Effect of the reduced pairing interaction on α-decay half-lives of multi-quasiparticle isomeric states

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(Received 19 June 2014; published 31 October 2014)

The role of the pairing interaction in multi-quasiparticle isomeric states was examined by calculating α-decay half-lives with the superfluid tunneling model. We found that this model reproduces the experimental α-decay half-lives with an accuracy typical of current α-decay models. In spite of the simplicity of the model, we are able to demonstrate how the reduction of pairing in multi-quasiparticle isomers has a remarkable effect on α-decay half-lives. Taking this effect into account may be important for spin and parity assignments of α-decaying multi-quasiparticle isomers.

DOI: 10.1103/PhysRevC.90.044324 PACS number(s): 21.10.Tg, 21.30.Fe, 23.60.+e, 23.35.+g

I. INTRODUCTION

Pair correlations, which play a crucial role in superconducting solids [1], also have an important role in the structure of atomic nuclei [2]. For example, it is well known that the ground states of even-even nuclei have $J^\pi = 0^+$ with no exception. These states correspond to a quasiparticle (QP) vacuum with the seniority quantum number $\nu = 0$. The lowest lying noncollective states in even-even nuclei are states with a broken pair of nucleons with two quasiparticles. These $\nu = 2$ states located above the pairing gap Δ, typically at around $E^* \approx 1.0$–1.3 MeV, result in a significant weakening of pairing correlation [3]. It can be quantified as a smaller pairing gap, which was estimated to be reduced by ≈20%–40% [4]. This reduction was observed, for example, in energies of multi-QP states [5] as well as moments of inertia of rotational bands based on these states [3].

Multi-QP isomers are expected to occur when one or more pairs of nucleons are broken and appropriate recoupling of the spins of the unpaired nucleons lead to the formation of states with high values of the total spin projection K onto the symmetry axis. Because of large ΔK values, the γ transitions from these relatively low-lying states are hindered, causing the isomerism. The configurations of these states are relatively pure making these states excellent sites to study the underlying single-QP structures. This is one reason why experimental studies on K isomers near $Z = 100$ have become an important method to investigate properties of the superheavy elements ($Z \geqslant 104$); see, for example, Refs. [6–21]. In addition, it was proposed that for the heaviest nuclei, such multi-QP isomers become generally longer lived than the corresponding ground states [22]. In fact, a multi-QP isomeric state having a lifetime longer than of the ground state was observed in ^{270}Ds ($Z = 110$) [23].

Nuclear α decay has gained a considerable amount of theoretical interest within the last decade (see, for example, Refs. [24–27] and references therein), partially because it is a common decay mode of the superheavy elements. The accuracy of different models used to calculate the α-decay half-lives was improved to the level, where the experimental half-lives are reproduced within a factor of 2–4 [25]. The formation of an α particle inside the nucleus involves a pair of neutrons and a pair of protons closely correlated in space, and is therefore sensitive to the neutron and proton pair correlations. By definition, the multi-QP states are formed by breaking a nucleon pair, and one can expect the α decays from these states being hindered compared to ground-state α decays.

In this work we study the effect of reducing the pairing interaction in multi-QP states on the α-decay half-lives. In addition, we discuss how the reduction of pairing can influence spin and parity assignments for multi-QP isomeric states.

II. THEORETICAL BACKGROUND

In this work the superfluid tunneling model of Ref. [28] is applied. This model was developed for cluster decays and is capable of reproducing experimental half-lives of α-decay, heavy-ion radioactivity, and nuclear fission [29,30]. It is generally somewhat difficult to calculate half-lives and therefore many calculations have order of magnitude differences in their predictions, especially for spontaneous fission [31–34]. The Hamiltonian of the model can be written as

$$\left(-\frac{\hbar^2}{2D}\frac{\partial^2}{\partial\xi^2}\right)\psi(\xi) = E_n\psi(\xi), \tag{1}$$

where ξ is a generalized deformation variable describing the path of the system in the multidimensional space of deformations. In the case when only quadrupole deformation is considered, the parameter ξ is proportional to the axial deformation parameter β. The nucleus is deformed in small steps, the parameter ξ going from 0 to 1, where $\xi = 0$ corresponds to the shape of the spherical or near-spherical initial nucleus, and $\xi = 1$ corresponds to the touching-sphere situation, where the residual nucleus and the α particle are touching each other just before breaking apart. This was illustrated in Fig. 1 of Ref. [29].

