

Forward sum rule for the 2γ -exchange correction to the charge-radius extraction from elastic electron scattering

Mikhail Gorchtein

Institut für Kernphysik, Universität Mainz, 55128 Mainz, Germany

(Received 13 June 2014; revised manuscript received 13 August 2014; published 24 November 2014)

Two-photon-exchange (TPE) contributions to elastic electron-proton scattering in the forward regime in leading logarithmic $\sim t \ln |t|$ approximation in the momentum transfer t are considered. The imaginary part of the TPE amplitude in the forward kinematics is related to the total photoabsorption cross section. The real part of the TPE amplitude is obtained from an unsubtracted fixed- t dispersion relation. This allows a clean prediction of the real part of the TPE amplitude at forward angles with the leading term $\sim t \ln |t|$. Numerical estimates are comparable with or exceed the experimental precision in extracting the charge radius from the experimental data.

DOI: [10.1103/PhysRevC.90.052201](https://doi.org/10.1103/PhysRevC.90.052201)

PACS number(s): 11.55.Hx, 13.40.Gp, 13.60.Fz, 13.60.Hb

Nucleon structure has been studied with elastic electron scattering since the 1950's. By means of the Rosenbluth separation the measurement of the unpolarized cross section allows us to extract the electromagnetic form factors of the nucleon. The interest in measuring the elastic cross section at low (negative) t is, e.g., the extraction of the slope of the electric Sachs form factor G_E that is related to the charge radius R_E as

$$G_E(t \rightarrow 0) = 1 + R_E^2 t / 6 + O(t^2). \quad (1)$$

A recent measurement at Mainz [1] led to the most precise ($\lesssim 1\%$) determination of the proton charge radius with electron scattering experiments to date,

$$R_E^p = 0.879 \pm 0.008 \text{ fm}, \quad (2)$$

where the uncertainty quoted above represents combined statistical, systematic, model-dependent, and group-dependent uncertainties defined in Ref. [1]. Proton charge radius is extracted from hydrogen spectroscopy data with even higher precision [2]

$$R_E^p = 0.8775 \pm 0.0051 \text{ fm}, \quad (3)$$

the two methods delivering results that are in a very nice agreement. The recent Lamb shift measurements in muonic hydrogen [3,4] led to an extraction of the proton charge radius that is ten times more precise,

$$R_E^p = 0.84087 \pm 0.00039 \text{ fm}, \quad (4)$$

and differs by seven standard deviations from the value obtained with electronic hydrogen and in scattering experiments. In the context of the ‘‘proton radius puzzle,’’ as this discrepancy was coined in the literature, nucleon structure-dependent corrections to the Lamb shift, most notably the two-photon exchange (TPE) correction, underwent a renewed scrutiny with two methods that provide a controlled estimate of the systematic uncertainty of such a calculation: the dispersion relations [5–7] and within effective theories [8–11]; however, the discrepancy is still present. For electron scattering, dispersion relations have the potential to provide model-independent calculations of the TPE effect [12–14], although these references only account for the ground-state

contribution to TPE. I refer the reader to a recent review of the TPE effects in electron scattering [15].

In this work, I reexamine the two-photon-exchange correction to elastic electron scattering at low momentum transfer. I consider the elastic electron-proton scattering process $e(k) + p(p) \rightarrow e(k') + p(p')$ for which I define $P = (p + p')/2$, $K = (k + k')/2$, and $\Delta = k - k' = p' - p$ and choose the invariants $t = \Delta^2 = -Q^2 < 0$ and $\nu = (PK)/M$ as the independent variables, where M denotes the nucleon mass, and the electron mass m_e is neglected. They are related to the Mandelstam variables $s = (p + k)^2$ and $u = (p - k')^2$ through $s - u = 4M\nu$ and $s + u + t = 2M^2$. The usual polarization parameter ε is related to the invariants ν and t as

$$\varepsilon = \frac{\nu^2 - M^2 \tau (1 + \tau)}{\nu^2 + M^2 \tau (1 + \tau)}, \quad (5)$$

with $\tau = -t/(4M^2)$. Elastic scattering of a massless electron off a spin-1/2 target in the Born (one-photon exchange, OPE) approximation is described by the familiar Dirac and Pauli form factors F_1 and F_2 , respectively,

$$T_B = \frac{e^2}{-t} \bar{u}(k') \gamma_\mu u(k) \bar{u}(p') \left[F_1 \gamma^\mu + F_2 \frac{i \sigma^{\mu\alpha} \Delta_\alpha}{2M} \right] u(p). \quad (6)$$

