Influence of electronic environment on α decay

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The influence of atomic and conductivity electrons in the metal matrices on the α-decay rate is analyzed. The Coulomb interaction of inner atomic electrons and free electrons with the nuclei is treated in the adiabatic approximation and as a shake-off process, respectively. The role of conductivity electrons is analyzed in the framework of strict collision theory. Simple formulas are derived for the exponential tunneling probability P in the approximation of a short tunneling path compared to characteristic electronic dimensions. It is shown that the pre-exponential factor in the decay rate depends on the energy absorbed by electrons during decay, whereas P is independent of it. The half-life of the nucleus surrounded by electrons is a bit larger ($\sim$1%) than that of the bare nucleus.

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I. INTRODUCTION

Influence of nuclear decay on the surrounding electrons as well as the opposite influence of the electrons on the nuclear processes have been analyzed in many papers. Migdal [1] and Feinberg [2] have been the first to predict the excitation of the inner atomic electrons by sudden alteration of the nuclear charge following its β decay. The most detailed description of such shake-off effect for the K and L electrons has been provided in Ref. [3] and that of the conductivity electrons of metals in Ref. [4]. Note that weakly bound electrons are mainly ejected from the crystal after sudden nuclear decay. Our theory [4] successfully reproduced the low-energy peak in the electron emission spectrum, which has been observed by Kovalik *et al.* [5]. Kondratyev and Bonasera [6], treating excitation of the conductivity electrons due to the electron capture or internal conversion, took into account exponential time dependence of the perturbation operator rather than the stepwise one used in the shake-off approach.

Another interesting phenomenon takes place in the opposite case when the influence of electrons on the nuclei is investigated. In particular, in ionized atoms with free electron levels there were discovered such effects as the bound internal conversion (BIC) (see, e.g., the review [7]) as well as the bound β decay [8,9], which may drastically change the nuclear half-life or even open the decay channels, being forbidden in neutral atoms by the energy conservation law. In a number of nuclei the nuclear excitation by electron transition (NEET) (see, e.g., Ref. [10]) was observed. The most complete theory of NEET is presented in Ref. [11], and its peculiarities in hot plasma are discussed in Ref. [12]. Influence of the electron screening on the Coulomb excitation of nuclei in hot plasma has been analyzed also in Ref. [13].

Numerous experiments on cold fusion (see, e.g., Refs. [14–16]) were successfully explained by the idea that the electronic screening of the Coulomb potential barrier leads to its narrowing and, as a consequence, facilitates penetration of charged particles through the barrier.

In recent years there has been intensive discussion concerning the role of the electronic environment in the β and α decay of the nuclei. Specifically, it was found experimentally that the half-life for the electron capture of ^{7}Be is longer by 0.8% [17], whereas it is 1.2% shorter for the β^{+} of ^{22}Na [18] and 4.0% longer for the β^{-} of ^{198}Au [19], when these nuclei were embedded in the metals cooled to $T = 12$ K (see also the review [20]).

As to the α decay half-life, Erma [21] predicted its decrease owing to atomic electrons. Using the plasma Debye model of the electronic screening, Kettner *et al.* [22] again suggested huge acceleration of the α decay of nuclei embedded in a metal cooled to low temperatures. Such a possibility, which could have great advantages in the processing of radioactive wastes, was verified in many experiments. Raiola *et al.* [23] reported results of the experiment on the α decay of ^{210}Po ($E_\alpha = 5.30$ MeV, $T_{1/2} = 138$ d) located in copper. They found the half-life of ^{210}Po to be shorter by 6.3 ± 1.4% at $T = 12$ K than that at room temperature. These optimistic results have been further confirmed by Dong *et al.* [24], who observed already 24 ± 8% decay acceleration of the ^{210}Po located inside Bi at $T = 4.2$ K. But all other experiments [24–28] on nuclei ^{210}Po, ^{221}Fr, ^{253}Es, ^{224}Rn, and ^{225}Ra, embedded in different metallic matrices, revealed no changes of their half-lives $T_{1/2}$. Recently Pöml *et al.* [29] again declared that $T_{1/2}$ of ^{210}Po implanted in a copper matrix at $T = 293$ K and $T = 4.2$ K remains the same.

It may seem that the conductivity electrons in metals are not significant since their energies are $\sim$1 eV, while for the emitted α particles $E_\alpha \sim 1$ MeV. However, like protons, they carry the unit charge and therefore the electron cloud near the nucleus may considerably distort the nuclear Coulomb potential barrier. Such a potential in the weak screening approximation is written in the form [30]

$$V_c^{(\mathrm{scr})}(r) = V_c(r)e^{-r/r_s}, \tag{1}$$

where $V_c(r) = 2(Z-2)e^2/r$ is the barrier produced by a bare nucleus and r_s represents the screening length. Within the barrier, where the tunneling path of α particle is much shorter than the screening radius r_s, the potential (1) may be approximated by the difference

$$V_c^{(\mathrm{scr})}(r) \approx V_c(r) - U_e, \tag{2}$$

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where the screening energy $U_e = 2(Z-2)e^2/r_s$. Such a representation of the potential is completely equivalent to replacement of the energy E by $E + U_e$ with unchanged potential $V_c(r)$. At first glance, this facilitates the α tunneling.

Zinner [31] noted that the binding energy of the α particle inside the parent nucleus, surrounded by electrons, lowers by the same quantity U_e, which compensates increasing of E. At the same time, the next expansion term of the screening exponent gives small positive correction to $V_c(r)$. Such corrections due to bound atomic electrons were analyzed earlier in Ref. [32]. In both these papers the screening was shown to provide small inhibition of the α decay but not its acceleration, but in both calculations the role of the kinetic energy of electrons has not been accounted for.

Contrary to Ref. [31], Patyk *et al.* [33] tried to compensate the screening energy by the difference of the binding energies of electrons in the fields of the parent and daughter nuclei (see Eq. (8) of Ref. [33]). In doing so they predicted acceleration of the α decay by surrounding electrons.

