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Stability of β-equilibrated dense matter and core-crust transition in neutron stars

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The stability of the β-equilibrated dense nuclear matter is analyzed with respect to the thermodynamic stability conditions. Based on the density dependent M3Y effective nucleon-nucleon interaction, the effects of the nuclear incompressibility on the proton fraction in neutron stars and the location of the inner edge of their crusts and core-crust transition density and pressure are investigated. The high-density behavior of symmetric and asymmetric nuclear matter satisfies the constraints from the observed flow data of heavy-ion collisions. The neutron star properties studied using β-equilibrated neutron star matter obtained from this effective interaction for a pure hadronic model agree with the recent observations of the massive compact stars. The density, pressure, and proton fraction at the inner edge separating the liquid core from the solid crust of neutron stars are determined to be $\rho_t = 0.0938$ fm^{-3}, $P_t = 0.5006$ MeV fm^{-3}, and $x_{p(t)} = 0.0308$, respectively.

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I. INTRODUCTION

The equation of state (EoS) of nuclear matter under exotic conditions is an indispensable tool for the understanding of the nuclear force and for astrophysical applications. This implies knowledge of the EoS at high isospin asymmetries and for a wide density range (both for subsaturation and suprasaturation densities). In order to ascertain our knowledge on the nature of matter under extreme conditions, neutron stars are among the most mysterious objects in the universe that provide a natural laboratory. Understanding their structures and properties has long been a very challenging task for both the astrophysics and the nuclear physics communities [1].

One of the most important predictions of an EoS is the location of the inner edge of a neutron star crust. Knowledge of the properties of the crust plays an important role in understanding many astrophysical observations [2–14]. The inner crust spans the region from the neutron drip point to the inner edge separating the solid crust from the homogeneous liquid core. While the neutron drip density ρ_d is relatively well determined to be about 4.3×10^{11} g cm^{-3} [15], the transition density ρ_t at the inner edge is still largely uncertain mainly because of limited knowledge on the EoS, especially the density dependence of the symmetry energy, of neutron-rich nuclear matter [6–8]. At the inner edge a phase transition occurs from the high-density homogeneous matter to the inhomogeneous one at lower densities. The transition density takes its critical value ρ_t when the uniform neutron-proton-electron (npe) matter becomes unstable with respect to the separation into two coexisting phases (one corresponding to nuclei, the other to a nucleonic sea) [8].

In general, the determination of the transition density ρ_t itself is a very complicated problem because the inner crust may have a very complicated structure. A well established approach is to find the density at which the uniform liquid first becomes unstable against small-amplitude density fluctuations, indicating the formation of nuclear clusters. This approach includes the dynamical method [2–5,16–20], the thermodynamical one [8,21–23], and the random phase approximation (RPA) [24,25]. It is worthwhile to mention here that both the dynamical and the thermodynamical methods give very similar results with the former giving a slightly smaller transition density than the latter and this is due to the fact that the former includes the density gradient and Coulomb terms that make the system more stable and lower the transition density. The small difference between the two methods implies that the effects of density gradient terms and the Coulomb term are unimportant in determining the transition density [19].

In the present work, using the EoS for neutron-rich nuclear matter constrained by the recent isospin diffusion data from heavy-ion reactions in the same subsaturation density range as the neutron star crust, the inner edge of neutron star crusts is determined. For the EoS used in the present work, which is obtained from the density dependent M3Y effective nucleon-nucleon interaction (DDM3Y), the incompressibility K_∞ for the symmetric nuclear matter (SNM), the nuclear symmetry energy $E_{\rm sym}(\rho_0)$ at saturation density ρ_0, the isospin dependent part K_τ of the isobaric incompressibility, and the slope L are all in excellent agreement with the constraints recently extracted from measured isotopic dependence of the giant monopole resonances in even-A Sn isotopes, from the neutron skin thickness of nuclei, and from analyses of experimental data on isospin diffusion and isotopic scaling in intermediate energy heavy-ion collisions [26,27]. The core-crust transition in neutron stars is determined by analyzing the stability of the β-equilibrated dense nuclear matter with respect to the thermodynamic stability conditions [28].