The unpolarized cross section is

$$\frac{d\sigma}{d\Omega_{\text{lab}}} = \frac{4\alpha^2 \cos^2 \frac{\Theta}{2}}{t^2} \frac{E'^3}{E} \sigma_R, \quad (7)$$

with Θ the electron laboratory scattering angle and $E(E')$ the incoming (outgoing) electron laboratory energy. The reduced cross section σ_R is expressed in terms of electric and magnetic Sachs form factors $G_E = F_1 - \tau F_2$ and $G_M = F_1 + F_2$, respectively, as

$$\sigma_R = \left[G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right] / (1 + \tau). \quad (8)$$

Before going on to discuss the two-photon exchange I wish to determine the level of accuracy that modern experiments set for this calculation. To this end, the reduced cross section taken in the Born approximation can be expanded in a Taylor series in powers of negative t . Keeping the linear terms in this

expansion, I write

$$\sigma_R^B = 1 + \frac{1}{3}R_E^2 t - \frac{t\mu_p^2}{4M^2\varepsilon} + \frac{t}{4M^2} + O(t^2), \quad (9)$$

with $\mu_p = G_M^p(0) \approx 2.793$ the proton magnetic moment in units of the nuclear magneton. Correspondingly, the 1% relative uncertainty in the charge radius is translated into the uncertainty in the reduced cross section

$$\delta\sigma_R^B = \frac{1}{3}R_E^2 \frac{2\delta R_E}{R_E} |t| \approx 0.120 \frac{|t|}{\text{GeV}^2}. \quad (10)$$

For the smallest values of $|t|$ accessed in the A1 experiment, $|t|_{\min} = 4 \times 10^{-3} \text{ GeV}^2$, the relative uncertainty of σ_R is of order 5×10^{-4} , similar to the natural size of the order α_{em} correction, $\sim \alpha_{\text{em}}/(4\pi)$. Most order α_{em} corrections can be calculated quite reliably, the exception being the two-photon exchange. The latter is included approximating the TPE graph by only the ground-state contribution that is furthermore approximated according to Mo and Tsai [16] or Maximon and Tjon [17], as well as the so-called Feshbach correction [18], leading to a generic result $\delta\sigma_R^{\text{OPE+R.C.}} = (1 + \delta)\sigma_R^B$ with the correction $\delta \sim \alpha_{\text{em}}$. We discuss the two corrections in more detail later. An inclusion of the general nucleon structure in the TPE is complicated and is only possible in forward kinematics as, e.g., in the calculation of the polarizability correction to the Lamb shift. It is possible to show that such inelastic contributions should vanish for $t = 0$, but can lead to the behavior $t \ln |t|$. This behavior was obtained in Ref. [19], which concentrated on the high-energy regime. Terms $\sim t \ln |t|$ introduce a substantial nonlinearity of the reduced cross section as a function of t at low t , the opposite to the OPE contribution in Eq. (9) that becomes more linear at lower t . The present work is dedicated to assessing this correction in the kinematics of the relevant experiments, from a few hundred MeV to a few GeV beam energy and $|t| \lesssim 0.1 \text{ GeV}^2$. The same approach is expected to be relevant for the measurement of the deuteron charge radius in elastic eD scattering, and the respective estimates will also be presented.

In the presence of the TPE effects, and in the approximation of small electron mass, the elastic ep -scattering amplitude is given by three scalar amplitudes $\tilde{F}_i(\nu, t)$,

$$T = \frac{e^2}{-t} \bar{u}(k') \gamma_\mu u(k) \times \bar{u}(p') \left[\tilde{F}_1 \gamma^\mu + \tilde{F}_2 \frac{i\sigma^{\mu\alpha} \Delta_\alpha}{2M} + \tilde{F}_3 \frac{\not{K} P^\mu}{M^2} \right] u(p), \quad (11)$$

In the OPE approximation, the known Dirac and Pauli form factors are recovered, $\tilde{F}_{1,2}^{\text{OPE}}(\nu, t) = F_{1,2}(t)$; while the third structure is absent, $\tilde{F}_3^{\text{OPE}} = 0$. I separate the TPE effects explicitly,

$$\begin{aligned} \tilde{F}_{1,2} &= F_{1,2} + \delta\tilde{F}_{1,2}, \\ \tilde{G}_{E,M} &= G_{E,M} + \delta\tilde{G}_{E,M}, \end{aligned} \quad (12)$$

where the generalization of the Sachs form factors was introduced, $\tilde{G}_E = \tilde{F}_1 - \tau\tilde{F}_2$ and $\tilde{G}_M = \tilde{F}_1 + \tilde{F}_2$. In the presence

