## <span id="page-0-0"></span>**Probing nuclear dissipation with particle multiplicity in heavy-ion-induced light fissioning systems**

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Based on the stochastic Langevin model, we study the effects of angular momentum  $(\ell)$  on the multiplicities of postsaddle neutrons, protons, and  $\alpha$  particles as a function of postsaddle friction strength ( $\beta$ ) for <sup>200</sup>Pb nucleus. It is shown that with increasing  $\ell$  the sensitivity of these particles to  $\beta$  is significantly enhanced. Moreover, we find that neutrons (charged particles) evaporated from light <sup>200</sup>Pb having high  $\ell$  and high excitation energy  $E^*$ exhibit a similar (greater) sensitivity to  $\beta$  as compared to the case of heavy <sup>251</sup>Es having low  $\ell$  and low  $E^*$ . Our findings suggest that on the experimental side, to accurately probe information of postsaddle dissipation with particle multiplicity, in particular with the multiplicity of charged particles, it is optimal to populate light fissioning systems with large angular momentum and high excitation energy.

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*Introduction*. Numerous measurements of particle multiplicity evaporated from an excited fissioning system exceed significantly that predicted by standard statistical models  $[1-8]$ . The discrepancy has been shown to be due to the neglect of friction effects in fission [\[9](#page-3-0)[–17\]](#page-4-0). A systematic investigation [\[18,19\]](#page-4-0) for the data of fission probabilities and particle multiplicities based on stochastic approaches to fission suggested a rising function of friction with deformation, i.e., a weak friction inside the fission barrier and a strong saddleto-scission friction. But a modified one-body dissipation was also used to reproduce fission data [\[13\]](#page-3-0). While the two different types of deformation-dependent friction give a similar strength of presaddle friction, the predicted postsaddle friction differs very much. It has been identified that the shape dependence of friction [\[20\]](#page-4-0) is a key input quantity of Langevin models when the model is applied to describe the fission process of a hot nucleus. Thus, how to accurately extract information of the saddle-to-scission friction becomes very ungent and necessary.

We note that the essential information of postsaddle friction obtained in Refs. [\[18,19\]](#page-4-0) is through an analysis of neutron multiplicity data of heavy decaying systems [\[21\]](#page-4-0). Making using of heavy nuclei is primarily because postsaddle particles rise with increasing size of fissioning nuclei. Furthermore, besides neutrons, multiplicities of light charged particles (LCPs), namely protons and  $\alpha$  particles, measured in coincidence with fission fragments from heavy fissioning systems  $(A > 250)$  [\[22\]](#page-4-0), were also employed to exploit fundamental properties of postsaddle dissipation, though the excitation energy of the formed heavy system is not very high ( $E^*$  < 70 MeV).

However, when yielding a heavy composite system  $(A \sim 240)$  by heavy ion collisions, both fusion-fission and quasifission channels appear  $[23]$ . They have contributions to the measured fission fragments, because the features of fragments produced in the two different types of reaction mechanisms have some overlaps. This causes a large uncertainty of determining nuclear friction in fusion-fission processes with multiplicity data of heavy systems, as clearly shown in Refs.  $[1,24-27]$  $[1,24-27]$ . As a consequence, to obtain fusion-fission process related friction parameters, one must correct experimental prescission multiplicity data by resorting

to a careful and complicated estimate for the contributions stemming from quasifission channels [\[28\]](#page-4-0) by employing dynamical models that account for entrance channel effects, such as Feldmeier's program HICOL [\[29\]](#page-4-0). In addition, the higher the incident energy (and hence the larger the excitation energy and the angular momentum of the formed composite system), the stronger the competition between the quasifission channel and the fusion-fission channel [\[30\]](#page-4-0). This further restricts the reliable use of particle emission data from heavy systems in exploring friction parameters related to fusionfission processes to a domain of low excitation energy and low angular momentum.