Equation (1) can be discretized on a grid of step $\Delta\xi = 1/n$ [35] and one finds the inertia of the system as

$$D = -\frac{\hbar^2}{2v}n^2, \tag{2}$$

where v is the transition matrix element between two successive steps and n is the number steps, approximated to

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be ≈ 4 for α decay [30]. This transition between successive steps is assumed to be governed by a pairing operator. It can be estimated by mean values of BCS wave functions [35] as

$$v = -\frac{\Delta_\nu^2 + \Delta_\pi^2}{4G}, \tag{3}$$

where $G = 25/A$ is the standard pairing strength [36] and Δ_π, Δ_ν are the pairing gap parameters for protons and neutrons, respectively. In the original papers of this model, Barranco and coworkers used the simple relation of Bohr and Mottelson $\Delta = 12/\sqrt{A}$ [36]. Here we have improved the accuracy of the model by calculating $\Delta = \Delta_\nu = \Delta_\pi$ via the relation,

$$\Delta = (1 - X \cdot I^2)\Delta^{(A)}, \tag{4}$$

where the neutron excess $I = (N - Z)/A$ and $\Delta^{(A)} = \frac{Y}{A^{1/3}}$ with X and Y fitted to experimental data (see Sec. III). Equation (4) was originally proposed in Ref. [37], in which the constants $X = 6.1, Y = 7.2$ were used. Different fits have been performed for these constants by several authors [38–41]. In this work, we have performed an independent fit (see Sec. III A for more details).

The decay constant λ of the α-decay process is calculated via the relation:

$$\lambda = f \cdot P \cdot T, \tag{5}$$

where $P = |\psi(\xi = 1)|^2$ is the α-particle formation probability at the nuclear surface, f is the assault frequency of the α particle hitting the barrier, and T is the transmission coefficient of the α particle through the barrier. To calculate P, we use the wave function describing the ground state of the harmonic oscillator,

$$\psi(\xi) = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{1}{2}\alpha^2\xi^2}, \tag{6}$$

where

$$\alpha^2 = \sqrt{\frac{C}{2|v|}}n, \tag{7}$$

with the potential energy parameter $C = 2V = 2(V_N + V_C - Q_\alpha)$ and V_N, V_C being the nuclear and Coulomb potentials, respectively. For the nuclear potential, we have used the Christensen-Winther potential [42],

$$V_N = -50.0\, R_{aA}\, e^{\frac{-r-R}{a}}, \tag{8}$$

where $a = 0.63$ fm and R_{aA} is the reduced radius with the parametrization $R_i = 1.233A_i^{1/3} - 0.98A_i^{-1/3}$ with $i = a, A$ [35]. The assault frequency f is calculated via

$$f = \frac{\omega}{2\pi}, \tag{9}$$

where $\omega = \sqrt{C/D}$, assuming motion of a particle with the inertia D in the ground state of a harmonic well. We have approximated the barrier penetrability T by the probability for an α particle tunneling through the Coulomb barrier (no nuclear potential involved) starting from the touching distance R_0 by the relation [43],

$$T = \frac{\rho}{F^2(\eta\rho) + G^2(\eta\rho)}, \tag{10}$$

where $\rho = R_0 k$ with $k = \sqrt{2Q_\alpha\mu}/\hbar$ and $R_0 = 1.2(A_D^{1/3} + A_\alpha^{1/3}) + 0.63$ fm and $\eta = 1/ka$, where $a = \hbar^2/e^2\mu Z_\alpha Z_D$. Here F and G are the regular and irregular Coulomb wave functions [44], which will also account for the effect of the orbital angular momentum L of the emitted α particle in the tunneling process. Only the lowest angular momentum transfer L of $|J_i - J_f| \leqslant L \leqslant J_i + J_f$ was considered. A change in parity increases L by one unit in some cases, i.e., only even values of L are allowed in case of no parity change and only odd values of L are allowed if there is a change in parity. As an approximation, the radius of the nuclear isomeric state was taken to be the same as that of the ground state.