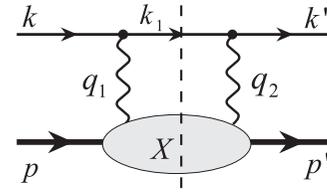


FIG. 1. Imaginary part of the 2γ -exchange diagram.

of TPE effects, reduced cross section σ_R reads

$$\begin{aligned} \sigma_R &= \frac{G_E^2 + \tau G_M^2/\varepsilon}{1 + \tau} + \frac{2G_E}{1 + \tau} \text{Re} \left(\delta\tilde{G}_E + \frac{\nu}{M} \tilde{F}_3 \right) \\ &+ 2 \frac{\tau}{\varepsilon(1 + \tau)} G_M \text{Re} \left(\delta\tilde{G}_M + \varepsilon \frac{\nu}{M} \tilde{F}_3 \right) \\ &= \frac{G_E^2 + \tau G_M^2/\varepsilon}{1 + \tau} + \frac{2G_E}{1 + \tau} \text{Re} \left(\delta\tilde{G}_E + \frac{\nu}{M} \tilde{F}_3 \right) + O(\alpha t^2). \end{aligned} \quad (13)$$

It is straightforward to see that the TPE effect on the unpolarized cross section at low t depends on the same combination of the amplitudes as the elastic amplitude averaged over nucleon spins,

$$\begin{aligned} \bar{T}_{2\gamma} &= \frac{e^2}{-t} \bar{u}(k') \gamma_\mu u(k) \\ &\times \frac{\text{Tr}(\not{p}' + M) [\delta\tilde{F}_1 \gamma^\mu + \delta\tilde{F}_2 \frac{i\sigma^{\mu\alpha} \Delta_\alpha}{2M} + \tilde{F}_3 \frac{K P^\mu}{M^2}]}{8M} (\not{p} + M) \\ &= \frac{e^2}{-t} \bar{u}(k') \not{p} u(k) \left[\delta\tilde{G}_E + \frac{\nu}{M} \tilde{F}_3 \right] \equiv \frac{e^2}{-t} \bar{u}(k') \not{p} u(k) \Phi(\nu, t). \end{aligned} \quad (14)$$

The imaginary part of the TPE diagram in Fig. 1 is given by the phase-space integral

$$2\text{Im}T_{2\gamma} = e^4 \int \frac{d^3\vec{k}_1}{(2\pi)^3 2E_1} \frac{\ell_{\mu\nu} \cdot \text{Im}W^{\mu\nu}}{(q_1^2 + i\epsilon)(q_2^2 + i\epsilon)}, \quad (15)$$

where the leptonic tensor is given by

$$\ell_{\mu\nu} = \bar{u}(k') \gamma_\nu (\not{k}_1 + m_e) \gamma_\mu u(k) \approx \bar{u}(k') \gamma_\nu \not{k}_1 \gamma_\mu u(k), \quad (16)$$

and the on-shell condition for the intermediate electron leads to $E_1 = (\vec{k}_1^2 + m_e^2)^{1/2} \approx |\vec{k}_1|$. The hadronic tensor can be split into elastic and inelastic contributions, $W^{\mu\nu} = W_{\text{el}}^{\mu\nu} + W_{\text{inel}}^{\mu\nu}$. This separation is possible because the former has a pole, $\text{Im}W_{\text{el}} \sim \delta((p + q_1)^2 - M^2)$, whereas the latter has a unitarity cut starting at the pion production threshold $(p + q_1)^2 = (M + m_\pi)^2$.

The imaginary part of the elastic part is due to the on-shell nucleon in the intermediate state,

$$\begin{aligned} \text{Im}W_{\text{el}}^{\mu\nu} &= 2\pi \delta((P + K - k_1)^2 - M^2) \\ &\times \bar{u}(p') \Gamma^{*\nu}(q_2) (\not{P} + \not{K} - \not{k}_1 + M) \Gamma^\mu(q_1) u(p), \end{aligned} \quad (17)$$

with $\Gamma^\mu(\Delta) = F_1(\Delta^2) \gamma^\mu + F_2(\Delta^2) i\sigma^{\mu\alpha} \Delta_\alpha / (2M)$ the on-shell nucleon electromagnetic vertex. It contains the infrared (IR) divergent part that is logarithmic in the fictitious photon mass, $\sim \ln \lambda^2$; the coefficient in front of it is model independent

and has been calculated in Refs. [16,17] using the soft photon approximation in the loop. Reference [16] used the approximation $q_1 \approx 0, q_2 \approx \Delta$ and vice versa both in the numerator and the denominator of the integral, the result simply factorizing the one-photon exchange (Born) amplitude as