II. INTRINSIC STABILITY OF THE NEUTRON STAR MATTER UNDER β EQUILIBRIUM

The inner edge of the neutron star crusts corresponds to a phase transition from the homogeneous matter at high densities to the inhomogeneous matter at low densities. In principle, the inner edge can be located by a detailed comparison of the relevant properties of the nonuniform solid crust and the uniform liquid core consisting mainly of the npe matter. However, this

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procedure is impracticable as the inner crust may contain nuclei having very complicated geometries, usually known as the nuclear pasta [1,11,12,29–31]. Moreover, the core-crust transition is expected to be a very weak first-order phase transition, and model calculations lead to very small density discontinuities at the transition [4,16,25,32]. In practice, therefore, a good approximation is to search for the density at which the uniform liquid first becomes unstable against small amplitude density fluctuations with clusterization. This approximation has been shown to produce a very small error for the actual core-crust transition density and would yield the exact transition density for a second-order phase transition [4,16,25,32]. Here, we use the thermodynamical method for analyzing the stability of the neutron star matter under β equilibrium.

A. The equation of state

The nuclear matter EoS is calculated using the isoscalar and the isovector [33,34] components of the M3Y interaction along with the density dependence. The density dependence of this DDM3Y effective interaction is completely determined from nuclear matter calculations. The equilibrium density of the nuclear matter is determined by minimizing the energy per nucleon. The energy variation of the zero range potential is treated accurately by allowing it to vary freely with the kinetic energy part $\epsilon^{\rm kin}$ of the energy per nucleon ϵ over the entire range of ϵ. This is not only more plausible, but also yields excellent results for the incompressibility K_∞ of the SNM which does not suffer from the superluminosity problem [35]. Details of the DDM3Y effective interaction is provided at the end as an Appendix.

The calculations are performed using the values of the saturation density $\rho_0 = 0.1533$ fm^{-3} [36] and the saturation energy per nucleon $\epsilon_0 = -15.26$ MeV [37] for the SNM obtained from the coefficient of the volume term of the Bethe-Weizsäcker mass formula which is evaluated by fitting the recent experimental and estimated atomic mass excesses from the Audi-Wapstra-Thibault atomic mass table [38] by minimizing the mean square deviation incorporating a correction for the electronic binding energy [39]. In a similar recent work, including the surface symmetry energy term, the Wigner term, the shell correction, and the proton form factor correction to the Coulomb energy also, a_v turns out to be 15.4496 MeV, and it is 14.8497 MeV when A^0 and $A^{1/3}$ terms are also included [40]. Using the usual values of $\alpha = 0.005$ MeV^{-1} for the parameter of energy dependence of the zero range potential and $n = 2/3$, the values obtained for the constants of density dependence C and β and the SNM incompressibility K_∞ are 2.2497, 1.5934 fm^2, and 274.7 MeV, respectively. The saturation energy per nucleon is the volume energy coefficient and the value of -15.26 ± 0.52 MeV covers, more or less, the entire range of values obtained for a_v for which now the values of $C = 2.2497 \pm 0.0420$, $\beta = 1.5934 \pm 0.0085$ fm^2, and the SNM incompressibility $K_\infty = 274.7 \pm 7.4$ MeV.

B. Intrinsic stability of a single phase under β equilibrium and the core-crust transition

The basic equation in neutron star matter research is the shape of the relationship between the pressure and energy density $P = P(\varepsilon)$, usually called the equation of state. At the zero temperature, the state of neutron star matter should be uniquely described by the quantities that are conserved by the process leading to equilibrium. Stable high density nuclear matter must be in chemical equilibrium for all types of reactions including the weak interactions, while the β decay and orbital electron capture takes place simultaneously. For the β-equilibrated neutron star matter we have free neutron decay $n \rightarrow p + \beta^- + \overline{\nu_e}$ which are governed by weak interaction and the electron capture process $p + \beta^- \rightarrow n + \nu_e$. Both types of reactions change the electron fraction and thus affect the EoS. Here we assume that neutrinos generated in these reactions leave the system. The absence of neutrinos has a dramatic effect on the equation of state and mainly induces a significant change on the values of the proton fraction x_p. The absence of neutrinos implies that

$$\mu = \mu_n - \mu_p = \mu_e, \tag{1}$$

where μ_e, μ_n, and μ_p are the chemical potentials for the electron, neutron, and proton, respectively.

