–(f) are based on random samples with spin-zero ground states, and (a′)–(f′) are based on those with spin-nonzero ground states. Correlations between R_6 and R_4 are labeled by using α, β, γ, δ, and ξ, respectively. Refer to the text for the definitions of these correlations.

Hamiltonian in Eq. (1) for the SO(8) symmetry is rewritten as [9]

$$\begin{aligned} H_{\mathrm{SO}(8)} &= H_0 + \nu_1 \Delta C_{\mathrm{SU}(2)} \\ &\quad + g_5 C_{\mathrm{SO}(5)} + g_6 C_{\mathrm{SO}(6)} + g\hat{I}^2, \\ \nu_1 &= G_0 - G_2, \quad g_5 = B_3 - B_2, \\ g_6 &= B_2 - G_2, \quad g = (B_1 - B_3)/5, \end{aligned} \tag{2}$$

or

$$\begin{aligned} H_{\mathrm{SO}(8)} &= H_0' + \nu_1' \Delta C_{\mathrm{SU}(2)} + g_5' C_{\mathrm{SO}(5)} + g_7' \Delta C_{\mathrm{SO}(7)} + g\hat{I}^2, \\ \nu_1' &= G_0 - B_2, \quad g_5' = B_3 - G_2, \quad g_7' = G_2 - B_2, \end{aligned} \tag{3}$$

where C_{G} is the quadratic Casimir invariant of a group G, and ΔC_{G} is the operator whose eigenvalues are excitation energies with respect to the vacuum. H_0 (H_0') is the rest part of the Hamiltonian, which is equal to a constant for all states of a given system, and is irrelevant to excitation energies of states. The Hamiltonian in Eq. (1) for the SP(6) symmetry is rewritten as [9]

$$\begin{aligned} H_{\mathrm{SP}(6)} &= H_0 + \nu_1 \Delta C_{\mathrm{SU}(2)} + s_3 C_{\mathrm{SU}(3)} + s\hat{I}^2, \\ s_3 &= B_2 - G_2, \quad s = 3(B_1 - B_2)/8. \end{aligned} \tag{4}$$

Let us begin with the vibrational limits. In the FDSM there are totally three dynamical symmetry limits which correspond to anharmonic spherical vibrators. For the SO(8) symmetry, the Hamiltonian, $H_{\mathrm{SO}(8)}$, has the SO(5)×SU(2) limit if $g_7' = 0$ in Eq. (3), and it has also the SO(7) limit if $\nu_1' = 0$. For the SP(6) symmetry, the Hamiltonian $H_{\mathrm{SP}(6)}$ has the SU(2) limit if $s_3 = 0$ in Eq. (4). The energy of I^+ states in these three vibrational limits can be written in a unified form as follows [8,9]:

$$\begin{aligned} E_{\mathrm{vib}} &= \varepsilon N_d + a N_d (N_d - 1)/2 + b(N_d - \tau)(N_d + \tau + 3) \\ &\quad + c[I(I+1) - 6N_d] \\ &= \varepsilon_0 N_d + a N_d (N_d - 1)/2 + b(N_d - \tau)(N_d + \tau + 3) \\ &\quad + cI(I+1). \end{aligned} \tag{5}$$

The definitions of ε, ε_0, a, b, c, N_d, and τ are presented in Table I. The reduction rules for the SO(5)×SU(2) limit of the SO(8) symmetry are written as follows:

$$\begin{aligned} \kappa &= N, N-1, \ldots, 1 \text{ or } 0, \\ \tau &= \kappa, \kappa - 2, \ldots, 1 \text{ or } 0, \\ n_\Delta &= [\tau/3], [\tau/3] - 1, [\tau/3] - 2, \ldots, 0, \\ \lambda &= \tau - 3n_\Delta, \\ I &= \lambda, \lambda + 1, \ldots, 2\lambda - 2, 2\lambda, \end{aligned} \tag{6}$$

where κ is half of the seniority number satisfying $\kappa \leqslant \Omega/2$; Ω is the pair degeneracy which is equal to 10 for the SO(8) symmetry and to 15 for the SP(6) symmetry; τ, n_Δ, and λ are additional quantum numbers. The reduction rules and additional quantum numbers for the SU(2) limit of the SP(6) symmetry and the SO(7) limit of the SO(8) symmetry are the same as those for the SO(5)×SU(2) limit, except for $\kappa \leqslant \Omega/3$ for the former case, and in the latter case κ is replaced by $\bar{\kappa}$, where $\bar{\kappa} = N - w/2$, and w is the number of nucleons that do not form D pairs.

