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Probing shape coexistence by α decays to 0^+ states

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We analyze the α-decay fine structure to excited 0^+_2 states in Hg and Rn isotopes. These states are described as minima in the potential energy surface (PES) provided by the standard deformed Woods-Saxon plus pairing approach. We also investigate α decay *from* the excited state $P(0^+_2)$ in the parent nucleus by evaluating the corresponding hindrance factor (HF). By analyzing the experimental HF's we find the remarkable property that the ground and excited states $D(0^+_1)$ and $D(0^+_2)$ in the daughter nuclei are occupied with almost equal probabilities if there is no excited $P(0^+)$ states in the parent nucleus. Moreover, if there exists an excited state $P(0^+_2)$ then the occupation probability of this state is 25%.

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The shape coexistence of nuclei is an old but still very exciting subject in nuclear physics [1]. The establishment of spherical and oblate isomers as well as triple shapes in light lead isotopes was reported in Refs. [2,3]. In a recent review [4] one important observation was emphasized, namely, "It appears that, without exception, low-lying excited 0^+ states are associated with shape coexistence." An interesting shape coexistence not only in $K = 0^+$ but also in $K = 2^+$ bands was detected in ^{194}Pt [5]. A very special triple coexistence of spherical, prolate, and oblate shapes in a neutron-defficient ^{186}Pb was evidenced in Ref. [6]. Shape coexistence in this nuclear region was further explored and confirmed by using α-decay probes [7].

Several approaches were used to describe excited 0^+ states in connection to the shape coexistence phenomenon. The proton-neutron interacting boson model (IBM-2) was used to explain the monotone decrease of the excitation energy versus decreasing neutron number, due to the quadrupole-quadrupole interaction between the proton and neutron systems [8]. More recently, the IBM-2 with configuration mixing was applied to describe the shape coexistence in neutron-deficient Pb [9], Pt [10,11], and Hg [12,13] isotopes.

There has been an intense study of 0^+ states and deformations in the lead region. Thus, the shape coexistence in Hg and Pb isotopes was discussed in Ref. [14] by using a nonaxial mean field, given by the Woods-Saxon potential, plus Strutinsky correction. A very important step needed to correctly describe shape coexistence is angular momentum projection. Thus, the neutron-deficient $^{182-194}$Pb isotopes were investigated in Ref. [15] within the generator coordinate method (GCM) by using configuration mixing of angular momentum and particle-number projected self-consistent mean-field states. Fluctuations about the equilibrium deformation of 0^+ states in deformed nuclei were described in Ref. [16], by using angular-momentum-projected states with different quadrupole deformations. Large-scale calculations by using projection before variation (VAMPIR code) [17] have been done in order to describe shape coexistence in several isotope chains [18]. The shape coexistence was studied within the relativistic Hartree-Bogoliubov approach in Ref. [19] and within the relativistic mean field formalism for even-even superheavy nuclei in Ref. [20].

α transition to excited states [21,22] is an important tool to probe nuclear structure. The α-decay spectroscopy was intensively used to investigate excited states (see Ref. [23] for a review). The α decay to excited states, or α-decay fine structure, is better analyzed in terms of the hindrance factor (HF), defined as the ratio between spectroscopic factors to ground and excited states [24]. Experimental HF's in nuclei were mainly investigated by using the quasiparticle random-phase approximation (QRPA) and coupled-channels approaches (for a review, see Ref. [25]). Excited low-lying 0^+ states have been found to be largely populated by α-decay probes in the ^{190,192,194}Pb isotopes [26,27]. It turns out that the HF of the α transition feeding the 0^+_2 state decreases as the excitation energy decreases, i.e., as one goes towards lighter lead isotopes, and it changes abruptly, from unity to about 80, when departing by a few protons from the $Z = 82$ closed shell [26,27]. In Ref. [28] gave a description of the HF of the first excited 0^+ states as a function of the neutron number in the region of Pb isotopes in terms of pairing vibrations. Transitions to excited 0^+_2 states in Pb and Po isotopes were analyzed in Ref. [29] in terms of the density-dependent cluster model.