The baryon number B is conserved by this type of reaction so the energy density ε and pressure P should be function of baryon number density ρ. We assume that the matter is electrically neutral and spatially homogeneous. The star as a whole is electrically neutral but the matter does not need to be locally neutral. So the thermodynamic state of a given phase is described by two quantities: baryon number B and charge Q where Q is the sum of all charges. The total energy U then becomes a function of $U(V,B,Q)$. To consider stability of a single phase, one needs to introduce local quantities $\epsilon = \frac{U}{B}$. The energy per particle ϵ then becomes a function of other local quantities taken per baryon number $v = \frac{V}{B}$ and $x = \frac{Q}{B}$. The first principle of thermodynamics takes the following form:

$$d\epsilon = -P dv - \mu dx, \tag{2}$$

where P is the pressure and μ is the chemical potential of an electric charge. The stability of any single phase, also called intrinsic stability, is ensured by convexity of $\epsilon(v,x)$. The thermodynamical inequalities allow us to express the requirement in terms of following inequalities:

$$-\left(\frac{\partial P}{\partial v}\right)_x > 0, \quad -\left(\frac{\partial \mu}{\partial x}\right)_P > 0. \tag{3}$$

One may find another pair of inequalities that are equivalent to above equations [21,22]:

$$-\left(\frac{\partial P}{\partial v}\right)_\mu > 0, \quad -\left(\frac{\partial \mu}{\partial x}\right)_v > 0. \tag{4}$$

The intrinsic stability conditions are equivalent to requiring the convexity of the energy per particle in the single phase [8] by ignoring the finite size effects due to surface and Coulomb energies as shown in following. Here the $P = P^b + P^e$ is the total pressure of the npe system with the contributions P^b and P^e from baryons and electrons, respectively. The proton fraction $x_p = \frac{\rho_p}{\rho}$ where $\rho = \rho_n + \rho_p$ and the asymmetry parameter $X = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$. Total energy $\epsilon = \epsilon_b(x_p) + \epsilon_e(\mu)$.

$$P = -\frac{\partial \epsilon}{\partial v} = \rho^2 \frac{\partial \epsilon}{\partial \rho}, \tag{5}$$

$$\left(\frac{\partial P}{\partial v}\right)_\mu = \frac{\partial P^b(\rho, x_p)}{\partial v} + \frac{\partial P^e(\mu)}{\partial v}. \tag{6}$$

Here $\frac{\partial P^e(\mu)}{\partial v} = 0$ because if β equilibrium is satisfied then $\mu = \mu_n - \mu_p = \mu_e$, and the electron contribution to P^e is only a function of the chemical potential μ and in that case $(\frac{\partial P^e(\mu)}{\partial v}) = 0$. Eventually $-(\frac{\partial P}{\partial v})_\mu > 0$ can be written as $-(\frac{\partial P^b}{\partial v})_\mu > 0$.

$$\left(\frac{\partial P}{\partial v}\right)_\mu = \frac{\partial P^b}{\partial \rho}\frac{\partial \rho}{\partial v} + \frac{\partial P^b}{\partial x_p}\frac{\partial x_p}{\partial v}$$
$$= -\rho^2 \frac{\partial P^b}{\partial \rho} - \rho^2 \frac{\partial P^b}{\partial x_p}\frac{\partial x_p}{\partial \rho}, \tag{7}$$

$$-\left(\frac{\partial P}{\partial v}\right)_\mu = \rho^2 \left(\frac{\partial P^b}{\partial \rho} + \frac{\partial P^b}{\partial x_p}\frac{\partial x_p}{\partial \rho}\right), \tag{8}$$

$$\mu = \mu_n - \mu_p = -\left(\frac{\partial \epsilon^b}{\partial x^p}\right)_\rho = -\frac{\partial \epsilon^b(\rho, x_p)}{\partial x_p}. \tag{9}$$

Differentiating the above equation with respect to x_p, we get

$$\frac{\partial \mu}{\partial x_p} = -\frac{\partial^2 \epsilon^b}{\partial x_p^2}. \tag{10}$$

From Eq. (5) we get

$$P^b = \rho^2 \frac{\partial \epsilon^b}{\partial \rho}, \tag{11}$$

and differentiating the above with respect to x_p, one obtains

$$\left(\frac{\partial P^b}{\partial x_p}\right) = \rho^2 \frac{\partial^2 \epsilon^b}{\partial x_p \partial \rho} = \rho^2 \epsilon^b_{\rho x_p}. \tag{12}$$