A more correct approach to the atomic electrons has been proposed by Karpeshin [34]. He described the decay process in the adiabatic approximation, using the fact that inner atomic electrons are much swifter than the emitted α particle. However, a number of details, such as the role of the recoil nucleus and applicability of the proposed monopole approximation, remained unclear.

In determining the temperature effect of the screening by conductivity electrons in metals, all authors for the screening length used the Debye-Hückel classical formula

$$r_{\mathrm{D}} = \sqrt{\frac{k_{\mathrm{B}}T}{4\pi e^2 n_0}}, \tag{3}$$

where n_0 is the average density of conductivity electrons. It is worth noting, however, that this formula has been derived in the high-temperature (weak coupling) approximation [35], when the ratio

$$e\varphi_p(r)/k_{\mathrm{B}}T \ll 1, \tag{4}$$

where $\varphi_p(r) = Ze/r$ is the potential of the parent nucleus. On the contrary, along the tunneling path of the α particle the ratio (4) is of the order of 10^6. Thus, the condition (4) is terribly violated and the Debye model cannot be exploited in the task about the screened α decay.

The Thomas-Fermi quasiclassical model, which in this respect looks much better, predicts the screening radius

$$r_{\mathrm{TF}} = \sqrt{\frac{\varepsilon_{\mathrm{F}}}{6\pi e^2 n_0}}, \tag{5}$$

where ε_{F} is the Fermi energy of electrons.

But there is one more shortcoming peculiar to both models at small distances. Namely, the exponential screening (1) is provided by the electronic charge distribution [13]

$$\rho_e(r) = -\left(\frac{Ze}{4\pi r_s^2}\right)\frac{1}{r}e^{-r/r_s}, \tag{6}$$

which divergences as $r \to 0$.

Therefore in this paper I suggest a straightforward description of the screening by conductivity electrons in metals. I describe such electrons by plane waves scattered by the radioactive defect in a crystal. By using the scattering theory I find the complete electron wave functions and respectively the charge density $\rho_e(r)$. It allows me to find the electric field created by the electron cloud and corresponding distortions of the nuclear Coulomb field.

The analysis of the effect of inner atomic electrons on the α decay is also given in more detail than in Ref. [34]. In particular, simple formulas are derived for the corrections to the effective potential barrier "seen" by the α particle.

In all previous papers it was assumed that the electrons may affect only the exponential tunneling probability through the Coulomb barrier. In the quasiclassical (WKB) approximation I show below that the pre-exponential factor also depends on the electronic environment.

II. α DECAY OF BARE NUCLEI

First I consider the α decay of a bare nucleus. It will give the opportunity to introduce some designations and to understand further the role of the electron environment. As usually the center-of-mass coordinate system of the parent nucleus is used. Let it have the charge and mass numbers Z and A. The interaction $V_{\mathrm{B}}(r)$ of the α particle with a bare nucleus in the range of the nuclear forces is approximated by a square potential well with the depth $-V_0$ and radius R, so that

$$V_{\mathrm{B}}(r) = \begin{cases} -V_0, & 0 \leqslant r < R, \\ V_c(r), & r > R, \end{cases} \tag{7}$$

where $\mathbf{r} = \mathbf{r}_\alpha - \mathbf{R}_d$ is the radius vector of the relative motion of the α particle and the daughter nucleus (d) ($V_{\mathrm{B}}(r)$ is drawn in Fig. 1). Following Ref. [36] I approximate R by a sum of the α particle and the daughter nucleus radii:

$$R = r_0[(A-4)^{1/3} + 4^{1/3}], \tag{8}$$

where the parameter $r_0 = 1.22$ fm.

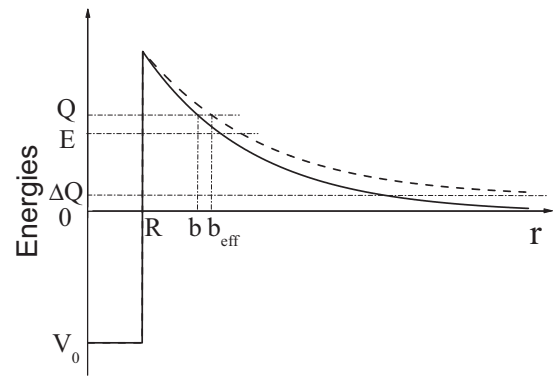

FIG. 1. Sketch of the α-decay energies. Drawn are the potential energy of the relative motion of the α particle and a bare daughter nucleus $V_{\mathrm{B}}(r)$ (full line) and the effective potential energy for the nucleus "dressed in the electron cloud" $V_{\mathrm{eff}}(r)$ (dashed). The Q and $E = Q - \Delta Q$ are the corresponding kinetic energies at infinity, and ΔQ is a part of the nuclear energy absorbed by electrons.

The Hamiltonian of the nuclear system $\hat{H}_{\rm B}$ in the α channel takes the form

$$\hat{H}_{\rm B}(r) = \hat{H}_{\rm in}^{(\alpha)} + \hat{H}_{\rm in}^{(d)} - \frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + V_{\rm B}(r), \tag{9}$$

where $\hat{H}_{\rm in}^{(\alpha)}$ and $\hat{H}_{\rm in}^{(d)}$ are the Hamiltonians for intrinsic motion of the α particle and the daughter nucleus, respectively, and $\mu = M_d M_\alpha/(M_d + M_\alpha)$ is their reduced mass with M_α and M_d being the masses of the α particle and the d nucleus. The eigenvalue of the operator (9), corresponding to ground states of the constituents, will be $(M_d + M_\alpha)c^2 + E$, where E is the kinetic energy of their relative motion. It is related to the kinetic energy of the emitted α particle E_α by $E = [1 + M_\alpha/(M_d + M_\alpha)]E_\alpha$. From the energy conservation law it follows that for the nucleus free of electrons

$$E = Q, \tag{10}$$

where the nuclear energy release $Q = (M_p - M_d - M_\alpha)c^2$ with M_p standing for the mass of the parent nucleus.