In this paper we study the 0^+ states within the two-level model proposed in Ref. [26] by using a microscopic description of the α-particle formation amplitude. Such description needs a more complex formalism than the simple phenomenological picture of a preformed α cluster penetrating through the Coulomb barrier with a preformation factor proportional to the fragmentation potential [30]. The R-matrix theory factorizes the decay width into the α-particle formation probability and the Coulomb penetrability [31]. The α-particle formation probability is given by the microscopic formation amplitude squared [23,32]. The investigation of α transitions to excited states is an important tool to probe nuclear structure because only single-particle (sp) states around the Fermi surface are involved in the formation amplitude. This is particularly favorable in the case of the pairing (BCS) evaluation of α decay in deformed nuclei, which has been shown to be very successful [23].

Below we study transitions from the kth level in the parent nucleus to the k'th level of the daugher nucleus, i.e.,

$$|P(0_k^+)\rangle \rightarrow |D(0_{k'}^+)\rangle + \alpha, k, k' = 1,2. \tag{1}$$

Assuming that the 0_k^+ states of axially deformed nuclei are described by BCS wave functions and assuming quadrupole deformations, the parent state is

$$|P(0_k^+)\rangle = |\psi^P_{\rm BCS}(\beta_2^{(k)})\rangle \equiv |\psi_k^P\rangle, \tag{2}$$

and similarly for the daughter nucleus. We assume that the excited state 0_2^+ corresponds to a minimum of the potential energy surface (PES) lying higher than the ground state (gs) minimum in both parent and daughter nuclei. This procedure determines the values of the deformation parameters [33]. In the laboratory system of coordinates the pairing interaction can be expanded in terms of monopole plus quadrupole components. Thus, the quadrupole pairing component, which is important for the description of deformed nuclei [4,34], is considered in our calculation with the same strength as the monopole part.

The α-particle formation amplitude for deformed nuclei can be microscopically evaluated by using the expansion of the parent wave function in terms of two-proton times two-neutron sp orbitals on the core provided by the daughter nucleus. The final result is a superposition of spherical harmonic oscillator (ho) wave functions depending on the α-particle center of mass (c.m.) radius (for details, see Ref. [23])

$$\mathcal{F}_{kk'}(\mathbf{R}) = \sum_{L_\alpha N_\alpha} W_{N_\alpha L_\alpha}(\beta_2^{(k)},\beta_2^{(k')})\Phi^{(4\lambda)}_{N_\alpha L_\alpha}(\mathbf{R}), \tag{3}$$

where $\lambda = M_N\omega/\hbar$ is the sp ho parameter. The W coefficient contains the parent-daughter expansion coefficients of the corresponding sp Nilsson wave functions and the overlap between parent-daughter wave functions. This overlap, written in terms of the BCS amplitudes, is

$$\langle\psi_k^P|\psi_{k'}^D\rangle = \sum_{m>0}[u_m^P(\beta_2^{(k)})u_m^D(\beta_2^{(k')}) + v_m^P(\beta_2^{(k)})v_m^D(\beta_2^{(k')})]. \tag{4}$$

Here m labels the deformed sp quantum numbers (energy, spin projection, and parity).

The HF is evaluated, as usual [28], as the ratio between the gs to gs and the gs to excited state α-particle formation amplitudes squared. Most of the formation amplitude is concentrated around the nuclear surface and practically vanishes beyond R_m= 12 fm [23]. Therefore we can characterize the α-particle formation amplitude squared by the expression