By Maxwell's relation

$$\left(\frac{\partial x_p}{\partial \rho}\right)_\mu = -v^2 \left(\frac{\partial x_p}{\partial v}\right)_\mu = v^2 \left(\frac{\partial P^b}{\partial \mu}\right)_{s,v}, \tag{13}$$

$$\frac{\partial P^b}{\partial \mu} = \frac{\frac{\partial P^b}{\partial x_p}}{\frac{\partial \mu}{\partial x_p}} = \frac{\rho^2 \frac{\partial^2 \epsilon^b}{\partial \rho \partial x_p}}{\frac{\partial \mu}{\partial x_p}} = -\frac{\rho^2 \frac{\partial^2 \epsilon^b}{\partial \rho \partial x_p}}{\frac{\partial^2 \epsilon^b}{\partial x_p^2}}. \tag{14}$$

Using Eqs. (13) and (14) we get

$$\left(\frac{\partial x_p}{\partial \rho}\right) = -v^2 \rho^2 \frac{\frac{\partial^2 \epsilon^b}{\partial \rho \partial x_p}}{\frac{\partial^2 \epsilon^b}{\partial x_p^2}} = -\frac{\frac{\partial^2 \epsilon^b}{\partial \rho \partial x_p}}{\frac{\partial^2 \epsilon^b}{\partial x_p^2}}. \tag{15}$$

From Eq. (11)

$$\frac{\partial P^b}{\partial \rho} = 2\rho \frac{\partial \epsilon^b}{\partial \rho} + \rho^2 \frac{\partial^2 \epsilon^b}{\partial \rho^2}. \tag{16}$$

Using Eqs. (12), (15), and (16) in Eq. (8) we get

$$-\left(\frac{\partial P^b}{\partial v}\right)_\mu = \rho^2 \left(2\rho \frac{\partial \epsilon^b}{\partial \rho} + \rho^2 \frac{\partial^2 \epsilon^b}{\partial \rho^2} - \rho^2 \frac{\epsilon^b_{\rho x_p} \epsilon^b_{\rho x_p}}{\epsilon^b_{x_p x_p}}\right), \tag{17}$$

where for brevity, the symbol $\epsilon^b_{\rho x_p}$ is used for $\frac{\partial^2 \epsilon^b}{\partial \rho \partial x_p}$ and the symbol $\epsilon^b_{x_p x_p}$ is used for $\frac{\partial^2 \epsilon^b}{\partial x_p^2}$. The quantity V_{thermal} which determines the thermodynamic instability region of neutron star matter at β equilibrium is given by $V_{\text{thermal}} = -(\frac{\partial P}{\partial v})_\mu$. Hence [19]

$$V_{\text{thermal}} = \rho^2 \left[2\rho \frac{\partial \epsilon^b}{\partial \rho} + \rho^2 \frac{\partial^2 \epsilon^b}{\partial \rho^2} - \rho^2 \frac{\left(\epsilon^b_{\rho x_p}\right)^2}{\epsilon^b_{x_p x_p}}\right]. \tag{18}$$

The condition for the core-crust transition is obtained by making $V_{\text{thermal}} = 0$. In the following we drop the superscript b and use ϵ for ϵ^b and P for P^b.

III. THEORETICAL CALCULATIONS

The β-equilibrated nuclear matter EoS is obtained by evaluating the asymmetric nuclear matter EoS at the isospin asymmetry X determined from the β-equilibrium proton fraction x_p [$= \frac{\rho_p}{\rho}$], obtained approximately by solving

$$\hbar c(3\pi^2 \rho x_p)^{1/3} = 4E_{\text{sym}}(\rho)(1 - 2x_p), \tag{19}$$

where $E_{\text{sym}}(\rho)$ is the nuclear symmetry energy. In general $E_{\text{sym}}(\rho)$ is defined as $\frac{1}{2}\frac{\partial^2 \epsilon(\rho, X)}{\partial X^2}|_{X=0}$. The higher-order terms in X are negligible and to a good approximation, $E_{\text{sym}}(\rho) = \epsilon(\rho, 1) - \epsilon(\rho, 0)$ [41] which represents a penalty levied on the system as it departs from the symmetric limit of equal numbers of protons and neutrons and can be defined as the energy required per nucleon to change the SNM to pure neutron matter (PNM).

The exact way of obtaining the β-equilibrium proton fraction is by solving Eq. (1) with $\mu_p - \mu_n$ being equal to $\frac{\partial \epsilon(\rho, x_p)}{\partial x_p}$ because neutron decays are always associated with proton productions whereas $\mu_e = \sqrt{p_e^2 c^2 + m_e^2 c^4} \approx p_e c = \hbar k_f c = \hbar c(3\pi^2 \rho x_p)^{1/3}$ where m_e and ρ_e being the rest mass and number density of the electron, respectively, and from charge neutrality $\rho_e = \rho_p = \rho x_p$. Thus

$$\hbar c(3\pi^2 \rho x_p)^{1/3} = -\frac{\partial \epsilon(\rho, x_p)}{\partial x_p} = +2\frac{\partial \epsilon}{\partial X}, \tag{20}$$

where the isospin asymmetry $X = 1 - 2x_p$.