The cases corresponding to the vibrational limits lead to the linear correlations α, β, γ, δ, and ξ. We exemplify this by using the case of $\nu_1 < 0$. In the SO(5)×SU(2), SO(7), and SU(2) limits, according to Table I, Ω and Ω' are large and positive, ε_0 is positive, and $|\varepsilon_0|$ is much larger than $|a|$ and $|b|$ in most of the cases statistically. Thus for a given I, the lowest energy state has $N_d = \tau = I/2$ in all three vibrational limits, and Eq. (5) gives

$$E_{\mathrm{vib}} = \varepsilon_0 I/2 + aI(I-2)/8 + cI(I+1),$$

which leads to

$$R_I = R_4 I(I-2)/8 - I(I-4)/4.$$

This gives the correlation α in Fig. 1 where $R_6 = 3R_4 - 3$.

By the same procedures, correlations β, γ, and δ can be derived from the SO(5)×SU(2) and SO(7) limits with $\nu_1 > 0$: For $N = 3$, one obtains the correlation β; for $N = 4$, one has

TABLE I. Definitions of ε, ε_0, a, b, c, N_d, and τ for vibrational limits in the FDSM [9]. The parameters ε, ε_0, a, b, and c are written in terms of Hamiltonian parameters in Eqs. (2)–(4). N_d and τ are quantum numbers in vibrational limits, and the reduction rules are presented in Eq. (6).

Parameter	SO(5)×SU(2)	SO(7)	SU(2)
ε	$-\nu_1\Omega+6g+4g_5$[a]	$g_7'\Omega'+6g+4g_5'$[b]	$-\nu_1\Omega+6s$
ε_0	$-\nu_1\Omega+4g_5$	$g_7'\Omega'+4g_5'$	$-\nu_1\Omega$
a	$2\nu_1+2g_5$	$2g_5$	$2\nu_1$
b	$-g_5$	$-g_5'$	0
c	g	g	s
N_d	κ ($\kappa\leqslant\Omega/2$)[c]	$\bar{\kappa}$[d]	κ ($\kappa\leqslant\Omega/3$)
τ	$\kappa,\kappa-2,\ldots,1$ or 0	$\bar{\kappa},\bar{\kappa}-2,\ldots,1$ or 0	$\kappa,\kappa-2,\ldots,1$ or 0

[a] Ω is the pair degeneracy which is equal to 10 for the SO(8) symmetry and to 15 for the SP(6) symmetry.
[b] $\Omega'=\Omega-2N+6$, where N is the pair number of many-body systems.
[c] κ is one-half of the seniority number.
[d] $\bar{\kappa}=N-w/2$ where w is the number of nucleons that do not form D pairs.

the correlation γ if $b<0$, and δ if $b>0$; for $N=5$, one obtains the correlation β if $b<0$, and δ if $b>0$.

In Figs. 1(a′) and 1(c′), one also obtains the correlation ξ, defined by $R_6=\frac{7}{5}R_4+1$. This correlation can be derived in the SO(5)×SU(2) and the SO(7) limits. In Eq. (5) if we take $N_d=2,3,2,3$ and $\tau=0,1,2,3$ for $I=0,2,4,6$, respectively, as well as $|c|\ll|b|$, the excitation energies of the 2^+, 4^+, and 6^+ states are written by

$$\begin{aligned}
E_{2^+}-E_{0^+}&=\varepsilon_0+2a+4b+6c\approx\varepsilon_0+2a+4b,\\
E_{4^+}-E_{0^+}&=-10b+20c\approx-10b,\\
E_{6^+}-E_{0^+}&=\varepsilon_0+2a-10b+42c\approx\varepsilon_0+2a-10b.
\end{aligned}$$

From the above formulas we arrive at the correlation $R_6=\frac{7}{5}R_4+1$. In Figs. 1(a′) and 1(c′) one sees that the results relevant to this correlation do not follow the straight line $R_6=\frac{7}{5}R_4+1$ very compactly.

