# Nuclear symmetry energy with strangeness in heavy-ion collisions

Hyun Kyu Lee[1,*] and Mannque Rho[2]
[1]*Department of Physics, Hanyang University, Seoul 133-791, Korea*
[2]*Institut de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette, France*
(Received 30 June 2014; published 1 October 2014)

The role of antikaons in the symmetry energy to be determined in heavy-ion collisions as, for instance, in such observables as the $\pi^-/\pi^+$ ratio is discussed using a simple chiral Lagrangian. It is shown, with some mild assumptions, that kaons, when present in the system, can affect the equation of state appreciably for both symmetric and asymmetric nuclear matter. For nuclear matter with small asymmetry, with which heavy-ion collisions are studied, it may be difficult to distinguish a stiff symmetry energy and the supersoft symmetry energy, even with kaons present. However, the effect of kaon is found to be significant such that $\mu_n - \mu_p \neq 0$ near $x = 1/2$, at which the chemical potential difference is 0 without kaon amplitude. We present arguments that, in order to make a more reliable calculation relevant for heavy-ion collisions, a much deeper understanding of how the strangeness degrees of freedom such as kaons, hyperons, etc., figure in dense baryonic matter than is presently available in the literature is needed.

DOI: 10.1103/PhysRevC.90.045201 PACS number(s): 21.65.−f, 25.75.−q, 25.80.Nv, 26.60.−c

## I. INTRODUCTION

The new degrees of freedom other than nucleons, such as kaons, hyperons, and strongly coupled quarks, in the dense baryonic matter are expected in heavy-ion collisions and at the core of compact stars. The nuclear symmetry energy, denoted $E_{\rm sym}$, in the literature figures importantly in nuclear physics and in compact-star physics. The nuclear symmetry energy plays an important role in determining the composition of dense baryonic matter and controlling the fate of compacts stars, which is one of the principal themes of our current theoretical research into baryonic matter at high densities. Although it is more or less controlled by experiments up to near the nuclear matter density $n_0$, it is almost completely unknown at high densities relevant to the interior of compact stars, going up to, e.g., $\sim 10n_0$ [1]. While there are a large number of theoretical predictions for $E_{\rm sym}$ that range widely above $n_0$, there has been practically no attention paid to the effect of new degrees of freedom, in particular, strangeness, on the nuclear symmetry energy. For a system of $n$ nucleon number density, the energy differences of states with different compositions of protons and neutrons are encoded in what we call the "asymmetry energy," $\mathcal{E}_{\rm asym}$, defined by subtracting the energy of the state with symmetric compositions, $n_p = n_n = n/2$, from the energy of the system composed of $n_p$ proton number density and $n_n$ neutron number density,

$$\mathcal{E}_{\rm asym}(n,x) \equiv E(n,x) - E(n,x = 1/2), \tag{1.1}$$

with $x = n_p/n$. Empirically it is found that it obeys a parabolic law in the asymmetry factor $\delta = (1 - 2x)$ as given by

$$\mathcal{E}_{\rm asym}(n,x) = S(n)\delta^2, \tag{1.2}$$

where $S(n)$ is what is referred to as the "symmetry energy" and conventionally denoted in the literature $E_{\rm sym}(n)$.[1]

In the presence of strange hadrons, the asymmetry energy should depend on the strangeness content in the baryonic matter, since there should be nontrivial interactions between nucleons and strange hadrons. Denoting the strangeness number fraction relative to the nucleon number density $x_S = n_S/n$, the asymmetry energy can be modified as

$$\mathcal{E}_{\rm asym}(n,x,x_S) \equiv E(n,x,x_S) - E(n,x = 1/2,x_S). \tag{1.3}$$

One of the immediate questions is whether the empirical parabolic law is valid in the presence of a strange degree of freedom, which is one of the motivations of this work.

