

Dynamic polarization potential and dynamical nonlocality in nuclear potentials: Nucleon-nucleus potential

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Background: Of the two sources of nonlocality in nucleon-nucleus and nucleus-nucleus interactions, knock-on exchange and dynamically generated, almost all papers referring to nonlocality mention only the first.

Purpose: Our purpose is threefold: to demonstrate a method for including dynamical nonlocality, for which a simple prescription (like the Perey factor for exchange nonlocality) is unknown, within distorted wave Born approximation (DWBA) calculations; to identify signatures of dynamic nonlocality and illuminate the extent to which the presence of such nonlocality can influence the extraction of spectroscopic information from direct reactions, and more generally, to increase our understanding of nucleus-nucleus interactions.

Methods: After reviewing existing indications of dynamically induced nonlocality, DWBA transfer calculations are presented which compare results involving dynamically nonlocal potentials with those involving their local equivalents. The dynamical nonlocal potentials are generated *in situ* by the presence of channel coupling and the local equivalents are generated by inversion of the corresponding coupled channel elastic S matrix. This method obviates the need for solving integro-differential equations for including nonlocal potentials in DWBA.

Results: The coupling of nucleons to collective states of the target nucleus induces dynamical nonlocality in the nucleon-nucleus interaction that has a significant effect on (p,d) reactions at energies relevant to spectroscopic studies.

Conclusions: A method for studying the contribution of dynamically induced nonlocality in nuclear interactions has been demonstrated. Dynamically induced nonlocality should not be overlooked in the analysis of direct reactions. The method can also be applied to dynamic nonlocality due to projectile excitation.

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I. INTRODUCTION

It is well known that the nucleon-nucleus potential is nonlocal as a consequence of knock-on exchange processes. The standard local optical model potential, OMP, has an energy dependence that is largely attributed to the fact that it is the local equivalent, i.e., giving the same elastic scattering S matrix S_{ij} as the theoretical potential that includes knock-on exchange explicitly. The phenomenological nonlocal OMP of Perey and Buck [1], that has no explicit energy dependence, fits nucleon-nucleus elastic scattering quite well over a considerable energy range. Furthermore, the exact local equivalent of the Perey-Buck potential has [2] just the energy dependence of the empirical OMP. The great importance of this nonlocality arises from the Perey effect [3], whereby the wave function of the nonlocal potential within the nucleus is about 15% smaller than the wave function of the equivalent local potential. Local potentials are central to most analyses of direct reactions that lead to spectroscopic information, and the Perey effect modifies both the bound state and scattering wave functions involved in the analysis. Hence, corrections for this are required in all analyses of direct reactions, especially those involving nucleons or deuterons [4]. The subject of

the present work is the incorporation into distorted wave Born approximation, DWBA, calculations of the effects of a different source of nonlocality: that generated by the coupling to inelastic or reaction channels. No prescription for this is known that is as simple as the introduction of the Perey correction factor. In this paper we demonstrate a method for incorporating dynamical nonlocality and show how, in a particular reaction, it can throw light on the significance of such nonlocality for direct reactions.

It may be the fact that exchange nonlocality accounts quite well for most of the energy dependence of the nucleon OMP that leads to the assumption that dynamically generated nonlocality is of little importance, particularly for the extraction of spectroscopic information. In this work we present a method for incorporating dynamical nonlocality that allows us to evaluate this assumption. We first briefly indicate the origin of what we have called dynamically generated nonlocality, a property of the dynamical polarization potential, DPP. To do this we survey the ways in which the DPP can be calculated and give an account of some of the indications that already exist for a significant dynamical nonlocality. This will lead to a presentation of calculations that show explicitly how dynamical nonlocality can influence the outcome of direct reaction calculations, with consequences for the extraction of spectroscopic information.

In Sec. II we discuss relevant general properties of the DPP, in Sec. III we review and contrast various ways in

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which the DPP can be evaluated, Sec. IV describes existing phenomenological evidence for the nonlocality of the DPP, Sec. V presents our evidence for the nonlocality of the DPP, showing its consequences for transfer reactions, and Sec. VI discusses the conclusions to be drawn. In this paper, we designate partial wave orbital angular momentum as l and bound nucleon or transfer angular momentum as L or J .

II. THE DYNAMICAL POLARIZATION POTENTIAL

In the background to the present work is the familiar Feshbach expression [5] for the nucleon OMP:

$$\mathcal{V}^P = PVP + PVQ \frac{1}{E+ - QHQ} QVP, \quad (1)$$

where P projects onto the elastic channel (assuming spin 0 or $\frac{1}{2}$) and $P + Q = 1$. This is already problematic since the nucleon-nucleon interaction entering into the first, PVP , term will not be the bare nucleon-nucleon interaction with its repulsive core but will be modified by particle-hole excitations. This raises questions of double counting that we shall not address here, but we note that the first term in Eq. (1) is often referred to as the folding model potential and the second term is the DPP. The folding model potential will contain exchange nonlocality but will not be expected to have dynamic nonlocality, and it is in this sense that we identify the DPP as the origin of dynamic nonlocality. In a complete treatment the DPP would also contribute to exchange nonlocality, but we do not consider that here.

