^{14}Be(g.s.) and single-particle energies in ^{13}Be

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Coupling two sd-shell neutrons to a pure p-shell ^{12}Be ground state (g.s.), rather than to the physical g.s., removes difficulties in applying a previous simple model to ^{14}Be. I have calculated the g.s. wave function in this simple model, and have estimated the $2s_{1/2}$ single-particle energy.

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I. INTRODUCTION

Recently, I analyzed selected 0^+ states in ^{10,12}Be and ^{14,16}C in terms of a simple model that assumed their structure was dominated by the configuration of two sd-shell neutrons coupled to p-shell cores [1]. There, I mentioned the problem of extending the model to ^{14}Be because of three difficulties: (1) The ^{12}Be core is not even close to being of pure p-shell character [2–4]. (2) The s and d single-particle energies (spe's) are not well known in ^{13}Be [5, and references therein]. (3) The s state in ^{13}Be is unbound, and unbound neutron s states are notoriously hard to handle. Here, I examine an approach that is aimed at overcoming these difficulties.

II. MODEL AND RESULTS

If E_s and E_d are, respectively, the s and d spe's relative to the physical ground state (g.s.) of ^{12}Be, then relative to a pure p-shell ^{12}Be(g.s.), the spe's are $E_s' = E_s - E_0$ and $E_d' = E_d - E_0$, where E_0 is the energy of ^{12}Be$_{1p}$(g.s.) relative to ^{12}Be$_{phys}$(g.s.). My favorite wave function [6] for ^{12}Be$_{phys}$(g.s.) has 68% of the configuration ^{10}Be$_{1p} \times (sd)^2$ and 32% of ^{12}Be$_{1p}$, with the excited 0^+ state at 2.24 MeV [7] having the orthogonal configuration (Table I). With these two wave functions, E_0 would be 1.52 MeV, but it will turn out that my final results do not depend on E_0. For any expected value of E_s, $E_s - E_0$ will be negative, so that the s state is bound relative to ^{12}Be$_{1p}$(g.s.).

Of course, coupling two sd-shell neutrons to the physical g.s. of ^{12}Be would have done violence to the Pauli principle, but coupling to ^{12}Be$_{1p}$ has no such problem. Single-particle energies are listed in Table II. I treat E_s as an unknown parameter to be determined later. I previously estimated $E_d - E_s$ in ^{13}Be [5] to be about 2.3 MeV. I arrived at that value by considering the trends in $N = 9$ and in $Z = 4$ nuclei. Recently, Hoffman *et al.* [8] analyzed $1/2^+$ and $5/2^+$ in several light nuclei. For ^{13}Be, they have $E_d - E_s = 1.5$ MeV, because they consistently used the lowest $1/2^+$ and $5/2^+$ states. However, I demonstrated earlier [5] that the first $5/2^+$ state in ^{13}Be is predominantly of $(sd)^3$ character. The value of 2.3 MeV is not inconsistent with the general trend [8] of s and d states in several nuclei. Using the same 0^+ two-body matrix elements as before [1], the resulting E_{2n} eigenvalues for the two 0^+ states of ^{14}Be are as listed in Table III.

TABLE I. Wave-function intensities for first two 0^+ states in ^{12}Be (Ref. [6]).

State	Energy (MeV)	^{10}Be$_{1p}x(sd)^2$	^{12}Be$_{1p}$
g.s.	0.00	0.68	0.32
exc 0^+	2.24	0.32	0.68
Energy centroid (MeV)		0.72	1.52

TABLE II. Single-particle energies in ^{13}Be.

Orbital	Relative to ^{12}Be$_{phys}$(g.s.)	Relative to ^{12}Be$_{1p}$(g.s.)
$2s_{1/2}$	E_s	$E_s - E_0$
$1d_{5/2}$	$E_d = E_s + 2.3$ MeV	$E_d - E_0$

The g.s. of ^{14}Be is bound by 1.27(13) MeV relative to ^{12}Be$_{phys}$(g.s.) $+2n$ [9]. If its structure is completely ^{12}Be$_{1p} \times (sd)^2$, equating the calculated 0_1^+ energy in Table III to this experimental energy results in a value of $E_s = 0.50(7)$ MeV. The extracted value of E_s is relatively insensitive to the assumed value of $E_d - E_s$. For example, changing the latter from 2.3 to 2.0 MeV changes E_s only from 0.50 to 0.55 MeV. If the ^{14}Be g.s. contains some $(sd)^4$ component, then the $(sd)^2$ energy will be above -1.27 MeV, so that my result becomes $E_s \geqslant 0.43$ MeV. Simply for illustrative purposes, let us assume the lowest 0^+ state that is predominately of $(sd)^4$ character is at 4-MeV excitation. Then, it is an easy matter to show that E_s would be given by $E_s = [0.50(7) + 2.0\beta^2]$ MeV, where β^2 is the intensity of $(sd)^4$ in the ^{14}Be(g.s.). Thus, e.g., if E_s is <0.7 MeV, we have $\beta^2 < 0.10$—quite a reasonable value.

III. DISCUSSION

I recently summarized the experimental and theoretical findings for ^{13}Be [5]. I demonstrated there that the lowest $5/2^+$ state is predominantly of ^{10}Be$_{1p} \times (sd)^3$ character rather than ^{12}Be$_{1p} \times d_{5/2}$ single particle. Of course, because

TABLE III. E_{2n} eigenvalues and wave functions for two $(sd)^2$ 0^+ states in ^{14}Be.