The pressure P of PNM and β-equilibrated neutron star matter are plotted in Fig. 1 as functions of ρ/ρ_0. The continuous line represents the PNM and the dashed line (almost merges with the continuous line) represents the β-equilibrated neutron star matter (present calculations) whereas the dotted line represents the same using the A18 model using the variational chain summation (VCS) of Akmal *et al.* [42] for the PNM. The areas enclosed by the continuous and the dashed lines in Fig. 1 correspond to the pressure regions for neutron matter consistent with the experimental flow data after inclusion of the pressures from asymmetry terms with weak (soft NM) and strong (stiff NM) density dependences, respectively [43]. Although the parameters of the density dependence of DDM3Y interaction have been tuned to reproduce ρ_0 and ϵ_0 which are obtained from finite nuclei, the agreement of the present EoS with the experimental flow data, where the high density behavior looks phenomenologically

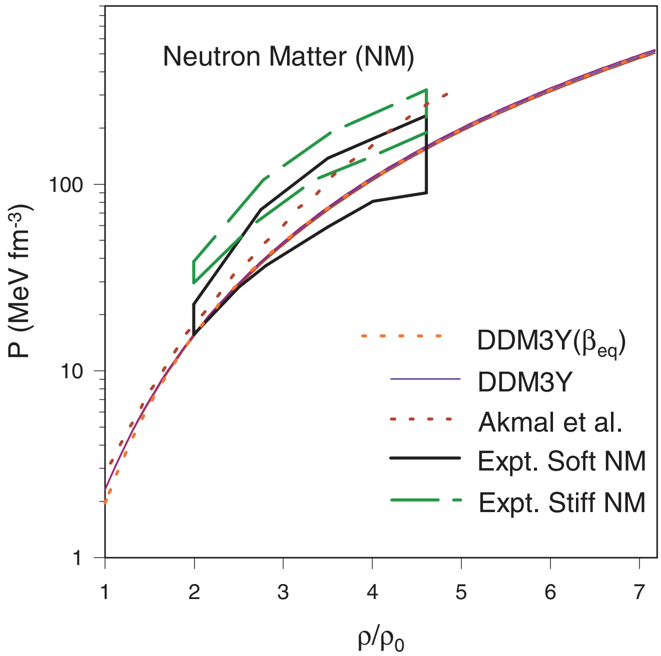

FIG. 1. (Color online) Plots for pressure P of dense nuclear matter as functions of ρ/ρ_0. The continuous line represents the pure neutron matter and the dashed line represents the β-equilibrated neutron star matter. The dotted line represents the same for A18 model using variational chain summation (VCS) of Akmal *et al.* [42]. The areas enclosed by the continuous and the dashed lines correspond to the pressure regions for neutron matter consistent with the experimental flow data after inclusion of the pressures from asymmetry terms with weak (soft NM) and strong (stiff NM) density dependences, respectively [43].

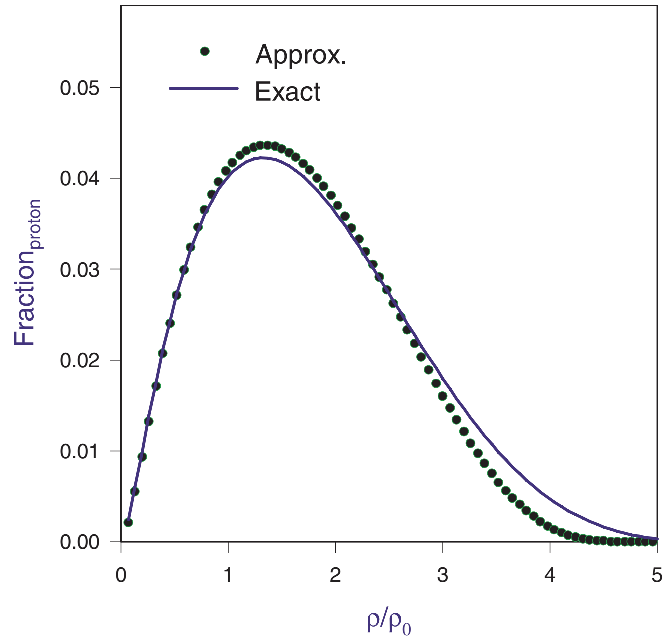

FIG. 2. (Color online) The β-equilibrium proton fractions obtained from the nuclear symmetry energy (approx.) and from exact calculations using the DDM3Y interaction are plotted as functions of ρ/ρ_0.