$$\text{Im}\Phi^{(a)} = -\frac{\alpha E_{\text{c.m.}}^2}{\pi} G_E(t) \int \frac{d\Omega_1}{q_1^2} = \alpha \ln\left(\frac{4E_{\text{c.m.}}^2}{\lambda^2}\right) G_E(t), \quad (18)$$

with c.m. energy of the electron $E_{\text{c.m.}} \approx (s - M^2)/2\sqrt{s}$, neglecting the electron mass. On the other hand, Ref. [17] applied the soft photon approximation in the numerator only, leading to

$$\text{Im}\Phi^{(b)} = \frac{-\alpha t E_{\text{c.m.}}^2}{2\pi} G_E(t) \int \frac{d\Omega_1}{q_1^2 q_2^2} = \alpha \ln\left(\frac{-t}{\lambda^2}\right) G_E(t). \quad (19)$$

The real part is obtained from a dispersion relation at fixed t ,

$$\text{Re}\Phi(v, t) = \frac{2v}{\pi} \mathcal{P} \int_{v_0^{\text{el}}}^{\infty} \frac{dv'}{v'^2 - v^2} \text{Im}\Phi(v', t), \quad (20)$$

with $v_0^{\text{el}} = t/(4M) \leq 0$ the threshold for the s -channel unitarity cut. The evaluation of the dispersion integral with the imaginary part of Eq. (19) yields

$$\text{Re}\Phi^{(b)} = \frac{\alpha}{\pi} \ln\left(\frac{-t}{\lambda^2}\right) G_E(t) \ln\left(\frac{4Mv + t}{4Mv - t}\right). \quad (21)$$

While the imaginary part of $\Phi^{(b)}$ behaves as $\ln(-t/\lambda^2)$, its real part is suppressed by an extra power of t coming from the second logarithm.

The result of Eq. (21) was used in the analysis of the low- t data from Mainz [1] (without the low- t approximation), and I use the IR part of the TPE amplitude in this form to define the IR finite part of the elastic box as

$$\Phi_F^{\text{el}} \equiv \Phi^{\text{el}} - \Phi^{(b)}, \quad (22)$$

which should be added to the full set of radiative corrections included in the experimental analysis. A straightforward calculation using the hadronic tensor of Eq. (17), the leptonic tensor of Eq. (16), and the definition of the amplitude Φ in Eq. (14), obtains

$$\begin{aligned} \text{Im}\Phi_F^{\text{el}}(v, t) &= \frac{-\alpha t E_{\text{c.m.}}^2}{2\pi} \int \frac{d\Omega_1}{q_1^2 q_2^2} \\ &\times \left\{ F_{11} F_{12} - F_1 + \frac{q_1^2 F_{12} F_{21} + q_2^2 F_{11} F_{22} - t F_2}{4M^2} \right. \\ &\left. + \frac{t - q_1^2 - q_2^2}{8M^2} \left[\mu_p^2 - 1 - \frac{4sM^2}{(s - M^2)^2 + st} \right] \right\}, \quad (23) \end{aligned}$$

where terms that cannot lead to $t \ln t$ -behavior were dropped. For compactness, the shorthand notation $F_{ij} = F_i(q_j^2)$ and

$F_i = F_i(t)$ was introduced. The above integral is IR finite because it depends on one master integral over the solid angle of the intermediate electron,

$$\int d\Omega_1 \frac{t - q_1^2 - q_2^2}{q_1^2 q_2^2} = \frac{2\pi}{E_{\text{c.m.}}^2} \ln\left(\frac{4E_{\text{c.m.}}^2}{-t}\right). \quad (24)$$

Expanding the form factors under the integral as $F_i(q^2) = F_i(0) + q^2 F_i'(0) + \dots$ produces

$$\text{Im}\Phi_F^{\text{el}} = \frac{\alpha t}{2} \ln\left(\frac{4E_{\text{c.m.}}^2}{-t}\right) \left[\frac{1}{4E_{\text{c.m.}}^2 + t} + \frac{R_E^2}{3} - \frac{\mu_p^2 - 1}{4M^2} \right], \quad (25)$$

where I used the relation $F_1'(0) = R_E^2/6 - F_2(0)/(4M^2)$. The real part is obtained according to Eq. (20),

$$\text{Re}\Phi_F^{\text{el}}(v, t) \approx \frac{\alpha\pi}{2} \left[\frac{\sqrt{-t}}{2v + \sqrt{-t}} + t \left(\frac{R_E^2}{3} - \frac{\mu_p^2 - 1}{4M^2} \right) \right]. \quad (26)$$