In this work, in contrast to the interior of compact stars, $x_S$ is considered a "probe parameter," similarly to $x$ and $n$. While inside the compact star one can assume weak equilibrium, which would enable us to determine the strangeness content, in heavy-ion collisions the transient time for the dense matter phase is too short to activate weak interactions. This difference between the hadronic matter in heavy-ion collisions and the hadronic matter in weak equilibrium of stellar matter renders the roles of the symmetry energy different from each other at some high density. However, heavy-ion collisions are expected to probe the same symmetry energy as in compact stars below the density at which kaons start condensing. Here the strange hadrons are produced via strong interactions. It is, therefore, natural that we cannot expect to have nonzero net strangeness number in an isolated hadronic system, since any process involving net strangeness number production is suppressed. Nevertheless, there may be possibilities of forming a lump of dense baryonic matter with strangeness with hadrons with compensating strangeness escaping from the lump such that the total strangeness produced is 0. One possible scenario is the production of $K^+$ and $K^-$. Suppose the $K^-$ is captured in a bound state in a nuclear matter lump due to $K^-N$ attractive interactions, but the $K^+$ escapes out of the lump carrying kinetic energies, thereby cooling the remaining baryon lump and forming a baryonic lump with a finite strangeness number. It is then expected that the nuclear symmetry energy of the system (lump) will be modified because of the $KN$

*hyunkyu@hanyang.ac.kr

[1]We use the notation $S(n)$ for the symmetry energy to avoid confusion with the "asymmetry energy," denoted $\mathcal{E}_{\rm asym}$.

 

interactions [2]. This process may or may not occur under the conditions provided by nature—some of the caveats are given below—but it illustrates in a clear way how the presence of strangeness can modify the nuclear asymmetry energy, even in neutron-rich systems.

In heavy-ion collisions, the initial neutron-proton asymmetry factor, $\delta$ $(=1-2x)$—which ranges from 0 for $^{12}$C to 0.198 for $^{197}$Au or 0.227 for $^{238}$U collisions [3]—does not change. It is very likely that the baryonic lump with bound kaons has a similar $n$-$p$ asymmetry when it is formed. Hence the nuclear symmetry energy around $x=1/2$ is particularly of interest and relevance to heavy-ion collisions. One may expect the extra energy to be carried along to cool down the system. We assume that the $s$-wave bound state of $K^-$ (that we simply refer to as a "kaon" unless otherwise noted) is feasible with the amplitude $K$ and energy $E_K$, similarly to kaon condensation:[2]

$$K^- = K\,\exp^{iE_K t}. \tag{1.4}$$

For nucleon-nucleon and kaon-nucleon interactions, we consider a simple—toy—model given by the Lagrangian [2]

$$\mathcal{L} = \mathcal{L}_{KN} + \mathcal{L}_{NN}, \tag{1.5}$$

where

$$\begin{aligned}\mathcal{L}_{KN} &= \partial_\mu K^- \partial^\mu K^+ - m_K^2 K^+ K^- \\ &\quad + \frac{1}{f^2}\Sigma_{KN}(\psi_n^\dagger\psi_n + \psi_p^\dagger\psi_p)K^+K^- \\ &\quad + \frac{i}{4f^2}(\psi_n^\dagger\psi_n + 2\psi_p^\dagger\psi_p)(K^+\partial_0 K^- - K^-\partial_0 K^+) \\ &\quad + \cdots,\end{aligned} \tag{1.6}$$

$$\begin{aligned}\mathcal{L}_{NN} &= \psi_n^\dagger i\partial_0\psi_n + \psi_p^\dagger i\partial_0\psi_p \\ &\quad - \frac{1}{2m_N}(\vec\nabla\psi_n^\dagger\cdot\vec\nabla\psi_n + \vec\nabla\psi_p^\dagger\cdot\vec\nabla\psi_p) - V_{NN} + \cdots,\end{aligned} \tag{1.7}$$

where the ellipses stand for higher derivatives, higher numbers of local fields, and the interaction with higher excitations such as $\Delta$ and hyperon degrees of freedom. $m_N$ ($m_K$) denotes the nucleon (kaon) mass, $f$ is a constant related to the pion decay constant $f_\pi$, $\Sigma_{KN}$ is the $KN$ $\Sigma$ term encoding the explicit breaking of chiral symmetry and $V_{NN}$ is an $NN$ potential that we need not specify for our purpose.

The Hamiltonian can be obtained,

$$\mathcal{H} = \mathcal{H}_{KN} + \mathcal{H}_{NN}, \tag{1.8}$$

where

$$\mathcal{H}_{KN} = \left[E_K^2 + m_K^2 - \frac{n}{f^2}\Sigma_{KN}\right]K^+K^-, \tag{1.9}$$

$$\mathcal{H}_{NN} = \frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} n + V(n) + n(1-2x)^2 S(n). \tag{1.10}$$

[2]Here we assume a uniform amplitude inside the lump but vanishing outside, and we neglect the surface effect.