In the SU(2) limit with $\nu_1>0$, all statistics in the Mallmann plot converge at $(R_6,R_4)=(7,10/3)$. However, because the value of s_3 in Eq. (4) is not precisely equal to 0 in random Hamiltonians, the SU(2) limit is not precisely (but approximately) satisfied, and

$$\begin{aligned}
E_{I^+}\approx{}&\varepsilon_0N_d+aN_d(N_d-1)/2+b(N_d-\tau)(N_d+\tau+3)\\
&+cI(I+1)+s_3\langle N\kappa\tau,I|C_{\mathrm{SU(3)}}|N\kappa\tau,I\rangle,
\end{aligned}\qquad(7)$$

where $|N\kappa\tau,I\rangle$ is the eigenstate of the SU(2) limit. The last term on the right-hand side of Eq. (7) leads to deviations from $(R_6,R_4)=(7,10/3)$.

We discuss the vibrational limits of the FDSM above, and Table II is a summary of the correlation between R_6 and R_4 in the vibrational limits. Below we discuss the rotational limits of the FDSM.

There are two dynamical symmetry limits corresponding to rotors: For the SO(8) symmetry, the Hamiltonian has the SO(6) limit if $\nu_1=0$ in Eq. (2); for the SP(6) symmetry, the Hamiltonian has the SU(3) limit if $\nu_1=0$ in Eq. (4). In the SO(6) limit of the SO(8) symmetry, the eigenenergies are given by [9]

$$E_{\mathrm{SO(6)}}=E_0+g_5\tau(\tau+3)+g_6\sigma(\sigma+4)+gI(I+1),\qquad(8)$$

where E_0 is a constant independent of I. For $N\leqslant\Omega/2$, the reduction rules for the SO(6) limit are written as follows:

$$\begin{aligned}
\sigma&=N,N-2,N-4,\ldots,0\text{ or }1,\\
\tau&=\sigma,\sigma-1,\sigma-2,\ldots,0,\\
n_\Delta&=[\tau/3],[\tau/3]-1,[\tau/3]-2,\ldots,0,\\
\lambda&=\tau-3n_\Delta,\\
I&=\lambda,\lambda+1,\ldots,2\lambda-2,2\lambda,
\end{aligned}\qquad(9)$$

where σ, τ, n_Δ, and λ are quantum numbers.

When $g_5>0$ and $g_6<0$, the yrast states have $\sigma=N$ and $\tau=I/2$, and Eq. (8) is reduced to $E_{\mathrm{SO(6)}}=E_0+g_5I(I+6)/4+g_6N(N+4)+gI(I+1)$, and one obtains $R_I=R_4I(I-2)/8-I(I-4)/4$ which is the correlation α; when $g_5>0$ and $g_6>0$, the yrast states have $\tau=I/2$, $\sigma=2[(I+2)/4]$ ($\sigma=2[I/4]+1$) for even (odd) number of nucleon pairs, in this case R_6 does not exhibit a linear correlation versus R_4; when $g_5<0$ and $g_6<0$, the correlation β arises if $N=3k$ (where k is an integer), and δ arises if $N\neq3k$; and when $g_5<0$ and $g_6>0$, the correlation β arises if $N=3$ and 5, δ arises if $N\neq3k$.

The eigenenergies of the SU(3) limit are given by [9]

$$\begin{aligned}
E_{\mathrm{SU(3)}}={}&E_0+\frac{s_3}{2}(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)\\
&+sI(I+1),
\end{aligned}\qquad(10)$$

TABLE II. Correlations between R_6 and R_4 in the vibrational limits of the FDSM [9]. ν_1 and b are parameters defined in Eqs. (2) and (5), respectively. N is the nucleon pair number of a system, and I is the spin of a state. N_d and τ are quantum numbers in vibrational limits, with the reduction rule given in Eq. (6). "+" means to take a positive value and "−" means a negative value.

