Rescattering effects for the ${}^{12}C(p,2p){}^{11}B$ reaction at 400 MeV/u

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Background: Single-particle occupancies for a wide range of magic and near-magic nuclei are significantly below the values predicted by independent-particle shell-model calculations.

Purpose: We aim in this paper to address the question: To what extent does the extracted structure information have uncertainties due to approximations involved in the reaction formalism and assumed dynamic model? **Method:** The proton knockout reaction ${}^{12}C(p,2p){}^{11}B$ at 400 MeV/u incident energy is analyzed using the few-body Faddeev–Alt-Grassberger-Sandhas reaction framework. A two-body model is used for ${}^{12}C$ which involves an inert ${}^{11}B(\frac{3}{2}^{-})$ core and a valence proton. The contributions of higher order terms beyond the single scattering approximation are studied, in particular the ones corresponding to the rescattering between ${}^{12}C$ constituents.

Results: Total cross section and core transverse momentum distribution are calculated. We have found that rescattering terms between the composite nucleus constituents contribute dominantly to the absorptive distortion. **Conclusions:** The accurate treatment of these rescattering terms in the reaction formalism is crucial if one aims to extract reliable structure information from the data. We have also found that work needs to be performed in the reaction theory in order to incorporate a complete structure description of the projectile nucleus and to bridge the proton knockout results with those obtained from electron scattering.

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I. INTRODUCTION

Electron scattering reactions (e, e'p) have been a longstanding tool to obtain information on single-nucleon structure of nuclei. Systematic studies have shown that for a wide range of magic and near-magic nuclei, the single-particle occupancies are significantly below the values predicted by independent-particle shell-model calculations (IPSM), suggesting a quenching factor of about 50% [1].

Occupancies have also been extracted using nuclear probes from transfer reaction experiments [1]. Single-nucleon (p, 2p)and (p, pn) knockout reactions at intermediate and high energies and in inverse kinematics have also been used to obtain information on single-particle occupancies of inner and outer shells [2,3]. In this approach, one calculates a theoretical cross section to a given final state of the nucleus from the sum of the single-particle removal cross sections of a nucleon with a given angular momentum configuration using a particular reaction formalism (traditionally Glauber eikonal [4] or distorted-wave impulse approximation (DWIA) [5] reaction frameworks), multiplied by a spectroscopic factor predicted by the IPSM. The ratio between the calculated theoretical and the experimental value should also evidence a reduction factor for the closed-shell nuclei compatible to the values obtained using electron scattering probes. However, all these attempts of extracting spectroscopic information are

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assumed dynamic model?

reliable only when one has a good understanding of the reaction mechanisms. It is timely to address the question: To what extent does the extracted structure information have uncertainties due to approximations involved in the reaction formalism and

In this work we aim to shed some light on the above question by getting insight on the importance of the contributions to nucleon knockout observables corresponding to the rescattering between constituents of the composite nucleus (called projectile or target correlations for brevity later on). Note that some of these terms are omitted in traditional nuclear reaction approaches. However, they were found to be quite important in the case of elastic proton-¹¹Be scattering at intermediate energies [6].

Here our working example is the proton knockout from the outer $1p_{3/2}$ shell of 12 C in the collision with a proton target at a beam energy of 400 MeV/u. For 12 C we assume a two-body model of a 11 B core coupled with a proton in a $1p_{3/2}$ shell. We use the Faddeev–Alt-Grassberger-Sandhas (Faddeev-AGS) [7,8] nonrelativistic three-body reaction formalism and its approximations.

II. REACTION

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Our present understanding for any direct reaction where a composite projectile (assumed to be well described by an inert

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core plus valence nucleon) collides with a target (in the present case a proton) is based on a three-body picture involving pairwise interactions and rescattering. Traditional reaction frameworks rely on the fundamental assumption that the valence nucleon reacts with the target, while the core provides the distortion. This picture gives rise to the distorted-wave Born approximation (DWBA) in pickup and stripping and the distorted-wave impulse approximation (DWIA) in breakup. The Faddeev-AGS reaction approach treats all three particles on an equal footing and all pairwise interactions are included.