confirmed, justifies its extrapolation to high density. It is interesting to note that the RMF-NL3 incompressibility for SNM is 271.76 MeV [44,45] which is about the same as the 274.7 $\pm$ 7.4 MeV obtained from the present calculation but the plot of P versus ρ/ρ_0 for PNM of RMF using the NL3 parameter set [44] does not pass through the pressure regions for neutron matter consistent with the experimental flow data [43].

In Fig. 2 it can be seen that the maximum of the β-equilibrium proton fraction $x_p \sim 0.0436$ calculated using the symmetry energy (approximate calculation) occurs at $\rho \sim 1.35\rho_0$ whereas the exact calculation yields a maximum of $x_p \sim 0.0422$ around the same density. Since the equilibrium proton fraction is always less than 1/9 [46], the calculated value of x_p forbids the direct URCA process. This feature is consistent with the fact that there are no strong indications [47,48] that fast cooling occurs. It was also concluded theoretically that an acceptable EoS of asymmetric nuclear matter shall not allow the direct URCA process to occur in neutron stars with masses below 1.5 solar masses [41]. Even recent experimental observations that suggest a high heat conductivity and an enhanced core cooling process indicating the enhanced level of neutrino emission were not attributed to the direct URCA process but were proposed to be due to breaking and formation of neutron Cooper pairs [49–52].

The intrinsic stability condition of a single phase for locally neutral matter under β equilibrium is determined, thermodynamically, by the positivity of the $V_{\rm thermal}$, under a constant chemical potential which is generally valid in our case. However, the limiting density that breaks these conditions will correspond to the core-crust (liquid-solid) phase transition. Thus the transition density ρ_t (with corresponding pressure P_t and proton fraction $x_{p(t)}$) is determined at which $V_{\rm thermal}$ becomes zero and goes to negative with decreasing density.

IV. RESULTS AND DISCUSSION

The stability of the β-equilibrated dense matter in neutron stars is investigated and the location of the inner edge of their crusts and core-crust transition density and pressure are determined using the DDM3Y effective nucleon-nucleon interaction. The results for the transition density, pressure, and proton fraction at the inner edge separating the liquid core from the solid crust of neutron stars are calculated and presented in Table I for $n = 2/3$. The symmetric nuclear matter incompressibility K_∞, the nuclear symmetry energy at saturation density $E_{\rm sym}(\rho_0)$, the slope L, and the isospin dependent part K_τ of the isobaric incompressibility are also tabulated since these are all in excellent agreement with the constraints recently extracted from the measured isotopic dependence of the giant monopole resonances in even-A Sn isotopes, from the neutron skin thickness of nuclei, and from analyses of experimental data on isospin diffusion and isotopic scaling in intermediate energy heavy-ion collisions.

It is recently conjectured that there may be a good correlation between the core-crust transition density and the symmetry energy slope L and it is predicted that this behavior

TABLE I. Results of the present calculations (DDM3Y) of the symmetric nuclear matter incompressibility K_∞, the nuclear symmetry energy at saturation density $E_{\rm sym}(\rho_0)$, the slope L, and the isospin dependent part K_τ of the isobaric incompressibility (all in MeV) [27] are tabulated along with the saturation density and the density, pressure, and proton fraction at the core-crust transition for β-equilibrated neutron star matter.

K_∞	$E_{\rm sym}(\rho_0)$	L	K_τ
274.7 ± 7.4	30.71 ± 0.26	45.11 ± 0.02	-408.97 ± 3.01
ρ_0 (fm^{-3})	ρ_t (fm^{-3})	P_t (MeV fm^{-3})	$x_{p(t)}$
0.1533	0.0938	0.5006	0.0308