SO(8)⊃ SO(5)×SU(2) or SO(8)⊃ SO(7)							
ν_1	b	N	N_d	τ	I	R_6–R_4	
−			$I/2$	$I/2$	0,2,4,6,...	$R_I = \frac{I(I-2)}{8}R_4 - \frac{I(I-4)}{4}$	α
+		3	N	3 1	0,4,6 2	$R_6 = \frac{21}{10}R_4$	β
+	−	4	N	4 2 0	6 2,4 0	$R_6 = \frac{9}{5}R_4 + 1$	γ
+	−	5	N	3 1	0,4,6 2	$R_6 = \frac{21}{10}R_4$	β
+	+	4	N	4 0	2,4,6 0	$R_6 = \frac{18}{7}R_4 - \frac{11}{7}$	δ
+	+	5	N	5 3	2,4,6 0	$R_6 = \frac{18}{7}R_4 - \frac{11}{7}$	δ

SP(6)⊃ SU(2)				
ν_1	N_d	τ	R_6-R_4	
−	$I/2$	$I/2$	$R_I = \frac{I(I-2)}{8}R_4 - \frac{I(I-4)}{4}$	α
+	N	$N, N-2, \ldots, 1$ or 0	$(R_I, R_4) = (\frac{I(I+1)}{6}, \frac{10}{3})$	

where E_0 is a constant independent of I. The reduction rules are as follows [9]:

$$\begin{aligned}(\lambda,\mu) &= (2N,0),(2N-4,2),\ldots,(0,N) \text{ or } (2,N-1),\\ &\quad (2N-6,0),(2N-10,2),\ldots,(0,N-3) \text{ or } (2,N-4),\\ &\quad (2N-12,0),(2N-16,2),\ldots,(0,N-6) \text{ or } (2,N-7),\ldots,\\ K_m &= \max(\lambda,\mu), \qquad K_0 = \min(\lambda,\mu), \qquad K = K_0, K_0-2,\ldots,0 \text{ or } 1,\\ I &= \begin{cases} K_m, K_m-2,\ldots,0 \text{ or } 1, & \text{if } K=0,\\ K, K+1,\ldots,K+K_m, & \text{if } K\neq 0.\end{cases}\end{aligned} \tag{11}$$

For $s_3 < 0$, the yrast states have $(\lambda,\mu) = (2N,0)$, and Eq. (10) is reduced to

$$E_{\mathrm{SU(3)}} = E_0 + s_3 N(2N+3) + sI(I+1).$$

We thus have $R_I = I(I+1)/6$, which leads to $(R_6,R_4) = (7,10/3)$. For $s_3 > 0$, the yrast states have $(\lambda,\mu) = (0,0),(2,2),(6,0),\ldots$ if $N = 3k$, and $(\lambda,\mu) = (2,0),(0,4),(4,2),\ldots$ or $(0,2),(4,0),(2,4),\ldots$ if $N \neq 3k$. Thus one obtains $R_6 = \frac{57}{28}R_4 + \frac{3}{14}$ if $N = 3k$, and $R_6 = 2R_4 + \frac{1}{3}$ if $N \neq 3k$. Table III is a summary of the correlation between R_6 and R_4 in the rotational limits.