The first term in the square bracket is the well-known Feshbach correction [18]. I see this calculation as a useful cross-check for the method of isolating the leading- t behavior (the Feshbach term was also found in a similar manner in Refs. [12,15]). The second term was recently found in Ref. [20] which, however, missed the third term. The missed term amounts to -0.075 fm^2 which is not small if compared to $R_E^2/3 \approx 0.255 \text{ fm}^2$. Moreover, due to the approximations made, the term $\sim t$ is not model independent: other terms without the logarithmic behavior were omitted, but they would contribute at the same order. Therefore, while I support the statement of Ref. [20] that the Feshbach correction alone is not enough to warrant the precision of the charge-radius extraction in Bernauer *et al.*, inclusion of the correction $\sim \alpha\pi r_E^2$ is also not sufficient. The main message to take home from this exercise is that the method allows one to obtain the leading- t behavior. In what follows I show that the term $\sim t \ln t$ that arises from the inelastic states in the box can be calculated model independently, as well.

Now I turn to the inelastic contribution and study first the two representative integrals over the solid angle,

$$\begin{aligned} I_1 &= \int \frac{d\Omega_1}{q_1^2 q_2^2} = \frac{2\pi}{-t(E_{1\text{c.m.}})^2} \ln\left(\frac{(E_{1\text{c.m.}})^2 - t}{(E_{\text{c.m.}} - E_{1\text{c.m.}})^2 m_e^2}\right), \quad (27) \\ I_2 &= -\int d\Omega_1 \left[\frac{1}{q_1^2} + \frac{1}{q_2^2} \right] = \frac{2\pi}{E_{\text{c.m.}} E_{1\text{c.m.}}} \ln\left(\frac{4E_{1\text{c.m.}}^2}{m_e^2}\right), \end{aligned}$$

where the c.m. energy of the intermediate electron is distinct from the external electron energy, $E_{1\text{c.m.}} = (s - W^2)/2\sqrt{s}$, and the invariant mass squared of the intermediate hadronic system, $W^2 = (p + q_1)^2$, lies above the pion production threshold, $W^2 \geq W_\pi^2 = (M + m_\pi)^2$. Due to this threshold, the IR divergence is absent. However, the collinear divergence (emission of an energetic real photon collinear to the electron

line) would be possible if the electron were massless. Keeping the finite mass of the electron makes the individual integrals finite, but a potential chiral divergence is introduced. It cannot appear in the final result, and one should expect this logarithmic dependence on the electron mass to vanish. I will be looking for the leading- t behavior that is expected to be $\sim \ln t$, and that behavior can only come from the integral I_1 . I will keep the integral I_2 to cancel the $\ln m_e^2$ dependence but neglect terms $\sim t$.

The spin-averaged part of the hadronic tensor with real photons in general (non-forward) kinematics is expressed in terms of two scalar amplitudes $f_{1,2}(P \cdot q_1, t)$ [21]

$$W^{\mu\nu} = f_1[(q_1 q_2)g^{\mu\nu} - q_1^\nu q_2^\mu] + f_2[(P q_1)^2 g^{\mu\nu} + (q_1 q_2)P^\mu P^\nu - (P q_1)(P^\mu q_1^\nu + P^\nu q_2^\mu)]. \quad (28)$$

Making use of the relation

$$\bar{u}(k')\not{k}_1 u(k) = \frac{t \binom{Pk_1}{PK} - q_1^2 - q_2^2}{4(PK)} \bar{u}(k')\not{P} u(k), \quad (29)$$

performing tensor contraction and consistently neglecting terms $\sim t q_{1,2}^2$ and $\sim q_1^2 q_2^2$ in the numerator, the imaginary part of the TPE amplitude can be cast in the form

$$\text{Im} T_{2\gamma} = e^4 \bar{u}(k')\not{P} u(k) \int \frac{d^3 \vec{k}_1}{(2\pi)^3 2E_1} \frac{1}{q_1^2 q_2^2} \times \frac{(PK)^2 + (Pk_1)^2}{2(PK)} \left[t \binom{Pk_1}{PK} - q_1^2 - q_2^2 \right] \text{Im} f_2, \quad (30)$$

while the amplitude f_1 does not contribute at the leading logarithm accuracy. According to the power counting used throughout this calculation, $\text{Im} f_2(P \cdot q_1, t, q_1^2, q_2^2)$ should be taken at $t = q_1^2 = q_2^2 = 0$. In these kinematics, the optical theorem relates this imaginary part to the total real photoabsorption cross section σ_T as

