Skyrme force for light and heavy hypernuclei

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We present a lambda-nucleon Skyrme force suitable for reproducing the observed binding energies of the whole range of known light and heavy single-lambda hypernuclei with $A = 5, \ldots, 208$. Notable exceptions such as ${}^{9}_{\Lambda}$Be, featuring a well-developed cluster structure, are identified and examined in comparison with a cluster approach for the hypernuclear structure.

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Introduction. An important goal of hypernuclear physics is to study multistrangeness systems including nucleons and hyperons. For this purpose, presently new experimental facilities at the Gesellschaft für Schwerionenforschung, the Thomas Jefferson National Accelerator Facility, the Japan Proton Accelerator Research Complex, etc. are becoming available, allowing a more detailed study of single- and double-Λ hypernuclei than has been possible so far [1,2]. This also requires continuing development of the theoretical tools to model and predict hypernuclear structure. The Skyrme–Hartree–Fock (SHF) approach is an efficient theoretical method that has been used with great success for ordinary nuclear structure calculations and predictions [3,4]. It has also been extended to the nucleon-hyperon sector for the description of single- and double-Λ hypernuclei and several sets of $N\Lambda$ Skyrme forces have been constructed [5–11]. However, all these forces are so far focused on the range of medium and heavy hypernuclei.

On the other hand, recently few-body calculation methods such as the Gaussian expansion method have been developed for light s-shell and p-shell Λ hypernuclei with $A = 3$ to 10, which allows us to extract new features such as the shrinkage effect, energy stabilization, etc. [12].

The SHF mean-field approach is of course not very suitable for modeling light (hyper)nuclei due to the importance of cluster and other finite-size effects in these small systems. However, as long as one is only interested in the Λ separation energy,

$$B_\Lambda = E({}^{A-1}Z) - E\left({}^{A}_{\Lambda}Z\right), \tag{1}$$

one can expect that a major part of the incorrect description of the *nuclear core* cancels out, as well as that other uncertainties of the SHF approach, such as center-of-mass (c.m.), pairing, deformation corrections, etc., become much less relevant. In any case we follow here a pragmatic approach and use the SHF model even for very light (He) Λ hypernuclei. We will therefore construct a Skyrme force suitable for the full range of light to heavy hypernuclei, identify the cases where this attempt definitely fails, and discuss them in detail in confrontation with a cluster model of hypernuclear structure.

For the same reasons, the B_Λ results depend only very slightly on the choice of the NN Skyrme force, and we use here the SLy4 force [13] without pairing interaction and with the standard treatment of c.m. corrections. We briefly review our formalism in the following.

Formalism. We use a model based on the one-dimensional (spherical) self-consistent SHF method [3], first extended to the description of hypernuclei in Ref. [5]. The fundamental SHF energy density functional is written as

$$\varepsilon = \varepsilon_N + \varepsilon_\Lambda, \tag{2}$$

where ε_N is the usual nucleonic part [4] and a standard parametrization for the hyperonic part in terms of the normal, kinetic, and spin densities ρ, τ, $\boldsymbol{J}$ is [5–11]

$$\begin{aligned}\varepsilon_\Lambda = {} & \frac{\tau_\Lambda}{2m_\Lambda} + a_0\rho_\Lambda\rho_N \\ & + a_1(\rho_\Lambda\tau_N + \rho_N\tau_\Lambda) - a_2(\rho_\Lambda\Delta\rho_N + \rho_N\Delta\rho_\Lambda)/2 \\ & + a_3\rho_\Lambda\rho_N^{1+\alpha} + a_3'\rho_\Lambda\left(\rho_N^2 + 2\rho_n\rho_p\right) \\ & - a_4(\rho_\Lambda\nabla\cdot\boldsymbol{J}_N + \rho_N\nabla\cdot\boldsymbol{J}_\Lambda) \\ & + c_0\rho_\Lambda^2 + c_1\rho_\Lambda\tau_\Lambda - c_2\rho_\Lambda\Delta\rho_\Lambda + c_3\rho_\Lambda^2\rho_N^\gamma,\end{aligned} \tag{3}$$

where two alternative parametrizations of nonlinear effects are indicated: the first one ($\sim a_3$) motivated from G-matrix calculations [8,10,11], and the second one ($\sim a_3'$) derived from a ΛNN contact force [5–7]. In symmetric matter the two choices are equivalent when $a_3 \equiv \frac{3}{2}a_3'$ and $\alpha = 1$. We use the first possibility in this work.

