Excitation of atomic nuclei in hot plasma through resonance inverse electron bridge

E. V. Tkalya*

Skobeltsyn Institute of Nuclear Physics Lomonosov Moscow State University, Leninskie gory, Moscow 119991, Russia and Nuclear Safety Institute of Russian Academy of Science, Bol'shaya Tulskaya 52, Moscow 115191, Russia

E. V. Akhrameev, R. V. Arutyunyan, L. A. Bol'shov, and P. S. Kondratenko

Nuclear Safety Institute of Russian Academy of Science, Bol'shaya Tulskaya 52, Moscow 115191, Russia

(Received 6 June 2014; revised manuscript received 24 July 2014; published 18 September 2014)

A process of nucleus excitation by photons under the mechanism of the inverse electron bridge (IEB) is examined provided the energies of atomic and nuclear transitions coincide. It is shown that in this case, the excitation of nuclei with EL [ML] transition with the energy $\omega_N \lesssim 10$ keV is strengthened relative to the process of photoabsorption by nucleus by a factor of $1/(\omega_N r_0)^{2(L+2)}$ [$e^4/(\omega_N r_0)^{2(L+2)}$], where r_0 is a typical size of domain in the ion shell for accumulation of electronic integrals. In the ^{84}Rb nuclei the IEB cross section for the 3.4 keV $M1$ transition $6^-(463.59\text{ keV}) \leftrightarrow 5^-(463.59\text{ keV})$ can exceed even a photoexcitation cross section for the 3.4 keV $E1$ transition with the reduced probability in the Weisskopf model $B_{W.u.}(E1) = 1$. This result can be important for understanding the mechanisms of atomic nucleus excitation in hot plasma. In particular, the considered process is capable to provide the existence of so called gamma luminescence wave or a nuclear isomer "burning" wave—an analog of self-maintaining process of triggered depopulation of nuclear isomer.

DOI: 10.1103/PhysRevC.90.034614   PACS number(s): 25.20.Dc, 32.90.+a, 52.50.Jm

I. INTRODUCTION

Experimental studies of atomic nuclei excitation in hot dense laser plasma [1–6] demand a detailed analysis of the mechanisms leading to transition of nuclei to isomeric state under the impact of plasma photons and electrons. The basic processes of nuclear excitation are well known, namely: the photoexcitation of nuclei by plasma thermal radiation [7,8]; inelastic scattering of plasma electrons at nuclei [9]; nuclear excitation by electronic transition (NEET) [10,11]; inverse electronic conversion (IEC) [12] which is better known in the modern literature under the name of nuclear excitation by electron capture (NEEC) [13]; multiphoton excitation of nuclei [14]; inverse electron bridge (IEB) [15], etc. The detailed review and classification of all these mechanisms in the frameworks of the perturbation theory for quantum electrodynamics is given in [16]. In recent years, a number of the most effective processes, such as photoexcitation, inelastic scattering of electrons, inverse electronic conversion, and NEET have been used in numerical calculations of excitation and deexcitation of nuclei in plasma and single atom or ion (see, for example, [6,13,17–26] and references therein).

Nuclear excitation in high-temperature dense plasma is mainly interesting from the possible applications point of view. Of them, the most essential, in our opinion, are the creation of inverse population in a matrix containing isomeric nuclei and observation: (a) of triggered depopulation of nuclear isomer, with accompanying γ emission [27]; or (b) even of γ-radiation wave ("combustion" wave—a self-maintaining process of triggered depopulation) in the system of isomeric nuclei with a short-living intermediate state that is close to the isomeric state [28]. In order to implement these processes in the experiment, the transition from the isomeric state (where a significant amount of nuclei can preliminary be accumulated) to the close short-living level (from which a working transition, say, to the basic nuclear state will be occurred later) should be an electric dipole transition [28]. Moreover, it is desirable that reduction of this $E1$ transition would be minimal, or, in other words, that its reduced transition probability in Weisskopf units would be close to unity.

Unfortunately, the last requirement contradicts the available experimental data on intensities of electric dipole transitions. In the low-energy part of the nuclear excitation spectrum, $E1$ transitions are usually suppressed because of structural factors. Although the number of known nuclides is large enough now, there is no possibility to find a nucleus satisfying all necessary criteria. Therefore, without the strengthening mechanism that could essentially increase the nuclear excitation cross section, in particular, for electric dipole transitions, it will be extremely difficult to have the processes such as the γ-radiation wave.

In the present work we will analyze in detail the photoexcitation of atomic nuclei through an atomic shell by the mechanism of inverse electron bridge. This process that was studied for the first time in [15] is interesting, because in the case of a resonance coincidence of atomic and nuclear transitions its cross section can significantly surpass the cross sections of all other mechanisms of nuclear excitation by photons and electrons of plasma. This basically solves the mentioned problem of small cross sections of excitation and opens certain prospects for the experimental study of the "combustion" process in the system of nuclear isomers with a close short-living intermediate state that was proposed in [28,29]. In the present work, we use the following system of units: $\hbar = c = k = 1$.

