Moments of nucleon structure functions at next-to-leading order in baryon chiral perturbation theory

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We obtain leading- and next-to-leading-order predictions of chiral perturbation theory for several prominent moments of nucleon structure functions. These parameter-free results turn out to be in overall agreement with the available empirical information on nearly all of the considered moments, in the region of low momentum transfer ($Q^2 < 0.3 \text{ GeV}^2$). Especially surprising is the situation for the spin polarizability δ_{LT} , which thus far was not reproducible in chiral perturbation theory for proton and neutron simultaneously. This problem, known as the " δ_{LT} puzzle," is not seen in the present calculation.

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I. INTRODUCTION

The recent advent of muonic hydrogen spectroscopy [1] is probing the limits of our understanding of the nucleon's electromagnetic structure. The unveiled discrepancy in the charge radius value between probing the nucleon with muons [1,2] or electrons [3,4] is only 4%, but is of great statistical significance (5 to 8 standard deviations) at the current level of precision. Interestingly enough, the accuracy of both muonic-hydrogen and electron-scattering measurements is limited by the knowledge of subleading effects of nucleon structure, entering through the two-photon exchange (TPE). The main aim of our present studies is to provide predictions for these contributions from first principles using a low-energy effective-field theory of QCD, referred to as the baryon chiral perturbation theory (B χ PT); see, e.g., [5].

In this endeavor we are primarily concerned with the doubly-virtual Compton scattering (VVCS) process which carries all the nucleon structure information of the TPE. Unitarity (optical theorem) relates the imaginary part of the forward VVCS amplitude to nucleon structure functions, and then the use of dispersion relations allows one to write the low-energy expansion of VVCS in terms of moments of structure functions [6]. The low-energy expansion of VVCS can, on the other hand, be directly computed in χ PT. Of course, not all of the moments enter the low-energy expansion of VVCS: either only odd or only even ones do, depending on the structure function. Here we shall present the leading-order (LO) and next-to-leading-order (NLO) B χ PT predictions for the following moments:

$$\alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M_N}{Q^4} \int_0^{x_0} dx \, x F_1(x, Q^2), \qquad (1a)$$

$$\alpha_L(Q^2) = \frac{4\alpha M_N}{Q^6} \int_0^{x_0} dx \, F_L(x, Q^2), \qquad (1b)$$

$$\gamma_0(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} dx \, x^2 g_{TT}(x, Q^2), \quad (1c)$$

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$$\delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} dx x^2 [g_1(x,Q^2) + g_2(x,Q^2)],$$
(1d)

$$\bar{d}_2(Q^2) = \int_0^{x_0} dx \, x^2 [2g_1(x,Q^2) + 3g_2(x,Q^2)], \qquad (1e)$$

$$I_A(Q^2) = \frac{2M_N^2}{Q^2} \int_0^{x_0} dx \, g_{TT}(x, Q^2), \tag{1f}$$

$$\Gamma_1(Q^2) = \int_0^{x_0} dx \, g_1(x, Q^2), \tag{1g}$$

where

$$F_L = -2xF_1 + \left(1 + 4M_N^2 x^2/Q^2\right)F_2,$$
 (2)

$$g_{TT} = g_1 - \left(4M_N^2 x^2/Q^2\right)g_2,\tag{3}$$

and $F_{1,2}$, $g_{1,2}$ are respectively the unpolarized and polarized inelastic structure functions, which depend on the photon virtuality Q^2 and the Bjorken variable $x = Q^2/(2M_N\nu)$, with M_N the nucleon mass and ν the photon energy; x_0 corresponds with an inelastic threshold, such as that of a pion production; α is the fine-structure constant.

These moments have already been the subject of intense experimental studies [7–13], including an ongoing experimental program at Jefferson Laboratory [14,15]; see Ref. [16] for a review. The first four moments have the interpretation of generalized nucleon polarizabilities [6], \bar{d}_2 at high Q^2 represents a color polarizability [17] or a color-Lorentz force [18], I_A is the generalized GDH integral, and Γ_1 is the Bjorken integral.