In Fig. 2, one sees that the correlation $R_6 = \frac{57}{28}R_4 + \frac{3}{14}$ is very close to the correlation β in the case of $N = 3$, and the correlation $R_6 = 2R_4 + \frac{1}{3}$ is very close to the correlations γ and β in the cases of $N = 4$ and 5, respectively. Scrutinizing the results of random ensembles in Fig. 2, one sees that the correlation $R_6 = \frac{57}{28}R_4 + \frac{3}{14}$ is less favored than the correlation β for $R_4 < -1$ and $N = 3$, and the correlation $R_6 = 2R_4 + \frac{1}{3}$ is more favored than the correlation γ for $R_4 > 4$ and $N = 4$ (or than the correlation β with $N = 5$). For $N = 3$ with $R_4 > -1$, the distribution of the (R_6, R_4) results for our random ensemble show no bias between correlation β and $R_6 = \frac{57}{28}R_4 + \frac{3}{14}$. Similarly, no noticeable bias is reflected between $R_6 = 2R_4 + \frac{1}{3}$ and γ (or β) for $R_4 < 4$ in the case of $N = 4$ (or 5), according to calculated results in Fig. 2.

As we relate R_6-R_4 correlation to dynamical symmetries in the FDSM, random samplings which do not follow these dynamical symmetries are of interest. This can be studied by setting the magnitudes of g_7', ν_1, ν_1', and s_3 comparably large in Eqs. (2)–(4). For the SO(8) symmetry, we define x_1 and x_2 as the smallest and the second smallest value among $|g_7'|$, $|\nu_1|$, and $|\nu_1'|$ in each random Hamiltonian, respectively. For the SP(6) symmetry, we define x_1 and x_2 as the smaller and the larger value between $|\nu_1|$ and $|s_3|$,

TABLE III. Same as Table II, except for the rotational limits of the FDSM [9]. g_5, g_6, and s_3 are parameters defined in Eqs. (2) and (4). σ and τ are quantum numbers of the SO(6) limit, with the reduction rule presented in Eq. (9). λ, μ, and K are quantum numbers of the SU(3) limit, with the reduction rule given in Eq. (11). k is an integer.

SO(8)⊃ SO(6)

g_5	g_6	N	σ	τ	I	R_6–R_4	
$+$	$-$		N	$I/2$	$0,2,4,6,\ldots$	$R_I = \frac{I(I-2)}{8}R_4 - \frac{I(I-4)}{4}$	α
$+$	$+$	$2k$	$2[\frac{I+2}{4}]$	$I/2$	$0,2,4,6,\ldots$		
$+$	$+$	$2k+1$	$2[\frac{I}{4}]+1$	$I/2$	$0,2,4,6,\ldots$		
$-$	$-$	$3k$	N	N $N-1$	$0,4,6,\ldots$ 2	$R_I = \frac{I(I+1)}{20}R_4$	β
$-$	$-$	$\neq 3k$	N	N $3[\frac{N}{3}]$	$2,4,6,\ldots$ 0	$R_I = \frac{(I-2)(I+3)}{14}R_4 - \frac{(I-4)(I+5)}{14}$	δ
$-$	$+$	3	$2[\frac{I}{4}]+1$ $2[\frac{I}{4}]+1$ 3 3	σ 0 3 2	$2,4,6$ 0 $0,4,6$ 2	$R_6 = \frac{21}{10}R_4$	β
$-$	$+$	4	$2[\frac{I+2}{4}]$ 4 4	$2[\frac{I+2}{4}]$ 4 3	 $2,4,6$ 0	$R_6 = \frac{18}{7}R_4 - \frac{11}{7}$	δ
$-$	$+$	5	$2[\frac{I}{4}]+1$ $2[\frac{I}{4}]+1$ 5 3	σ 0 5 3	$2,4,6$ 0 $2,4,6$ 0	$R_6 = \frac{21}{10}R_4$ or $R_6 = \frac{18}{7}R_4 - \frac{11}{7}$	β δ

SP(6)⊃ SU(3)

s_3	N	(λ,μ)	K	I	R_6-R_4
$-$		$(2N,0)$	0	$0,2,4,6,\ldots$	$(R_I,R_4) = (\frac{I(I+1)}{6},\frac{10}{3})$
$+$	$3k$	$(0,0)$ $(2,2)$ $(2,2)$ $(6,0)$	0 $0,2$ 2 0	0 2 4 6	$R_6 = \frac{57}{28}R_4 + \frac{3}{14}$
$+$	$3k+1$	$(2,0)$ $(0,4)$ $(4,2)$	0 0 2	$0,2$ 4 6	$R_6 = 2R_4 + \frac{1}{3}$
$+$	$3k+2$	$(0,2)$ $(4,0)$ $(2,4)$	0 0 2	$0,2$ 4 6	$R_6 = 2R_4 + \frac{1}{3}$