Then one obtains the corresponding SHF mean fields:

$$\begin{aligned}V_\Lambda = {} & a_0\rho_N + a_1\tau_N - a_2\Delta\rho_N - a_4\nabla\cdot\boldsymbol{J}_N + a_3\rho_N^{1+\alpha} \\ & + 2c_0\rho_\Lambda + c_1\tau_\Lambda - 2c_2\Delta\rho_\Lambda + 2c_3\rho_\Lambda\rho_N^\gamma,\end{aligned} \tag{4}$$

$$\begin{aligned}V_N^{(\Lambda)} = {} & a_0\rho_\Lambda + a_1\tau_\Lambda - a_2\Delta\rho_\Lambda - a_4\nabla\cdot\boldsymbol{J}_\Lambda \\ & + a_3(1+\alpha)\rho_\Lambda\rho_N^\alpha + c_3\gamma\rho_\Lambda^2\rho_N^{\gamma-1},\end{aligned} \tag{5}$$

and a Λ effective mass

$$\frac{1}{2m_\Lambda^*} = \frac{1}{2m_\Lambda} + a_1\rho_N + c_1\rho_\Lambda. \tag{6}$$

The relation to the standard $N\Lambda$ (t_i^N) and $\Lambda\Lambda$ (t_i^Λ) Skyrme parameters is

$$a_0 = t_0^N\left(1 + x_0^N/2\right), \quad c_0 = t_0^\Lambda/4, \tag{7}$$

$$a_1 = \left(t_1^N + t_2^N\right)/4, \quad c_1 = \left(t_1^\Lambda + 3t_2^\Lambda\right)/8, \tag{8}$$

$$a_2 = \left(3t_1^N - t_2^N\right)/8, \quad c_2 = 3\left(t_1^\Lambda - t_2^\Lambda\right)/32, \tag{9}$$

$$a_3' = t_3^N/4, \quad c_3 = t_3^\Lambda/4. \tag{10}$$

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TABLE I. Parameters of different $N\Lambda$ Skyrme forces, Eq. (3), compatible with an energy density given in MeV fm^{-3} and densities in fm^{-3}. ΔE is the rms deviation from the experimental B_Λ values of the chosen data set.

	a_0	a_1	a_2	a_3	α	ΔE(MeV)
SLL4	−316.0	23.25	13.88	650.0	1	0.40
SLL4′	−326.0	20.50	20.75	705.0	1	0.35
RAY13 [5]	−280.7	0	−16.25	750.0	1	1.02
YBZ6 [6]	−352.3	45.00	27.70	750.0	1	0.75
SKSH2 [7]	−290.0	0.35	10.68	693.75	1	0.66
YMR [10]	−1056.2	26.25	35.00	1054.2	1/8	0.47
HPΛ2 [11]	−302.8	23.72	29.85	514.25	1	1.00

The parametrization comprises the 11 parameters $a_0, a_1, a_2, a_3, a_4, c_0, c_1, c_2, c_3, \alpha, \gamma$. The c_i parameters, listed here for completeness, are usually only used for double- and multi-Λ hypernuclei [9] because they represent unphysical hyperon self-interactions in the single-Λ case [14]. In any case we found that they can hardly improve the fit to single-Λ nuclei presented below. The same is true for varying α and γ from their standard value 1, and for introducing isospin-breaking effects, i.e., different $n\Lambda$ and $p\Lambda$ interactions. We therefore consider here only the four fit parameters a_0, a_1, a_2, a_3. The parameter a_4 for the very small $N\Lambda$ spin-orbit force is also disregarded here and can be determined by separate considerations [10,15].