The α decay rate is usually written as [36]

$$\lambda = \nu s P, \tag{11}$$

where ν is referred to as the assault frequency of the α particle on the potential barrier and s as the preformation probability. The tunneling probability through the barrier $V_c(r)$

$$P_{\rm B} = e^{-2S(E)} \tag{12}$$

depends on the action determined by the well-known WKB formula [36,37]

$$S(E) = \frac{1}{\hbar}\int_R^b \sqrt{2\mu(V_c(r) - E)}dr, \tag{13}$$

where the outer turning point $b = 2(Z-2)e^2/E$.

III. BASIC EQUATIONS

Now a more careful analysis of the problem, taking into account the electronic environment, will be provided. I follow a conventional description of the atoms, where the electrostatic energy for interaction between the nucleus and electrons is treated as a potential energy, entering the Schrödinger equation for electrons.

While free electrons with the energies $\sim$1 eV are moving slowly compared to the α particle, the inner electrons of the atom with characteristic energies $\sim$10 keV may be treated as a fast subsystem with respect to this particle. Thus, the velocities of free electrons v_e', α particle v_α and internal atomic electrons v_e satisfy the inequality

$$v_e' \ll v_\alpha \ll v_e. \tag{14}$$

In the α channel the Hamiltonian of the whole system (nuclei + bound electrons of the radioactive atom embedded in the metal + free electrons) is represented by the following sum:

$$\hat{\mathcal{H}} = \hat{H}_{\rm B}(r) + \hat{H}_a(\mathbf{r}_e,\mathbf{r}) + \hat{H}_{\rm fe}(\mathbf{r}_e',\mathbf{r}), \tag{15}$$

where $\hat{H}_a(\mathbf{r}_e,r)$ and $\hat{H}_{\rm fe}(\mathbf{r}_e'\mathbf{r})$ are respectively the Hamiltonians of the atomic electrons and free electrons of the conductivity band with coordinates $\mathbf{r}_e$ and $\mathbf{r}_e'$.

The first of them is given by

$$\hat{H}_a(\mathbf{r}_e,\mathbf{r}) = \hat{K}_e + V_{\rm int}(\mathbf{r}_e,\mathbf{r}), \tag{16}$$

where $\hat{K}_e$ is the kinetic energy operator of the electrons and

$$V_{\rm int}(\mathbf{r}_e,\mathbf{r}) = -\sum_e \left(\frac{(Z-2)e^2}{|\mathbf{r}_e - \mathbf{R}_d|} + \frac{2e^2}{|\mathbf{r}_e - \mathbf{r}_\alpha|}\right) \tag{17}$$

is the Coulomb interaction of electrons with the nuclei.

The second operator will be

$$\hat{H}_{\rm fe}(\mathbf{r}_e',\mathbf{r}) = \hat{K}_e + U_{\rm cr}(\mathbf{r}_e') + V_{\rm int}(\mathbf{r}_e',\mathbf{r}), \tag{18}$$

where $U_{\rm cr}(\mathbf{r})$ represents a periodical crystal field.

The wave function which describes the initial state of the system is

$$\Psi_i = g_p \phi_a^{(0)}(\mathbf{r}_e)\chi^{(0)}(\mathbf{r}_e'), \tag{19}$$

where g_p describes the initial state of the parent nucleus and the factors $\phi_a^{(0)}(\mathbf{r}_e)$ and $\chi^{(0)}(\mathbf{r}_e')$ are the eigenfunctions of the operators $\hat{H}_a^{(0)}(\mathbf{r}_e) = \hat{H}_a(\mathbf{r}_e,0)$ and $\hat{H}_{\rm fe}^{(0)}(\mathbf{r}_e') = \hat{H}_{\rm fe}(\mathbf{r}_e',0)$, respectively. Specifically, for free electrons

$$\hat{H}_{\rm fe}^{(0)}(\mathbf{r}_e')\chi^{(0)}(\mathbf{r}_e') = \epsilon\chi^{(0)}(\mathbf{r}_e'). \tag{20}$$

The corresponding energy of the system equals

$$\mathcal{E}_i = M_p c^2 + E_a + \epsilon, \tag{21}$$

where E_a represents the initial energy of atomic electrons and ϵ the initial energy of all electrons of the conductivity band. At $r \to \infty$ one easily finds the eigenvalue of the operator (15), i.e., the final energy $\mathcal{E}_f$ of the whole system:

$$\mathcal{E}_f = (M_d + M_\alpha)c^2 + E_a' + \epsilon' + E, \tag{22}$$

where $E_a' = E_a + \Delta E_a$ and $\epsilon' = \epsilon + \Delta\epsilon$ are the final energies of bound and free electrons, respectively. From the energy conservation law $\mathcal{E}_f = \mathcal{E}_i$, it follows that

$$E = Q - \Delta Q, \tag{23}$$

where the energy absorbed by electrons

$$\Delta Q = \Delta E_a + \Delta\epsilon. \tag{24}$$

In view of the inequality (14) emission of the α particles may be treated as a shake-off process with respect to slow conductivity electrons [4]. As a result, during the α decay the wave function of free electrons $\chi^{(0)}(\mathbf{r}_e')$ remains the same, so that the wave function of the whole system in the α channel is

$$\Psi_f = g_d g_\alpha \Phi(\mathbf{r}_e,\mathbf{r})\chi^{(0)}(\mathbf{r}_e'), \tag{25}$$

where g_d and g_α describe intrinsic motion of the daughter nucleus and the α particle, and the factor $\Phi(\mathbf{r}_i,\mathbf{r})$ describes the atomic electrons coupled to the nucleus and α particle.