II. RESULTS AND DISCUSSION

We have computed the VVCS amplitude to next-to-nextto-leading order (NNLO) in the χ PT expansion scheme with pion, nucleon, and $\Delta(1232)$ degrees of freedom, where the Δ -nucleon mass difference $\Delta = M_{\Delta} - M_N \simeq 300$ MeV is an intermediate small scale, viz., the " δ expansion" [19,20].



FIG. 1. One- πN -loop graphs contributing to Compton scattering at $\mathcal{O}(p^3)$. Graphs obtained from these by crossing and time reversal are not shown, but are evaluated too.

This allows us to obtain the LO [i.e., $\mathcal{O}(p^3)$] and NLO [i.e., $\mathcal{O}(p^4/\Delta)$] contributions to the moments listed above. The diagrams we needed to evaluate these two orders are shown in Figs. 1 and 2 respectively. Their detailed description can be found in Ref. [21], where they are worked out for the case of real Compton scattering, i.e., $Q^2 = 0$. The extension to VVCS done in this work is rather tedious and will be discussed elsewhere [22]. Here we only note that the extension to finite Q^2 for the Δ -isobar contributions, arising here at NLO, follows closely Ref. [23]; in particular, the magnetic $\gamma N\Delta$ coupling g_M , entering the first graph of Fig. 2, acquires a dipole form factor. Further details are available in a MATHEMATICA notebook in Ref. [24]. As in [21], there are no free parameters to fit at these orders, hence this calculation is "predictive."

The resulting predictions for the moments of interest are shown in Table I for $Q^2 = 0$, and in Figs. 3–6 as functions of Q^2 . In the figures, the LO B χ PT is given by the red solid curves, while the complete result, including the NLO and the uncertainty estimate (cf. Ref. [23]), is given by the blue bands.

In all the plots, the black dotted curves represent the empirical evaluation using the 2007 version of the Mainz online partialwave analysis of meson electroproduction (MAID) [25,26]. Some of the plots contain data points described in the legends. Other curves represent previous χ PT evaluations, as will be discussed further.

The scalar polarizabilities of the proton and the neutron are shown in Fig. 3. Here the blue dashed lines denote the LO of heavy-baryon (HB) χ PT. It exactly corresponds with the static-nucleon approximation of the LO B χ PT. Given the large differences between the two (HB vs B: blue dashed vs red solid lines), we conclude that the static-nucleon approximation does not work well in any of these cases. The HB result happens to be in remarkable agreement with the data at $Q^2 = 0$, but much less so at finite Q^2 . Furthermore, the agreement is lost in HB when the Δ resonance is included [34], whereas the relativistic result leaves the room for a natural accommodation of the Δ contribution [21]. Comparing the LO and NLO B χ PT results, we see that the Δ contributions are very significant in



FIG. 2. Graphs contributing at $O(p^4/\Delta)$. Double lines denote the propagator of the Δ isobar. Graphs obtained from these by crossing and time reversal are evaluated too.

TABLE I. The NLO $B\chi PT$ predictions for the forward VVCS polarizabilities (at $Q^2 = 0$) compared with the available empirical information. Where the reference is not given, the empirical number is provided by the MAID analysis [25,26], with unspecified uncertainty.

	Proton		Neutron	
	This work	Empirical	This work	Empirical
$\overline{\alpha_{E1}+\beta_{M1}}$	15.12(82)	13.8(4)	18.30(99)	14.40(66)
(10^{-4} fm^3)		Ref. [27]		Ref. [28]
α_L	2.31(12)	2.32	3.21(17)	3.32
(10^{-4} fm^5)		[MAID]		[MAID]
γ_0	-0.93(5)	-1.00(8)(12)	0.05(1)	-0.005
(10^{-4} fm^4)		Ref. [8]		[MAID]
δ_{LT}	1.35(7)	1.34	2.20(12)	2.03
(10^{-4} fm^4)		[MAID]	. ,	[MAID]

the combination $\alpha_{E1} + \beta_{M1}$, but not in α_L . It is known that the $\Delta(1232)$ is not as easily excited by longitudinal photons as it is by magnetic ones; see, e.g., [20].