It is often of interest to determine the contribution to the DPP of a specific subset of the excited target states, for example, highly collective states across a region of varying collectivity. Formally, we can partition the target states so that $\pi = P + p$ projects onto the ground state plus selected excited states, with q projecting onto all the others, so that $Q = p + q$ and $P + p + q = 1$. All the usual projection operator expressions apply, e.g., $\pi^2 = \pi$, $pq = 0$, etc. It can then be shown [6] that the effective potential for the enlarged π space is

$$\mathcal{V}^\pi = \pi V \pi + \pi V q \frac{1}{E+ - qHq} q V \pi. \quad (2)$$

The formal expression for the DPP that is generated in the elastic, P , channel, by the coupling within the enlarged $\pi = P + p$ space, can be shown [6] to be just

$$\Delta V = \mathcal{V}^P - \mathcal{V}^\pi. \quad (3)$$

The DPP ΔV will be both l -dependent and nonlocal but is commonly calculated as a local and l -independent potential that is S -matrix equivalent to ΔV , as described in Sec. III. Note that the interaction Hamiltonian that occurs in \mathcal{V}^π involves all the interchannel coupling within the p space and this implies, especially where q represents a numerically small number of states, that almost the full complex potential should be involved in the diagonal and nondiagonal coupled channel, CC, calculation within the π space. This also suggests the use of complex potentials whenever coupling between excited states is omitted in calculations of the OMP involving a ‘‘complete’’ set of excitations, as in cases mentioned below.

In particular cases, it may be feasible to include the entire space of excited states. For example, with a simple projectile such as a deuteron, the excited state space can, in principle, be included within the continuum discretized coupled channel, CDCC, framework, based on phenomenological interactions between the components of the projectile and the target nucleus. The DPP arising from the breakup of projectiles has been calculated in this way and provides examples of some of the properties described in a subsequent paper.

Feshbach’s theory did not explicitly consider the contribution of reaction channels. The explicit inclusion of these in CC calculations has revealed very significant contributions but also raises difficult questions of double counting and nonorthogonality, the second of which can now be allowed for. The explicit inclusion of particular strongly coupled reaction channels does reveal effects that do not seem to be represented by folding models of the kind that depend smoothly upon target mass and involve the local density approximation, LDA. Such folding models do not lead to l -dependence [7], dynamic nonlocality, or other consequences of the finite extension of the nucleus. These effects can be well represented with the second of the approaches described in Sec. III.

III. CALCULATING THE DPP

The DPP arising from channel coupling can be established using two general approaches:

- (1) The explicit evaluation of the Feshbach expression [5,8], or some approximation to it; see, for example, Refs. [9–11].
- (2) The inclusion of specific channels (the p channels) in a CC calculation, followed by a derivation of a potential that in some way (see below for a list of ways) incorporates the effects of the coupling of those specific channels to the elastic channel. Subtracting the ‘‘bare’’ potential of the CC calculation leads to a form of the DPP using one of the procedures to be listed below. (We use ‘‘CC’’ to include coupled reaction channel, CRC, and continuum-discretized coupled-channel, CDCC.) An earlier variation of this approach involved refitting the observables rather than the S -matrix produced by the CC code [6,12,13], but this procedure was superseded with the advent of S -matrix-to-potential inversion techniques [14–19].

The first approach, applying Feshbach’s formalism directly, has generally been applied to the calculation of the full OMP rather than the DPP due to selected channels. Quite extreme approximations were necessary, although suggestive results [10] have been obtained that supported some results of the second approach, in particular the l -dependence of the equivalent local potentials and the requirement that the propagation in the intermediate states must be in a complex potential and that plane wave propagators, as in some early calculations, are severely deficient. Other results from the calculations of Ref. [11], which are in some ways more realistic, have a bearing on the nonlocality considerations of this paper.

The second approach, “CC plus inversion,” has revealed that CC contributions lead to local-equivalent DPPs that are far from smooth in form and certainly do not represent a uniform normalization of the bare (folding model) potential. Calculations of this kind (i) facilitate the evaluation of processes (such as coupling to transfer channels) that do not appear to be well represented within folding models based on the LDA; (ii) produce corrections to global OMPs arising from the particular collectivities of specific nuclei; and (iii) provide a means of determining shell corrections to global potentials.

Coupling to collective states and transfer processes in particular have been found to lead to dynamic nonlocalities and l -dependencies outside the scope of the LDA. CC plus inversion can also be applied to projectiles as well as target nuclei: It has given valuable insight concerning the interaction of light nuclei, such as deuterons, in which something approaching a complete description of the projectile excited states can be included. Such calculations also provide suggestive insights concerning the general nature of DPPs.