By contrast, the population of light compound systems can be free from the evident interference from quasifission, a prominent advantage for experimentally obtaining precise information of prescission particles coming from fusion-fission processes alone. This greatly favors a more stringent constraint on the determination of postsaddle friction by comparing multiplicity measurements with model calculations. To more effectively utilize the opportunity provided by light compound nuclei in pining down the properties of postsaddle dissipation, the present work is devoted to studying under which conditions the sensitivity of particle emission of light fissioning systems to postsaddle friction can be enhanced. To this end, we will survey the influences of angular momentum and excitation energy on the sensitivity in the framework of Langevin models. The stochastic approach  $[9-11,13-19]$  $[9-11,13-19]$  has been shown to successfully reproduce many observables, including particle multiplicities and evaporation residue cross sections for a lot of compound nuclei over a broad range of excitation energy, angular momentum, and fissility.

*Theoretical model*. A description of the combined Langevin equations coupled with a statistical decay model (CDSM) is given. We refer the reader to Refs.  $[18,19]$  for more details. The dynamic part of the CDSM is described by the Langevin equation that is expressed by entropy. We employ the following one-dimensional overdamped Langevin equation [\[18\]](#page-4-0) to perform the trajectory calculations:

$$
\frac{dq}{dt} = \frac{T}{M\beta} \frac{dS}{dq} + \sqrt{\frac{T}{M\beta}} \Gamma(t).
$$
 (1)

Here  $q$  is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus. M is the inertia parameter  $[31]$ . As is usual in the literature, the reduced friction coefficient (also called the dissipation strength)  $\beta = \gamma/M$  is used, which is the ratio of the friction coefficient  $\gamma$  to the inertia M. So, Eq. [\(1\)](#page-0-0) is numerically solved with the reduced friction coefficient  $\beta$ . The temperature in Eq. [\(1\)](#page-0-0) is denoted by T, and  $\Gamma(t)$  is a fluctuating force whose average and correlation function are  $\langle \Gamma(t) \rangle = 0$  and  $\langle \Gamma(t) \Gamma(t') \rangle$  $= 2\delta(t - t')$ , respectively. The driving force of the Langevin equation is calculated from the entropy:

$$
S(q, E^*) = 2\sqrt{a(q)[E^* - V(q)]},
$$
 (2)

where  $E^*$  is the total internal energy of the system.  $a(q)$  is deformation-dependent level density parameter and reads as  $a(q) = a_1A + a_2A^{2/3}B_s(q)$  with A being the mass number.  $a_1 = 0.073$  and  $a_2 = 0.095$  are taken from Ignatyuk *et al.* [\[32\]](#page-4-0). Equation (2) is constructed from the Fermi-gas expression with a finite-range liquid-drop potential  $[33,34]$   $V(q)$  that includes  $q$ -dependent surface, Coulomb, and rotation energy terms. In our dynamical calculations we use  $\{c, h, \alpha\}$  [\[35\]](#page-4-0) parametrization of the compound nucleus shape. Since only symmetrical fission is considered, the parameter describing the asymmetry of the shape is set to  $\alpha = 0$  [\[19,36\]](#page-4-0). The deformation coordinate q is obtained by the relation  $q(c,h)$  =  $(3c/8){1 + \frac{2}{15}[2h + (c-1)/2]c^3}$  [\[18,37\]](#page-4-0), where c and h correspond to the elongation and neck degrees of the freedom of the nucleus, respectively

In the CDSM, light-particle evaporation is coupled to the fission mode by a Monte Carlo procedure. The emission width of a particle of kind  $\nu$  is given by [\[38\]](#page-4-0)

$$
\Gamma_{\nu} = (2s_{\nu} + 1) \frac{m_{\nu}}{\pi^2 \hbar^2 \rho_c(E^*)}
$$
  
 
$$
\times \int_0^{E^*-B_{\nu}} d\varepsilon_{\nu} \rho_R(E^* - B_{\nu} - \varepsilon_{\nu}) \varepsilon_{\nu} \sigma_{\text{inv}}(\varepsilon_{\nu}), \quad (3)
$$

where  $s_v$  is the spin of the emitted particle  $v$ , and  $m_v$  its reduced mass with respect to the residual nucleus. The level densities of the compound and residual nuclei are denoted by  $\rho_c(E^*)$  and  $\rho_R(E^* - B_\nu - \varepsilon_\nu)$ .  $B_\nu$  are the liquid-drop binding energies.  $\varepsilon$ is the kinetic energy of the emitted particle and  $\sigma_{\text{inv}}(\varepsilon_v)$  the inverse cross sections [\[38\]](#page-4-0).