In this section we describe the employed few-body Faddeev-AGS reaction framework and discuss how projectile rescattering terms are taken into account in the reaction dynamics. We also analyze the approximations made in other standard reaction approaches used for the description of (p, 2p)reactions and attempt to enlighten their possible shortcomings.

A. Formalism

1. Faddeev-AGS

The Faddeev-AGS [7,8] is a nonrelativistic three-body reaction formalism treating all allowed reactions on equal footing.

We use here the odd-man-out notation for the three interacting particles (1,2,3), which means, for example, that the potential within the pair (2,3) is denoted as v_1 . The Faddeev-AGS formalism is given in terms of the transition operators whose on-shell matrix elements are the transition amplitudes. Three-body operators are obtained by solving the three-body AGS integral equations [8,9]

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_{\gamma} G_0 U^{\gamma\alpha}, \qquad (1)$$

with $\alpha, \beta, \gamma = (1, 2, 3)$, ($\beta = 0$ in the final breakup state). Here, $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$ and the two-body transition operator is

$$t_{\gamma} = v_{\gamma} + v_{\gamma} G_0 t_{\gamma}, \qquad (2)$$

with the free propagator $G_0 = (E + i0 - H_0)^{-1}$, where *E* is the total energy of the three-particle system in the center of mass (c.m.) frame. The solution of the Faddeev-AGS equations at higher energies can be found by iteration

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_{\gamma} \bar{\delta}_{\gamma\alpha} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_{\gamma} \sum_{\xi} G_0 \bar{\delta}_{\gamma\xi} t_{\xi} \bar{\delta}_{\xi\alpha} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_{\gamma} \sum_{\xi} G_0 \bar{\delta}_{\gamma\xi} t_{\xi} \sum_{\eta} G_0 \bar{\delta}_{\xi\eta} t_{\eta} \bar{\delta}_{\eta\alpha} + \cdots .$$
(3)

The successive terms of this series can be considered as terms of zero order (which contribute only for rearrangement transitions), first order (single scattering), second order (double scattering), and so on in the transition operators t_{γ} . The breakup series up to third order is represented diagramatically in Figs. 1–3, where the upper particle, taken as particle 1, is scattering from the bound state of the pair (23).



FIG. 1. Single-scattering diagrams for breakup in the Faddeev-AGS scattering framework.

In the Faddeev-AGS reaction framework the single scattering terms are viewed as the plane wave impulse approximation (PWIA) and include in this formalism the term where proton scatters from the knockout particle and in addition the term where proton scatters from the inert core. This is a core recoil term. The higher order terms will represent in this formalism the effect of distortion. For a weakly bound particle the net contribution from this core recoil term is expected to be small and suppressed by some higher order distortion terms of the Faddeev-AGS expansion. For a tightly bound particle like a particle from an outer shell from ¹²C this may not be the case, and the effect of its contribution needs to be investigated. We notice that in the DWIA reaction formalism the PWIA corresponds only to the single scattering contribution where the proton scatters from the knockout particle. The single scattering contribution where the proton scatters from the core is not included and is assumed to be exactly canceled by higher order terms. We return to this point later.

The breakup observables are calculated from the on-shell matrix elements of the AGS operators, $T^{0\alpha} = \langle \mathbf{q}\mathbf{p}|U^{0\alpha}|\psi_{\alpha}\rangle$, where particle α is the spectator in the initial state (in our case α is the proton) and $\mathbf{q}\mathbf{p}$ are the Jacobi momenta in the final state as described in Ref. [10].