II. INVERSE ELECTRON BRIDGE

The electron bridge is a process of decay of the excited nuclear state through the atomic shell described by the diagram

*tkalya@srd.sinp.msu.ru

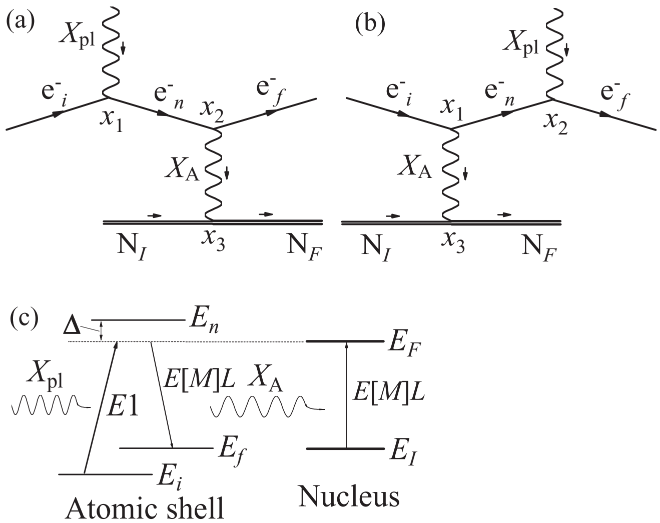

FIG. 1. Diagrams of nucleus excitation by photons through the atomic shell under the mechanism of inverse electron bridge: (a) direct diagram, (b) exchange diagram. (c) Scheme of the process corresponding to the direct diagram.

of the third order of the constant of electromagnetic interaction e [30]. Accordingly, the inverse process, i.e., excitation of a nucleus through the atomic shell [see Figs. 1(a)–1(b)] has obtained the name of the inverse electron bridge. We will consider here how the inverse electron bridge works in one particular but important case—when the energy mismatch Δ shown in Fig. 1(c) tends to zero, and the excitation of the nucleus is reduced to a sequence of the two following processes: photoexcitation of the ion shell and NEET.

In QED, the cross section, as a rule, decreases with an increase of the order of the diagram describing the process. This cannot be true, if the bound electronic states, whose wave functions are localized at an atom or an ion, are involved in the process. In this case, the domain of the accumulation of integrals in electronic matrix elements r_0 is limited by the size of the atom or the electronic shell of the ion. There is a spherical Hankel function of the first kind of electronic coordinate r and nuclear transition energy ω_N in the multipole expansion [31] of the photon propagator in the frequency-coordinate presentation $D_{\mu\nu}(\omega_N;\mathbf{r}-\mathbf{R}) = -g_{\mu\nu}\exp(i\omega_N|\mathbf{r}-\mathbf{R}|)/|\mathbf{r}-\mathbf{R}|$:

$$D_{\mu\nu}(\omega_N;\mathbf{r}-\mathbf{R}) \propto \sum_{L,M} h_L^{(1)}(\omega_N r) Y_{LM}(\Omega_r) j_L(\omega_N R) Y^*_{LM}(\Omega_R)$$

[$j_L(\omega_N R)$ are the spherical Bessel function of nuclear coordinate R [32], $Y_{LM}(\Omega)$ are the spherical harmonics]. If the nuclear transition energy (here $\omega_N = E_F - E_I$, where $E_{I,F}$ are the energies of the initial and final nuclear states in Fig. 1) does not exceed some kiloelectronvolts, the argument of $h_L^{(1)}$ satisfies the condition $\omega_N r \ll 1$ if the electron coordinate $r \leqslant r_0$, and in the area of the effective accumulation of integral, the Hankel function is large, since it has a pole $h_L^{(1)}(\omega_N r) \sim -i(2L-1)!!/(\omega_N r)^{L+1}$. The specified behavior of the Hankel function compensates the smallness defined by the additional electron-photon vertexes to the amplitude and the process cross section.

In the case of nuclear transitions with energy $\omega_N \lesssim 1$ keV, the condition $\omega_N r_0 \ll 1$ is always true even in atoms, since $r_0 < a_B$, where a_B is the Bohr radius. Atoms in plasma with temperature $1 < T \lesssim 10$ keV turn into multicharge ions with $Z_{pl} \simeq 30$. For such ions, the condition $\omega_N r_0 \ll 1$ will be valid for transitions with energy $\omega_N \lesssim T$. This is connected to the fact that in ions of atoms with $A \simeq 150$–200, transitions of the given energy occur between L and M subshells, where the vacancies are formed in hot plasma. In the electronic matrix element, the effective domain of the accumulation of integrals will be defined, mainly, by the sizes of a subshell of the "lower" state (in this case the L state), characteristic sizes of which are much less than Bohr radius a_B. Besides, for multicharge ions, the binding energies increase at internal L and M shells. This leads to a reduction of their sizes as well as promotes observation of the condition $\omega_N r_0 \ll 1$.

Note that the mechanism of amplification considered above and long lifetime of the plasma (and as a consequence the interaction time available for the plasma electrons to excite the nuclei) explain in particular the numerical result obtained in Ref. [26]. In this work it was shown that the number of the ^{93}Mo nucleus excited in the $E2(4.85\,\text{keV})21/2^+(2424.95\,\text{keV}) \rightarrow (17/2)^+(2429.80\,\text{keV})$ transition by electrons in the IEC-NEEC process in plasma, produced by an x-ray free-electron laser beam interacted with solid state target, exceeds the number of the nucleus excited directly by the laser x rays.