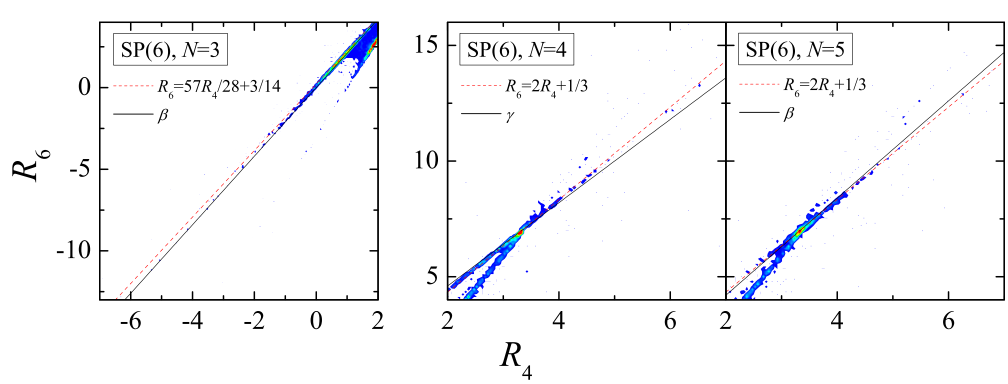

FIG. 2. (Color online) Distribution of (R_6, R_4) for the SP(6) symmetry, with the focus of $R < 2$ for $N = 3$ and of $R > 2$ for $N = 4,5$, respectively. Calculated results of the entire random ensemble, regardless of their ground state spins, are included. One sees that two correlations in each panel, i.e., $R_6 = \frac{57}{28}R_4 + \frac{3}{14}$ and β correlation for $N = 3$, $R_6 = 2R_4 + \frac{1}{3}$ and γ correlation for $N = 4$, $R_6 = 2R_4 + \frac{1}{3}$ and β correlation for $N = 5$, are very close to each other.