$$\langle S_{kk'}\rangle = \frac{S_{kk'}}{R_m}, \tag{5}$$

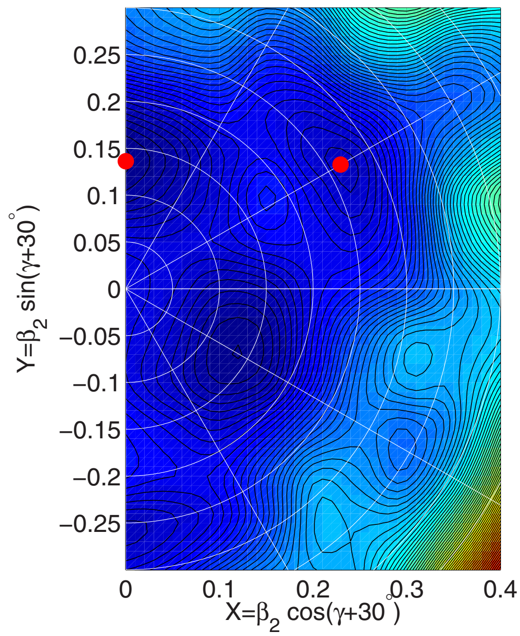

FIG. 1. (Color online) PES of ^{182}Hg. Local minima for the ground state (gs) ($\beta_2 = -0.13$) and excited 0_2^+ state ($\beta_2 = 0.27$) are shown by circles.

where $S_{kk'}$ is the spectroscopic factor given by

$$S_{kk'} = \int_0^{R_m} R^2 dR \int d\hat{R}\, |\mathcal{F}_{kk'}(\mathbf{R})|^2 \approx \sum_{N_\alpha L_\alpha} W^2_{N_\alpha L_\alpha}(\beta_2^{(k)},\beta_2^{(k')}). \tag{6}$$

The matrix element connecting the states in the parent and daughter nuclei by the α-transition operator T is

$$\langle\psi_{k'}^D|\hat{T}|\psi_k^P\rangle \equiv T_{k'k} = \sqrt{\langle S_{kk'}\rangle}. \tag{7}$$

Therefore the theoretical HF is given by the ratio

$$\mathrm{H}_{th}(k) = \left|\frac{\langle\psi_1^D|\hat{T}|\psi_k^P\rangle}{\langle\psi_2^D|\hat{T}|\psi_k^P\rangle}\right|^2 \equiv \left|\frac{T_{1k}}{T_{2k}}\right|^2. \tag{8}$$

To obtain the sp spectra we use the universal parametrization of the Woods-Saxon mean field [35]. As mentioned above, the deformation parameters are provided by PES calculation. In Fig. 1 we give an example of such calculations corresponding to the nucleus ^{182}Hg. One can see in this figure that there are two minima in the PES. The deepest one, i.e., the gs, corresponds to an oblate spheroid with deformation parameter $\beta_2^{(1)} = -0.12$. The 0_2^+ state carries a deformation $\beta_2 = 0.27$, indicating that in this state the nucleus is prolate. We thus confirm a shape coexistence in this case [36]. We determined the wave function amplitudes by solving the BCS equations with the experimental gaps as input parameters.

With these amplitudes we evaluated the spectroscopic factors. We found that these quantities depend strongly upon the structure of the states, that is, upon the deformation parameters. This can be seen in Fig. 2 for the transition ^{180}Hg$(\beta_2^{(1)}) \rightarrow {}^{176}Pt(\beta_2^{(2)}) + \alpha$. Assuming that the overlap