III. RESULTS AND DISCUSSION

A. Pairing gap parameter Δ

Before calculating α decays of multi-QP states, the pairing gap parameter Δ was optimized for $\nu = 0$ and 1 states by using the data set of 341 partial half-lives of ground state to ground-state α decays from Ref. [25]. Three odd-odd nuclei ($^{176}_{77}$Ir, $^{206}_{85}$At, and $^{218}_{91}$Pa), were removed from the data set of Ref. [25] because of a relatively large difference between the theoretical and experimental half-life values. This may be from inaccuracies in the experimental data, as already suggested for the two latter nuclei in Ref. [45]. The free parameters (X, Y) of Eq. (4) were varied to minimize the root-mean-square (RMS) deviation:

$$\sigma = \left\{\frac{1}{(n-1)}\sum_{i=1}^{n}\left[\log_{10}T_{1/2}^{\text{calc.}}(i) - \log_{10}T_{1/2}^{\text{exp.}}(i)\right]^2\right\}^{1/2}. \tag{11}$$

The reduction of pairing from a blocking of an odd particle was taken into account by performing separate fits for even-even and odd nuclei, the latter including both odd-A and odd-odd nuclei. This resulted in values of $X = 1.88, Y = 5.92$ for even-even nuclei and $X = 9.3, Y = 6.06$ for odd-A and odd-odd nuclei. The accuracy of the fit could only be slightly improved by performing separate fits for even-odd, odd-even, and odd-odd nuclei, respectively. The resulted RMS errors of the decimal logarithm of the α-decay half-lives obtained in the framework of the superfluid tunneling model and other models [24,25,45–48] are shown in Table I. The simple model used in this work can reproduce the experimental half-lives within an order of magnitude (within a factor of 2.5 for even-evens), which is comparable to other α-decay models. The decimal logarithms of the ratios between the theoretical and experimental half-lives are presented in Fig. 1, which shows also the comparison between the theoretical values of this work and the values calculated with the semiempirical formula for even-even nuclei proposed by Royer [45]. The pairing gap parameters calculated by using Eq. (4) and the fitted coeffients X, Y are shown in Fig. 2. This shows that the simple model

TABLE I. RMS errors of the decimal logarithms of α-decay half-lives values for different models. The four lowest rows have been taken from Ref. [25]. The quantity shown in brackets is the number of studied decays.

Even-even	Odd-A	Odd-odd	
0.383 (136)	0.890 (160)	0.514 (45)	This work
0.328 (136)	0.613 (160)	0.429 (45)	[45]
0.267 (157)	0.285 (231)	0.435 (79)	[24]
0.3088 (136)	0.7816/0.7621 (84/76)	0.7546 (48)	[25]
0.5165 (136)	1.1611/1.3348 (84/76)	1.2568 (48)	[46]
0.3712 (136)	1.5425/1.3541 (84/76)	1.3307 (48)	[47]
1.2928 (136)	1.4300/1.5607 (84/76)	1.2751 (48)	[48]

contains the essential ingredients to describe features of α decay.

B. Pairing reduction in multi-quasiparticle states

The main focus of this work is to study how large an impact the reduction in pairing has on the α-decay half-lives of multi-QP isomeric states. The α decays of these states differ from ground-state α decays in three ways. (i) The Q_α(i.s.→g.s.) value compared to the Q_α(g.s.→g.s.) value is larger; (ii) the angular momentum difference between the states gives rise to a large L barrier; and (iii) the magnitude of the pairing interaction is reduced. Of these effects the first two are well known. The first has the $1/\sqrt{Q_\alpha}$ dependence on α-decay half-lives [49,50], making the α decay faster and the second, the L barrier, introduces an additional hindrance factor, making the α decay slower as demonstrated in Fig. 2 of Ref. [51]. To our knowledge, the effect of the pairing reduction was not explicitly examined in earlier studies. To illustrate this effect, a dramatic example shown in Fig. 3, we have taken the reduction in pairing into account by modifying the pairing gap Δ and calculating the factors $f,P,T,$ and $t_{1/2}$ for the α decay of element $^{294}118$ assuming a hypothetical, $K = 10^+$