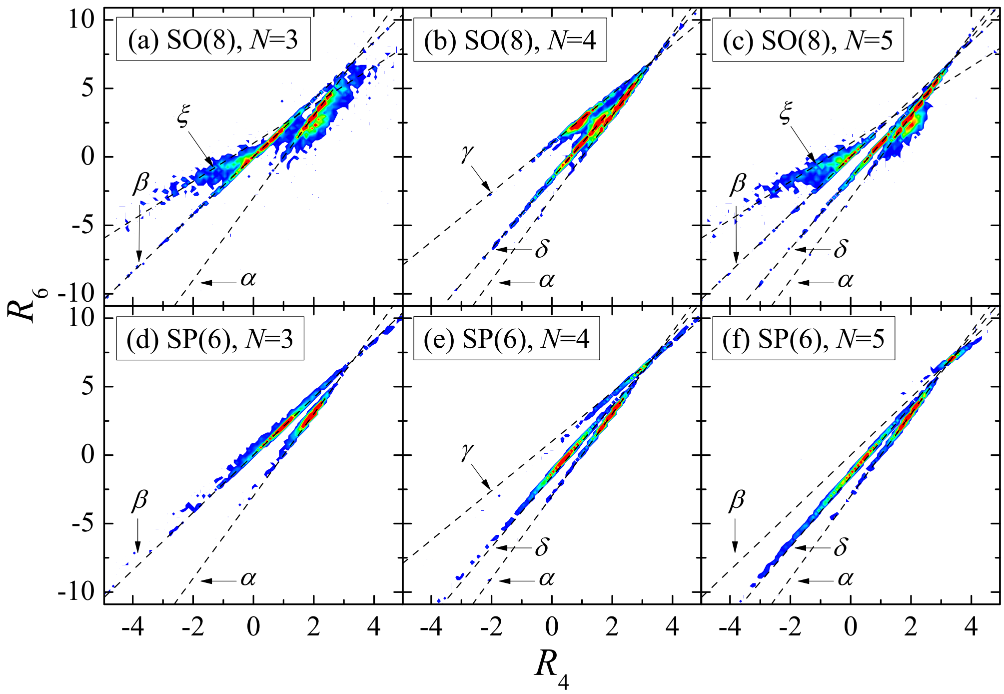

FIG. 3. (Color online) Distribution of (R_6, R_4) for random samplings which do not well follow dynamical symmetry limits in the FDSM. Calculated results of the random samplings, regardless of their ground state spins, are considered here. One sees that correlations α, β, γ, δ, and ξ are as remarkable as in Fig. 1.

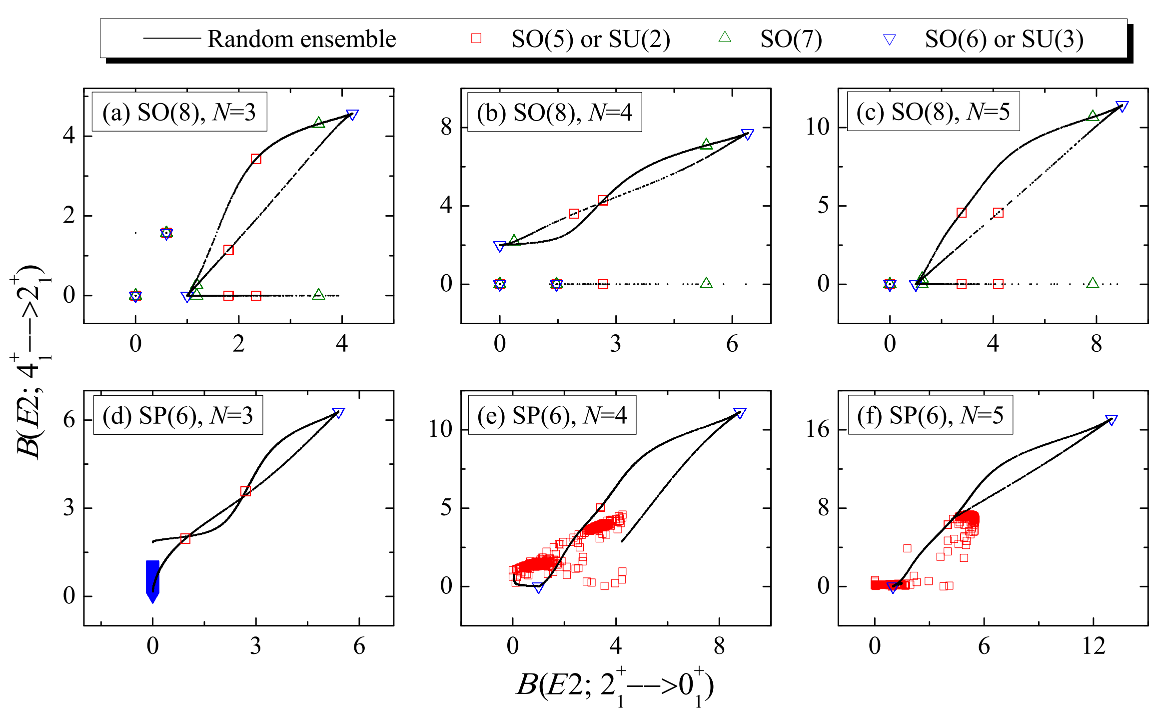

FIG. 4. (Color online) Distribution of $[B(E2;4_1^+ \longrightarrow 2_1^+), B(E2;2_1^+ \longrightarrow 0_1^+)]$ for the random ensemble of the FDSM. The solid trajectories in black are given by the ensemble with the Hamiltonian of Eq. (1); squares in red correspond to random samplings with the SO(5)×SU(2) or the SU(2) limit; triangles in green correspond to those with the SO(7) limit; and inverse triangles in blue correspond to those with the SO(6) or the SU(3) limit.