The CC plus inversion approach to determining the DPP can be realized in different ways, each of which produces a different representation of it. These are the following:

- (1) **S-matrix inversion.** The elastic channel S -matrix, S_l , S_{lj} , or, for spin-1, S_{lj}^j , is inverted to produce a local potential that reproduces it exactly. The resulting potential is never of a smooth form and is always far from being a factor times the bare potential. This immediately calls into question the practice of including a uniform normalization factor when applying a folding model. There is now a substantial literature recording the results of CC-plus-inversion calculations. There are no limitations on the nature of the excitations that may be included: These may be inelastic channels, reaction channels, and projectile breakup channels.
- (2) **Exact TELP.** Franey and Ellis [20] derived independent potentials for each partial wave by determining algebraically the radial potential that produces in a single-channel Schrödinger equation the radial wave function calculated in the multichannel calculation. This is the TELP, trivially equivalent local potential. It can vary considerably from partial wave to partial wave and will become singular if the channel wave function has zeros. Coulter and Satchler [10] also introduced a somewhat different TELP for their nonlocal Feshbach DPPs; these were strongly l -dependent.
- (3) **Weighted TELP.** The strong l -dependence of the exact TELP can be tamed somewhat by a suitable weighting over contiguous partial waves to yield an l -independent potential [21]. This is the form of TELP incorporated in the CC code FRESKO [22]. The resulting potential is generally qualitatively similar to that derived by exact inversion, but can be significantly different in detail; see, for example, Ref. [23] for a comparison in the case of ${}^6\text{Li}$ breakup.
- (4) **ψ -potential.** An alternative form of TELP is the ψ -potential [24–28], the l -independent potential that exactly reproduces the elastic scattering wave function

as a function of two coordinates on the scattering plane in the interaction region.

- (5) **l -dependent factor.** The bare potential, or some approximate inverted smooth potential $V^R(r) + iV^I(r)$, is renormalized by independent functions of l for the real and imaginary parts, $N^R(l)V^R(r) + iN^I(l)V^I(r)$, so as to reproduce exactly the CC elastic channel S -matrix; see Ref. [29].
- (6) **Analytic approximations.** For certain specific kinds of collectivity there exist analytical procedures for calculating approximate DPPs; see Refs. [30,31].

It is the first of the above, S -matrix inversion, that yields a potential that is most directly related to local OMP phenomenology. In this way it offers the possibility of explaining, for example, departures from global behavior of a local OMP that has been fitted to particular nuclei. At the same time, the results from the other procedures suggest the possible pitfalls of the unconsidered application of local OMPs in the analysis of direct reactions, for reasons to be described below.

IV. EXISTING INDICATIONS OF SIGNIFICANT DYNAMIC NONLOCALITY

The theoretical DPP according to Feshbach’s formalism is both nonlocal and l -dependent (l -dependence is here to be distinguished from the parity dependence that arises with certain non-knock-on exchange processes.) It is not straightforward to identify the contribution of dynamical nonlocality to the properties of the local and l -independent S -matrix equivalent potential found by $S_l \rightarrow V(r)$ inversion. This is in part because *any* explicitly l -dependent local potential has an l -independent S -matrix equivalent having undulatory features which often resemble the undulatory features that arise in precision fits to elastic scattering data. In general, local-equivalent DPPs are quite undulatory, with the waviness having a point-by-point magnitude that can be disproportionate to their contribution to the volume integral of the potential. This might explain why the associated dynamic nonlocality or l -dependence contributes little to the global energy dependence of the OMP in the way that exchange nonlocality does, but it does not imply that it is not important.

The first ψ -potential calculations [24] studied the nature of the DPP that arises from the coupling to rotational channels. Features of the DPP that appear near $r = 0$ in a conventional representation appear instead at a finite impact parameter on the scattering plane. Over a substantial radial range, the wave function for the nonlocal potential (the elastic channel wave function of the CC calculation) exceeded the local wave function found by S -matrix inversion; both wave functions are identical in the asymptotic radial region. This is, in effect, an “anti-Perey” effect. If such effects are found to be a common consequence of dynamically generated nonlocality, this is likely to be significant for spectroscopic analyses involving direct reactions. Another suggestive phenomenon revealed by these calculations is the occurrence of an emissive region in the “shadow” edge of the nucleus, a clear indication of flux being returned to the elastic channel, a feature identified by Austern [32] as characteristic of nonlocality.

Subsequent ψ -potential calculations [26] evaluated the DPP generated by coupling the proton elastic channel to the same 10 phonon states that were employed by Coulter and Satchler [10]. A comparison on the scattering plane of the CC elastic channel wave function $\psi_{\text{nonloc}}(r, \theta)$ and the wave function of the local potential $\psi_{\text{loc}}(r, \theta)$ with the same S -matrix, also shows a distinct “anti-Perey” effect. The standard local DPP found by inversion, $\Delta V(r)$, exhibited deep undulations in both the real and imaginary parts, with substantial regions of emissivity in the imaginary part. As a check, the spatial distribution of the “generalized Perey factor”

$$R(r, \theta, \phi) = \frac{|\psi_{\text{nonloc}}|}{|\psi_{\text{loc}}|} \quad (4)$$

was calculated for a Perey-Buck nonlocal potential and it was consistent with the usual Perey effect, being close to spherical and roughly 0.85 within the nucleus. However, with the coupling to the ten phonons, the nature of $R(r, \theta)$ was very different. There was a complicated pattern of regions where R was < 1 (“Perey”) and regions where R was > 1 (“anti-Perey”). The coupling to deuteron channels led to extensive anti-Perey regions [but nonorthogonality corrections were not included in these ($p \leftrightarrow d$) calculations.] The phonon coupling calculations suggest that dynamical nonlocality is significant for the scattering of nucleons and other light ions from nuclei. It is not obvious what would be the effect of this dynamical nonlocality upon direct reactions involving nucleons, but there is now a strong motivation for a direct evaluation of this. It is likely that transfer reactions would depend sensitively upon where the maxima of the spectroscopic functions occur with respect to the features that we have described.