A formula suggested by Fröbrich and Gontchar  $[18]$  $[18]$  is used to evaluate the deformation dependence of the charged-particle emission barriers:

$$
V_c(q) = V_{\nu} \times B_c(q). \tag{4}
$$

Here,  $V_v = \frac{(Z - Z_v)Z_v K_v}{R_v + 1.6}$  with  $K_v = 1.32$  for  $\alpha$ , and 1.15 for protons.  $R_v = 1.21[(A - A_v)^{1/3} + A_v^{1/3}] + (3.4/\varepsilon_v^{1/2})\delta_{v,n}$ , where  $A_{\nu}$  and  $\varepsilon_{\nu}$  is the mass number and the kinetic energy of the emitted particle  $v = n, p, \alpha$ .

In the CDSM, the formulas used to calculate the surface energy  $B_s(q)$  and the Coulomb energy  $B_c(q)$  in Eq. (4) are taken from Ref. [\[39\]](#page-4-0).

Deformation affects Coulomb emission barrier of LCPs. It also modifies particle binding energy [see Fig.  $3(a)$ ], because the mass formula [\[34\]](#page-4-0) contains the deformation-dependent

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surface energy term and Coulomb energy term. So, the particle binding energy  $B_i$   $(i = n, p, \alpha)$  is also a function of q [\[11,](#page-3-0)[18,40,41\]](#page-4-0) and is expressed as  $B_i(q) = M_p(q) M_d(q) + M_i$ . Here,  $M_i$   $(i = n, p, \alpha)$  is the mass of the emitted particles.  $M_p(q)$  and  $M_d(q)$  are the masses of the mother and daughter nuclei, respectively.

The CDSM describes the fission process as follows. At early times, the decay of the system is modeled by means of the Langevin equation. After the fission probability flow over the fission barrier attains its quasistationary value, the decay of the compound system is described by a statistical branch. In the statistical branch we calculate the decay widths for particle emission and the fission width and use a standard Monte Carlo cascade procedure with the weights  $\Gamma_i/\Gamma_{\text{tot}}$  with  $(i = \text{fission}, n, p, \alpha)$  and  $\Gamma_{\text{tot}} = \sum_i \Gamma_i$ . This procedure allows for multiple emissions of light particles and higher chance fission. In case fission is decided there, one switches again to the Langevin equation for computing the evolution from saddle to scission. Prescission particle multiplicities are calculated by counting the number of corresponding evaporated particle events registered in the dynamic and statistical branch of the CDSM.

*Results and discussion*. In the present study the presaddle friction strength is set as  $3 \times 10^{21}$  s<sup>-1</sup>, in accordance with recent theoretical and experimental results [\[13](#page-3-0)[,14,18,42–44\]](#page-4-0). Moreover, to better probe the postsaddle friction strength  $(\beta)$ with particle emission, in this work dynamical calculations are performed considering different values of  $β$ . To accumulate sufficient statistics,  $10<sup>7</sup>$  Langevin trajectories are simulated.

Shown in Fig. 1 are the calculated postsaddle neutrons  $(M_n)$  and LCPs  $(M_p$  and  $M_\alpha)$  at various  $\beta$  for two different angular momenta of  $10\hbar$  and  $50\hbar$  for  $200$ Pb nucleus. The most prominent feature observed from Fig. 1 is that with increasing  $\ell$ , all light particles exhibit a larger sensitivity to  $\beta$ . This is because at high  $\ell$ , fission barriers become smaller, which not only increases fission probability but also decreases transient time. Both factors lower presaddle neutrons, an important decay channel competing with fission inside the fission barrier. A decreasing presaddle neutron number makes more



FIG. 1. (Color online) Postsaddle multiplicities of neutrons (a), protons (b), and α particles (c) of <sup>200</sup>Pb systems as a function of  $β$ at excitation energy  $E^* = 120 \text{ MeV}$  and at critical angular momenta  $\ell = 10\hbar$  (squares) and 50 $\hbar$ (triangles), respectively.