$$\text{Im} f_2(P \cdot q_1, 0, 0, 0) = -2\sigma_T / [(P q_1) e^2]. \quad (31)$$

Using the definition of Eq. (14) and identifying the solid angle integrals in Eq. (30) with the previously introduced $I_{1,2}$ in Eq. (27), one can express the leading logarithm contribution to the imaginary part of the elastic ep -scattering amplitude near the forward direction as

$$\text{Im} \Phi = \frac{-t}{4\pi^2} \int_{E_\pi}^E \frac{d\omega}{\omega} \sigma_T(\omega) \ln \left(\frac{4\omega_{\text{c.m.}}^2}{-t} \right) \left[1 - \frac{\omega}{E} + \frac{\omega^2}{2E^2} \right], \quad (32)$$

with $\omega = (W^2 - M^2)/2M$ the laboratory real photon energy, and $\omega_{\text{c.m.}} = M\omega/\sqrt{s}$ the c.m. photon energy. The dispersion integral starts from the pion threshold,

$$\text{Re} \Phi(E, t) = \frac{2E}{\pi} \mathcal{P} \int_{E_\pi}^{\infty} \frac{dE'}{E'^2 - E^2} \text{Im} \Phi(E', t). \quad (33)$$

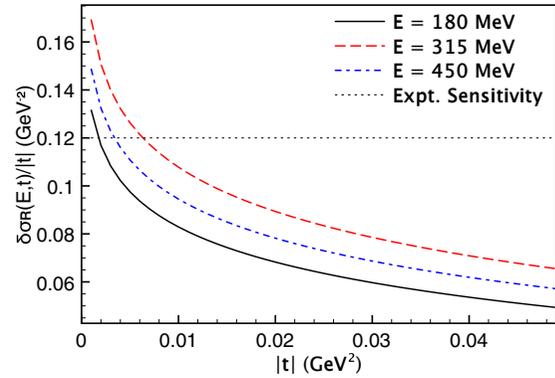


FIG. 2. (Color online) Results for the TPE effect on the reduced cross section $\delta\sigma_R(E, t)/|t|$ for the proton, as a function of $|t|$ in GeV^2 for three values of the laboratory beam energies: 180, 315, and 450 MeV. The experimental sensitivity is also shown.

The principal value integral can be done analytically by changing the order of integration, and I obtain

$$\text{Re} \Phi(E, t) = \frac{-t}{4\pi^3} \int_{E_\pi}^{\infty} \frac{d\omega}{\omega} \sigma_T(\omega) \ln \left(\frac{4\omega_{\text{c.m.}}^2}{-t} \right) \times \left[\left(1 + \frac{\omega^2}{2E^2} \right) \ln \left| \frac{E + \omega}{E - \omega} \right| + \frac{\omega}{E} \ln \left| 1 - \frac{E^2}{\omega^2} \right| - \frac{\omega}{E} \right]. \quad (34)$$

This is the master formula that is a more general result than that of Ref. [19] where the high-energy approximation for the cross section was made.

The integral of Eq. (34) can be evaluated numerically using the phenomenological fit [22] of the world data on real photoabsorption on the proton target [23]. In Figs. 2–4, I present results for the quantity

$$\delta\sigma_R^{\text{TPE}}/|t| = 2 \text{Re} \Phi(E, t)/|t|, \quad (35)$$

which features the logarithmic behavior at low $|t|$, at three values of the electron beam energy relevant for the Mainz A1 experiment with the proton [1] and the deuteron [24] target, the latter being currently under analysis. It is compared with the experimental sensitivity of the Mainz experiments, which

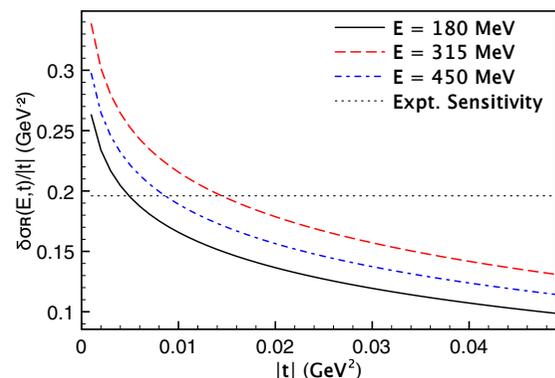


FIG. 3. (Color online) Same as in Fig. 2 for the deuteron.