The semi-inclusive differential cross section, where one of the emitted particles is detected (in our working example the ${}^{11}B$ core) and is given by

$$\frac{d^3\sigma}{d\Omega_C dE_C} = \mathcal{C} \int d^2 \hat{p} \,\mathcal{F} \,|T^{0\alpha}|^2. \tag{4}$$

In inverse kinematics

$$\mathcal{C} = (2\pi)^4 \frac{m_p}{2} \frac{m_p + m_C}{K_{\text{LAB}}} m_C \mathcal{K}_C, \qquad (5)$$



FIG. 2. Double-scattering diagrams for breakup in the Faddeev-AGS scattering framework.



FIG. 3. Triple scattering diagrams for breakup in the Faddeev-AGS scattering framework.

with m_p and m_C being the masses of the proton and core, \mathcal{K}_C being the momentum of the detected core in the exit system, and \mathcal{F} being a kinematical factor,

$$\mathcal{F} = \sqrt{2\mu E - \frac{\mu}{\overline{\mu}} \left(\mathcal{K}_C^2 + \frac{m_C^2}{M^2} \mathcal{K}_{\text{TOT}}^2 - 2\frac{m_C}{M} \mathcal{K}_C \cdot \mathcal{K}_{\text{TOT}} \right)},$$
(6)

where $M = m_p + m_p + m_C$ and $\mathcal{K}_{\text{TOT}} = K_{\text{LAB}}$ (laboratory frame) or $\mathcal{K}_{\text{TOT}} = 0$ (c.m. frame).

By integrating this cross section with respect to the core final energy or the core final angle we obtain the angular distribution and the energy spectrum, respectively.

From the semi-inclusive cross section one can calculate the transverse momentum distributions of the detected particle as described in Ref. [10].

Higher order terms (second, third, and so on in the pair interaction transition operators) beyond the single scattering contain rescattering (or composite nucleus correlations) terms between the pair (23) of the composite nucleus. To third order, these rescattering terms are included in diagrams (b) and (d) of Fig. 2, and in diagrams (b)–(d) and (f)–(h) of Fig. 3. The relative importance of these terms to the reaction dynamics needs to be assessed and treated properly in order to extract reliable structure information from the data.

2. DWIA

The DWIA approximation has been used to analyze the knockout reactions A(a,ab)B where an incident particle a knocks out a nucleon or a bound cluster b in the target nucleus A, resulting in three particles (a,b,B) in the final state, with B being the heavy fragment or core. In particular, it has been used extensively to study (p,2p) and (p,pn) knockout reactions both at medium and at high energies. This formalism has been described, for example, in the work of Chant and Roos [5].

It is an approximate framework and relies on the fundamental assumption that no hard collision occurs between the projectile a and the core B that results in the breakup of Ainto B + b. This assumption is based primarily upon intuition. Attempts to connect this reaction framework with a multiple scattering series (MSS) have been done in the early days and shown to be part of a complete series [11]. Recently, the validity of the DWIA was also addressed at intermediate energies for fully exclusive observables [12].

In the present work we also analyze to what extent the approximations made in the DWIA reaction approach might affect the inclusive observables at high energies.

For completeness we revise here the DWIA approach. According to this formalism the transition amplitude for the knockout reaction A(a,ab)B is given by

$$T_{AB} = \langle \eta_{Bab}^{(-)} | t_{ab} | \phi_{Bb} \eta_{aA}^{(+)} \rangle, \tag{7}$$

where ϕ_{Bb} is the (Bb) bound-state wave function, and $\eta_{aA}^{(+)}$ and $\eta_{Bab}^{(-)}$ describe the relative motion of the particles in the entrance and exit channels, respectively, neglecting the interaction V_{ab} . Therefore in the entrance channel $\eta_{aA}^{(+)}$ satisfies

$$(T_{aA} + V_{aA} - V_{ab} - \epsilon_{aA})\eta_{aA}^{(+)} = 0,$$
(8)