For our problem, the cross section of nucleus excitation through the inverse electron bridge can be estimated in a single-level approach [16]. Really, at the second stage of the process—the nucleus excitation in electronic transition—not only the closeness of electronic and nuclear transition energies is required, but their multipolarities should also coincide. This reduces essentially the number of intermediate states. In practice, generally, we can leave only one level with the dominating contribution to the amplitude of the process (a single-level approach). At that, the direct diagram [that is shown in Fig. 1(a)] plays the dominant role in the excitation process, and the cross-section formula factorizes [33] and can be written in the form of two-factor product. One of them corresponds to the excitation of atom to an intermediate state by a plasma photon, and another to the subsequent process of nucleus excitation during the transition of atom from intermediate to the final state [see Fig. 1(c)].

Let us consider this reduction in detail. According to the QED rules [34] the amplitude of the IEB process can be written as

$$\begin{aligned} S_{fi}^{(3)} = i\int & d^4x_1 d^4x_2 d^4x_3\, J^{\rho}_{FI}(x_3)\bar{\psi}_f(x_2) e\gamma^{\nu} \\ & \times G(x_2-x_1) e\gamma^{\mu}\psi_i(x_1)[A_{\mu}(x_1)D_{\nu\rho}(x_3-x_2) \\ & + A_{\nu}(x_2)D_{\mu\rho}(x_3-x_1)], \end{aligned} \tag{1}$$

where $x_{1,2} = (t_{1,2},\mathbf{r}_{1,2})$ and $x_3 = (t_3,\mathbf{R})$. The first term in the square brackets corresponds to the direct diagram [see Fig. 1(a)] and the second term to the exchange diagram [see Fig. 1(b)].

In Eq. (1) e is the proton charge, γ^μ are the Dirac matrices, $A_\mu(x) = e^{-i\omega_{pl}t} A_\mu(\mathbf{r};\omega_{pl})$, where $A_\mu(\mathbf{r};\omega_{pl}) = \sqrt{2\pi/\omega_{pl}}\xi_\mu \exp(i\mathbf{q}_{pl}\mathbf{r})$ describes the plasma photon with the energy ω_{pl}, momentum $\mathbf{q}_{pl}$, and polarization ξ_μ, $\psi_{i,f}(x) = e^{-i(E_{i,f}-i\Gamma_{i,f}/2)t}\psi_{i,f}(\mathbf{r};E_{i,f})$ is the electron wave function, $E_{i,f}$ and $\Gamma_{i,f}$ are the binding energies and widths of the initial (i) and final (f) electron states, $D_{\nu\rho}(x_3 - x_{1,2})$ and $G(x_2 - x_1)$ are the photon and electron propagators, $J^\rho_{FI}(\mathbf{R},t_3) = e\Psi^+_F(\mathbf{R},t_3)\hat{J}^\rho\Psi_I(\mathbf{R},t_3)$ is the nuclear current between the long-lived isomeric state I and the final state F with the energies $E_{I,F}$ and the widths $\Gamma_{I,F}$ $\Psi_{I,F}(\mathbf{R},t_3) = e^{-i(E_{I,F}-i\Gamma_{I,F}/2)t_3}\Psi_{I,F}(\mathbf{R})$, and $\hat{J}^\rho$ is the operator of the nuclear electromagnetic transition. We suppose that $\Gamma_I \ll \Gamma_F$ and put the width $\Gamma_I = 0$ in this work.

The photon and electron propagators are calculated according to the relations [34]

$$D_{\mu\nu}(x_3 - x_{1,2}) = \int \frac{d\omega}{2\pi} e^{i\omega(t_3 - t_{1,2})} D_{\mu\nu}(\omega;\mathbf{R} - \mathbf{r}_{1,2}),$$

and

$$G(x_2 - x_1) = -\psi_n(\mathbf{r}_1)\bar{\psi}_n(\mathbf{r}_2)e^{-\Gamma_n t_1}\int \frac{dE}{2\pi}\frac{e^{iE(t_2-t_1)}}{E + E_n - i\Gamma_n/2}.$$

Here E_n and Γ_n are the electron binding energy and the total width of the intermediate state n which satisfies the resonance condition $\Delta \to 0$ in Fig. 1(c). For example, we use the one-level approximation in the electron propagator and neglect the others electron states, where the resonance condition is not fulfilled.