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TABLE I. The multiplicities of postsaddle neutrons  $M_n$ , protons  $M_p$  (multiplied by 10<sup>3</sup>), and  $\alpha$  particles  $M_\alpha$  (multiplied by 10<sup>3</sup>) of <sup>200</sup>Pb and <sup>251</sup>Es systems at various  $\beta$  at  $E^* = 60$  MeV and  $\ell = 40\hbar$ .

$\beta$ (10 <sup>21</sup> s <sup>-1</sup> )	200Pb			$^{251}Es$		
	$M_{n}$	$M_{p}$	$M_{\alpha}$	$M_n$	$M_{p}$	$M_{\alpha}$
7	0.087	0.1890	0.233	0.900	3.45	2.84
10	0.117	0.2190	0.308	1.163	4.39	3.57
12	0.135	0.2580	0.356	1.319	4.84	3.89
15	0.160	0.3070	0.401	1.526	5.42	4.33
20	0.195	0.3520	0.466	1.822	6.25	4.92
25	0.223	0.3850	0.484	2.073	6.84	5.22

energy left for postsaddle emission. Consequently, postsaddle neutrons become larger [Fig.  $1(a)$ ], leading to a rise of its sensitivity to  $\beta$ . Analogously, LCPs ( $M_p$  and  $M_\alpha$ ) are also observed to depend more sensitively on  $\beta$  at high  $\ell$  [Figs. [1\(b\)](#page-1-0) and  $1(c)$  $1(c)$ ]. The results obtained from Fig. 1 thus indicate that in experiments, populating a light compound system with high spin can apparently enhance postsaddle emission and its sensitivity to  $\beta$ .

A decaying nucleus with a small size has a short saddle-toscission descent. This constrains the particle number emitted from the postsaddle region that is unfavorable for better determination of  $\beta$ . The expectation is confirmed in Table I, where it is shown that at the same low  $E^*$  and  $\ell$ , different types of light particles evaporated from heavy  $251$  Es illustrate a more sensitive variation with friction than those of light <sup>200</sup>Pb.

However, the experimental prescission particles in heavyion-induced heavy system fission come from both quasifission and fusion-fission channels, and the former channel will have a greater contribution when the bombarding energy of projectiles is increased. But light fissioning systems can be well produced by fusion mechanism. So, experimentally fragment particle angular correlations measured for light fissioning nuclei and, correspondingly, the extracted prescission particle multiplicity is little contaminated by quasifission. In addition to delivering a high spin ( $\sim$ 75 $\hbar$  [\[45\]](#page-4-0)), heavy ion collisions also deposit more energy (up to 200 MeV [\[46–49\]](#page-4-0)) into the light decaying system.

Besides angular momentum, excitation energy is also a key parameter that controls the de-excitation mode of a hot nucleus. The number of emitted particles in fission is an increasing function of excitation energy. It implies that to better reveal friction effects, a high excitation-energy condition resulting from a large incident energy is required for a precise determination of  $\beta$ .

In Fig. 2(a), we compare the sensitivity of neutrons to  $\beta$ between light  $^{200}$ Pb with high spin and excitation energy [\[46\]](#page-4-0), a typical characteristic of the formed light compound system in heavy ion reactions, and heavy <sup>251</sup>Es with low  $E^*$  (<70 MeV) and  $\ell$  (<45 $\hbar$ ), provided also in heavy-ion collision experiments  $[22]$ . We observe from the figure that the  $M_n$  of both  $200Pb$  and  $251Es$  demonstrate a similar and marked rise with the variation of β. It indicates that neutrons evaporated from light systems are also an equally good information source probing postsaddle dissipation as heavy systems provided that the light



FIG. 2. (Color online) Comparison of sensitivity of postsaddle neutrons to  $\beta$  in the absence (a) and in the presence (b) of deformation effects between light system <sup>200</sup>Pb (circles) at  $E^* = 150$  MeV and  $\ell = 70\hbar$  and heavy system <sup>251</sup>Es (triangles) at  $E^* = 70$  MeV and  $\ell = 45\hbar$ .

system is of a large spin and excitation energy. In other words, when using neutrons to explore postsaddle friction, besides heavy nuclei, producing light compound nuclei with high spin and high excitation energy is also an alternative experimental approach.