Regarding the data to fit we consider the B_Λ values given in Ref. [16] for the following 18 light nuclei observed in emulsion experiments: ${}^{5,6,8}_{\ \ \Lambda}$He, ${}^{7\text{–}9}_{\ \Lambda}$Li, ${}^{7,8,10}_{\ \ \Lambda}$Be, ${}^{9\text{–}12}_{\ \ \Lambda}$B, ${}^{12\text{–}14}_{\ \ \Lambda}$C, ${}^{15,16}_{\ \ \Lambda}$N, supplemented by the recent $(e,e'K^+)$ result $B_\Lambda^{\text{expt}}({}^7_\Lambda\text{He}) = 5.68 \pm 0.03 \pm 0.25$ MeV [17]. The notable exceptions not included in the analysis are ${}^9_\Lambda$Be and ${}^{16}_{\ \Lambda}$O, which will be discussed in detail soon. For heavier nuclei, we consider the 16 s,p,d,f,g Λ separation energies of ${}^{28}_{\ \Lambda}$Si, ${}^{89}_{\ \Lambda}$Y, ${}^{139}_{\ \ \Lambda}$La, and ${}^{208}_{\ \ \Lambda}$Pb, as published in Ref. [1].

Results. We perform fits of the four parameters a_0, a_1, a_2, a_3, and the resulting optimal values are listed in Table I, termed SLL4, together with the corresponding parameters of some other selected $N\Lambda$ Skyrme forces given in the literature (computed employing also the compatible originally-used NN Skyrme forces and the proper a_3 [YMR,HPΛ2] or a_3'-contribution [RAY13, YBZ6, SKSH2]. In the latter cases, we give $a_3 \equiv \frac{3}{2}a_3'$ in the table for better comparison), and the value of the rms deviation ΔE of fit and all 35 data points B_Λ mentioned before. Since the other forces are adjusted only to heavier hypernuclei, their ΔE values are higher than for the SLL4. One notes that all forces have similar a_0, a_3 parameters, more or less constrained by the Λ-well depth in nuclear matter, $V_\Lambda \to a_0\rho_0 + a_1\tau_0 + a_3\rho_0^{1+\alpha}$, while there are no such constraints on the parameters a_1, a_2, which exhibit bigger variations between the forces.

Figure 1 shows the theoretical and experimental Λ separation energies for the light emulsion hypernuclei used in the fit. One notes an overall satisfactory reproduction apart from mainly two cases, ${}^9_\Lambda$Be and ${}^{16}_{\ \Lambda}$O, that turn out to be impossible to describe within the current SHF framework with a global set of parameters. We discuss them separately:

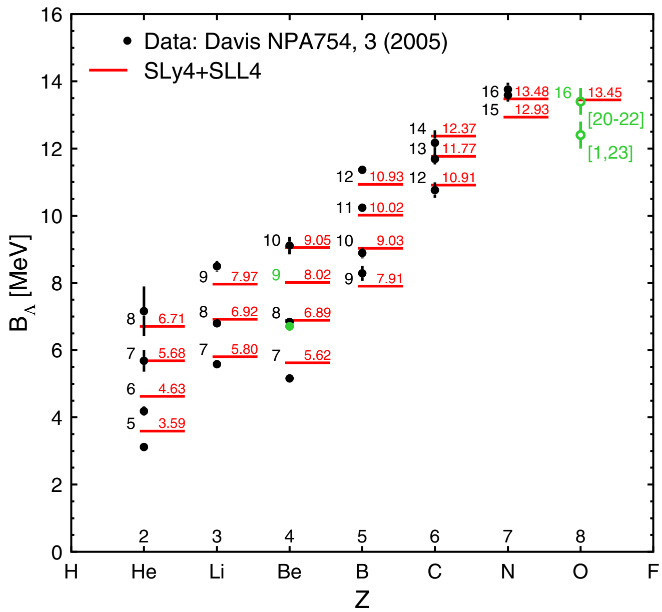

FIG. 1. (Color online) Binding energies of light single-Λ hypernuclei. Red bars and numbers (in MeV) show SHF predictions and full dots indicate experimental data from Refs. [16] and [17] (for ${}^7_\Lambda$He). Open dots show conflicting experimental results for ${}^{16}_{\ \Lambda}$O from Refs. [20–22] or [1,23]; see text. The green dots are excluded from the fitting procedure; see discussion in text.