respectively. In order to skip typical vibrational and rotational cases among the random ensemble, our random Hamiltonians are restricted to cases of $x_1/x_2 > 0.7$. The Mallmann plot of these restricted random ensemble, plotted in Fig. 3, *remains* to exhibit the remarkable correlations α, β, γ, δ, as well as ξ. Therefore these correlations are very robust with respect to random interactions. We also note that in Figs. 3(a) and 3(c), the correlation α is relatively less pronounced, and many more samples are found at $(R_6,R_4) \approx (2.5,2)$. The previous statistical peak at $(R_6,R_4) = (7,3.33)$ almost disappears in Figs. 3(d) and 3(e).

We further consider electric quadrupole transition probability, $B(E2)$, between yrast states under random interactions. Here the quadrupole operator is taken to be $P^{(2)}$. The correlation of $B(E2;4_1^+ \longrightarrow 2_1^+)$ versus $B(E2;2_1^+ \longrightarrow 0_1^+)$ is shown in Fig. 4. Interestingly, these two $B(E2)$ values follow very compact trajectories with "exceptions."

We diagonalize random Hamiltonians which are restricted to the SO(5)×SU(2), SO(7), SU(2), SO(6), and SU(3) limits, and calculate the $B(E2)$ values. In Fig. 4, one sees that $B(E2)$ results take specific values in the plane of $B(E2;4_1^+ \longrightarrow 2_1^+)$ and $B(E2;2_1^+ \longrightarrow 0_1^+)$ for the SO(5)×SU(2), SO(7), SU(2), SO(6), and SU(3) limits. This is understandable, as the wave functions of yrast states are predetermined according to the dynamical symmetry limits. The "exceptions" arise in the case of $N = 4$ and 5 in the SU(2) limit of the SP(6) symmetry [see Figs. 4(e) and 4(f)], in which the distribution of $B(E2;4_1^+ \longrightarrow 2_1^+)$ versus $B(E2;2_1^+ \longrightarrow 0_1^+)$ is smeared and does not follow the trajectories of the entire random ensemble. These "exceptions" originate from level degeneracies of yrast I =2 and 4 states. According to Table II, the lowest energy state for given I has $N_d = \kappa = N$ and $\tau = N, N-2, \ldots, 1$ or 0, in the SU(2) limit with $\nu_1 > 0$. For $N = 4$, we have $N_d = 4$ and $\tau = 4$, 2, or 0; for $N = 5$, we have $N_d = 5$ and $\tau = 5$, 3, or 1. Because N_d represents the number of D pairs, the number of states is two for both I =2 and 4, thus both states are twofolded degenerate, and numerical calculations present wave functions given by random combinations of two configurations with different τ, yielding the smeared distribution of $B(E2;4_1^+ \longrightarrow 2_1^+)$ versus $B(E2;2_1^+ \longrightarrow 0_1^+)$ in Figs. 4(e) and 4(f). These "exceptions" also occur for $N = 3$ in the SU(3) limit of the SP(6) symmetry [see Fig. 4(d)]. In this case $s_3 > 0$ in Eq. (4), and there are two degenerate spin-2 states which are lowest in energy, with $(\lambda,\mu) = (2,2)$ and $K = 0$ or 2. Clearly, all $B(E2)$ results follow the trajectories given here once the above degeneracy is broken.

To summarize, in this paper we study the correlation of yrast states in the FDSM, under random interactions. Here we consider *the entire random ensemble*. We find that R_6–R_4 exhibit strong linear correlations, regardless of the ground state spin. We have shown that these simple correlations can be obtained in the vibrational [SO(5)×SU(2), SO(7), and SU(2)] and the rotational [SO(6) and SU(3)] limits in the FDSM. We have also shown that these correlations survive for random samplings which do not obey these dynamical symmetries. We discern the strong correlation between $B(E2;4_1^+ \longrightarrow 2_1^+)$ and $B(E2;2_1^+ \longrightarrow 0_1^+)$ for the random ensemble in the FDSM, and future consideration of its origin is warranted.

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