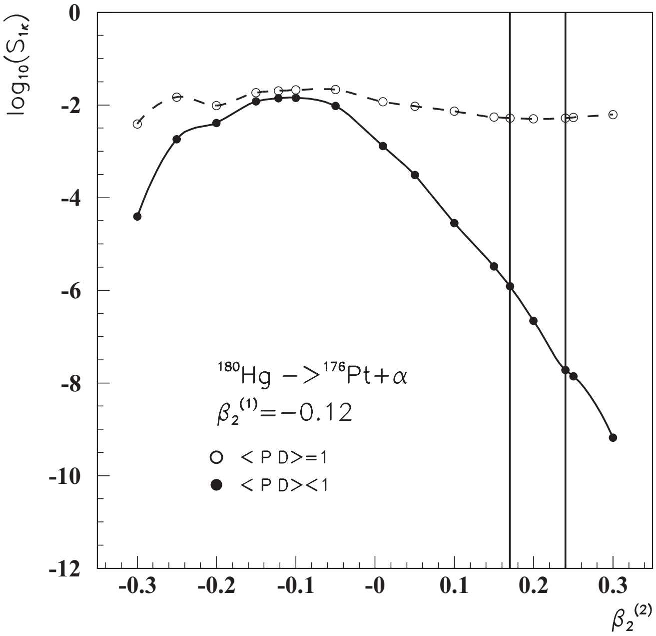

FIG. 2. Spectroscopic factor for the transition ^{180}Hg(gs) $\rightarrow$ ^{176}Pt(0_2^+) + α between BCS states with $k = 1$ in Eq. (4). The parent (initial) state corresponds to an oblate deformation with $\beta_2^{(1)}B = -0.12$. The daughter (final) state carries a deformation $\beta_2^{(2)}$ as given in the abscissa. The dashed curve was obtained by considering the overlap between the parent and daughter BCS states as unity (i.e., $\langle P|D\rangle = 1$) while for the full curve that overlap is the one provided by our calculation. Vertical lines denote the deformations provided by the PES calculation for the minima corresponding to the 0_1^+ ($k' = 1$) and 0_2^+ ($k' = 2$) states.

between initial and final states is $\langle P|D\rangle = 1$, the spectroscopic factor (dashed line in the figure) is nearly a constant, while by using the value of $\langle P|D\rangle$ provided by our calculation there is a huge difference, of more than six orders of magnitude, between the largest and smallest values of the spectroscopic factor in the range of deformations shown in the figure. This difference will also be found in the HF, since it is $\log_{10}\mathrm{H}_{th} = \log_{10}S_{11} - \log_{10}S_{12}$.

The important conclusion of this study is that the investigation of hindrance factors in α-decay processes is a powerful tool to determine even small features in the structure of the states involved in the decay.

But there is even more that one can obtain from the analysis of the experimental values of hindrance factors. To see this we start by describing the parent ($A = P$) and daughter ($A = D$) nuclei as in Refs. [26,28], i.e.,

$$\begin{aligned} |\varphi_1^A\rangle &= X_A|\psi_1^A\rangle + Y_A|\psi_2^A\rangle, \\ |\varphi_2^A\rangle &= -Y_A|\psi_1^A\rangle + X_A|\psi_2^A\rangle,\ \ A = P,D, \end{aligned} \tag{9}$$

with the normalization condition $X_A^2 + Y_A^2 = 1$, i.e.,

$$\begin{Bmatrix} X_A^2 \\ Y_A^2 \end{Bmatrix} = \frac{1}{2} \pm \delta_A. \tag{10}$$

TABLE I. Quadrupole deformation of the 0_1^+ (third column) and 0_2^+ state (fourth column). Experimental HF (fifth column) and theoretical HF for 0_1^+ (sixth column) and 0_2^+ states (seventh column). In the last column is given the mixing parameter (10). The first line for each case corresponds to parent and the second line to daughter nucleus.

No.	Parent Daughter	$\beta_2^{(1)}(0_1^+)$ $\beta_2^{(2)}(0_1^+)$	$\beta_2^{(1)}(0_2^+)$ $\beta_2^{(2)}(0_2^+)$	$\mathrm{H}_{\exp}(1)$	$\mathrm{H}_{th}(1)$	$\mathrm{H}_{th}(2)$	δ_P δ_D
1	^{180}Hg	−0.12		16.1	64.0		0.5
	^{176}Pt	0.17	0.24				0.03
2	^{182}Hg	−0.13	0.27	4.8	0.1	1.1	0.25
	^{178}Pt	0.25	0.18				0
3	^{184}Hg	−0.13	0.25	1.9	0.06	1.7	0.25
	^{180}Pt	0.26	0.18				0
4	^{202}Rn	0.09		25.2	810.9		0.5
	^{198}Po	0.07	−0.15				0.007

The mixing coefficients δ_A can be determined by using the experimental HF's, i.e.,

$$\mathrm{H}_{\exp}(k) = \left|\frac{\langle\varphi_1^D|\hat{T}|\varphi_k^P\rangle}{\langle\varphi_2^D|\hat{T}|\varphi_k^P\rangle}\right|^2. \tag{11}$$

We present in Table I all available experimental data concerning α transitions to excited 0_2^+ states [37]. We give the quadrupole deformations, experimental and theoretical HF's, together with the mixing parameter δ. The first line corresponds to the parent, while the second line corresponds to the daughter nucleus for each case.