The DPP for heavier ions can be studied with the same methods, and in Ref. [27] the DPP generated in the $^{16}\text{O} + ^{12}\text{C}$ interaction by rotational excitations of ^{12}C was studied for 139.2 MeV laboratory energy ^{16}O . The inverted local DPP, the Franey-Ellis TELP, and the ψ -potential were compared, but the generalized Perey factor was not calculated. The DPP was found to have a strong emissive region in the surface, as well as surface repulsion. It was found that the coupling had the counterintuitive effect of increasing $|S_l|$ for higher values of l , so that the coupling actually reduced the partial reaction cross section for those partial waves. This phenomenon occurs in various circumstances and for a general discussion see Refs. [33,34]; see also Ref. [35] and below. For this particular case, the surface emissivity and increases in $|S_l|$ for higher values of l both occurred only if the imaginary component of the OMP was deformed [27], but there are strong reasons for believing that it should be.

More generally, the DPP calculated by the CC-plus-inversion technique applied to the S -matrix from a wide range of coupled channel calculations generally has varying degrees of undulatory character. We mention just a few recent examples. In Ref. [34] the effect upon deuteron elastic scattering of breakup to the continuum was studied at 56, 79, and 120 MeV. The DPPs were strongly undulatory, exhibiting emissive regions in the imaginary terms. The undulations and emissivity were greatest at the lower energies, where also the tendency for the coupling to increase $|S_l|$ was most marked.

It was noticed [23] that following CDCC calculations of projectile breakup in $^6\text{Li} + ^{12}\text{C}$ at 90, 123.5, 168.6, 210, and 318 MeV, there was a tendency for the local DPP due to breakup coupling to be strongly undulatory within the nucleus (an effect not shown by the weighted TELP). The undulations included emissive radial regions at 123.5 and 90 MeV and the potential was somewhat wavy in the surface for the 90 MeV case at which energy $|S_l|$ was almost doubled for several values of l . This is the same counterintuitive effect noted for deuteron scattering under the influence of deuteron breakup [34]. Arguments have been made [33,35] that this behavior is indicative of nonlocal effects, with the emissive features being the consequence of representing an underlying nonlocal DPP with a local equivalent potential.

The explicit evaluation of the Feshbach potential by Rawitscher, yielding potentials that are strongly nonlocal and also l -dependent, suggests that the relevant question now is the following: is there a reason, such as some kind of cancellation, why OMPs should not have these properties? We note that in cases where precise and wide angular range elastic scattering data are precisely fitted using model-independent methods, undulatory potentials tend to appear, a signature of underlying l -dependence. Moreover, potentials exhibiting substantial emissive features in their imaginary parts are found, suggestive of nonlocality. Examples for nucleon-nucleus potentials are given in Refs. [36] and [37]; the second of these exhibits an imaginary term with a large emissive feature while the potentials in Ref. [36] were more undulatory, probably because emissivity was explicitly excluded by the search procedure. Note also that the final potentials of Ref. [37] were for protons on ^{16}O for which the OM potential is parity dependent [38]; enforcement of parity independence leads to a more undulatory structure. An example of a highly undulatory potential that is required to give a precise fit to deuteron scattering data is given in Ref. [39]. This is a case where parity dependence is not expected.

There is another line of argument concerning the existence of substantial dynamic nonlocality arising from channel coupling. If there are two distinct nonlocal terms in the potential, $V_1(\mathbf{r}_1, \mathbf{r}_2)$ and $V_2(\mathbf{r}_1, \mathbf{r}_2)$ then the local equivalents $\tilde{V}_1(r)$ and $\tilde{V}_2(r)$ do not add linearly. That is, the local equivalent of $V_1(\mathbf{r}_1, \mathbf{r}_2) + V_2(\mathbf{r}_1, \mathbf{r}_2)$ is not $\tilde{V}_1(r) + \tilde{V}_2(r)$. This can be seen, for example, from the iterative nature of the method by which Perey and Buck [1] determined the local equivalent. This inequality applies to the nonlocal DPPs generated by coupling to channels that have no mutual coupling. This was studied in Ref. [40] where the influence on proton scattering of specific pickup reaction channels that were not mutually coupled was evaluated. The sum of local equivalent DPPs for two such reaction channels was not equal to the local equivalent of the two channels when both were coupled.

Finally, we note that various properties described here, such as the occurrence of emissivity and anti-Perey effects, suggest that simple prescriptions, along the lines of a Perey factor for exchange nonlocality, are unlikely to be effective for accounting for dynamically generated nonlocality.