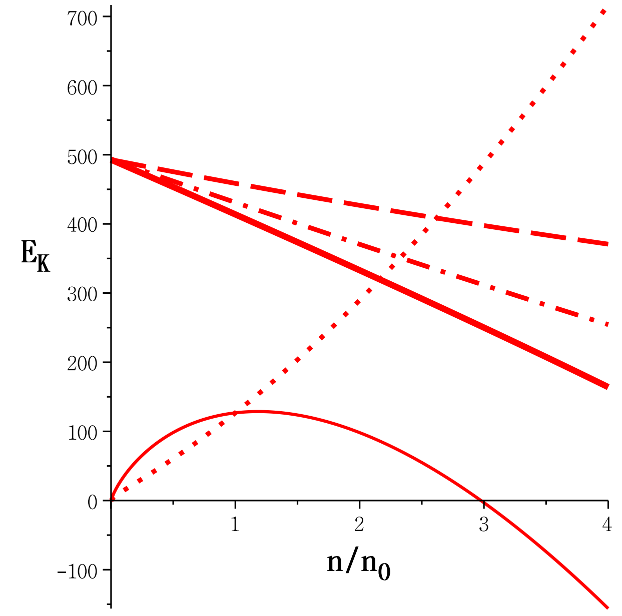


FIG. 1. (Color online) Density dependence of $E_K$, with $\Sigma_{KN} = 0, 2f$, and $3.2f$ denoted by dashed, dot-dashed, and thick solid lines, respectively, for $x=0$. $4S(n)$ (dotted line) and $4S_{\rm ss}(n)$ (thin solid line), roughly the corresponding electron chemical potentials, are plotted together to show the kaon condensation threshold densities in weak equilibrium. With $4S_{\rm ss}(n)$ there is no kaon condensation.

The kaon number density (equivalently, the strange number density, $-n_S$) is given by

$$n_K = \left[2E_K + \frac{(1+x)n}{2f^2}\right]K^2. \tag{1.11}$$

The kaon energy, $E_K$, is determined by the $KN$ interaction given by

$$m_K^2 - E_K^2 - E_K\frac{(1+x)n}{2f^2} - \frac{n}{f^2}\Sigma_{KN} = 0. \tag{1.12}$$

Then we get

$$\begin{aligned}E_K &= -\frac{(1+x)n}{4f^2} \\ &\quad + \frac{1}{2}\sqrt{\left(\frac{(1+x)n}{2f^2}\right)^2 - 4\left(\frac{\Sigma_{KN}}{f^2}n - m_K^2\right)},\end{aligned} \tag{1.13}$$

and the Hamiltonian density in a simplified form given by

$$\mathcal{H}_{KN} = E_K n_K. \tag{1.14}$$

The density dependence for a given $x$ of $E_K$ is determined by the $KN$ $\Sigma$ term, $\Sigma_{KN}$, and the pion decay constant $f \approx 93$ MeV. It is shown in Fig. 1 for $\Sigma_{KN} = 0, 2f$, and $3.2f$ by the dashed, dot-dashed, and thick solid lines,[3] respectively, for the pure neutron matter $x=0$.

[3]The current lattice calculations put a bound on $\Sigma_{KN} \lesssim 250$ MeV [4].

Note the steeper slope for larger values of $\Sigma_{KN}$. The extreme case is for $\Sigma_{KN} = 0$, where only the Weinberg-Tomozawa term, the last term in Eq. (1.6), is effective.[4] Note that the $x$ dependence of $E_K$ is strongly controlled by the Weinberg-Tomozawa term for densities far below the critical condensation density, $n_{\rm th} = m_K^2 f^2/\Sigma_{KN}$, which comes out to be $\sim 6n_0$ if one uses the lattice value for the $\Sigma$ term.[5] This is because the first term in the square root in Eq. (1.13), is much smaller than the second term, $-4(\frac{\Sigma_{KN}}{f^2}n - m_K^2)$ (which is non-negative). For $\Sigma_{KN}$ considered in our model, the densities to be probed by heavy-ion machines, $n = 2 \sim 3n_0$, are far from $n_{\rm th}$.