For the time integration in the vertexes of the diagrams we used the well-known relation [34]

$$e^{i(\mathcal{E}-i\varepsilon)t} = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\Omega \frac{e^{i\Omega t}}{\Omega + \mathcal{E} - i\varepsilon}.$$

Integration in Eq. (1) over the times t_1,t_2,t_3 and over the energies of the intermediate states E and ω [these integrals were evaluated as $\int dE \to 2\pi i \mathrm{Res}(E = -(E_n - i\Gamma_n/2)]$ and $\int d\omega \to 2\pi i \mathrm{Res}(\omega = E_n - E_f - i(\Gamma_n + \Gamma_f)/2)$ for the direct diagram and $\int d\omega \to 2\pi i \mathrm{Res}(\omega = -(E_n - E_i + i(\Gamma_n + \Gamma_i)/2)$ for the exchange diagram) gives

$$\begin{aligned} S^{(3)}_{fi} = i &\int d^3r_1 d^3r_2 d^3R\, J^\rho_{FI}(\mathbf{R}) \\ &\times \bar{\psi}_f(\mathbf{r}_2)e\gamma^\nu\psi_n(\mathbf{r}_2)\bar{\psi}_n(\mathbf{r}_1)e\gamma^\mu\psi_i(\mathbf{r}_1) \\ &\times \left[\frac{A_\mu(\omega_{pl},\mathbf{r}_1)D_{\nu\rho}(\omega_{nf};\mathbf{R}-\mathbf{r}_2)}{\left(\omega_{pl} - \omega_{ni} - i\frac{\Gamma_i+\Gamma_n}{2}\right)\left(\omega_N - \omega_{nf} + i\frac{\Gamma_f+\Gamma_n+\Gamma_N}{2}\right)}\right. \\ &\left. - \frac{A_\nu(\omega_{pl},\mathbf{r}_2)D_{\mu\rho}(\omega_{ni};\mathbf{R}-\mathbf{r}_1)}{\left(\omega_{pl} + \omega_{nf} - i\frac{\Gamma_f+\Gamma_n}{2}\right)\left(\omega_N + \omega_{ni} + i\frac{\Gamma_i+\Gamma_n+\Gamma_N}{2}\right)}\right]. \end{aligned} \tag{2}$$

In Eq. (2) we used the notations $\omega_{ni} = E_n - E_i$ and $\omega_{nf} = E_n - E_f$ for the energies of atomic transitions, and $\omega_N = E_F - E_I$ for the energy of nuclear transition.

The first term in square brackets in Eq. (2) which corresponds to the direct diagram contains two resonance conditions for the energy of photons $\omega_{pl} = \omega_{ni}$ and $\omega_N = \omega_{nf}$. The second term and, as a consequence, the exchange diagram do not have such resonance conditions and give considerably smaller contribution to the process near the resonances. Correspondingly, one can neglect the exchange diagram in this case and work further only with the direct diagram.

Let us introduce two new amplitudes H_{ex} and H_{int} in Eq. (2). The amplitude H_{ex} corresponds to the interaction between the electronic current $j^\mu_{ni}(\mathbf{r}_1) = -e\bar{\psi}_n(\mathbf{r}_1)\gamma^\mu\psi_i(\mathbf{r}_1)$ and the plasma photon $A_\mu(\omega_{pl};\mathbf{r}_1)$ in the process of excitation of the atomic shell:

$$H_{ex} = \int d^3r_1 j^\mu_{ni}(\mathbf{r}_1)A_\mu(\mathbf{r}_1;\omega_{pl}).$$

The amplitude H_{int} describes the interaction between the electronic current $j^\nu_{fn}(\mathbf{r}_2) = -e\bar{\psi}_f(\mathbf{r}_2)\gamma^\nu\psi_n(\mathbf{r}_2)$ and the nuclear current $J^\rho_{F,I}(\mathbf{R})$ in the NEET process [see Figs. 1(a) and 1(c)]:

$$H_{\mathrm{int}} = \int d^3r_2 d^3R\, j^\nu_{fn}(\mathbf{r}_2)D_{\nu\rho}(\omega_{nf};\mathbf{R}-\mathbf{r}_2)J^\rho_{F,I}(\mathbf{R}).$$

Now the square modulus of the amplitude (2) can be written as

$$\left|S^{(3)}_{fi}\right|^2 = \frac{|H_{ex}|^2}{(\omega_{pl} - \omega_{ni})^2 + (\Gamma_i + \Gamma_n)^2/4} \times \frac{|H_{\mathrm{int}}|^2}{(\omega_N - \omega_{nf})^2 + (\Gamma_f + \Gamma_n + \Gamma_N)^2/4}. \tag{3}$$

It is evident that the cross section of the IEB process which is obtained by averaging of the amplitude in Eq. (3) over the initial states and summation over the final ones can be presented as a product of the cross section of the excitation of atom and the probability of the NEET process.

Really, the excitation cross section of the atomic level n has the form [34]

$$\sigma_{ex}(\omega_{pl}) = \delta(\omega_{pl} - \omega_{ni})\frac{\lambda^2_{pl}}{4}\Gamma^{\mathrm{rad}}_A(\omega_{pl}; i \to n), \tag{4}$$

where $\lambda_{pl} = 2\pi/\omega_{pl}$ and $\Gamma^{\mathrm{rad}}_A(\omega_{pl}; i \to n)$ is the radiative width or probability of the atomic transition $i \to n$

$$\Gamma^{\mathrm{rad}}_A(\omega_{pl}; i \to n) = \frac{2}{\pi}\omega^2_{pl}\frac{1}{2}\frac{1}{2j_i + 1}\sum_{m_i,m_n}|H_{ex}|^2 \tag{5}$$