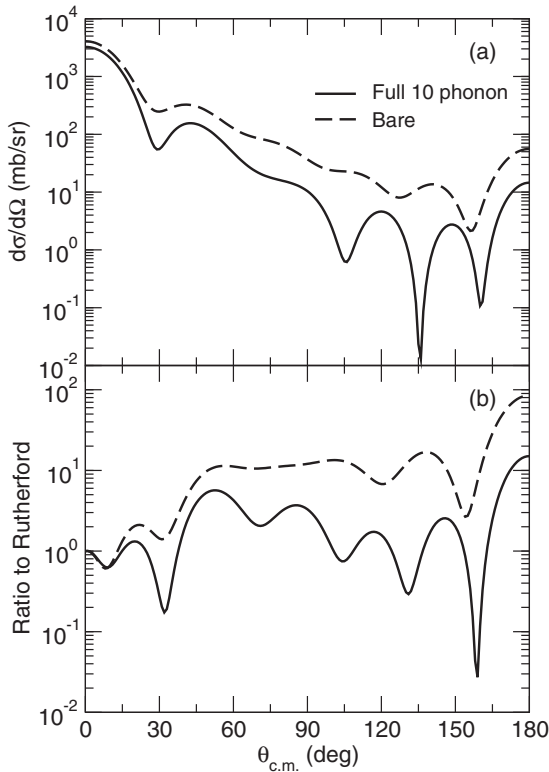


FIG. 1. Angular distributions for 30.3 MeV (a) neutrons and (b) protons on ^{40}Ca . The dashed lines are for the bare potential, and the solid lines are for coupling to the 10 collective states [10,41].

V. DIRECT EVIDENCE FOR SIGNIFICANT DYNAMIC NONLOCALITY

Section IV surveyed some of the evidence for dynamical nonlocality in nucleon and other (mostly) light-ion interactions, but it is not clear how important this almost universally ignored effect might be when local potentials are applied in the analysis of reactions. Here, we approach this by addressing the following question concerning the nucleon optical potentials that are employed in the analysis of (p,d) and (n,d) DWBA nucleon transfer reactions: What effect does the dynamical nonlocality, that might reasonably be expected to be a property of the proton or neutron potentials, have on the differential cross sections and other observables from standard DWBA calculations? For example, is the effect of sufficient magnitude to affect the extraction of spectroscopic factors?

To answer these questions, we study the effect of dynamical nonlocality by comparing DWBA (p,d) and (n,d) calculations employing dynamically nonlocal nucleon optical potentials, with calculations that employ local nucleon potentials. The local potentials in each comparison will be S -matrix equivalent to the corresponding nonlocal potential, hence yielding identical elastic scattering observables. The nonlocal potential will be generated *in situ* by coupling the nucleon elastic channel to a set of excited states of the target nucleus. The states chosen are expected to contribute to the experimental OMP for that projectile. The corresponding local potential will be determined by $S \rightarrow V$ inversion to have exactly

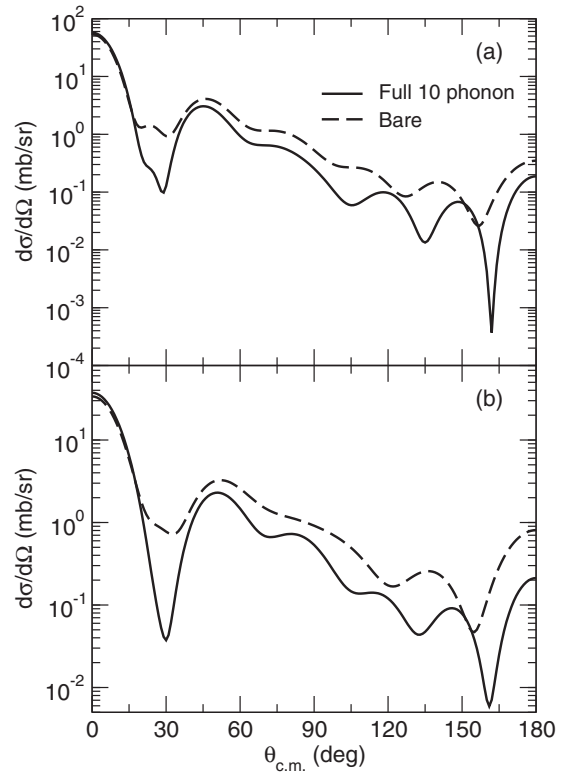


FIG. 2. Angular distributions for (a) (n,d) pickup and (b) (p,d) pickup, of $L = 0$ nucleons from ^{40}Ca for 30.3 MeV incident nucleons: The dashed lines are the ADs with the bare nucleon potential and the solid lines are the ADs with induced nonlocality due to coupling to the 10 collective states.

the same same S -matrix as the elastic channel S -matrix of the CC calculation. The nonlocal DWBA calculation will be carried out by exploiting the ability of the CC code FRESKO [22] to include a deuteron partition simultaneously with the inelastic nucleon scattering CC calculation. Backward coupling between partitions is excluded, so the only coupling to the deuteron channels is one way, from the ground state of the target. How this works will be explicit in the examples to follow.