The function (25) satisfies the Schrödinger equation

$$\hat{\mathcal{H}}\Psi_f = \mathcal{E}_i\Psi_f, \tag{26}$$

where the final energy $\mathcal{E}_f$ is replaced by $\mathcal{E}_i$ having the same value. By multiplying both sides of Eq. (26) on the left by $\chi^{(0)}(\mathbf{r}_e')^*$, then integrating over $\mathbf{r}_e'$ and taking into account

Eq. (20), one arrives at the equation for the factor Φ:

$$\left\{-\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + V_{\rm B}(r) + \hat{H}_a(\mathbf{r}_e,\mathbf{r}) + \delta\epsilon(r)\right\}\Phi(\mathbf{r}_e,\mathbf{r}) = (Q + E_a)\Phi(\mathbf{r}_i,\mathbf{r}), \quad (27)$$

which contains an additional potential energy of the α particle

$$\delta\epsilon(r) = \langle\chi^{(0)}|\delta V(\mathbf{r}'_e,\mathbf{r})|\chi^{(0)}\rangle, \quad (28)$$

where

$$\delta V(\mathbf{r}'_e,\mathbf{r}) = V_{\rm int}(\mathbf{r}'_e,\mathbf{r}) - V_{\rm int}(\mathbf{r}'_e,0). \quad (29)$$

Equation (27) can be solved in the adiabatic approximation (see, e.g., Ref. [38]), when the function Φ is looked for as a product

$$\Phi(\mathbf{r}_e,\mathbf{r}) = \phi_a(\mathbf{r}_e,\mathbf{r})\psi_\alpha(\mathbf{r}), \quad (30)$$

where the atomic wave function $\phi_a(\mathbf{r}_e,\mathbf{r})$ depends on $\mathbf{r}$ as on the parameter. First by fixing the coordinates of the α particle, i.e., omitting its kinetic energy operator, the Schrödinger equation for fast atomic electrons is solved:

$$\hat{H}_a(\mathbf{r}_e,r)\phi_a(\mathbf{r}_e,\mathbf{r}) = E_a(r)\phi_a(\mathbf{r}_e,\mathbf{r}). \quad (31)$$

Next the energy $E_a(r)$ serves as an additional potential energy in the following equation for the function $\psi_\alpha(r)$:

$$\left\{-\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + V_{\rm eff}(r) - Q\right\}\psi_\alpha(\mathbf{r}) = 0, \quad (32)$$

where the effective potential energy

$$V_{\rm eff}(r) = V_{\rm B}(r) + \delta E_a(r) + \delta\epsilon(r). \quad (33)$$

Here I introduced the notation $\delta E_a(r) = E_a(r) - E_a$.

When $r \to \infty$, the effective potential approximates the constant ΔQ. Then Eq. (32) reduces to

$$-\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}}\psi_\alpha(\mathbf{r}) = E\psi_\alpha(\mathbf{r}) \quad (34)$$

with the kinetic energy E as a difference of Q and ΔQ. Thus, again Eq. (23) is derived.

It is clear that the tunneling probability $P_{\rm at}$ of the α particle through such an effective potential barrier is determined by the same formula (12) but with the action

$$S_{\rm eff}(E) = \frac{1}{\hbar}\int_R^{b_{\rm eff}} \sqrt{2\mu(V_{\rm eff}(r) - Q)}dr \quad (35)$$

with the outer turning point $b_{\rm eff}$ determined by the equation $V_{\rm eff}(b_{\rm eff}) = Q$.

The potential energy can be determined with the accuracy up to arbitrary constant. Therefore one can replace the effective potential by the potential $V(r) = V_{\rm eff}(r) - \Delta Q$, which everywhere is lower than $V_{\rm eff}(r)$ and tends to zero at infinity (see also Ref. [31]).

IV. ROLE OF ATOMIC ELECTRONS

At small r it is convenient to rewrite the operator $\hat{H}_a(\mathbf{r}_e,\mathbf{r})$ as

$$\hat{H}_a(\mathbf{r}_e,r) = \hat{H}_a^{(0)}(\mathbf{r}_e) + \delta V(\mathbf{r}_e,\mathbf{r}), \quad (36)$$

where the perturbation δV is given by Eq. (29) with $\mathbf{r}'_e \to \mathbf{r}_e$. It is convenient to rewrite it as a sum of two terms

$$\delta V_\alpha(\mathbf{r}_e,\mathbf{r}) = -2e^2\sum_e\left(\frac{1}{|\mathbf{r}_e - \mathbf{r}|} - \frac{1}{\mathbf{r}_e}\right) \quad (37)$$

and

$$\delta V_d(\mathbf{r}_e,\mathbf{r}) = -(Z-2)e^2\sum_e\left(\frac{1}{|\mathbf{r}_e - \mathbf{R}_d|} - \frac{1}{\mathbf{r}_e}\right), \quad (38)$$

associated with the α particle and the daughter nucleus, respectively.

The energy shift $\delta E_a(r)$ may be calculated in the first order of the perturbation theory,

$$\delta E_a(r) = \left\langle\phi_a^{(0)}(\mathbf{r}_e)\middle|\delta V(\mathbf{r}_e,\mathbf{r})\middle|\phi_a^{(0)}(\mathbf{r}_e)\right\rangle, \quad (39)$$

on the eigenfunctions $\phi_a^{(0)}(\mathbf{r}_e)$ of the unperturbed operator $\hat{H}_a^{(0)}$. It is done by applying the multipole expansion

$$\frac{1}{|\mathbf{r}_e - \mathbf{r}|} = \sum_{L=0}^{\infty}\frac{r_<^L}{r_>^{L+1}}P_L(\cos\theta). \quad (40)$$