($j_{i,n,f}, m_{i,n,f}$ are the total angular momentum and its projection for the electronic states). We can generalize the expression in Eq. (4) to the case, when the atomic levels i and n have finite widths. Using the relation [35]

$$\delta(\omega) = \lim_{\varepsilon\to 0}\frac{1}{2\pi}\frac{\varepsilon}{(\omega^2 + \varepsilon^2)}, \tag{6}$$

the cross section (4) near the resonance can be written as

$$\sigma_{ex}(\omega_{pl}) = \frac{\lambda^2_{pl}}{4}\Gamma^{\mathrm{rad}}_A(\omega_{pl}; i \to n) \times \frac{1}{2\pi}\frac{(\Gamma_i + \Gamma_n)/2}{(\omega_{pl} - \omega_{ni})^2 + ((\Gamma_i + \Gamma_n)/2)^2}. \tag{7}$$

If we substitute into this formula the width of the radiation transition $\Gamma^{\mathrm{rad}}_A(\omega_{pl}; i \to n)$ from Eq. (5), we get in fact the first factor in the formula (3).

As for the second factor in Eq. (3), it is equivalent to the relative probability of the NEET process, P_{NEET}, which can be calculated by the expression [11]:

$$P_{\text{NEET}} \simeq \frac{E_{\text{int}}^2(L;\omega_N;n\to f,I\to F)}{(\omega_{nf}-\omega_N)^2+(\Gamma_n+\Gamma_f+\Gamma_N)^2/4}, \quad (8)$$

where E_{int} is the energy of nuclear and electron current interaction in the electron transition $n\to f$ with multipolarity L, coinciding with the multipolarity of nuclear transition $I\to F$:

$$E_{\text{int}}^2 = \frac{1}{2j_n+1}\frac{1}{2J_I+1}\sum_{m_n,m_f}\sum_{m_I,m_F}|H_{\text{int}}|^2. \quad (9)$$

Here $J_{I,F}, m_{I,F}$ are the total angular momentum and its projection for the nuclear states.

Comparing the expressions in Eqs. (3)–(9) it is easy to obtain the formula for the cross section of the IEB process

$$\sigma_{\text{IEB}}^{(3)}(\omega_{pl}) = \frac{1}{2}\frac{1}{2j_i+1}\frac{1}{2J_I+1}\sum_{\xi_\mu}\sum_{m_i,m_f}\sum_{m_I,m_F}\frac{|S_{fi}^{(3)}|^2}{T}$$
$$= \frac{\lambda_{pl}^2}{4}\Gamma_A^{\text{rad}}(\omega_{pl};i\to n)P_{\text{NEET}}\delta(\omega_{ni}-\omega_{pl}), \quad (10)$$

where the process progresses in the time interval $T\simeq 1/\sum\Gamma_{i,n,f,N}$.

III. EFFICIENCY OF EXCITATION PROCESSES

Let us compare the efficiency of excitation under the mechanism of the inverse electron bridge with the efficiency of the photo-excitation process of nuclei by plasma radiation.

If the density of plasma photons as a function of their energy is $n(\omega)d\omega$ in the energy interval $d\omega$, and τ is time of plasma existence, a fraction of the excited nuclei, or efficiency of the excitation process under the mechanism of the inverse electron bridge can be calculated from the relation

$$\zeta_{\text{IEB}}^{(3)} = \int_0^\infty d\omega_{pl}\sigma_{\text{IEB}}^{(3)}(\omega_{pl})\tau n(\omega_{pl})$$
$$\approx \frac{\lambda_{in}^2}{4}\Gamma_A^{\text{rad}}(\omega_{in};i\to n)P_{\text{NEET}}\,\tau n(\omega_{in}). \quad (11)$$

The photoexcitation cross section for nuclei can be easily calculated according to the standard QED rules. The cross section features a strong resonance [16]:

$$\sigma_\gamma^{(1)}(\omega_{pl}) = \frac{\lambda^2}{4}\Gamma_N^{\text{rad}}(\omega_{pl};I\to F)\delta(\omega_N-\omega_{pl}),$$

where $\Gamma_N^{\text{rad}}(\omega;I\to F)$ is the radiation width of nuclear transition. The efficiency of the photo-excitation process can be calculated similar to Eq. (11) using the expression

$$\zeta_\gamma^{(1)} \approx \frac{\lambda_N^2}{4}\Gamma_N^{\text{rad}}(\omega_N;I\to F)\tau n(\omega_N). \quad (12)$$

For further calculations, we shall use the following expression for the interaction energy in Eq. (8) [11]:

$$E_{\text{int}}^2 = \frac{1}{4}\Gamma_A^{\text{rad}}(E[M]L;\omega_N;n\to f)$$
$$\times\Gamma_N^{\text{rad}}(E[M]L;\omega_N;I\to F)\left(1+\frac{1}{\delta^2}\right).$$