While  $M_n$  becomes larger in the presence of deformation effects because of a reduction of neutron binding energy [see Fig.  $3(a)$ ], the conclusion drawn in Fig.  $2(b)$  is still the same as that in Fig.  $2(a)$ , where deformation effects are ignored.

Apart from neutrons, LCPs of a heavy fissioning system [\[22\]](#page-4-0) have been utilized to analyze the properties of nuclear dissipation in fission. One can see from Fig.  $4(a)$  that the slope of the curve of  $M_p$  versus  $\beta$ , which reflects the sensitivity of the proton emission to friction, is steeper for <sup>200</sup>Pb at  $E^*$  = 150 MeV and  $\ell = 70\hbar$  than for <sup>251</sup>Es at  $E^* = 70$  MeV and  $\ell = 45\hbar$ ; that is,  $M_p$  displays a greater sensitivity to  $\beta$  for the light <sup>200</sup>Pb system.

In contrast with neutrons, deformation effects decrease  $M_p$ [see Figs.  $4(a)$  and  $4(b)$ ]. It is due to the result of competition between the rapid rise of the proton binding energy [Fig.  $3(a)$ ] and a drop in its emission barrier [Fig.  $3(b)$ ] with increasing deformation. Moreover, Fig.  $4(b)$  shows that deformation effects reduce the sensitive dependence of  $M_p$  on  $\beta$ . A heavy nucleus will experience a larger deformation when it fissions. This causes a stronger suppression of proton emission in comparison with the case of a light nucleus; as a result, the magnitude of  $M_p$  of heavy <sup>251</sup>Es is insensitive to a change in  $\beta$ .

A picture like protons is seen for  $\alpha$  particles [Figs. [4\(c\)](#page-3-0) and  $4(d)$ ]. In addition, Fig.  $4(d)$  shows that the sensitivity of  $M_{\alpha}$  to  $\beta$  almost disappears after deformation effects are accounted for; that is,  $\alpha$  particles of heavy fissioning nuclei are not a suitable tool of postsaddle dissipation.

When a compound system proceeds from equilibrium ground state to scission configuration, deformation effects on

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FIG. 3. (Color online) (a) Change in neutron, proton, and  $\alpha$ -particle binding energies  $\Delta B$  as a function of deformation coordinate  $q$  relative to the spherical binding energies for <sup>251</sup>Es. (b) Emission barrier ( $V_c$ ) of protons and  $\alpha$  particles of the <sup>251</sup>Es system as a function of deformation coordinate  $q$ .

particle evaporation along the fission path are an important factor that needs to be taken into account in calculation, because it has a quite strong constraint on the probe of postsaddle friction with protons and  $\alpha$  particles, especially for the case of heavy fissioning systems, as shown in Fig. 4. In this regard, when employing LCPs as an observable, in experiments, populating light decaying systems that have high spin and excitation energy is a preferable choice of investigating postsaddle friction.

*Conclusions*. In the framework of Langevin models coupled to a statistical decay model, it has been found that raising



FIG. 4. (Color online) Comparison of sensitivity of postsaddle protons (top panel) and  $\alpha$  particles (bottom panel) to the postsaddle friction strength  $\beta$  in the absence  $[(a)$  and  $(c)]$  and in the presence  $[(b)$ and (d)] of deformation effects between light system  $^{200}Pb$  (circles) at  $E^* = 150 \text{ MeV}$  and  $\ell = 70\hbar$  and heavy system <sup>251</sup>Es (triangles) at  $E^* = 70$  MeV and  $\ell = 45\hbar$ .

angular momentum of a fissioning system can apparently increase the sensitivity of various particle multiplicities to postsaddle friction. We further find a similar sensitivity of neutrons to friction for light <sup>200</sup>Pb formed at high  $E^*$  and  $\ell$ and for heavy <sup>251</sup>Es formed at low  $E^*$  and  $\ell$  as well as a greater sensitivity of LCPs to  $\beta$  for the former system than for the latter one. These results suggest that to determine the postsaddle dissipation strength more accurately by measuring particle multiplicity, in particular the LCPs multiplicity evaporated in the fission process, it is best to yield light compound nuclei with higher spins and larger excitation energies.

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