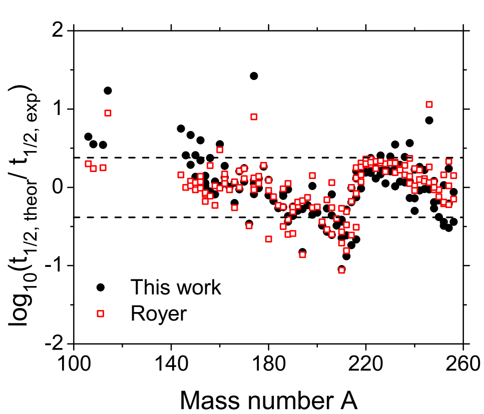

FIG. 1. (Color online) Decimal logarithms of the ratios between theoretical and experimental half-lives of even-even α emitters. The theoretical values have been calculated with the tunneling theory and the Royer formula [45]. The dashed line shows the RMS deviation of $\sigma = 0.383$. The effect of $Z = 82$ and $N = 126$ on α-particle preformation factors is clearly visible in both theories.

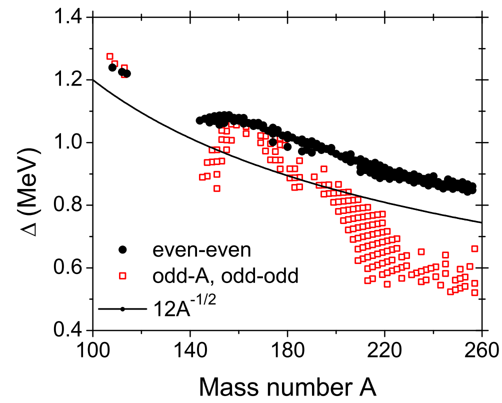

FIG. 2. (Color online) Pairing gaps extracted from the experimental α-decay half-lives by using the superfluid tunneling model. The even-even nuclei are shown as circles (black) and odd-A and odd-odd nuclei in squares (red). The standard average gap is shown as a solid curve. See text for more details.

2-QP isomeric state at $E = 1.05$ MeV, which corresponds to $Q_\alpha = 12.87$ MeV. To form an α particle, a pair of neutrons and a pair of protons need to be coupled together. Therefore, the reduction in the pairing interaction has a remarkable effect on the α-particle preformation factor, which is clearly seen in Fig. 3.

We would like to comment here that the measured half-life of the ground state of element $^{294}118$ is $t_{1/2} = 0.69$ ms [52], which is 1–2 orders of magnitudes shorter than of the hypothetical isomeric state (see Fig. 3). Therefore, assuming a stiff axial deformation to persist, we can expect the heaviest elements to have isomeric states with longer half-lives than their ground states.

Determination of pairing reduction factor for multi-quasiparticle states

We have calculated the α-decay half-lives for 15 α-decaying multi-QP isomeric states in odd-A and or even-even nuclei with known $t_{1/2}, L,$ and Q_α with the pairing gap varied between 0.6–1 of the $\Delta_{g.s.}$. Only the α transitions to the ground states

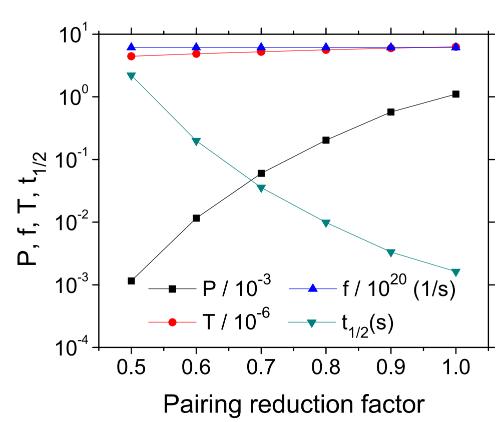

FIG. 3. (Color online) Contribution of the pairing reduction on different factors $f,P,T,$ and $t_{1/2}$ on the α decay of a hypothetical 2-QP isomeric state in element 118. See text for more details.