## II. RESULTS

The chemical potentials of a neutron and a proton with bound $s$-wave kaons are given by

$$\mu_n = \mu_n^0 - \left[\frac{E_K}{2f^2} + \frac{\Sigma_{KN}}{f^2}\right]K^2, \tag{2.1}$$

$$\mu_p = \mu_p^0 - \left[\frac{E_K}{f^2} + \frac{\Sigma_{KN}}{f^2}\right]K^2, \tag{2.2}$$

where

$$\mu_n^0 - \mu_p^0 = 4\left(1 - 2\frac{n_p}{n}\right)S_N(n). \tag{2.3}$$

One can note that the effect of the kaon on the neutron and proton chemical potentials has two parts, one from the Weinberg-Tomozawa term and the other from the $\Sigma_{KN}$ term. The latter contributes equally to the chemical potentials since it does not depend on the $n$-$p$ asymmetry,[6] while the former depends on the $n$-$p$ asymmetry and induces different contributions. Then we see that the chemical potential difference $\mu_n^0 - \mu_p^0$ gets an additional contribution from the $s$-wave kaon amplitude $K$ through the Weinberg-Tomozawa term as given by

$$\mu_n - \mu_p = 4(1 - 2x)S_N(n) + \frac{E_K}{2f^2}K^2. \tag{2.4}$$

It should be noted that it is this quantity that one hopes to determine in heavy-ion collisions (at an energy high enough to produce kaon pairs), as, for example, in the ratio of $\pi^+/\pi^-$ and, in the future, in the $K^+/K^0$ ratio. (See [6]). Given this quantity, then the asymmetry energy per nucleon with bound $s$-wave kaons can be obtained in the form

$$\mathcal{E}_{\rm asym}(n,x,x_K) = (1-2x)^2 S_N(n) + [E_K(n,x) - E_K(n,1/2)]x_K, \tag{2.5}$$

where $x_K = n_K/n$ is the kaon number fraction.

As a rough estimate of what is involved, we assume that $S_N(n)$ does not get a substantial modification due to the presence of kaons.[7] Then we may make use of the energy density and symmetry energy factor $S$ of phenomenological models. We take one simple approach, called the "momentum-independent interaction" (MID), used by Li *et al.* [7].[8] We can rewrite the symmetry energy, $S(n)$, from [7] as

$$S(n) = (2^{2/3} - 1)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} + F\frac{n}{n_0} + (18.6 - F)\left(\frac{n}{n_0}\right)^C. \tag{2.6}$$

The parameters $A$, $B$, and $\gamma$ determined by experiments are $A = -298.3$ MeV, $B = 110.9$ MeV, $\gamma = 1.21$, and $E_F^0 = 37.2$ MeV, and we take $F = 3.673$ and $C = 1.569$. For comparison we consider also a supersoft symmetry energy given by

$$S_{\rm ss}(n) = 13.1\left(\frac{n}{n_0}\right)^{2/3} + 107\frac{n}{n_0} - 88.4\left(\frac{n}{n_0}\right)^{1.25}. \tag{2.7}$$

In Fig. 2, the $x$ dependence of the chemical potential difference, Eq. (2), at $n = 2n_0$ are shown for two symmetry energy factors. The results are very similar for $n = 3n_0$. The range of densities $(2\text{–}3)n_0$ is appropriate for the purpose because it comes before kaons could condense, and also the baryonic matter could be considered to be in a Fermi-liquid state.[9]

One can see that the effect of the kaon is significant near $x = 1/2$, at which the chemical potential difference, the first term in Eq. (2.4), is 0 without the kaon amplitude. Thus it will affect the ratios $\pi^-/\pi^0$, $K^+/K^0$, etc. In the above calculation, we took $x_K = 1$ for simplicity, which is, of course, too big. To

[4]For kaon condensation in star matter in this case, we need a relatively stiffer symmetry energy.

[5]This simple formula based on Eq. (1.5) for $n_{\rm th}$ is perhaps too naive even near—not to mention far away from—the equilibrium density of nuclear matter. We are assuming that kaons condense from the state of matter that can be described as a Fermi liquid. To do better, one should approach it with the hidden local symmetry Lagrangian in the mean field as mentioned in Sec. III. Such an approach—in a highly oversimplified form—is discussed in [5], where $n_{\rm th}$ was found to be $\sim 3n_0$.