I use the fact that the tunneling path of α particle is much shorter than the radii of the electronic orbits, $b \ll a_0/Z$, where $a_0 = \hbar^2/me^2$ is the Bohr radius. This allows me to calculate $\delta E_a(r)$ in the lowest order of the small parameter $\xi = Zb/a_0$. The monopole term of the expansion for $\delta V_\alpha(\mathbf{r}_e,\mathbf{r})$ takes the form

$$\delta V_\alpha(r_e,r) \approx -2e^2\sum_e\left(\frac{1}{r} - \frac{1}{r_e}\right)\theta(r - r_e), \quad (41)$$

where the Heaviside step function

$$\theta(x) = \begin{cases}1, & x > 0,\\ 0, & x < 0.\end{cases} \quad (42)$$

The value of $\delta E_a(r)$ should be calculated in narrow region $R \leqslant r \leqslant b_{\rm eff}$, where only the s-electrons have considerable density. Inserting their relativistic radial wave functions $f_{ns}(r)$, $g_{ns}(r)$, given in Ref. [39], and the operator (41) in the Eq. (39) one has

$$\delta E_a(r)_s = -4e^2\sum_{n_r}\int_0^r\left(\frac{1}{r} - \frac{1}{r_e}\right)\left(f_{ns}^2(r_e) + g_{ns}^2(r_e)\right)^2 r_e^2 dr_e, \quad (43)$$

where the radial quantum number n_r runs the values $0,1,\ldots,$ and the principal quantum number $n = n_r + 1$. At $r_e \to 0$ the s-wave functions behave as

$$f^2(r_e)_s + g^2(r_e)_s \sim r_e^{2\gamma-2}, \quad (44)$$

where

$$\gamma = \sqrt{1 - \alpha^2 Z^2} \quad (45)$$

and $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant. By means of such functions one finds

$$\delta E_a(r)_s = \sum_{n_r} \frac{\Gamma(2\gamma + n_r + 1)}{\Gamma(2\gamma+1)\Gamma(2\gamma+2)\gamma n_r!} \times \frac{n_r^2+(N+1)^2-2n_r(N+1)\varepsilon}{N(N+1)} \left(\frac{2Z}{Na_0}\right)^{2\gamma+1} e^2 r^{2\gamma}, \quad (46)$$

where $\Gamma(z)$ is the Γ function,

$$N = \sqrt{1+n_r^2+2\gamma n_r}, \quad \varepsilon = \frac{1}{\sqrt{1+\left(\frac{\alpha Z}{\gamma+n_r}\right)^2}}. \quad (47)$$

Here the following property of the Γ functions is used:

$$\Gamma(z+1) = z\Gamma(z). \quad (48)$$

Similar to the case of internal conversion, the main effect here is due to K electrons, for which Eq. (46) simplifies as

$$\delta E_a(r)_{1s} = \frac{2}{\Gamma(2\gamma+2)\gamma}\left(\frac{2Z}{a_0}\right)^{2\gamma+1} e^2 r^{2\gamma}. \quad (49)$$

For $r \leqslant b_{\rm eff}$ the $\delta E_a(r)_s$ calculated above is of the order $\xi^{2\gamma}$. The next terms of the expansion (40) with $L > 0$ and $r > r_e$ are omitted because they produce a correction of higher order in ξ.

Let us consider now the region $r < r_e$. The expansion terms $-2e^2 r^L \sum_e P_L(\theta_e)/r_e^{L+1}$ of (37) with $L = 1,2$ might be most important. However, their averaging on filled electronic shells $|jlm\rangle$ gives nothing since

$$\langle jlm|Y_{L0}(\theta\varphi)|jlm\rangle \sim \sum_{\mu=-l}^{l} \langle Y_{l\mu}(\theta\varphi)|Y_{L0}(\theta\varphi)|Y_{l\mu}(\theta\varphi)\rangle = 0, \quad (50)$$

where $m = \mu + \sigma$, while μ and σ are projections of the orbital angular momentum and spin of the electron on the quantization axis. As to the terms $L > 2$, they are too small due to the inequality $\xi \ll 1$.

The multipole expansion for the operator $\delta V_d(\mathbf{r}_e,\mathbf{r})$, provided by recoil of the daughter nucleus, is also reduced to the monopole term

$$\delta V_d(r_e,r) = -(Z-2)e^2 \sum_e \left(\frac{1}{R_d} - \frac{1}{r_e}\right)\theta(R_d - r_e), \quad (51)$$

where the displacement of the daughter nucleus $R_d = M_\alpha r/(M_d + M_\alpha)$. It can be shown that its contribution into $\delta E_a(r)$ is of the order of $(Z-2)(M_\alpha/M_d)^2 \ll 1$ and hence will be neglected.

Since $\delta E_a(r)$ is always positive, the effective potential barrier in "dressed" atomic nuclei becomes higher than the Coulomb barrier of bare nuclei, which leads to reduction of the tunneling probability, $P_{\rm at} < P_{\rm B}$. The relative change of the corresponding tunneling probabilities is defined by the parameter [34]

$$Y = P_{\rm B}/P_{\rm at} - 1. \quad (52)$$

By using Eq. (49) I calculated Y_{1s} for those nuclei, which have been analyzed previously by Karpeshin [34] and Patyk *et al.* [33]. My results as well as those of Refs. [34] and [33] are presented in Table I. Note that Patyk *et al.* [33] calculated the ratio $\delta\lambda/\lambda \approx -Y$, and therefore all positive values of $\delta\lambda/\lambda$ given in Ref. [33] are listed here with the negative sign.

TABLE I. Reduction of the α decay rate (in %) due to atomic electrons calculated in Refs. [33,34] and here.