where ϵ_{aA} is the relative kinetic energy and T_{aA} is the relative kinetic energy operator. In standard DWIA calculations one makes the following approximations: (i) one takes $V_{aA} - V_{ab} \sim V_{aB}(\vec{r}_{aA})$ and thus the distorted wave in the incident channel satisfies

$$(T_{aA} + V_{aB}(\vec{r}_{aA}) - \epsilon_{aA})\tilde{\eta}_{aA}^{(+)} = 0,$$
 (9)

and (ii) for the exit channel one assumes as a good approximation to write the 3-body wave function $\eta_{Bab}^{(-)}$ as a factorized product

$$\eta_{Bab}^{(-)} \sim \tilde{\eta}_{aB}^{(-)} \tilde{\eta}_{bB}^{(-)}, \tag{10}$$

where

$$(T_{aB} + V_{aB}(\vec{r}_{aB}) - \epsilon_{aB})\tilde{\eta}_{aB}^{(+)} = 0$$
(11)

and

$$(T_{bB} + V_{bB}(\vec{r}_{bB}) - \epsilon_{bB})\tilde{\eta}_{bB}^{(+)} = 0.$$
(12)

The potentials V_{aB} and V_{bB} are taken to be the optical potentials which describe the a + B and b + B scattering at energies ϵ_{aB} and ϵ_{bB} respectively. Therefore one writes

$$T_{AB} \sim \langle \tilde{\eta}_{aB}^{(-)} \tilde{\eta}_{bB}^{(-)} | t_{ab} | \phi_{Bb} \tilde{\eta}_{aA}^{(+)} \rangle.$$
(13)

In Ref. [12] the DWIA transition amplitude was written in terms of a multiple scattering expansion that we refer here as DWIA-MS. It was shown in Ref. [12] that the DWIA-MS leads to an incomplete multiple scattering series. First, the single scattering term only contains the contribution due to the scattering between the valence particle b and the incident particle a represented in Fig. 4. Thus no hard collision occurs between the projectile and the core. The distortion in the exit



FIG. 4. Single scattering diagram in the DWIA scattering framework.



FIG. 5. Diagrams due to the distortion in the exit channel in the DWIA scattering framework.

and entrance channels takes approximately into account the double and triple scattering diagrams represented in Figs. 5 and 6 respectively. Therefore the double scattering diagram (b) of Fig. 2 and the triple scattering diagrams (b), (c), (e), (g), and (h) of Fig. 3 are not taken into account in the DWIA-MS formalism. We note that according to DWIA, diagrams where the transition amplitude for the collision between the incident and knockout particle t_{ab} does not occur or appears more than once are not taken into account.

We aim here to examine to what extent the missing terms simulate the cancellations between the single scattering and higher order terms, implicit within DWIA, in high-energy ${}^{12}C(p,2p)$ reaction.

We also aim to estimate to what extent spectroscopic factors extracted by comparing the calculated DWIA cross sections and the data have uncertainties due to missing scattering diagrams. This can be assessed by performing Faddeev-AGS multiple scattering calculations.

3. Eikonal

We consider the scattering of a projectile (system 1) from a few-body composite target consisting of *n* subsystems weakly bound to each other. The subsystems $\mathcal{I}, \mathcal{J}, \ldots$ are assumed to be stable and can be either composite nuclei or nucleons. The number of subsystems *n* can be any. Therefore we abandon in this section the odd-man-out notation, which is suitable for three-body problems only. The interaction between the projectile and the subsystem \mathcal{I} are denoted by $v_{\mathcal{I}}$.