This expression binds E_{int}^2 with radiation widths of atomic (Γ_A^{rad}) and nuclear (Γ_N^{rad}) transitions by the factor $\delta\equiv \text{Re}[\mathcal{M}_L^{E[M]}(\omega_N)]/\text{Im}[\mathcal{M}_L^{E[M]}(\omega_N)]$ that is well known from the theory of internal electron conversion. This factor is the ratio of the real and imaginary parts of the matrix element $\mathcal{M}_L^{E[M]}(\omega_N)$. For the electric and magnetic types of transitions, the electron matrix elements are calculated using the equations [11]

$$\mathcal{M}_L^E(\omega_N) = \int_0^\infty drr^2\bigg(h_L^{(1)}(\omega_N r)[g_n(r)g_f(r)+f_n(r)f_f(r)]$$
$$-\frac{h_{L-1}^{(1)}(\omega_N r)}{L}[(\kappa_n-\kappa_f-L)g_n(r)f_f(r)$$
$$+(\kappa_n-\kappa_f+L)g_f(r)f_n(r)]\bigg),$$
$$\mathcal{M}_L^M(\omega_N) = \frac{\kappa_n+\kappa_f}{L}\int_0^\infty drr^2h_L^{(1)}(\omega_N r)$$
$$\times[g_n(r)f_f(r)+g_f(r)f_n(r)],$$

where $g(r)$ and $f(r)$ are, correspondingly, large and small components of the Dirac wave functions of electron in atomic shell, $\kappa=(l-j)(2j+1)$, where l is the orbital angular momentum quantum number of the electronic state.

From Eq. (11) one can conclude that at excitation of nuclei by the mechanism of the inverse electron bridge, the width of the part of the spectrum responsible for the resonance transition to the intermediate state in atomic shell near $\omega_{pl}=\omega_{in}$ equals to $\Gamma_A^{\text{rad}}(\omega_{in};i\to n)$. Thus, the density of the resonance photons is $n(\omega_{in})\Gamma_A^{\text{rad}}(\omega_{in};i\to n)$. This is essentially higher (especially for the atomic transition $E1$) than the similar value $n(\omega_N)\Gamma_N^{\text{rad}}(E[M]L;\omega_N;I\to F)$ in Eq. (12), characterizing the density of the resonance plasma photons causing the nucleus excitation in the first-order process—direct photoabsorption. Denoting by Δ the value of mismatch between the energy of atomic transition $n\to f$ and ω_N in the NEET process [$\Delta\equiv(E_n-E_f)-\omega_N$, see in Fig. 1], we shall obtain the following expression for the ratio of excitation efficiencies:

$$\frac{\zeta_{\text{IEB}}^{(3)}}{\zeta_\gamma^{(1)}} \approx \frac{1}{4\delta^2}\frac{\Gamma_A^{\text{rad}}(E1;\omega_{in};i\to n)}{\Gamma_N^{\text{rad}}(E[M]L;\omega_N;I\to F)}$$
$$\times\frac{\Gamma_A^{\text{rad}}(E[M]L;\omega_N;n\to f)\Gamma_N^{\text{rad}}(E[M]L;\omega_N;I\to F)}{\Delta^2+\Gamma_n^2/4}. \quad (13)$$

Assume that for experimental studies we managed to select an ideal object with so small mismatch value, that the following condition is satisfied: $\Delta \leqslant \Gamma_n$. (Such examples, though uncommon, are known. The NEET process close to resonance takes place in ^{197}Au and ^{193}Ir [33]. We shall discuss this situation in greater detail below.) It is clear, that the radiation width $\Gamma_A^{\rm rad}(E1;\omega_{in};n \to i)$ gives the basic contribution to the width of the intermediate state Γ_n (Auger process in the upper shells of a highly ionized atom is unlikely). The formula (13) for the ratio of excitation efficiencies is simplified, and under the condition $\omega_{in} \approx \omega_{nf} \approx \omega_N$ has the following form:

$$\frac{\zeta^{(3)}_{\rm IEB}}{\zeta^{(1)}_{\gamma}} \approx \left(\frac{\mathrm{Im}\big[\mathcal{M}_L^{E[M]}(\omega_N;n \to f)\big]}{\mathrm{Re}\big[\mathcal{M}_1^{E}(\omega_N;n \to i)\big]}\right)^2. \quad (14)$$

The small parameter $\omega_N r_0 \ll 1$ expansion of the atomic matrix elements in Eq. (14) gives $|\mathrm{Im}[\mathcal{M}_L^{E[M]}(\omega_N r_0)]| \simeq 1/(\omega_N r_0)^{L+1}$ for the EL transition, and $\simeq e^2/(\omega_N r_0)^{L+1}$ for the ML transition (we took into account here that by an order of magnitude the small component of the Dirac bispinor can be estimated using the equation $f(x) \simeq e^2 g(x)$ [34]), and $\mathrm{Re}[\mathcal{M}_1^E(\omega_N r_0)] \simeq (\omega_N r_0)^1$. It allows writing the ratio of efficiencies in the form

$$\frac{\zeta^{(3)}_{\rm IEB}}{\zeta^{(1)}_{\gamma}} \approx \begin{cases} 1/(\omega_N r_0)^{2(L+2)}, & \text{for} \quad EL \text{ nuclear transition} \\ e^4/(\omega_N r_0)^{2(L+2)}, & \text{for} \quad ML \text{ nuclear transition} \end{cases}. \quad (15)$$

It follows from Eq. (15) particularly that in the case of nuclear $E1$ transition of relatively small energy ω_N (some kiloelectronvolts) in high-temperature plasma, when $\omega_N r_0 \approx 1/10$, the efficiency of the excitation process through the electron bridge can surpass by several orders the efficiency of direct photoexcitation of nuclei by thermal plasma radiation.