[6]For the density we consider, we ignore the difference between $\Sigma_{Kn}$ and $\Sigma_{Kp}$.

[7]There is a caveat to this in the case of kaon condensation. As mentioned in Sec. III, condensed kaons—perhaps relevant in compact stars, though not in heavy-ion collisions—could significantly modify the baryon sector.

[8]The objective of heavy-ion experiments is to probe the symmetry energy in the range of densities $n \gtrsim (2-3)n_0$. At such densities and beyond, the detailed structure of interactions (i.e., tensor forces, short-range repulsions, etc., the degrees of freedom involved, e.g., half-skyrmions, kaons, hyperons, strongly coupled quarks, etc.) is crucial for understanding the physics of dense compact-star matter. The simple model form we take here, though fit for experiments up to $n_0$, is just a parametrization and contains no information of what takes place in the highly correlated matter involved at high densities. We use it just to gain a rough idea of what might be going on in two extreme cases.

[9]This is to avoid the possible half-skyrmion phase predicted in the skyrmion crystal model, since heavy-ion measurements for the meson ratio are not to probe the regime in which a topology change could intervene.

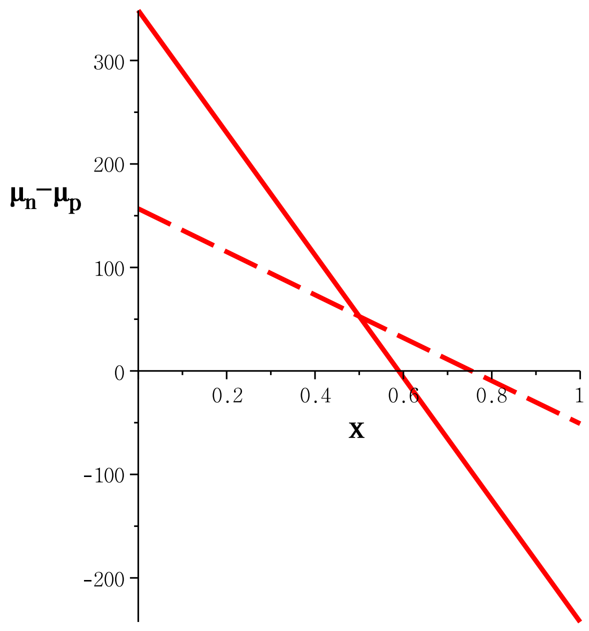


FIG. 2. (Color online) Chemical potential difference $\mu_n - \mu_p$ vs $x$ for $S(n)$ (solid line) and $S_{ss}(n)$ (dotted line). Here, for illustration, we have taken $n = 2n_0$ and the extreme case of $x_K = 1$.

be realistic, we need to fix it dynamically with the collision condition using a more sophisticated theory as mentioned below. The simplest possibility is the charge-neutral baryon lump, which would give $x_K \sim x$ near $x \sim 1/2$.

The asymmetry energy, $\mathcal{E}_{\rm sym}$, at $n = 2n_0$ for $x_K = 1$ is shown, for illustrative purposes, for two symmetry energy factors $S(n)$ and $S_{ss}(n)$ in Fig. 3.

We see that the neutron-proton permutation symmetry in the nucleon sector, characterized by $(1–2x)^2$, is significantly