should not depend on the relation between L and K_τ [53]. On the contrary, no correlation of the transition pressure with L was obtained [53]. In Table II, variations of different quantities with parameter n which controls the nuclear matter incompressibility are listed. It is worthwhile to mention here that the incompressibility increases with n. The standard value of $n = 2/3$ used here has a unique importance because then the constant of density dependence β has the dimension of cross section and can be interpreted as the isospin averaged effective nucleon-nucleon interaction cross section in the ground state symmetric nuclear medium. For a nucleon in the ground state nuclear matter $k_F \approx 1.3$ fm^{-1} and $q_0 \sim \hbar k_F c \approx 260$ MeV, and the present result for the "in medium" effective cross section is reasonably close to the value obtained from rigorous Dirac-Brueckner-Hartree-Fock [54] calculations corresponding to such k_F and q_0 values which is ≈12 mb. Using the value of $\beta = 1.5934$ fm^2 along with the nucleonic density 0.1533 fm^{-3}, the value obtained for the nuclear mean free path λ is about 4 fm which is in excellent agreement with that obtained using another method [55]. Moreover, comparison of the theoretical values of symmetric nuclear matter incompressibility and isobaric incompressibility with the recent experimental values for $K_\infty = 250$–270 MeV [56,57] and $K_\tau = -370 \pm 120$ MeV [58] further justifies importance for our choice of $n = 2/3$. It is interesting to mention here that the present EoS for $n = 2/3$ provides the maximum mass for the static case is 1.92 $M_\odot$ with a radius ∼9.7 km and for the star rotating with Kepler's frequency it is 2.27 $M_\odot$ with an equatorial radius ∼13.1 km

TABLE II. Variations of the core-crust transition density, pressure, and proton fraction for β-equilibrated neutron star matter, the symmetric nuclear matter incompressibility K_∞, and the isospin dependent part K_τ of isobaric incompressibility with parameter n.

n	ρ_t (fm^{-3})	P_t (MeV fm^{-3})	$x_{p(t)}$	K_∞ (MeV)	K_τ (MeV)
Expt. values				250–270	-370 ± 120
1/6	0.0797	0.4134	0.0288	182.13	−293.42
1/3	0.0855	0.4520	0.0296	212.98	−332.16
1/2	0.0901	0.4801	0.0303	243.84	−370.65
2/3	0.0938	0.5006	0.0308	274.69	−408.97
1	0.0995	0.5264	0.0316	336.40	−485.28

[59]. However, for stars rotating with a maximum frequency limited by the r-mode instability, the maximum mass turns out to be 1.95 (1.94) $M_\odot$ corresponding to a rotational period of 1.5 (2.0) ms with a radius about 9.9 (9.8) km [60] which reconcile with the recent observations of the massive compact stars ∼2 $M_\odot$ [61,62].

V. SUMMARY AND CONCLUSION

In summary, the stability of the β-equilibrated dense nuclear matter is analyzed with respect to the thermodynamic stability conditions. The proton fraction obtained using nuclear symmetry energy does not affect seriously the results of an exact calculation. Since the higher-order symmetry-energy coefficients are needed to describe reasonably well the proton fraction of the β-stable (npe) matter at high nuclear densities and the core-crust transition density [63], exact calculations are performed using the density dependent M3Y effective nucleon-nucleon interaction for investigating the proton fraction in neutron stars and the location of the inner edge of their crusts and their core-crust transition density and pressure.

The nucleon-nucleon effective interaction used in the present work, which is found to provide a unified description of elastic and inelastic scattering, various radioactivities, and nuclear matter properties, also provides an excellent description of the β-equilibrated neutron star matter which is stiff enough at high densities to reconcile with the recent observations of the massive compact stars [59,60,64] while the corresponding symmetry energy is supersoft as preferred by the FOPI/GSI experimental data. The density, the pressure, and the proton fraction at the inner edge separating the liquid core from the solid crust of the neutron stars determined to be $\rho_t = 0.0938$ fm^{-3}, $P_t = 0.5006$ MeV fm^{-3}, and $x_{p(t)} = 0.0308$, respectively, are also in close agreement with other theoretical calculations [63] corresponding to high nuclear incompressibility and with those obtained using the SLy4 interaction [65].