Note that Eq. (15) assumes a multipolar exchange in the process of excitation through the electron bridge. The first stage—excitation of atom (ion)—always occurs as a result of $E1$ transition from the initial to the intermediate state. Thus, Eq. (15) is true for nucleus excitation through an inelastic electron bridge. The less probable process—elastic electron bridge—is not considered in this paper. The exception makes a case of the nuclear $E1$ transition, for which Eq. (15) is true for both types of electron bridge.

It is necessary to understand that the numerical assessments based on Eq. (15) shall be considered with some caution. The equations help to understand only the most general qualitative regularities existing between the various processes of nucleus excitation. Equation (14) is much more precise, but it requires the calculation of electron matrix elements that, in turn, requires the corresponding computer codes.

IV. DE-EXCITATION OF ^{84}RB NUCLEUS

Let us consider, as an example, the induced by the plasma photons decay of the isomeric state 6^-(463.59 keV, 20.26 m) in the ^{84}Rb nucleus (see Fig. 2) through the state 5^-(466.64 keV, 9 ns). In hot plasma with temperature $T \simeq \omega_N = 3.4$ keV (see Table I), excitation of a short-living state

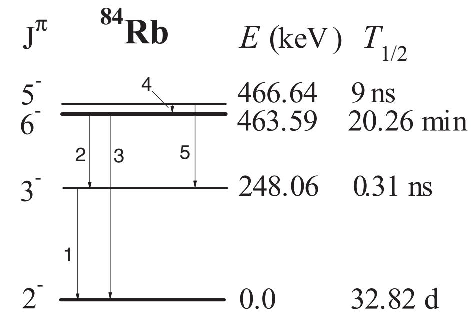

FIG. 2. Decay scheme of ^{84}Rb.

5^-(466.64 keV) will occur both as a result of photoabsorption by isomeric nuclei ^{84}Rb^m (photoexcitation), and in the course of an inverse electron bridge through the Rb ion shell.

Parameters of nuclear transitions shown in Fig. 2 are given in Table I (see Ref. [36]). Note that a clarification is needed for the energy of transition 6^-(463.59 keV) $\to$ 5^-(466.64 keV) from [36]. The tabular value 3.4 keV is considerably larger than the difference of energy levels $E_{6^-} - E_{5^-} = 3.05$ keV.

Modern numerical codes do not allow calculating energies of atomic states with accuracy that would allow making a conclusion regarding coincidence or discrepancy of energies of electronic and nuclear transitions within the width of the atomic line. We have calculated the atomic shells and multicharge Rb ions using the code RAINE [37]. The obtained assessments: (i) allow tracking the tendency of electronic transition energy change as the atomic shell is ionized; and (ii) indicate a possibility in principal to observe the IEB process for ^{84}Rb. (Analogous calculations for the $L_3 \to K$ transition in the ^{237}Np ions were done in [25].)

We will consider the energies of $M1$ transitions $5S_{1/2} \to 2S_{1/2}$ and $5P_{1/2} \to 2P_{1/2}$ in the electronic shell of the Rb ion as an example. In the Rb atom, the values of these energies are, correspondingly, 2 keV and 1.8 keV. While degree of atomic ionisation increases, the energies of the specified transitions also increase. Figure 3 shows these changes in the Rb^{25+} – Rb^{33+} ions. One can see that the assumed area of nuclear transition 3.05—3.4 keV overlaps in the Rb^{28+}–Rb^{32+} ions.

NEET probability and the cross section of nuclear excitation in the IEB process in the Rb^{32+} ion are estimated for the typical and not the most optimum case of electronic transition $2S_{1/2} \xrightarrow{E1} 5P_{1/2} \xrightarrow{M1} 2P_{1/2}$ given in Fig. 4. (Note in brackets that, for example, the cross section of the nuclear

TABLE I. Properties of ^{84}Rb transitions. N is a transition number from Fig. 2.

N	Energy, keV	Multi-polarity	Conversion coefficient	Reduced probability of transition
1	248.02	$E2+M1$	≈0.0343	$B_{W.u.}(M1) \approx 0.00020$ $B_{W.u.}(E2) \approx 82$
2	215.61	$M3+E4$	1.08	$B_{W.u.}(M3) = 0.00091$ $B_{W.u.}(E4) = 72$
3	463.62	$E4$	0.0391	$B_{W.u.}(E4) = 0.132$
4	(3.4)	$(M1)$	361	$B_{W.u.}(M1) \approx 0.08$
5	218.3	$E2$	0.0556	$B_{W.u.}(E2) = 3.1$