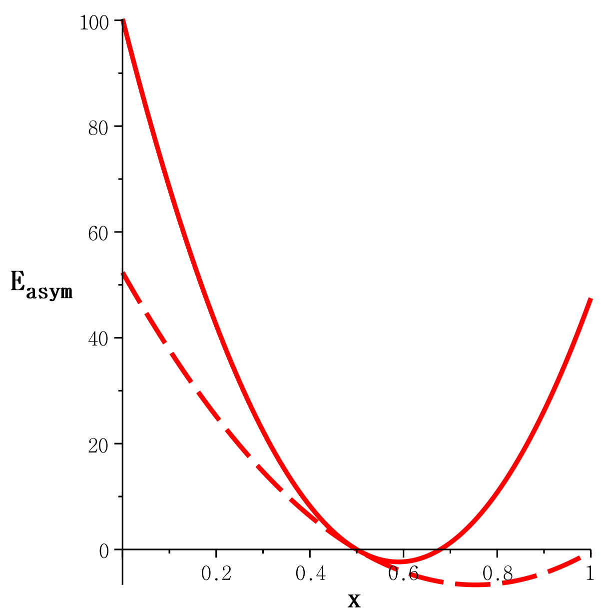


FIG. 3. (Color online) Asymmetry energy per nucleon, $\mathcal{E}_{\rm asym}(n,x,x_K = 1)$, vs $x$ for $S(n)$ (solid line) and $S_{ss}$ (dotted line) for $n = 2n_0$ and $x_K = 1$.

distorted in the presence of kaons. The effect is more prominent in the supersoft case. It is interesting to note that the minima, $x_{\rm min}$, in Fig. 3 are shifted toward $x_{\rm min} > 1/2$, which is equivalent to the proton-rich configuration.[10] So far the weak equilibrium condition applicable in compact stars has not been used, so the equilibrium threshold density for kaons has no meaning. Kaons are produced by strong interactions. This is the reason we can consider kaon amplitude even at a lower density, $n \lesssim 3n_0$, below the weak-equilibrium kaon threshold.

## III. REMARKS AND CONCLUSION

A few remarks concerning the reliability and relevance of what is discussed above are in order. The model Lagrangian adopted here, (1.6), is limited to the chiral order $O(p^2)$ of a chiral Lagrangian that arises from a flavor SU(3) chiral Lagrangian that contains, in addition to octet baryons and pseudo-Goldstone bosons, U(3) vector mesons, when the hyperons and vector mesons are integrated out and only the leading terms (in both derivatives and number of fields) are retained. To that order, the kaon number density is quadratic in the kaon-field amplitude. Now one may wonder what happens if higher kaon fields figure in the Lagrangian as expected in the derivative expansion in the integrating-out. They would enter in $n_K$. The question can be raised whether kaon interactions mediated through coupling with nucleons would not generate repulsion that would be limited to and saturated at a finite $n_K$ [8].

This question can be addressed with a simple renormalization-group argument with (1.6) for heavy-ion systems, where the effective kaon mass cannot go down as much. In terms of chiral perturbation theory, the leading-order term in the $KN$ interactions is the Weinberg-Tomozawa term as used in the literature [8]. This term is "irrelevant" in the renormalization-group flow [9]. Thus, at least in the perturbative sense—which should be reliable for densities not as high above $n_0$—higher kaon field operators cannot do much.

For compact star matter, in contrast to the heavy-ion collision process, the weak interaction becomes sufficiently active to drive the star matter to be stabilized in weak equilibrium. In weak equilibrium, the chemical potential of kaons, $\mu_K$, should be identical to the electron chemical potential, $\mu_e$:

$$\mu_e = \mu_K = \mu_n - \mu_p. \tag{3.1}$$

When $\mu_K$ ($=\mu_e$) crosses $E_K$ in Fig. 1, kaon condensation can occur and it defines the threshold density for kaon condensation, $n_{\rm th}$. The strange number fraction after kaon condensation threshold should range from 0 (condensation threshold) to 1/2 (massless kaon) for locally neutral star matter in weak equilibrium. Since the kaon number fraction is not larger than 1/2, the attractive nature of $KN$ interactions may remain dominant over the repulsion between kaons such that it might not be strong enough to destroy the $KN$ attraction for

[10]In the realistic case, the minimum of $\mathcal{E}_{\rm asym}$ is expected to be further modified when the electromagnetic interaction is included.

kaon condensation. This could also be the case in heavy-ion collisions for the formation of a kaon-bound baryon lump.[11]

It is possible that the dynamics involved could very well be inaccessible by the mean-field approach even at a density that is not too high as suggested in [10] and [11]. For instance, the smooth transition at about $\sim 2n_0$ from hadronic matter to strongly interacting quark matter in compact star matter that can accommodate $\sim 2M_\odot$ stars [12] resembles the kaon condensation scenario associated with the topology change that takes place at $\sim 2n_0$ as suggested in [13] and [14] and mentioned below. Furthermore, there is a strong indication that kaon condensation and hyperon appearance should be considered on the same footing [15]. In the literature, the two processes have been treated separately with no connection between the two. This point may also be pertinent for the process in heavy-ion collisions discussed here.