We compare angular distributions (ADs) for pickup calculated with dynamically nonlocal potentials with the ADs calculated using local nucleon potentials that are S -matrix equivalent. The dynamical nonlocality is that which is induced by coupling to collective states of the target nucleus. Specifically, the collective states (phonons) were the collection of 10 that were included by Coulter and Satchler [10] in their calculation of the imaginary part of the optical potential for 30.3 MeV protons scattering from ^{40}Ca . The collective states cover a wide range of multipolarities and excitation energies, with deformation lengths judged to give a realistic account of the various multipole strengths. The local potential giving the same elastic scattering S -matrix is calculated using $S_l \rightarrow V(r)$ inversion. An account of the relevant inversion calculations is given in Ref. [41], which includes volume integrals and other characteristics of the local DPPs generated by the coupling. The local DPPs are the inverted potentials

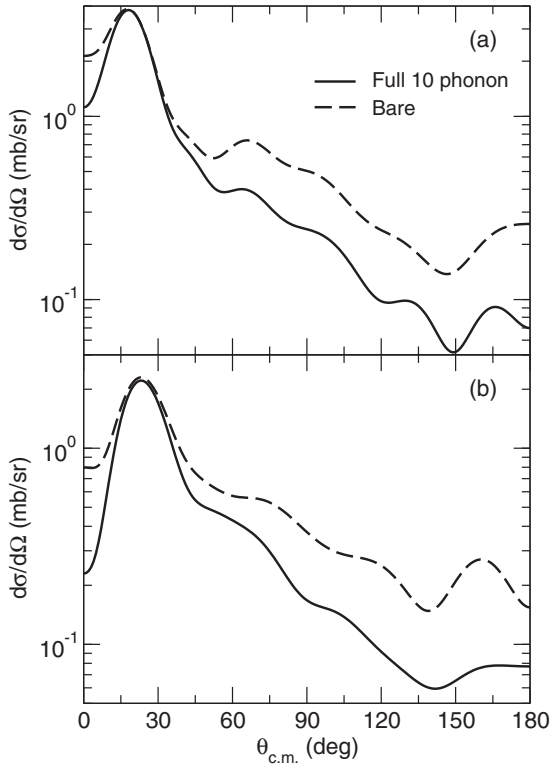


FIG. 3. Angular distributions for (a) (n,d) pickup and (b) (p,d) pickup, of $L = 2$ nucleons from ^{40}Ca for 30.3 MeV incident nucleons: The dashed lines are the ADs for the bare nucleon potential and the solid lines are the ADs with induced nonlocality due to coupling to the 10 collective states.

with the bare potentials of the CC calculations subtracted. The present calculations employ the same set of excitations of ^{40}Ca and the local equivalent potentials presented in Ref. [41] are those employed herein. We remark that the true DPPs are l -dependent as well as nonlocal, so that this work is actually studying the impact of both l -dependence and nonlocality that is generated by the coupling to collective states. We also remark that there are phenomenological grounds for l -dependence of the nucleon OMP [7]. Nevertheless, in what follows we generally refer to this work simply as a study of nonlocal effects, except where specific effects of l -dependence appear to be present.

For present purposes, a comparison of (p,d) and (n,d) ADs calculated in the DWBA, it is sufficient to employ a deuteron wave function calculated following Rawitscher [42] with a Gaussian n - p binding potential which is amenable to full finite-range interaction. Use of the DWBA implies that the spectroscopic factors are immaterial and were chosen to be unity. The radial wave function of the transferred nucleon is relevant and for both proton and neutron transfer, three cases were considered. These were $L = 0$ and $L = 2$ nucleons with binding energies appropriate to the respective ground-state-to-ground-state experimental Q values, and $L = 2$ nucleons bound 5 MeV more deeply, like the $\frac{5}{2}^+$ nucleons in a realistic mass 40 nucleus. These choices allow a comparison of effects for different L transfers independently of binding,

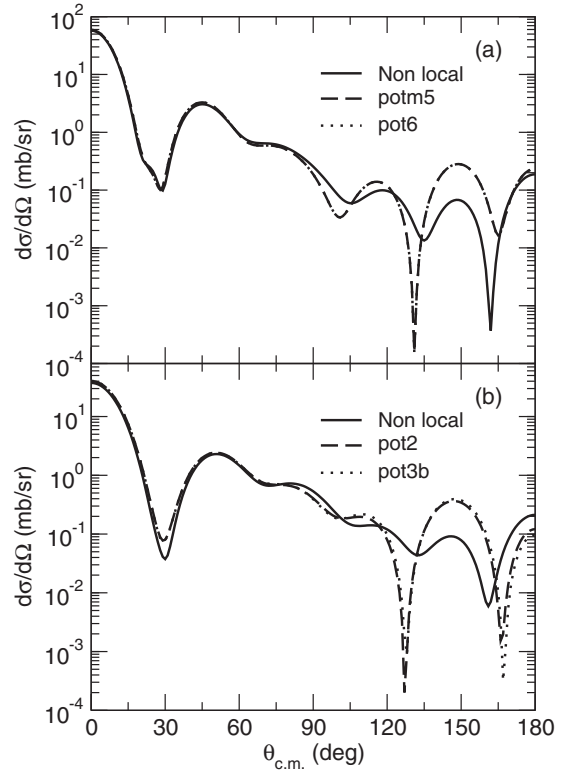


FIG. 4. Angular distributions for (a) (n,d) pickup and (b) (p,d) pickup, of $L = 0$ nucleons from ^{40}Ca for 30.3 MeV incident nucleons: The solid lines show the ADs for the nonlocal nucleon optical potential, and the dotted and dashed lines are for alternative local equivalent potentials found by inversion, as explained in the text.

and also an evaluation of the dependence upon binding energy for $L = 2$ transfer. The transferred nucleons were bound in local Woods-Saxon potentials with conventional parameters $r_0 = 1.2$, $a = 0.65$, and for protons $r_c = 1.3$. Since we are not considering the effects of deuteron breakup in this work, we used the global deuteron potential of Daehnick *et al.* [43] in the exit channel.