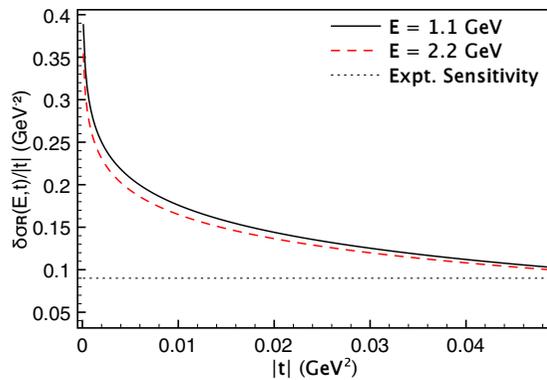


FIG. 4. (Color online) Same as in Fig. 2 for the proton and for two values of the laboratory beam energies: 1.1 and 2.2 GeV corresponding to the kinematics of the proposed JLab experiment [26].

is obtained as

$$\delta\sigma_R^{R_E, \text{expt}}/|t| = -R_E^2/3(1 \pm \delta R_E/R_E). \quad (36)$$

For the proton, the experimental result of $R_E = 0.879(8)$ fm translates into

$$\delta\sigma_R^{R_E, \text{expt}}/|t| = -6.61(12) \text{ GeV}^{-2}. \quad (37)$$

This experimental sensitivity is compared in Fig. 2 with the numerical evaluation of Eq. (34) in the kinematics of the Mainz A1 experiment [1]. The energy dependence (difference between the solid, dashed, and dash-dotted lines) reflects the energy dependence of the photoabsorption cross section around the $\Delta(1232)$ region. For the deuteron the projected precision of 0.25% [24] together with the recent global extraction of the deuteron radius from scattering and spectroscopy data $R_E^d = 2.1424(21)$ fm [2] leads to

$$\delta\sigma_R^{R_E, \text{expt}}/|t| = -39.278(196) \text{ GeV}^{-2}, \quad (38)$$

the uncertainty corresponding to a 0.25% projected precision of the scattering experiment. A somewhat smaller value of 2.130(3) fm was extracted from the electron scattering data alone in Ref. [25]; however, the difference is of no numerical importance for the analysis presented here. A comparison of this sensitivity to the TPE correction in the kinematics of the Mainz A1 experiment is displayed in Fig. 3. The result for a higher energy relevant for the proposed Jefferson Lab experiment [26] at higher energies is shown in Fig. 4. As already mentioned, the TPE results in the leading logarithm approximation are model-independent modulo a constant $\times t$ offset that translates into a constant in Figs. 2–4. Therefore, what really matters is not the absolute value of the TPE correction but rather the difference between the lowest and highest values of t (i.e., nonlinearity). This nonlinearity is close to the experimental precision for the proton and the deuteron at moderate energies, as in the Mainz A1 kinematics, but is seen to be roughly three times the experimental sensitivity for the energy in the range of a few GeV and $|t|$ between 10^{-4} and $5 \times 10^{-2} \text{ GeV}^2$ as in the proposed measurement at JLab. This suggests that the leading logarithm TPE correction has to be included in the experimental analyses that aim at extracting the charge radius from electron scattering with an

accuracy below 1%. It is seen that the inclusion of the TPE correction leads to a stronger t dependence at low momentum transfer. Upon subtracting the positive-definite $|t| \ln(4E^2/|t|)$ correction from the experimental data, the extracted value of the charge radius will necessarily decrease. The subleading corrections $O(\alpha t)$, not included in this calculation, can also affect the extracted value of the charge radius. The second term in Eq. (26) contributes $\sim 1\%$ to the radius [20]. However, further corrections $O(\alpha t)$ may reduce this number as, e.g., the third term in Eq. (26) does.

The TPE effect for the deuteron is somewhat larger than for the proton in comparison with the respective experimental sensitivity. This can be understood by recalling that the total photoabsorption cross section for the deuteron is roughly twice that for the proton in the hadronic range. On the other hand, the quantity $R_E^2(\delta R_E/R_E)$ is only about 1.5 times larger for the deuteron giving a larger relative effect. Nuclear effects were neglected in this estimate. Moreover, the deuteron quasielastic breakup was not included: the derivation is based on treating t as small compared to all other scales, an approximation that would not be valid if t were to be compared to the characteristic scale $MB_d \approx 2 \times 10^{-3} \text{ GeV}^2$, with the deuteron binding energy $B_d \approx 2.224 \text{ MeV}$. An exact calculation would be needed to account for the nuclear part of the photoexcitation of the deuteron.