In this work, we have investigated the effect of the strangeness degree of freedom on the symmetry energy assuming baryon lumps with kaons bound, which might be produced in heavy-ion collisions. One possible scenario is that the $K^-$ is captured in a bound state in a nuclear matter lump due to $K^- N$ attractive interactions, but the $K^+$ escapes out of the lump carrying kinetic energies, thereby forming a baryonic lump with a finite strangeness number. We take the simplest Lagrangian to describe this system, Eq. (1.5). It is then expected that the nuclear symmetry energy of the system (lump) will be modified because of the $KN$ interactions. It is found that even in the presence of kaons, there is little difference in the asymmetry energies with $S$ and $S_{\rm ss}$ near $x \sim 1/2$, which is roughly the initial condition of heavy-ion collisions. Pertinent to experimental efforts, it should be kept in mind that if our result is correct, the pion ratio could not distinguish between a stiff symmetry energy and a supersoft symmetry energy. The presence of kaons, however, distorts the Fermi levels of neutrons and protons via the Weinberg-Tomozawa term such that $\mu_n - \mu_p \neq 0$ at $x = 1/2$, which should vanish without kaons. It will affect the particle spectrum including the pion ratio in the heavy-ion collision.

There are a few caveats. The scenario in this work is that $K^-$ is bound to the nuclear matter, but $K^+$ escapes from the nuclear matter. Of course, the crucial question is then how to control the kaon number, a problem yet to be worked out. Since we take the simplest form of the Lagrangian in this work, the possible roles of higher derivatives, higher dimension field operators, and, moreover, the interaction with higher excitations, such as $\Delta$, $\Lambda(1405)$, and hyperons, should be discussed in detail, which remains for future work.

A topic relevant to the above discussion is the possibility and the effect of kaon condensation on compact star matter. One possibility is that kaon condensation would produce instability in the Fermi-liquid structure of the baryonic matter. This phenomenon may be relevant perhaps only very near the critical condensation density $n_{\rm th}$, but it indicates the possibility of a variety of subtleties in the equation of state as one goes above a few times the normal density $n_0$.

[11] In molecules or solids, most electrons are bound to their parent nuclei, although interactions between electrons inside an atom or in different atoms are repulsive electromagnetically. More speculatively, as conjectured by Yamazaki [10], it may be that shared kaons mediate more attractions between nucleons.

## ACKNOWLEDGMENTS

We are grateful for discussions with Bao-An Li and Won-Gi Paeng. This work was partially supported by the WCU project of National Research Foundation of Korea (R33-2008-000-10087-0).

[1] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, Nature **467**, 1081 (2010); J. Antoniadis *et al.*, Science **340**, 448 (2013).
[2] H. K. Lee and M. Rho, Int. J. Mod. Phys. E **22**, 1330005 (2013).
[3] R. Cavagnoli, C. Providência, and D. P. Menezes, Phys. Rev. C **83**, 045201 (2011).
[4] See, e.g., C. Alexandrou, arXiv:1404.5213 [hep-lat].
[5] G. E. Brown and M. Rho, Nucl. Phys. A **596**, 503 (1996).
[6] Z.-G. Xiao *et al.*, Eur. Phys. J. A **50**, 37 (2014).
[7] B.-A. Li, L.-W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).
[8] See, for example, A. Gal, Int. J. Mod. Phys. E **19**, 2301 (2010).
[9] W.-G. Paeng and M. Rho, arXiv:1407.1611 [nucl-th].
[10] T. Yamazaki, Hyperfine Interact. **211**, 69 (2012).
[11] Y. Akaishi and T. Yamazaki, Phys. Rev. C **65**, 044005 (2002).
[12] K. Masuda, T. Hatsuda and T. Takatsuka, Astrophys. J. **764**, 12 (2013).
[13] H. K. Lee and M. Rho, Eur. Phys. J. A **50**, 14 (2014); H. K. Lee *et al.*, in *The Multifaceted Skyrmion*, edited by G. E. Brown and M. Rho (World Scientific, Singapore, 2010).
[14] M. Rho, Nucl. Phys. A, in press (2014); arXiv:1401.1741 [nucl-th].
[15] H. K. Lee and M. Rho, arXiv:1301.0067 [nucl-th].