(i) ${}^9_\Lambda$Be is well known for its 2α-cluster structure [18,19], which leads to an anomalously large size, and a low average nucleon density. This is turn causes a weak $N\Lambda$ mean field and a relatively low Λ separation energy [16]: $B_\Lambda^{\text{expt}}({}^9_\Lambda\text{Be}) = 6.71 \pm 0.04$ MeV, even less than $B_\Lambda^{\text{expt}}({}^8_\Lambda\text{Be}) = 6.84 \pm 0.05$ MeV for the lighter isotope. The SHF mean-field approach is not able to realize this cluster structure and therefore overestimates B_Λ.

This is clearly illustrated in Fig. 2, which compares the nuclear density distributions of ${}^9_\Lambda$Be in the present SHF approach and in the α-cluster model of Ref. [19]. One notes the slightly larger extension and peak structure in the latter approach (thin vs thick solid red curves), which implies a marked reduction of the nucleonic central density, where the Λ is located (red dashed curve), thus causing the alluded effect. In order to verify this supposition, we have recalculated the $N\Lambda$ energy contribution $\int d^3r\varepsilon_\Lambda$, Eq. (3), by replacing the self-consistent nucleonic Skyrme densities with those of the cluster model shown in the figure, and find indeed a reduction of about 0.7 MeV, which would bring the SHF result much closer to the experimental value.

It is also interesting to compare with the case of the ${}^{13}_{\ \Lambda}$C nucleus (black curves in Fig. 2), which also has a strong 3α-cluster structure [25]. However, here the effect of the depletion of the central nuclear density is compensated by an increase in the range of $r \approx 1$ to 2 fm (thin vs thick solid black curves), where still a substantial fraction of the Λ resides, and which in the end causes a Λ binding of similar size in both the SHF and cluster approach, and thus a good agreement with the

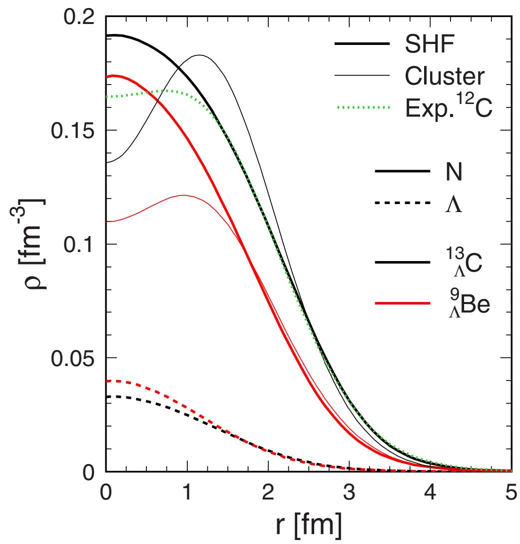

FIG. 2. (Color online) Density distributions of nucleons (solid curves) and Λ (dashed curves) in $^{9}_{\Lambda}$Be (red) and $^{13}_{\Lambda}$C (black) obtained within the SHF (thick) and cluster (thin) approaches. The dotted green curve shows twice the experimental charge density of ^{12}C from Ref. [24].

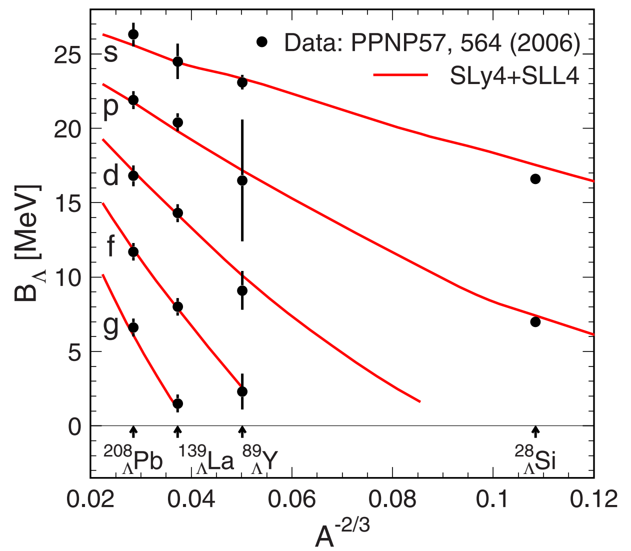

FIG. 3. (Color online) Λ removal energies of heavy single-Λ hypernuclei. Experimental data are from Ref. [1].