The spin polarizabilities γ_0 and δ_{LT} are shown in Fig. 4. These quantities deserve a more extensive discussion since they were traditionally hard to reproduce in χ PT. In the case of δ_{LT} this problem became known as the " δ_{LT} puzzle." Obviously our complete result (blue bands) is in a reasonable agreement with the empirical information, so where is the problem?

The δ_{LT} puzzle was first observed in the HB variant of χ PT [31,33,34], which invokes an additional semi-relativistic expansion, in the inverse nucleon mass. Evidently, this expansion works poorly for these quantities; compare the HB

(blue dashed) curves, which only for δ_{LT} are within the scale of the figure, with the corresponding B χ PT calculation (blue bands). First attempts to go beyond HB were done in the infrared-regularized (IR) version of B χ PT [35], which has an incorrect analytic structure (unphysical branch cuts), leading to results shown by the red bands [32]. Having the relativistic result with unphysical analytic structure obviously did not solve the problem—the disagreement of the red bands with the data or the MAID is too large.

More recently, a first B χ PT calculation has appeared [30], shown by the grey bands in the figure. As one can see, for γ_0 it works much better than the HB and IR counterparts. In the lower panel, it seems to resolve the δ_{LT} puzzle for the neutron, albeit at the expense of introducing it for the proton. Indeed, despite having presently no experimental data for the proton, we anticipate them to follow closely to the MAID result, shown by the black dotted line. Again, δ_{LT} would not be reproduced simultaneously for the proton and neutron.

In contrast, the present calculation (blue bands) shows no puzzle in either the proton or the neutron, and hence the question of what exactly is the difference between the two $B\chi$ PT calculations is to be addressed. At the level of πN loops they are equivalent, however the inclusion of the Δ isobar is done in different counting schemes: " δ counting" here vs the "small-scale expansion" in Ref. [30]. In the latter case, more graphs with Δ are included, particularly those with photons coupling to the Δ in the loops. They are the only good candidates to account for the difference between the two calculations. We have checked that our result for the Δ -isobar contribution to δ_{LT} agrees with the expectation from the MAID analysis, where a separate estimate of this contribution can be



FIG. 3. (Color online) Scalar polarizabilities of proton and neutron. Red solid lines and blue bands represent, respectively, the LO and NLO results of this work. Blue dashed line is the LO result in the HB limit. Black dotted lines represents the empirical result of MAID2007 [26]. The data points at $Q^2 = 0$ correspond with Refs. [27] and [28] (red and purple points, respectively) for the proton, and [28] for the neutron. The data point in the left upper panel at $Q^2 = 0.3 \text{ GeV}^2$ is from Ref. [29].



FIG. 4. (Color online) Generalized spin polarizabilities of proton and neutron. Red solid lines and blue bands represent, respectively, the LO and NLO results of this work. Black dotted lines represent MAID2007. Grey bands are the covariant B χ PT calculation of Ref. [30]. The blue dashed line is the $\mathcal{O}(p^4)$ HB calculation [31], off the scale in the upper panels. The red band is the IR calculation [32]. The data points for the proton γ_0 at finite Q^2 are from Ref. [7] (blue dots), and at $Q^2 = 0$ from [8] (purple square). For the neutron all the data are from Ref. [9].

obtained. The corresponding effect in Ref. [30], measured by the difference between the grey and red curves in the figure for δ_{LT} of the proton, is about an order of magnitude larger and has an opposite sign.

the experimental data where available. Going to NLO (i.e., including the Δ) does not change the picture qualitatively in our B χ PT calculation (blue bands). The effect of the Δ is appreciably larger again for the proton in the B χ PT calculation of Bernard *et al.* [30] (grey bands). The $\mathcal{O}(p^4)$ HB χ PT result without explicit Δ 's (blue dashed lines) is in disagreement