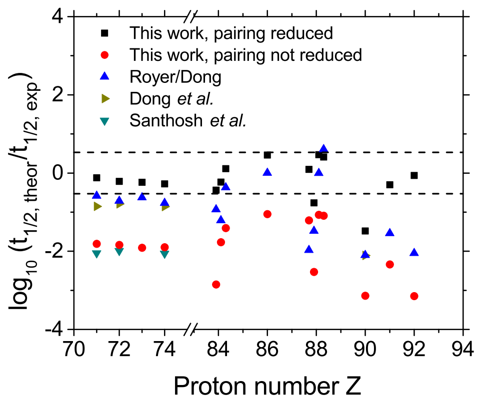

FIG. 4. (Color online) α-decay half-lives for multi-QP isomers: ^{155}Lu^{m2}, ^{156}Hf^{m}, ^{157}Ta^{m}, ^{158}W^{m}, ^{211}Po^{m}, ^{212}Po^{m}(6^+,8^+), ^{214}Rn^{m}, ^{213}Ra^{m}, ^{214}Ra^{m}, ^{216}Ra^{m}(8^+,10^+), ^{216}Th^{m}, ^{217}Pa^{m}, and ^{218}U^{m} calculated with and without the pairing reduction and compared with the experimental partial half-lives [53] as well as the calculated $t_{1/2}$ values found in literature [51,54]. The blue triangles have been calculated by formulas from Ref. [51] for unfavored decays. The dashed line shows the RMS deviation of $\sigma = 0.53$ for all 15 cases.

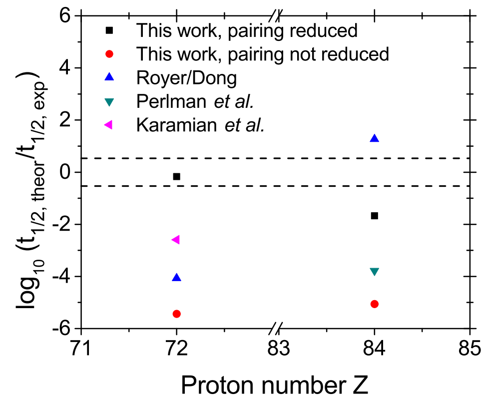

FIG. 5. (Color online) α-decay half-lives for 4-QP isomers ^{178}Hf^{m2} and ^{212}Po^{m} calculated with and without the pairing reduction and compared with the values found in literature [56,57]. The blue triangles have been calculated by formulas from Ref. [51] for unfavored decays. The dashed line shows the RMS deviation of $\sigma = 0.53$ extracted from the $\Delta v = 2$ decays.

have been considered. The pairing reduction of 0.6 (40%) was observed to reproduce best the experimental half-life values. This is in agreement with the simple estimate for the seniority ν states $\Delta_\nu \cong \sqrt{\Delta_0(\Delta_0 - \nu \cdot d)}$ [3], giving $\Delta \approx 0.58$–0.66 Δ_0 for the 13 studied cases. Here the average level spacing $d = 50/A$ [2] and $\Delta_0 = 12/\sqrt{A}$ have been used.

With the reduction factor of 0.6 common for all known cases, the 15 partial half-lives of multi-QP ($\Delta v = 2$) α decays were reproduced within a factor of 3.4 ($\sigma = 0.53$). The only exception here is ^{216}Th^{m}, which is a factor of $\approx$30 off. The calculated isomeric half-life values compared to the experimental partial half-lives are presented in Fig. 4. For a comparison, the half-life values calculated with the Royer's formulas [55] with the L hindrance corrected by a formula proposed by Dong [51] have larger deviation, $\sigma = 1.25$, although the Royer's formulas [45,55] for even-even nuclei reproduce the ground state to ground-state decays slightly better compared to our model.

There exist two α-decaying 4-QP isomers in literature, ^{178}Hf^{m2} and ^{212}Po^{m} [56,57]. For those isomers, we have calculated half-lives for $\Delta v = 4$ α transitions by using the reduction factor of $0.6^2 = 0.36$. For ^{178}Hf^{m2} only the total half-life was measured, and the theoretical decay constant was calculated as sum of the decay constants to all states with $I \leqslant 16$ in the ground state band of ^{174}Yb. For ^{212}Po^{m} the partial half-life of the transition to the ground state was calculated. A comparison with the experimental half-lives is shown in Fig. 5.