The eikonal approach can be viewed as a multiple scattering expansion, where the total transition amplitude T for the projectile-target is

$$T = \sum_{\mathcal{I}=2}^{\mathcal{N}} \tau_{\mathcal{I}} + \sum_{\mathcal{I}=2}^{\mathcal{N}} \sum_{\mathcal{J}\neq\mathcal{I}} \tau_{\mathcal{I}} \hat{G} \tau_{\mathcal{J}} + \sum_{\mathcal{I}=2}^{\mathcal{N}} \sum_{\mathcal{J}\neq\mathcal{I}} \sum_{\mathcal{K}\neq\mathcal{J}} \tau_{\mathcal{I}} \hat{G} \tau_{\mathcal{J}} \hat{G} \tau_{\mathcal{K}} + \cdots$$
(14)



FIG. 6. Diagrams due to the distortion in the entrance channel in the DWIA scattering framework.

In here, $\mathcal{N} = n + 1$, and the projectile- \mathcal{I} subsystem transition amplitude $\tau_{\mathcal{I}}$ is

$$\tau_{\mathcal{I}} = v_{\mathcal{I}} + v_{\mathcal{I}} \hat{G} \tau_{\mathcal{I}}, \qquad (15)$$

and \hat{G} is the target resolvent

$$\hat{G}(z) = \left(z - H_0 - \sum V_{\mathcal{I}\mathcal{J}}\right)^{-1}.$$
(16)

In this equation V_{IJ} is the interaction between subsystems I and J.

One notes that the eikonal multiple scattering series differs from the Faddeev-AGS expansion. In fact, the single scattering contribution contains target correlations because the $\tau_{\mathcal{I}}$ scattering amplitude is not the free two-body transition amplitude embedded in *n*-body space.

Within the eikonal approach these target correlations are taken approximately by replacing the propagator

$$\hat{G}(z) \to G^{\text{eik}} = (z - K_P - \bar{H})^{-1}.$$
 (17)

This means that target rescattering terms between the subsystem of the composite particles are taken into account in an approximate way and the validity of this approximation needs to be assessed if one wants to extract accurate structure information. Insight on this issue can be gained by estimating the importance of the valence-core rescattering terms.

B. Quenching factors

The theoretical breakup cross section for the knockout of the valence proton from a given shell, calculated using a simple core + valence model, has to be multiplied by the corresponding spectroscopic factor S to compare with the experimental data. This enables extracting a renormalization factor

$$\mathcal{R} = \frac{\sigma_t^{\exp}}{\mathcal{S} \times \sigma_t},\tag{18}$$

whose deviation from 1 is attributed to structure effects. However, this ratio might include combined effects due to structure and approximations in the reaction formalism, namely due to the uncertainty of the interactions, an incomplete treatment of the rescattering between projectile constituents, or inadequacy of the few-body model.

III. STRUCTURE

The dynamical input to the Faddeev-AGS equations are three pair interactions: the NN valence proton-target proton (p_v-p) , the valence proton-¹¹B core $(p_v-^{11}B)$, and the target proton-¹¹B core $(p-^{11}B)$. We take the realistic nucleon-nucleon CD Bonn potential [13] for the p_v-p pair. The NN transition amplitude is a sum of partial wave components that satisfy the generalized Pauli principle (L + S + T = odd) and thus different configurations are allowed in the case of p-p and n-p. The p-p scattering occurs only in isospin T = 1, while n-p scattering exists in isospin T = 0 and T = 1 with equal weight. It is crucial to include a proper NN partial wave decomposition to describe the (p,2p) reaction.

To describe the interaction between the proton target and the ¹¹B core we use a phenomenological nuclear optical potential

of the form

$$V_{\text{opt}}(r) = -V_0 f(r, R_0, a_0) - i W_W f(r, R_W, a_W) + i W_S 4a_S \frac{d}{dr} f(r, R_S, a_S),$$
(19)

where f(r, R, a) is the usual Woods-Saxon form factor

$$f(r, R, a) = 1/\{1 + \exp[(r - R)/a]\},$$
(20)

and $R_i = r_i A^{\frac{1}{3}}$ with parameters taken from the Bauhoff optical potential parametrization [14] evaluated at the appropriate c.m. energy; the spin-orbit interaction is neglected. Given that this parametrization is for proton elastic scattering from ¹²C, there is an uncertainty associated with this pair interaction. The conclusions presented here should be independent of the choice of this interaction.