Following Coulter and Satchler [10] and Ref. [41], we treat nucleons as spinless, omitting spin-orbit interactions; a study of the effect of nonlocality on analyzing powers must await later studies. Consistently with this, we also omit spin in our treatment of the stripping process and this, of course, includes omitting the D state of the deuteron. The collective coupling, by design, produces a large part of the absorption of a realistic OMP for this case. However, transfer contributions were omitted [41] so the CC calculation employed a bare potential with a nonzero imaginary component. The effect of the coupling on the elastic scattering angular distribution is large, as can be seen in Fig. 1, with the neutron case in the upper panel and the proton case below. In both cases the dashed line is the AD for the bare potential, without coupling to the collective states, and the solid line is the AD with full coupling.

The substantial effect of the coupling is reflected in the change in the (n,d) and (p,d) angular distributions for the transfer of an $L = 0$ nucleon when the channel coupling effects

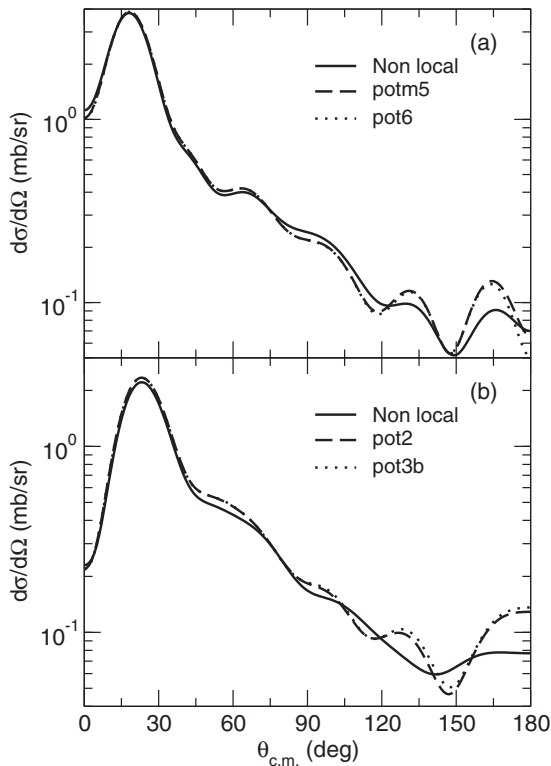


FIG. 5. Angular distributions for (a) (n,d) pickup and (b) (p,d) pickup, of $L = 2$ nucleons from ^{40}Ca for 30.3 MeV incident nucleons: The solid lines show the ADs for the nonlocal nucleon optical potential, and the dotted and dashed lines are for alternative local equivalent potentials found by inversion, as explained in the text.

are switched on; see Fig. 2. These figures compare the DWBA ADs for the bare nucleon potential with the DWBA ADs when the nonlocal DPP due to the collective coupling is switched on. The same comparison for the transfer of an $L = 2$ nucleon is shown in Fig. 3.

These figures show that the transfer differential cross section close to the main peak is modified very little by changes in the nucleon OMP that have a very large effect on the elastic scattering ADs. The question now, of course, is the following: How closely do the ADs calculated with nucleon potentials that are local equivalents to the nonlocal potentials approach the ADs for the nonlocal potentials? The difference will represent the effect of the dynamical nonlocality on the transfer angular distributions and thus give a measure of the significance of dynamical nonlocality.

The answer to this question is provided for $L = 0$ transfer in (n,d) pickup and (p,d) pickup in Fig. 4 and for the transfer of an $L = 2$ nucleon in Fig. 5.

In Figs. 4 and 5, the two alternative ADs for the local equivalent potentials are for alternative solutions [41] to the $S_l \rightarrow V(r)$ inversion [17–19]. As the iterative inversion process converges, the resulting potentials may become somewhat oscillatory and although the two potentials shown both give elastic scattering ADs that are indistinguishable from the elastic channel AD from the CC calculation, these figures