In summary, I have considered elastic electron-proton (deuteron) scattering at low momentum transfer t and in the presence of the two-photon exchange (TPE). I calculated the TPE effect on the unpolarized cross section in the limit of low t . For the TPE effect with just the nucleon degrees of freedom inside the loop (elastic contribution), the leading behavior $\sim \sqrt{-t}$ is given by the model-independent Feshbach correction. For the TPE effect with inelastic states, the leading low- t behavior is $t \ln t$, and the coefficient in front of this term is model independent and given by a weighted integral over the total photoabsorption cross section. This integral was evaluated numerically using the recent parametrization of world total photoabsorption data on the proton and the deuteron, and the result was compared with the experimental accuracy in extracting the charge radius from electron scattering data at low t in the kinematics of recent and upcoming experiments. I found that while at the beam energy of a few hundred MeV, as in Mainz A1, the nonlinearity introduced by the $t \ln t$ TPE correction is comparable to the experimental precision. At higher energies of 1–2 GeV corresponding to the experiment planned at Jefferson Lab, this effect becomes about three times larger than the experimental precision for extracting the proton charge radius and must be included.

The author is grateful to M. Vanderhaeghen for pointing out that Eq. (34) was first derived in [27] and applied to the e^+/e^- cross section ratio at high energies, and to M. Distler, V. Pascalutsa, O. Tomalak, and S. Karshenboim for useful discussions. This work was supported by the Deutsche Forschungsgemeinschaft DFG through the Collaborative Research Center “The Low-Energy Frontier of the Standard Model” (SFB 1044) and the Cluster of Excellence “Precision Physics, Fundamental Interactions and Structure of Matter” (PRISMA).

- [1] J. C. Bernauer *et al.* (A1 Collaboration), *Phys. Rev. Lett.* **105**, 242001 (2010).
- [2] P. J. Mohr, B. N. Taylor, and D. B. Newell, *Rev. Mod. Phys.* **84**, 1527 (2012), and references therein.
- [3] R. Pohl *et al.*, *Nature* **466**, 213 (2010).
- [4] A. Antognini *et al.*, *Science* **339**, 417 (2013).
- [5] K. Pachucki, *Phys. Rev. A* **60**, 3593 (1999).
- [6] C. E. Carlson and M. Vanderhaeghen, *Phys. Rev. A* **84**, 020102 (2011).
- [7] M. Gorchtein, F. J. Llanes-Estrada, and A. P. Szczepaniak, *Phys. Rev. A* **87**, 052501 (2013).
- [8] D. Nevado and A. Pineda, *Phys. Rev. C* **77**, 035202 (2008).
- [9] R. J. Hill and G. Paz, *Phys. Rev. Lett.* **107**, 160402 (2011).
- [10] M. C. Birse and J. A. McGovern, *Eur. Phys. J. A* **48**, 120 (2012).
- [11] J. M. Alarcon, V. Lensky, and V. Pascalutsa, *Eur. Phys. J. C* **74**, 2852 (2014).
- [12] D. Borisyuk and A. Kobushkin, *Phys. Rev. C* **75**, 038202 (2007).
- [13] D. Borisyuk and A. Kobushkin, *Phys. Rev. C* **78**, 025208 (2008).
- [14] O. Tomalak and M. Vanderhaeghen, *Phys. Rev. D* **90**, 013006 (2014).
- [15] J. Arrington, P. G. Blunden, and W. Melnitchouk, *Prog. Part. Nucl. Phys.* **66**, 782 (2011).
- [16] L. W. Mo and Y.-S. Tsai, *Rev. Mod. Phys.* **41**, 205 (1969).
- [17] L. C. Maximon and J. A. Tjon, *Phys. Rev. C* **62**, 054320 (2000).
- [18] W. A. McKinley and H. Feshbach, *Phys. Rev.* **74**, 1759 (1948).
- [19] M. Gorchtein, *Phys. Lett. B* **644**, 322 (2007).
- [20] R. N. Lee and A. I. Milstein, arXiv:1402.3054.
- [21] R. Tarrach, *Nuovo Cim. A* **28**, 409 (1975).
- [22] M. Gorchtein, T. Hobbs, J. T. Londergan, and A. P. Szczepaniak, *Phys. Rev. C* **84**, 065202 (2011).
- [23] J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).
- [24] H. Merkel (unpublished). See <http://www1.kph.uni-mainz.de/A1/publications/proposals/MAMI-A1-01-2012.pdf>
- [25] I. Sick and D. Trautmann, *Nucl. Phys. A* **637**, 559 (1998).
- [26] A. Gasparian *et al.* (unpublished). See http://www.jlab.org/exp_prog/proposals/12/C12-11-106.pdf
- [27] R. W. Brown, *Phys. Rev. D* **1**, 1432 (1970).