In Ref. [3] a geometric dependence of the pairing on seniority ν, $\Delta_\nu \approx 0.75 \cdot \Delta_{\nu-2}$ was derived based on calculations with the Lipkin-Nogami method [58,59]. That pairing reduction factor is somewhat larger than extracted from the α-decay half-lives, which may reflect that the pairing reduction factor of 0.6 may be folding in also some additional structure hindrance. In other words, our simple picture of associating the longer half-life to only a reduction in pairing is complicated in reality by the interplay of other effects such as changes in the barrier height from shell effects.

C. Determination of the spin and parity of α-decaying isomeric states

In many cases, especially for superheavy nuclei, the only experimental information available are the measured α energies and half-lives. In those cases, calculated half-life values for given E_α and different L are used to estimate the spin and parity of the α-decaying state according to the selection rules discussed in Sec. II. However, if the α-decaying state has a multi-QP character, the pairing is reduced and if not taken into account may lead to an incorrect conclusion. As already shown earlier, this effect changes the calculated half-life values by 1–2 orders of magnitude and L by 1–3 units. A good example is ^{217}Pa, which has an α-decaying isomeric state at 1.85 MeV with a partial half-life of 2.1 ms for the α transition to the ground state of ^{213}Ac ($J^\pi = 9/2^-$) [60]. Based on systematics, the most probable assignments for spin and parity for ^{217}Pa^{m} are either $23/2^-$ or $29/2^+$, which corresponds to $L = 8$ and $L = 11$ transitions, respectively. All three experimental works [60–62] proposed the $29/2^+$, based, however, on calculations performed with different models [63–65]. We calculate $t_{1/2} = 1.1$ ms for the $23/2^-$ and $t_{1/2} = 60$ ms for the $29/2^+$ assignment, clearly favoring the $23/2^-$ configuration. Our assignment is supported by a recent paper [66], in which the authors arrived at the same conclusion based on the systematics of the fully aligned $(\pi h_{9/2})^2(\pi f_{7/2})^1$ configuration in $N = 126$ isotones.

IV. CONCLUSIONS

The influence of a reduced pairing interaction on α-decay half-lives of multi-QP isomeric states was studied. We have used the superfluid tunneling model of Ref. [28] and used this to extract pairing gap parameter Δ values from the α-decay half-lives. These pairing gap parameter values are comparable to the values calculated from the atomic masses. Despite the simplicity of the model, the experimental partial half-lives of multi-QP isomeric states are reproduced within a factor of ≈ 3.4 by introducing a pairing reduction of 40%, common for all 15 cases found in literature. This pairing reduction corresponds to hindrance factors in the range 30–10^3. We assume that large fraction of that hindrance factor is from the pairing reduction, although some component of the hindrance may also originate from nuclear structure effects. The additional hindrance caused by reduced pairing may increase the possibility that the heaviest elements form long-living α-decaying isomeric states having half-lives longer than their ground states.

The model can also be used to aid the determination of the spins and parities of α-decaying isomers based on the orbital angular momentum L of the emitted α particle. We propose that the pairing reduction has to be taken into account when calculating the α-decay half-lives of multi-QP isomeric states. In the model used, the reduction of pairing has a remarkable effect on the α-particle preformation factor. Many other α-decay models, including, for example, the generalized liquid drop model (GLDM) [51], the density-dependent cluster model (DDCM) [24,67], and the R-matrix theory [68] approximate the α-particle preformation factor as constant, being of the order of 0.1–0.01, and differing only between even-even, odd-A, and odd-odd nuclei. It would be of great interest to see whether the modification of the pairing strength proposed in this work can be included in those models.

ACKNOWLEDGMENTS

We are thankful to H. L. Crawford, J. M. Gates, and K. E. Gregorich for illuminating discussions and critical reading of the manuscript and to O. Tarvainen for helping with figures.

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