The inclusion of the Coulomb interaction requires special care. For reactions with two charged clusters in the final state the method of screening and renormalization [15] can be used in the Faddeev-AGS scattering framework. However, it is not applicable for three charged clusters in the final state as in the present ${}^{12}C(p,2p){}^{11}B$ case. We therefore include only the screened Coulomb interaction $\omega_R(r)$ in the form of Ref. [15], and only for the target-core $(p{}^{-11}B)$ pair; Coulomb is neglected for valence-core $(p_v{}^{-11}B)$ and proton-proton $(p_v{}^{-}p)$ pairs. Even for reactions with two charged particles the method of screening and renormalization is well defined only for exact solutions of the Faddeev-AGS equations, but not for incomplete scattering series. Thus, the results of single, double, and triple scattering presented below refer to the sum of the optical potential and the screened Coulomb $p{}^{-11}B$ potentials

$$V_{\text{tot}}^{\text{p}}(r) = V_{\text{opt}}(r) + \omega_R(r), \qquad (21)$$

and, unless already converged in terms of multiple scattering expansion, depend on the screening radius R which is taken 8 fm in the present calculations. Thus, also the conclusions of this work on the importance of various MS contributions refer, strictly speaking, to nuclear plus p^{-11} B screened Coulomb interaction, but qualitatively are expected to be independent of R since the Coulomb effect at these high energies is irrelevant for considered breakup observables. This has been checked by performing calculations without Coulomb.

To describe the structure of 12 C we assume a two-body model. The interaction between the valence proton and the 11 B core is local with central and spin-orbit terms

$$V(r) = -V_c f(r, R_0, a_0) + 4\vec{L} \cdot \vec{S} V_{\rm SO} \frac{1}{r} \frac{d}{dr} f(r, R_{\rm SO}, a_{\rm SO}).$$
(22)

In this equation \vec{S} is the total spin of the core and valence proton, that is $\vec{S} = \vec{S}_C + \vec{S}_{p_v}$. The Coulomb interaction was neglected, the interaction assumed to be *L* dependent, and for standard geometry parameters the depths are adjusted to reproduce the energy spectrum. The *p*-wave potential was taken from Ref. [16] and adjusted to generate a bound state with energy $E(1p_{\frac{3}{2}}) = -15.3$ MeV.

In order to assess the contribution of other partial waves in the p_v -¹¹B interaction beyond the one that is responsible for the single particle configuration ¹¹B(3/2⁻) $\otimes vp_{3/2}$ leading to the 0⁺ ground state of ¹²C, we take in the other partial waves a weaker potential with a depth of 80% of the potential in the *p* wave. This choice has no other reason than lack of a more enlightened alternative. This interaction also supports deep $1s_{\frac{1}{2}}$ and $1p_{\frac{1}{2}}$ bound states. Both are projected out by moving them to large positive energy, although only the $1s_{\frac{1}{2}}$ should be Pauli blocked.

In order to estimate the effects on the calculated observables due to the uncertainty associated with the p_v -¹¹B interations we also consider a potential with an imaginary component. We took this potential from the Koning-Delaroche parametrization [17].

Although the calculated results suffer as well from the uncertainty associated with the incompleteness of the Coulomb interaction, this should not affect the estimates and qualitative conclusions presented below.

IV. RESULTS AND DISCUSSION

The three-body Faddeev-AGS integral equations, Eq. (1), are solved in momentum space after partial wave decomposition and discretization of all momentum variables.

We include p-p partial waves with relative orbital angular momentum $L \leq 6$, p-¹¹B with $L \leq 16$, and p_v -¹¹B with $L \leq$ 6. Three-body total angular momentum is included up to J =50, although we found that for the considered observables J = 20 is practically sufficient.