We next turn to I_A and \bar{d}_2 moments shown in Fig. 5. The LO result here (red solid line) is already in agreement with



FIG. 5. (Color online) Generalized GDH integral and inelastic part of the d_2 moment. The legend is the same as in the previous figure, except for the $\mathcal{O}(p^4)$ HB result (blue dashed line) which here is from Ref. [33], and the data points which are from Ref. [10] for I_A and Ref. [11] for \bar{d}_2 .



FIG. 6. (Color online) Left panel: $I_1 = (2M_N^2/Q^2) \Gamma_1$ for the proton. Right panel: isovector part of the Bjorken integral. Legend for the curves is as in Fig. 4. Data points for I_1^p are from [7], for Γ_1^{p-n} from [12] (squares) and [13] (dots).

with the experimental data, and in worse agreement with the empirical picture from MAID.

Note that by means of the GDH sum rule, $I_A(0) = -\kappa^2/4$, with κ the anomalous magnetic moment of the nucleon. The χ PT calculations are (at $Q^2 = 0$) fixed to this value due to renormalization, while in the MAID evaluation it comes out differently. This difference can perhaps serve as a rough uncertainty estimate of the MAID evaluation.

The last moment in Eq. (1), Γ_1 , is the first Cornwall-Norton moment of the inelastic spin structure function g_1 , i.e., the inelastic part of the Bjorken integral. The isovector (proton minus neutron) combination for this moment is shown in the right panel of Fig. 6. Here the HB χ PT, the previous [30] and the present B χ PT calculations compare fairly well with the experimental data of Refs. [12,13]. The MAID analysis is in worse agreement.

In the left panel of Fig. 6 we show $I_1 = (2M_N^2/Q^2)\Gamma_1$ for the proton. Here the discrepancy of the B χ PT calculations with the experimental data is most appreciable. At $Q^2 = 0$, this quantity is expressed in terms of the anomalous magnetic moment of the proton: $I_1^p(0) = I_A^p(0) = -\kappa_p^2/4$. The empirical result of MAID is not entirely consistent with this constraint, just as in the case of I_A . However it is consistent with experimental data, leaving one to wonder whether in either of them the integral I_1 is evaluated accurately.

III. CONCLUSION

We conclude by making the connection to the charge radius problem mentioned in the beginning. In a recent paper [5] we presented the leading-order predictions for the proton *polarizability* effect in the Lamb shift of muonic hydrogen. It is based on the same B χ PT framework and the same VVCS amplitude as the present work. The magnitude of the effect turned out to be in agreement with models based on dispersion relations, but not with the results of HB χ PT [36,37]

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which indicate a substantially larger effect. Given that the longitudinal response of the nucleon is predominant in the atoms, we focus on the polarizabilities α_L and δ_{LT} and observe that the difference between B and HB χ PT results is substantial indeed (cf., lower panels in Figs. 3 and 4). It is especially large in the scalar polarizability α_L which is relevant to the Lamb shift; the spin polarizability δ_{LT} may only affect the hyperfine splitting. Thanks to the available empirical information, provided by the MAID analysis, we conclude that the longitudinal response of the nucleon is largely overestimated in HB χ PT.

In overall the B χ PT predictions presented here are in good (within 3 standard deviations) agreement with the empirical information on the moments of nucleon structure functions. The most appreciable disagreement of the present B χ PT calculation with experiment is observed in the integral I_1 . For the first time, the spin polarizability δ_{LT} is reproduced for both the proton and the neutron within a parameter-free (predictive) χ PT calculation, thus potentially closing the issue of the " δ_{LT} puzzle." The latter statement relies of course on the empirical results of MAID for the proton δ_{LT} . The forthcoming measurement at Jefferson Laboratory is expected to provide the data for that observable, hence putting to the test the MAID and present χ PT results.

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