Nucleus	Z	Q(MeV)	[34]	[33]	Y_{1s}	$\Delta\mathcal{R}$
^{144}Nd	60	1.905	0.24	−1.6	0.10	0.76
^{214}Rn	86	9.208	0.02		0.03	0.30
^{222}Rn	86	5.59		−0.6	0.09	0.53
^{222}Rn	86	6.39		−0.5	0.03	0.42
^{226}Ra	88	4.871	0.23		0.26	0.81
^{252}Cf	98	6.217	0.28		0.35	0.85
^{241}Es	99	8.320	0.12		0.17	0.55
^{210}Po	84	5.42			0.13	0.57
^{212}Po	84	8.95		−0.3	0.03	0.30
^{147}Sm	62	2.31		−1.4	0.002	0.58
^{148}Sm	62	1.99		−1.6	0.002	0.67
^{213}Fr	87	6.91		−0.5	0.03	0.40
^{220}Fr	87	6.80		−0.5	0.03	0.41

V. ROLE OF CONDUCTIVITY ELECTRONS

I undertake here a tedious but straightforward analysis of the screening by the conductivity electrons. They will be treated in the framework of the simplest model as independent particles moving in the rectangular potential well, whose volume $V = L^3$ coincides with the volume of the crystal (see Ref. [37]). These electrons are described by the functions $|\mathbf{q}\rangle = V^{-1/2}e^{i\mathbf{q}\mathbf{r}'_e}$, where the components of the wave vectors $\mathbf{q}$ run discrete values $2\pi n/L$ (n is an integer), imposed by the periodic boundary conditions. The conductivity electrons are scattered by a local potential well

$$V_a(r) = -\frac{Ze^2}{r}e^{-r/r_s}, \quad (53)$$

created by the radioactive defect. Here the screening is ensured mainly by a dense cloud of the bound atomic electrons, whose characteristic radius according to the Thomas-Fermi model is $r_a = a_0 Z^{-1/3}$ [40]. Therefore following Ref. [13] I set $r_s = r_a$.

The incident wave $|\mathbf{q}\rangle$ gives rise to a scattered spherical outgoing wave. So the complete wave function of such an electron will be $V^{-1/2}\psi^+_{\mathbf{q}}(\mathbf{r}'_e)$, where the function $\psi^+_{\mathbf{q}}(\mathbf{r}'_e)$ may be written as an expansion in terms of the partial waves (see, e.g., [41]),

$$\psi^+_{\mathbf{q}}(\mathbf{r}'_e) = 4\pi i^l e^{i\delta_l(q)} \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \frac{F_l(qr'_e)}{qr'_e} Y^*_{lm}(\vartheta,\varphi)Y_{lm}(\theta,\phi), \quad (54)$$

where ϑ,φ and θ,ϕ are spherical angles of the vectors $\mathbf{q}$ and $\mathbf{r}'_e$, respectively, and $\delta_l(q)$ is the phase shift for the lth partial wave. The radial function $F_l(x)$ with $x = qr'_e$ satisfies the Schrödinger equation

$$\frac{d^2F_l(x)}{dx^2} - \left[\frac{l(l+1)}{x^2} + \frac{2\eta}{x}g(x) - 1\right]F_l(x) = 0, \quad (55)$$

where the screening factor

$$g(x) = \exp(-x/x_0), \quad x_0 = qr_a, \tag{56}$$

the dimensionless Coulomb parameter

$$\eta = -\frac{mZe^2}{\hbar^2 q}, \tag{57}$$

and m is the electron mass.

When $x \ll 1$ the radial function

$$F_l(x) \approx C_l(q)x^{l+1}, \tag{58}$$

where $C_l(q)$ is the amplitude of the wave. In the opposite case, when $x \gg x_0$ and the screened Coulomb potential vanishes,

$$F_l(x)/x \approx \cos\delta_l[j_l(x) - \tan\delta_l n_l(x)], \tag{59}$$

where $j_l(x)$ and $n_l(x)$ are the spherical Bessel and Neumann functions, respectively [41]. The equation (55) with the boundary condition (59) has been solved numerically by iterations (for some details, see Ref. [13]).

The wave function of all the conductivity electrons $\chi^{(0)}(\mathbf{r}'_e)$ is a properly symmetrized product of the functions $V^{-1/2}\psi_{\mathbf{q}}(r'_e)$. By substituting them into Eq. (7) and performing the statistical averaging over the conductivity band one has

$$\delta\epsilon(\mathbf{r}) = -\frac{4e^2}{V}\sum_{\mathbf{q}} \bar{\nu}(\mathbf{q})\langle\psi^+_{\mathbf{q}}|1/|\mathbf{r}-\mathbf{r}'_e| - 1/\mathbf{r}'_e|\psi^+_{\mathbf{q}}\rangle, \tag{60}$$

where

$$\bar{\nu}(q) = \frac{1}{e^{(\varepsilon(q)-\mu)/k_BT}+1}, \tag{61}$$

is the Fermi-Dirac distribution, $\varepsilon(q) = \hbar^2q^2/2m$ is the electron kinetic energy, and μ is the chemical potential. In Eq. (60) I took into account that every level $\mathbf{q}$ can be occupied by two electrons with different spin projections.

The summation over $\mathbf{q}$ in Eq. (60) is replaced further by integration:

$$\frac{1}{V}\sum_{\mathbf{q}} \to \int \frac{d\mathbf{q}}{(2\pi)^3}. \tag{62}$$

As a result, the crystal volume V disappears.

Note that $V_{\rm int}(\mathbf{r}'_e,\mathbf{r})$, defined by Eq. (17), is the potential energy of the α particle in the point $\mathbf{r}$ with the instantaneous electric potential created by the conductivity electrons $-e\sum_e 1/|\mathbf{r}-\mathbf{r}_e|$. Then one can rewrite Eq. (60) as

$$\delta\epsilon(\mathbf{r}) = 2e[\varphi_e(r) - \varphi_e(0)], \tag{63}$$

where $\varphi_e(r)$ is the field averaged over the electron distribution

$$n_e(r) = \frac{1}{4\pi^3}\int d\mathbf{q}\bar{\nu}(q)|\psi_{\mathbf{q}}(\mathbf{r})|^2. \tag{64}$$

Inserting here the expansion (54) and using the equality

$$\sum_{m=-l}^{l}|Y_{lm}(\theta,\phi)|^2 = \frac{2l+1}{4\pi}, \tag{65}$$

one has

$$n_e(r) = \frac{1}{\pi^2}\int_0^\infty dq\,\bar{\nu}(q)\sum_{l=0}^{\infty}(2l+1)\frac{F_l^2(qr)}{r^2}. \tag{66}$$