We present now the observables calculated using the above described interactions. The results have an associated quantitative uncertainty due to the pair interactions $p_v^{-11}B$ and $p^{-11}B$ and also because we are not including the Coulomb interaction in all pairs. The main conclusions are expected to be independent of this uncertainty. We have found that the Coulomb interaction in the $p^{-11}B$ pair gives a very small contribution to the calculated converged results.

In Fig. 7 we show the ¹¹B core transverse momentum distribution for the ${}^{12}C(p,2p){}^{11}B$ reaction at 400 MeV/u. The curves represent the contributions of the different multiple



FIG. 7. (Color online) ¹¹B core transverse momentum distribution for the ${}^{12}C(p,2p){}^{11}B$ reaction at 400 MeV/u. The curves represent the contribution of the different multiple scattering components as described in the text.

TABLE I. Total cross section for several multiple scattering approximations for the knockout of a valence proton from the $1p_{3/2}$ shell as described in the text.

MS	σ_t	$\sigma_t^{\rm Nresc}$ (mb)
Single	34.03	
Single-pn	10.70	
Double	6.59	26.12
Triple	5.26	25.74

scattering components: The dashed line includes the single scattering (SS) contributions of SS-*pn* and SS-*p*-core (note, however, that this sum would oscillate logarithmically with increasing screening radius). The dashed-double dotted line includes only the contribution of SS-*pn*. It follows that the SS-*p*-core is dominant. The observable calculated with the double scattering contribution (thick solid line) is not far from the converged third order (dash-dotted line) result.

The results show that higher order multiple scattering terms are responsible for a subtle cancellation of the SS-*p*-core contribution. In addition, these terms reduce the SS-*pn* contribution even further.

When the terms involving the rescattering of the ¹²C constituents are neglected the magnitude of the observable calculated including multiple scattering terms up to second order (thin solid line) increases significantly, getting closer to the single scattering result. This result evidences that the projectile correlation terms give a major contribution to the multiple scattering expansion.

In Table I we show the calculated total cross sections assuming a knockout of a proton from the p shell and a unit spectroscopic factor. The results show that higher order multiple scattering induce strong cancellations and reduce significantly the calculated cross section as compared to the single scattering p-n approximation.

In order to demonstrate the importance of the composite nucleus constituent rescattering terms we define the ratio between the total cross sections calculated with (σ_t) and without (σ_t^{Nresc}) these rescattering terms. We have obtained $\mathcal{R}^{\text{resc}} = \frac{\sigma_t^{\text{Nresc}}}{\sigma_t} \sim 5$.

To estimate the uncertainties associated with the valencecore interaction we have compared the scattering observables using a complex potential taken from the Koning-Delaroche parametrization with the real parametrization defined in the text. The relative effect of modifying the nucleon-core interaction is about $\sim 7\%$. We have also found that the effects of the rescattering terms remain important in the same way.

In Fig. 8 we show the ¹¹B core transverse momentum distribution in the ${}^{12}C(p,2p){}^{11}B$ reaction assuming a knockout of a proton from the *p* shell and a unit spectroscopic factor. We compare results calculated to second and third orders in the multiple scattering expansion using all the Faddeev-AGS terms and the ones obtained from a truncated series as in the DWIA-MS reaction approach. The total cross sections obtained via integration of the momentum distributions are 6.59, 5.26, and 5.34 mb, respectively. Thus, the DWIA-MS result underestimates the Faddeev-AGS second-order result by about 20% but agrees well with the converged third-order result.



FIG. 8. (Color online) ¹¹B core transverse momentum distribution for the ¹²C(p,2p)¹¹B reaction at 400 MeV/u. The curves represent the observable calculated to second and third orders in the multiple scattering expansion using all the Faddeev-AGS terms and with a truncated series as in the DWIA reaction approach.