experimental value; see Fig. 1. In fact the experimental nuclear (charge) density distribution of the core nucleus ^{12}C [24] (dotted green curve in the figure) lies somewhat in between the predictions of the two extreme, mean-field and cluster, theoretical models. We finally note that the central Λ density for these light nuclei is about 0.03 to 0.04 fm^{-3}, slightly higher in the smaller $^{9}_{\Lambda}$Be nucleus.

(ii) The experimental situation regarding $^{16}_{\Lambda}$O appears still unclear. A recent experiment [20] reports $B_\Lambda = 13.4 \pm 0.4$ MeV, which is consistent with Refs. [21] (13.3 ± 0.4) and [22] (13.4 ± 0.4), and also with the value 13.76 ± 0.16 obtained for the mirror nucleus $^{16}_{\Lambda}$N [26]; but is in contrast with the value 12.4 ± 0.4 of Refs. [1,23]. We refer also to the discussion in Ref. [26]. Figure 1 illustrates the contrasting results of those references. Although this nucleus has been excluded from the fitting analysis, we note that our model strongly favors the recent value 13.4 MeV, since $^{16}_{\Lambda}$O is not supposed to have a strong α-cluster structure. More precise experimental results seem necessary in order to finally settle this issue.

This discussion is in fact also related to the B_Λ results of (π^+, K^+) reactions reported in Ref. [1] for heavier hypernuclei, which have been used in our fitting procedure. These values are now thought to be underestimated [27] by about 0.6 MeV due to the fact that an inaccurate emulsion result for $^{12}_{\Lambda}$C [16] was used for normalization in the data analysis. We therefore provide another parameter set SLL4′, also listed in Table I, for the data set modified in this way. Indeed the quality of the fit is substantially improved in this case, $\Delta E = 0.35$ MeV.

The various unmodified separation energies for the selected heavier hypernuclei are shown in Fig. 3 together with the prediction of the SLL4 force. One notes a good reproduction of all energies, such that the SLL4 force provides a satisfactory modeling of hypernuclei over the whole periodic table with a unique set of four parameters.

We have not considered the lightest observed hypernuclei $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H in our SHF model, but we finally remark on the possible existence of a bound $^{6}_{\Lambda}$H hypernucleus, which was recently claimed in Ref. [28], but not confirmed in Ref. [29]. Indeed the SLL4 and SLL4′ forces predict a very slightly bound nucleus within our calculation, (the $1p3/2$ neutrons are just bound by about 0.2 MeV due to the added Λ) with a total mass of 5802.0 MeV (the experimental claim is 5801.4 ± 1.1 MeV). Obviously, this small binding is outside the predictive power of our model with $\Delta E = 0.4$ MeV, and furthermore we cannot for consistency calculate the unbound ^{5}H core within this simple Skyrme model. Therefore this cannot be considered a reliable prediction for this very delicate system.

Summary. In conclusion, we examined how well a mean-field approach is able to reproduce Λ separation energies of even very light hypernuclei and thus serve as a basis for comparison with more sophisticated theoretical methods.

We presented a new parameter set for a standard $N\Lambda$ Skyrme force, which has been derived with particular attention to light hypernuclei. In fact all known emulsion data with $A > 4$ are included in the fit, with the exception of $^{9}_{\Lambda}$Be and $^{16}_{\Lambda}$O. The peculiarities of these cases were discussed in detail. In order to perform a more profound theoretical analysis and to draw firm conclusions, it appears important to clarify the remaining experimental ambiguities regarding these nuclei and also $^{12}_{\Lambda}$C, which is used to normalize the (π^+, K^+) spectra of heavy hypernuclei.

The new parameter set may be used for simple estimates of hypernuclear structure over the whole range of the periodic table. In the future, the availability of better data for double-Λ hypernuclei will allow us to extend the fit also to the $S = -2$ sector.

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