The potential $\varphi_e(r)$ is related with the spherically symmetric electron distribution by

$$\varphi_e(r) - \varphi_e(0) = 4\pi e\int_0^r \frac{dr'}{r'^2}\int_0^{r'}[n_e(r'') - n_0]r''^2dr''. \tag{67}$$

Having extracted here the average electron density n_0 we impose the boundary condition $\varphi_e(\infty) = 0$ (otherwise $\varphi_e(r)$ diverges at $r \to \infty$). The Eq. (67) can be verified by substitution $\varphi_e(r)$ in the Poisson equation

$$\triangle\varphi_e(r) = 4\pi e[n_e(r) - n_0]. \tag{68}$$

Numerical calculations show that the Fermi distribution can be replaced by its value corresponding to $T = 0$. The resulting error is of the order of the ratio k_BT/ε_F, being much less than unity at room temperature. Thus, the final expression for $\delta\epsilon(\mathbf{r})$ takes the form

$$\delta\epsilon(\mathbf{r}) = \frac{8e^2}{\pi}\int_0^{q_F}dq\int_0^{qr}\frac{dy}{y^2}\int_0^y dx \times\left\{\sum_{l=0}^{\infty}(2l+1)F_l^2(x) - \frac{1}{3}\left(\frac{q_F}{q}\right)^2x^2\right\}, \tag{69}$$

where the Fermi vector q_F is related to the average electron density by [37]

$$q_F = (3\pi^2n_0)^{1/3}. \tag{70}$$

From here the energy shift $\Delta\epsilon = \delta\epsilon(\infty)$ can be immediately obtained . For ^{210}Po solved in copper with density of the conductivity electrons $n_0 = 8.48\times10^{22}$ cm^{-3} and the Fermi energy $\varepsilon_{\rm F} = 7$ eV I got $\Delta\epsilon = 0.96$ keV.

The correction to the effective potential barrier $\delta\epsilon(\mathbf{r})$ at $r \ll b$ is ensured mainly by s wave. In this case Eq. (69) reduces to

$$\delta\epsilon(r) = \frac{4\pi}{3}e^2[n_e(0) - n_0]r^2, \tag{71}$$

where $n_e(0)$ stands for the electron density at the nucleus:

$$n_e(0) = \frac{1}{\pi^2}\int_0^{g_F}C_0^2(q)q^2dq. \tag{72}$$

Numerical estimations show that the energy correction $\delta\epsilon(r)$ results in a small increase of the parameter Y. For ^{210}Po in copper I get $Y = 0.14\%$ instead of the value $Y_s = 0.13\%$, provided only by the atomic s electrons.

And for completeness, I follow a standard approach. The electric potential of the parent nucleus, screened by surrounding electrons, is written as [30]

$$\varphi_p^{(\rm scr)}(r) = \frac{Ze}{r}e^{-r/r_s}, \quad r > R. \tag{73}$$

This potential is a sum of the nuclear potential $\varphi_p(r) = Ze/r$ and the electronic potential

$$\varphi_e(r) = \frac{Ze}{r}(e^{-r/r_s} - 1). \tag{74}$$

Expanding it in power series in r one finds such contribution to the effective potential:

$$\delta\epsilon(r) = \frac{Ze^2}{r_s^2}r, \quad r \leqslant b \ll r_s. \tag{75}$$

The corresponding energy shift at infinity will be

$$\Delta\epsilon = \frac{2Ze^2}{r_s}. \tag{76}$$

Equation (75) gives incorrect linear dependence of $\delta\epsilon(r)$ on the radius in accordance with the remarks following Eq. (6).

For ^{210}Po solved in copper, where $r_{\rm TF} = 5.51 \times 10^4$ fm, the energy shifts $\Delta\epsilon_{\rm TF} = 4.39$ keV and $\Delta Y_{\rm TF} = 0.004\%$, which is close to the exact numerical result $\Delta Y = 0.01\%$. Accepting the Debye model at $T = 300$ K one has much more less screening length $r_{\rm D} = 4.1 \times 10^3$ fm, which results in an energy shift $\Delta\epsilon_{\rm D} = 59.0$ keV that is too large.

VI. COMPLETE WKB CALCULATIONS

In all previous works it was suggested that electrons only affect the penetration probability through the Coulomb barrier. In order to understand the role of other factors I shall trace here the derivation of the α decay width $\Gamma_\alpha = \hbar\lambda$. It is proportional to the squared overlapping integral of the wave function g_p, which describes a doorway state of the parent nucleus, and the wave function $\psi_\alpha(\mathbf{r})$ of the continuous spectrum [42]:

$$\Gamma_\alpha \sim k|\langle\psi_\alpha(\mathbf{r})|g_p\rangle|^2, \tag{77}$$

where k stands for the wave vector of the relative motion of the α particle and the daughter nucleus at infinity, i.e.,

$$k = \sqrt{2\mu E}/\hbar \tag{78}$$

with $E = Q - \Delta Q$. For simplicity, we regard α transitions between ground states of the parent and the daughter nuclei. Then the corresponding s-wave function takes the form

$$\psi_\alpha(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\frac{R_0(r)}{r} \tag{79}$$

and the Scrödinger equation (32) transforms to

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + V_{\rm eff}(r) - Q\right\}R_0(r) = 0. \tag{80}$$

Its regular solution inside the square potential well ($0 \leqslant r < R$) is

$$R_0(r) = \frac{C_\alpha}{\sqrt{\hbar K}}\sin(Kr), \tag{81}$$

where $C_\alpha/\sqrt{\hbar K}$ designates the amplitude of the wave and

$$K = \sqrt{2\mu(V_0 + Q)}/\hbar \tag{82}$$

the wave vector inside the potential well. By making use of standard WKB rules we easily find the $R_0(r)$ under the barrier ($R < r < b_{\rm eff}$) to be a sum of attenuating and growing exponents:

$$R_0(r) = \frac{C_\alpha}{|p(r)|}\left[\frac{1}{2}\sin\alpha\exp\left(-\int_R^r|p(r)|\frac{dr}{\hbar}\right) - \cos\alpha\exp\left(\int_R^r|p(r)|\frac{dr}{\hbar}\right)\right], \tag{83}$$

where the following designations are used:

$$\alpha = KR - \frac{\pi}{4}, \quad p(r) = \sqrt{2\mu(Q - V_{\rm eff}(r))}. \tag{84}$$

If $KR = (n + 3/4)\pi$, where $n = 0, 1, \ldots$, then $\cos\alpha = 0$, and the nucleus has a quasistationary state, characterized by large amplitude of the wave function inside the well and single attenuating exponent under the barrier [42]. Such quasilevel has a very narrow width $\sim e^{-2S}$ and therefore there is too small a probability to find it at low excitation energies of the parent nucleus.