APPENDIX

In a Fermi gas model of interacting neutrons and protons, with isospin asymmetry $X = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$, $\rho = \rho_n + \rho_p$, where ρ_n, ρ_p, and ρ are the neutron, proton, and nucleonic densities respectively, the energy per nucleon for isospin asymmetric nuclear matter can be derived as [35]

$$\epsilon(\rho, X) = \left[\frac{3\hbar^2 k_F^2}{10m}\right] F(X) + \left(\frac{\rho J_v C}{2}\right)(1 - \beta\rho^n), \qquad \text{(A1)}$$

where m is the nucleonic mass, $k_F = (1.5\pi^2\rho)^{\frac{1}{3}}$ which equals the Fermi momentum in the case of SNM, the kinetic energy per nucleon $\epsilon^{\rm kin} = [\frac{3\hbar^2 k_F^2}{10m}]F(X)$ with $F(X) = [\frac{(1+X)^{5/3} + (1-X)^{5/3}}{2}]$, and $J_v = J_{v00} + X^2 J_{v01}$, with J_{v00} and J_{v01} representing the volume integrals of the isoscalar and the isovector parts of the M3Y interaction. The isoscalar t_{00}^{M3Y} and the isovector t_{01}^{M3Y} components of the M3Y interaction potential are given by

$$t_{00}^{M3Y}(s,\epsilon) = +7999\frac{\exp(-4s)}{4s} - 2134\frac{\exp(-2.5s)}{2.5s} + J_{00}(1 - \alpha\epsilon)\delta(s),$$

$$t_{01}^{M3Y}(s,\epsilon) = -4886\frac{\exp(-4s)}{4s} + 1176\frac{\exp(-2.5s)}{2.5s} + J_{01}(1-\alpha\epsilon)\delta(s), \quad \text{(A2)}$$

where s represents the relative distance between two interacting nucleons, $J_{00} = -276$ MeV fm^3, $J_{01} = +228$ MeV fm^3, and the energy dependence parameter $\alpha = 0.005$ MeV^{-1}. The strengths of the Yukawas were extracted by fitting their matrix elements in an oscillator basis to those elements of the G matrix obtained with the Reid-Elliott soft core NN interaction, and the ranges were selected to ensure OPEP tails in the relevant channels as well as a short-range part which simulates the σ-exchange process [66]. The density dependence is employed to account for the Pauli blocking effects and the higher order exchange effects [67]. Thus the DDM3Y effective NN interaction is given by $v_{0i}(s,\rho,\epsilon) = t_{0i}^{M3Y}(s,\epsilon)g(\rho)$ where the density dependence $g(\rho) = C(1-\beta\rho^n)$ [35] with C and β being the constants of density dependence.

The Eq. (A1) can be differentiated with respect to ρ to yield an equation for $X = 0$:

$$\frac{\partial\epsilon}{\partial\rho} = \frac{\hbar^2k_F^2}{5m\rho} + \frac{J_{v00}C}{2}[1-(n+1)\beta\rho^n] - \alpha J_{00}C[1-\beta\rho^n] \times \left[\frac{\hbar^2k_F^2}{10m}\right]. \quad \text{(A3)}$$

The equilibrium density of the cold SNM is determined from the saturation condition. Then Eqs. (A1) and (A3) with the saturation condition $\frac{\partial\epsilon}{\partial\rho} = 0$ at $\rho = \rho_0, \epsilon = \epsilon_0$ can be solved simultaneously for fixed values of the saturation energy per nucleon ϵ_0 and the saturation density ρ_0 of the cold SNM to obtain the values of β and C. The constants of density dependence β and C, thus obtained, are given by

$$\beta = \frac{\left[(1-p)+\left(q-\frac{3q}{p}\right)\right]\rho_0^{-n}}{\left[(3n+1)-(n+1)p+\left(q-\frac{3q}{p}\right)\right]}, \quad \text{(A4)}$$

where $p = \frac{10m\epsilon_0}{\hbar^2k_{F_0}^2}$, $q = \frac{2\alpha\epsilon_0 J_{00}}{J_{v00}^0}$, $J_{v00}^0 = J_{v00}(\epsilon_0^{\rm kin})$ implying J_{v00} at $\epsilon^{\rm kin} = \epsilon_0^{\rm kin}$, the kinetic energy part of the saturation energy per nucleon of SNM, $k_{F_0} = [1.5\pi^2\rho_0]^{1/3}$, and

$$C = -\frac{\left[2\hbar^2k_{F_0}^2\right]}{5mJ_{v00}^0\rho_0\left[1-(n+1)\beta\rho_0^n - \frac{q\hbar^2k_{F_0}^2(1-\beta\rho_0^n)}{10m\epsilon_0}\right]}, \quad \text{(A5)}$$

respectively. It is quite obvious that the constants of density dependence C and β obtained by this method depend on the saturation energy per nucleon ϵ_0, the saturation density ρ_0, the index n of the density dependent part, and the strengths of the M3Y interaction through the volume integral J_{v00}^0.

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