The experimental HF's were estimated as the ratio between the decay widths and the corresponding Coulomb penetrabilities at the touching radius [23]. These values are close to those given by the Rasmussen method [27]. We calculated the theoretical HF's by using Eq. (8) with deformation parameters as in Table I. The HF_{th} thus calculated are larger than the corresponding experimental values $\mathrm{HF}_{\exp}$ in the first and last cases, while in the other cases they are smaller. The differences between theory and experiment are very large, as expected given the great sensitivity of the HF to deformations. This is the great feature of HF studies, namely that one can explore tiny features of the wave function which may induce appreciable effects upon the HF.

In our case the experimental values $\mathrm{HF}_{\exp}(1)$ were measured only from the gs of the parent nuclei. The corresponding calculated HF should be very sensitive to the values of the amplitudes in Eq. (9). Assuming in this equation a pure parent nucleus, i.e., assuming $X_P = 1, Y_P = 0$, one obtains $\delta_P = 0.5$. This is exactly the case in the first and last lines of Table I, indicating that in those cases the parent nuclei in their ground states consist of a pure 0_1^+ BCS state.

One can go farther in this analysis by noticing that for pure parent states the daughter mixing parameter can be written as

$$\delta_D = \frac{h}{h^2 + 1},\ \ h \equiv \sqrt{\mathrm{H}_{th}(1)H_{\exp}(1)}, \tag{12}$$

and one sees in Table 1 that indeed for the cases that we discuss here the two states in the daughter nucleus are almost equally shared [confirming Eq. (9)] with $\delta_D \approx 0$.

The transition probabilities from the excited states $P(0_2^+)$ in the second and third cases of Table I are not measured. In order to determine the mixing parameter δ_P in the parent nucleus for these cases we assume a similar situation for daughter states as in the previous two cases, i.e., $\delta_D = 0$. It is worthwhile to mention in this context that in Fig. 12 of Ref. [13] a strong increase of the coexistence not only in the case studied here but also in all the two lowest-lying states in the $^{180-184}$Hg isotopes is seen.

We obtain from Eqs. (9) and (11) the relation

$$\delta_P = \frac{1}{4}\frac{C_2^2 - C_1^2}{C_2^2 + C_1^2},$$
$$C_k = T_{1k}[\sqrt{\mathrm{H}_{\mathrm{exp}}(1)} + 1] - T_{2k}[\sqrt{\mathrm{H}_{\mathrm{exp}}(1)} - 1]. \quad (13)$$

From Table I, and as already pointed out, one sees that in its gs the decaying nucleus is oblate, while the deformation of the excited $P(0_2^+)$ state is prolate and similar to the deformations of the corresponding daughter states $D(0_k^+)$. Therefore the spectroscopic factor for transitions from gs $P(0_1^+)$ is much smaller than for transitions from the excited state $P(0_2^+)$. Thus, one has $C_1 << C_2$ and therefore $\delta_P \approx 0.25$; i.e., in these cases the mixing ratio of parent states $P(0^+)$ is two times smaller than for the previously investigated pure parent cases. This explains why the experimental HF's for pure parent states (cases 1 and 4) are larger by a factor of 4 than the mixed parent states (cases 2 and 3).

In conclusion, we analyzed the α-decay fine structure to excited 0^+ states in Hg and Rn isotopes. We described these states as minima of the PES as provided by a deformed Woods-Saxon plus pairing approach. We estimated the HF of excited states relative to ground states by using the corresponding α-decay spectroscopic factors. We computed the α-decay formation amplitude within the same Woods-Saxon mean field plus pairing potentials as used for the PES calculation. We described the 0_k^+ ($k = 1,2$) mother and daughter wave functions in terms of the corresponding BCS states $|\varphi_k^A\rangle$ [Eq. (9)]. By comparison with experimental HF's we concluded that the states $|\varphi_1^A\rangle$ and $|\varphi_2^A\rangle$ are occupied with equal probabilities in both the ground and 0_2^+ states in the daughter nucleus if the parent nucleus has no excited 0_2^+ state. We also made the important prediction that if the two components are equally shared in the daughter nuclues, then the excited 0_2^+ state in the parent nucleus should have an occupation probability of about 25%.

We have also shown that the investigation of α-decay fine structure is a very powerful tool to probe deformations and shape coexistence in nuclei.

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