The WKB solution in the outer region ($r > b_{\rm eff}$) is obtained as a superposition of the incoming and outgoing waves. I first match it with the function (83) and then compare it with the asymptotic expression for the wave function [41]

$$R_0(r) \approx \sin(kr + \delta_0), \tag{85}$$

valid in the region $r \gg r_s$. Here δ_0 is the phase shift of the s wave.

The latter procedure enables me to get the amplitude $C_\alpha/\sqrt{\hbar K}$ of the wave function. It can be shown that far from the extremely sharp resonance, when $\cos\alpha \neq 0$, it is given by the expression

$$C_\alpha = \frac{(\hbar k)^{1/2}}{2\cos\alpha}\exp\{-S_{\rm eff}(Q)\}. \tag{86}$$

By substituting Eq. (86) into Eq. (77), one gets for the "dressed" nucleus

$$\Gamma_\alpha \sim (k^2/K)\exp(-2S_{\rm eff}(Q)). \tag{87}$$

For a bare nucleus the $S_{\rm eff}(Q))$ is to be replaced by $S(Q)$ as well as E by Q in Eq. (78).

Thus, the decay rate $\lambda \sim E$, where E is a sum of the kinetic energies of the daughter nucleus and α particle at infinity. Respectively, the ratio of half-periods of the nucleus in the electronic environment and the bare nucleus takes the form

$$\mathcal{R} = \frac{T^{\rm at}_{1/2}}{T^{\rm B}_{1/2}} = (1 + Y)\left(\frac{Q}{Q - \Delta Q}\right), \tag{88}$$

where Y is defined by Eq. (52). Since $Y \ll 1$ and $\Delta Q \ll Q$, the relative change of the half-lives $\Delta\mathcal{R} = \mathcal{R} - 1$ becomes

$$\Delta\mathcal{R} \approx Y + \Delta Q/Q. \tag{89}$$

So $\Delta\mathcal{R}$ contains one more contribution $\Delta Q/Q$ due to pre-exponential factors. Using experimental values of the atomic binding energies [43] I calculated ΔQ and $\Delta\mathcal{R}$ for a few nuclei. The values of $\Delta\mathcal{R}$ are presented in Table I, where it is seen that $\Delta Q/Q$ exceeds Y.

VII. CONCLUSION

The internal energy of the parent nucleus $M_p c^2$ is shared between the emitted α particle, daughter nucleus, and electrons. From the energy conservation law it follows that the sum of kinetic energies of the α particle and the daughter nucleus E lowers compared to the nuclear energy release Q by the energy ΔQ absorbed by electrons during the decay. This energy shift is $\Delta Q = \Delta E_a$ in insulators and $Q = \Delta E_a + \Delta\epsilon$ in metallic matrices. For polonium with the atomic energy $E_a = -379.72$ keV decaying to lead with $E'_a = -355.93$ keV [43] the difference ΔE_a amounts 23.78 keV, while the difference of energies of free electrons of copper in the final and initial

states $\Delta\epsilon \approx 1$ kev. Thus, the energy of α particles emitted by the nuclei embedded in a metal is less by small quantity $\Delta\epsilon$ than their energy in the case of a dielectric target. Such energy reduction can, in principle, be observed in those crystals, which demonstrate the Mott transition from the insulating state to metallic one with decreasing temperature [37].

The energy shift ΔQ disappears from Eq. (32) for the wave function of the α particle $\psi_\alpha(\mathbf{r})$. This result completely agrees with the conclusion of Zinner [31]. Note that Denisov and Khudenko [44] earlier a priori inserted Q instead of E in the formula for the action (13), which allowed them to improve agreement with the experiment.

I have shown that the pre-exponential factor in the decay constant λ occurs to be connected with the electronic environment. However strange it may seem, the nucleus starting the decay already "knows" the final stage of the decay process. More definitely, it "knows" which kinetic energy E the α particle together with the recoil nucleus will have at infinity. This "knowledge" is reflected in dependence of the decay constant λ on E and respectively on ΔQ. Just such a dependence provides a main contribution into the relative change $\Delta\mathcal{R}$ of the half-lives owing to atomic electrons.

Another contribution into $\Delta\mathcal{R}$, which was previously discussed in Ref. [34], is ensured by corrections $\delta E_a(r)$ to the Coulomb barrier $V_c(r)$, which are small along the tunneling path. In the adiabatic approximation I derived simple analytic formulas (46) and (49) for such corrections caused by atomic electrons. Besides, strict calculations in the framework of the collision theory have shown that the conductivity electrons only provide small additions $\delta\epsilon(r)$. Its temperature dependence is of the next order of smallness since $k_B T \ll \epsilon_F$. Thus, there is no possibility to speed up the α decay in a cooled metal matrix. Such a statement supports the experimental results [24–28,45]. At the same time, the situation may alter in dense stars, when density of free electrons at the nucleus is high. In this case just free electrons become responsible for inhibition of the nuclear decay.

Thus, one can conclude that the electronic environment slightly decreases the α-decay rate (typical relative change of the half-lives for the "dressed" and bare nuclei is somewhat less than 1%).

I have also shown that standard Debye-Hückel and Thomas-Fermi models cannot be applied for description of the α decay.

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