In Table II we show the spectroscopic factors for the $J^{\pi} = 3/2^{-}$ state, obtained from the IPSM [3]. The theoretical spectroscopic factors predicted by this work for the ground (3/2-) and excited 2.125 (1/2-) and 5.020 (3/2-) MeV states are 3.16, 0.58, and 0.19, respectively. The small remainder corresponds to states above 10-MeV excitation. We also show the spectroscopic factors extracted from electron scattering data [18] and from transfer reaction [1].

Also shown in Table II is the total cross section calculated as a product of the Faddeev-AGS prediction $\sigma_t = 5.26$ mb and the corresponding spectroscopic factor for the ¹¹B ground state. These results should be viewed only as an estimate to the calculated observable due to the uncertainties associated with the nucleon-core interaction and relativity.

Quasifree studies of ${}^{12}C(p,2p)$ reaction at intermediate and high energies in coincidence with γ ray and composite particle decay have revealed *p*-hole contributions from ${}^{11}B$ ground (3/2-) and excited 2.125 (1/2-) and 5.020 (3/2-) MeV states, and an *s*-hole state in high excited energy region above 10 MeV [19–21]. The detailed structure of this *s*-hole state is still not well known.

Standard (p,2p) reaction calculations describe the structure of the composite nucleus (in our example ¹²C) using an Hilbert space that includes only configurations based on a mean field picture of an inert core, that can be either in the ground state (which was the only case under consideration in the present study) or in the excited state.

A timely issue is to have insight on the effects of using an incomplete structure picture of the composite nucleus when extracting information from comparison of the calculated observables with the experimental data. Such a comparison using results of Refs. [20,21] would favor a spectroscopic factor close to the prediction of the independent shell model. In addition, assuming that the calculated total cross section for the excited states would lead to a value close to that obtained for the $J^{\pi} = 3/2^{-}$ ground state, one can estimate that the IPSM would predict a total contribution for the *p*-hole states

TABLE II. Spectroscopic factors for the $J^{\pi} = 3/2^{-}$ ground state and calculated total cross section. The latter is obtained as a product of the Faddeev-AGS prediction $\sigma_t = 5.26$ mb and the corresponding spectroscopic factor for the ¹¹B ground state.

Reaction	S	Reference	$S \times \sigma_t \text{ (mb)}$
IPSM (WBP)	3.16	Brown <i>et al.</i> [3]	16.61
(e, e'p)/transfer	1.72	Van Der Steenhoven <i>et al.</i> [18], Kramer <i>et al.</i> [1]	9.0
QFS	2.694	Kramer <i>et al.</i> [1]	14.2

of about 20.67 mb. However, in these calculations all possible configurations are not included in the Hilbert space for the reaction calculation.

More work needs to be done to incorporate structure model beyond the mean field picture into the reaction framework and to bridge the proton knockout with electron scattering.

V. CONCLUSIONS

We used a two-body model for ¹²C which involves an inert ${}^{11}B(\frac{3}{2}^{-})$ core and a valence proton. We have calculated core transverse momentum distributions and the total cross section for the ${}^{12}C(p,2p){}^{11}B$ reaction at 400 MeV/u using the fewbody Faddeev-AGS reaction framework.

The results show that higher order multiple scattering terms induce strong cancellations of the single scattering contributions and reduce significantly the cross section obtained in the single scattering p-n approximation.

We have found that rescattering terms between the composite nucleus constituents contribute dominantly to the absorptive distortion. The accurate treatment of these terms in the reaction formalism is therefore crucial if one aims to extract reliable structure information from the data.

In addition, we have found that to second order the total cross section evaluated using the DWIA-MS reaction approach underestimates the second-order Faddeev-AGS result by about $\sim 20\%$ but agrees well with converged third-order result.

Work needs to be performed in the reaction theory in order to incorporate a complete structure description of the projectile nucleus and to bridge the proton knockout results with those obtained from electron scattering.

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