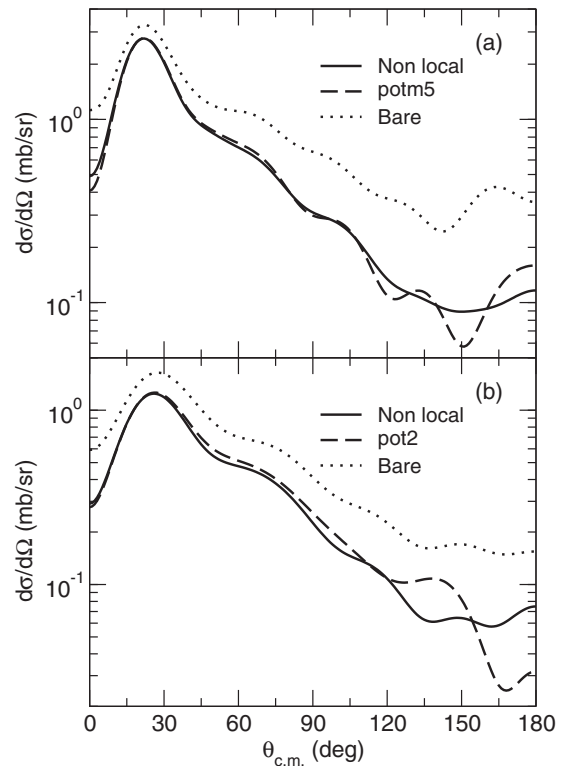


FIG. 6. Angular distributions for (a) (n,d) pickup and (b) (p,d) pickup, of $L = 2$ nucleons from ^{40}Ca for 30.3 MeV incident nucleons leading to a 5 MeV hole state. In each case, the dotted line is the AD with the bare nucleon optical potential, the solid line is with induced nonlocality due to coupling to the 10 collective states and the dashed line is for the local equivalent nucleon OMP. The labels on the dashed lines refer to particular inverted potentials; see Ref. [41].

verify that they also give very good agreement when applied in transfer reactions.

Except for the $L = 2$ neutron pickup case, the nonlocality effect shown in Figs. 4 and 5 is very small close to the main peak at forward angles. However, the detailed angular distributions are considerably modified at larger angles, for both $L = 0$ and $L = 2$. While this suggests that dynamic nonlocality does not substantially undermine the extraction of spectroscopic factors from fits to pickup, it is clear that a precise fit to the entire angular distribution requires consideration of the dynamic nonlocality. Certainly this motivates extending these calculations to include spin degrees of freedom since J -dependence depends on the angular distribution beyond the main peak.

We have examined the effect of a nonlocal nucleon OMP by comparing the pickup of $L = 2$ nucleons that are more bound by 5 MeV, calculated with nonlocal and local-equivalent nucleon OMPs. In Fig. 6 we present the angular distributions for pickup corresponding to hole states in ^{39}K and ^{39}Ca at 5 MeV excitation. Unlike the previous cases, here the AD near the main peak does depend on the presence of one or the other representation of the channel coupling effect. Moreover, for the proton case, the AD does respond significantly to the nonlocality quite close to the main peak.

The last figure suggests there is a variable response to the nonlocality of the nucleon potential, and it is clear that there is much to learn from calculations along the lines described here.

VI. DISCUSSION AND CONCLUSIONS

When a nucleon interacts with a target nucleus, the possible excitations of the nucleus generate a nonlocal and l -dependent dynamical polarization potential. Although such dynamical nonlocality must exist together with the well-known exchange nonlocality, it is almost never considered in the standard DWBA analysis of direct reactions involving nucleons. Nonlocal potentials generally require the solution of integrodifferential equations. For the particular case of exchange nonlocality there is a simple prescription, the Perey correction factor, that allows this to be avoided. However, in Sec. IV we gave reasons why dynamical nonlocality is unlikely to be representable by such simple means. In this paper we have shown how the need for solving Schrödinger's equation for a complicated nonlocal potential can be obviated. This involves the wave function being generated directly and employed *in situ* by means of an application of a coupled channel code.

The procedure was applied to study the dynamical nonlocality of the proton-nucleus potential due to coupling to collective vibrational states. The effect is appreciable for the case of nucleons scattering from ^{40}Ca , as revealed by an analysis of (p,d) reactions. Although a wide range of energies and target nuclei remain to be studied, the results allow us to conclude that conventional DWBA analysis of a (p,d) reaction,

with conventional local optical potentials, omits a significant contribution. This will affect the fitting of pickup angular distributions and perhaps the determination of spectroscopic factors.

Much remains to be done in applying the method developed here to study the effects of dynamical nonlocality. We have not studied the effect upon analyzing powers in pickup reactions and have studied only cases with low-angular-momentum transfer. It is likely that the effect will be larger for cases where the projectiles penetrate further into the nucleus. An obvious and important extension would be to include spin so that the effect on analyzing powers can be studied as well as J -dependence for which the details of the angular distribution are important. As far as the cases studied here are concerned, we note that the effect close to the main peak is not large for the strongly excited states studied, but the details away from the main peak, important for identifying J -dependence, are modified.

The method we have developed is quite general. For example, other sources of nonlocality such as reaction channel coupling remain to be explored. When deuterons or heavier composite nuclei interact with nuclei, the excitation of these projectiles also generates nonlocality in their effective interaction with the target. Hence, the same general approach that we have applied to nonlocality in proton channels will also yield information concerning a quite separate additional contribution to reactions involving deuterons. We refer to dynamic nonlocality in the deuteron channel due to breakup of the projectile. This is relevant to (d,p) , (d,t) , $(d,^6\text{Li})$, etc., reactions and will be the subject of a subsequent paper.

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