State	Relative to ^{12}Be$_{1p}$ (g.s.)	Relative to ^{12}Be$_{phys}$ (g.s.)	s^2	d^2
0_1^+	$2(E_s - E_0)$ -2.26 MeV	$2E_s - 2.26$ MeV	0.85	0.15
0_2^+	$2(E_s - E_0) + 2.54$ MeV	$2E_s + 2.54$ MeV	0.15	0.85

^{12}Be$_{phys}$(g.s.) contains appreciable ^{10}Be$_{1p}\times(sd)^2$ component [6], this lowest $5/2^+$ state has a large spectroscopic factor to ^{12}Be$_{phys}$(g.s.). I also estimated that the s,d spe splitting is about $E_d - E_s = 2.3$ MeV in ^{13}Be. Two papers [10,11] that have appeared since that summary have served partly to further confuse the issue. I will return to this point below.

As demonstrated above, in the simple model used here, if the g.s. of ^{14}Be is pure ^{12}Be$_{1p}\times(sd)^2$, then E_s in ^{13}Be is 0.50(7) MeV. Any component of $(sd)^4$ in ^{14}Be(g.s.) causes an increase in this spe. Thus, it would appear that the present results rule out all prior suggestions of an s state near threshold in ^{13}Be. The present approach removes all three difficulties mentioned in the Introduction.

Several workers [12–17] have treated ^{13}Be as ^{12}Be + n, and ^{14}Be as ^{12}Be + n + n, and have used the known $2n$ separation energy of ^{14}Be(g.s.) to deduce properties of the low-lying resonances of ^{13}Be. Bertsch and Esbensen [12] and Thompson and Zhukov [13] found that they needed an s state just above threshold to reproduce S_{2n}. Reference [12] had a $d_{5/2}$ state at 2.4 MeV, whereas Ref. [13] found they needed $E_d = 1.3$ or 1.0 MeV. Labiche *et al.* [14] concluded that to fit the known $2n$ separation energy of ^{14}Be(g.s.) and to have a d state near 2 MeV in ^{13}Be, the g.s. of ^{13}Be should not be $1/2^+$, but rather it was necessary to have a $1/2^-$ resonance near 0.3 MeV as the g.s. of ^{13}Be. Pacheco and Vinh Mau [15] concluded the $s_{1/2}$, $p_{1/2}$ ordering in ^{12,13}Be was the same as in ^{11}Be, as did Blanchon *et al.* [16]. Hamomoto [17] suggested the two lowest states in ^{13}Be might both be $1/2^+$. Reference [5] summarizes several other results.

TABLE IV. Recently reported resonance energies (MeV) in ^{13}Be.

J^π	Randisi *et al.*[a]	Aksyutina *et al.*[b]	Present
$1/2^+$	0.40(3)	0.46	>0.43
$1/2^+$	–	2.9	–
$5/2^+$	$0.85^{+.15}_{-.11}$	–	–
$5/2^+$	2.35(14)	2.0	$E_s + 2.3$
$1/2^-$	<1	0.45	–

[a]Reference [10].
[b]Reference [11].

Returning to two recent papers [10,11] on ^{13}Be, Table IV lists resonance energies reported by them, compared with present results. Consistency can be noted for the first $1/2^+$ energy, but agreement for $5/2^+$ is less clear.

Finally, the ^{14}Be(g.s.) wave function obtained here has 85% s^2, whereas analysis of matter radius gave a result for s^2 probability of 72(7)% [18]. An earlier, somewhat different analysis of matter radius provided $P(s^2) = 0.55(30)$ [19]. These would all appear to be consistent.

[1] H. T. Fortune, Phys. Rev. C **89**, 067302 (2014).
[2] D. E. Alburger, D. P. Balamuth, J. M. Lind, L. Mulligan, K. C. Young, R. W. Zurmuhle, and R. Middleton, Phys. Rev. C **17**, 1525 (1978).
[3] D. E. Alburger, S. Mordechai, H. T. Fortune, and R. Middleton, Phys. Rev. C **18**, 2727 (1978).
[4] H. T. Fortune, G.-B. Liu, and D. E. Alburger, Phys. Rev. C **50**, 1355 (1994).
[5] H. T. Fortune, Phys. Rev. C **87**, 014305 (2013).
[6] R. Sherr and H. T. Fortune, Phys. Rev. C **60**, 064323 (1999).
[7] S. Shimoura *et al.*, Phys. Lett. B **560**, 31 (2003).
[8] C. R. Hoffman, B. P. Kay, and J. P. Schiffer, Phys. Rev. C **89**, 061305(R) (2014).
[9] M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu, and B. Pfeiffer, Chin. Phys. C **36**, 1603 (2012).
[10] G. Randisi *et al.*, Phys. Rev. C **89**, 034320 (2014).
[11] Yu. Aksyutina *et al.*, Phys. Rev. C **87**, 064316 (2013).
[12] G. F. Bertsch and H. Esbensen, Ann. Phys. (NY) **209**, 327 (1991).
[13] I. J. Thompson and M. V. Zhukov, Phys. Rev. C **53**, 708 (1996).
[14] M. Labiche, F. M. Marqués, O. Sorlin, and N. Vinh Mau, Phys. Rev. C **60**, 027303 (1999).
[15] J. C. Pacheco and N. Vinh Mau, Phys. Rev. C **65**, 044004 (2002).
[16] G. Blanchon, N. Vinh Mau, A. Bonaccorso, M. Dupuis, and N. Pillet, Phys. Rev. C **82**, 034313 (2010).
[17] Ikuko Hamamoto, Phys. Rev. C **77**, 054311 (2008).
[18] T. Moriguchi *et al.*, Nucl. Phys. A **929**, 83 (2014).
[19] H. T. Fortune and R. Sherr, Eur. Phys. J. A **48**, 103 (2012).