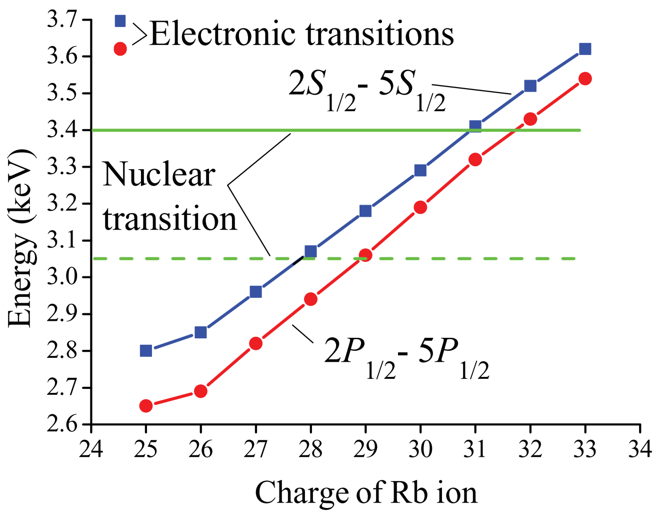

FIG. 3. (Color online) Energy of $M1$ transitions $5S_{1/2} \to 2S_{1/2}$ and $5P_{1/2} \to 2P_{1/2}$ in the Rb ion shell as a function of the ion charge.

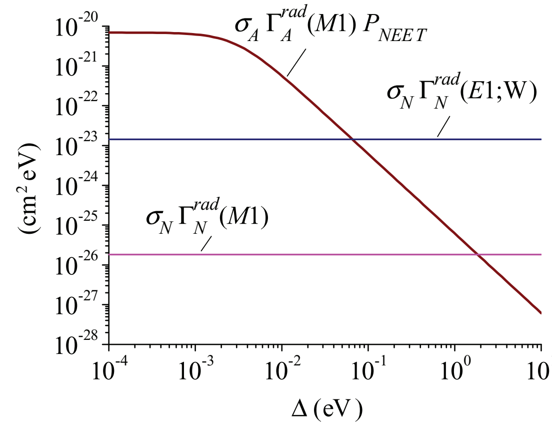

FIG. 5. (Color online) Function $(\lambda_A^2/4)\Gamma_A^{\rm rad}(2S_{1/2}\to 5P_{1/2})P_{\rm NEET}$ for the IEB process and functions $(\lambda_N^2/4)\Gamma_N^{\rm rad}(\omega_N; M1, 6^- \to 5^-)$ and $(\lambda_N^2/4)\Gamma_N^{\rm rad}(\omega_N; E1; W)$ for the process of direct photoexcitation of ^{84}Rb nucleus in the $M1$ transition studied in the present work and $E1$ transition of the same energy with the reduced transition probability in the Weisskopf model equal to 1.

excitation appears essentially higher in the electronic transition $2P_{1/2} \xrightarrow{E1} 5S_{1/2} \xrightarrow{M1} 2S_{1/2}$.) The value of the nuclear transition energy $6^- \to 5^-$ $\omega_N = 3.4$ keV recommended in [36] is used in calculations.

Energy of interaction $E_{\rm int}$ of electronic and nuclear currents in the course of energy exchange between electronic shell and the nucleus is calculated using the known equation from [11]:

$$E_{\rm int}^2 = 4\pi e^2 \frac{\omega_N^{2(L+1)}}{[(2L+1)!!]^2}\left(C^{j_f 1/2}_{j_i 1/2 2L0}\right)^2 \left|\mathcal{M}_L^{E/M}(\omega_N)\right|^2 B(E/ML; J_i \to J_f). \qquad (16)$$

For the scheme shown in Fig. 4, the calculation with electronic wave functions $2P_{1/2}$ and $5P_{1/2}$ gives the value $E_{\rm int} \simeq 2\times10^{-4}$ eV. The obtained value is comparable with the radiation widths of electronic $E1$ transitions with energies 3–4 keV in multicharge Rb ions. For example, the radiation width of the $E1$ transition $5S_{1/2} \to 2P_{1/2}$ is approximately equal to 2×10^{-4} eV, and of the transition $5P_{1/2} \to 2S_{1/2}$—to 6×10^{-3} eV. Therefore, in the resonance condition, when detuning between the energies of atomic and nuclear transitions

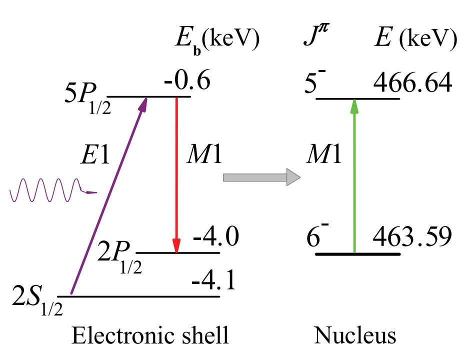

FIG. 4. (Color online) One of possible schemes of the IEB process in the ^{84}Rb^{32+} ion.

Δ does not exceed 10^{-3}–10^{-2} eV, the probability $P_{\rm NEET}$ from Eq. (8) can reach the values of an order of 10^{-2} in ^{84}Rb^{32+}.

The efficiency ζ of nucleus excitation of Eqs. (11)–(12) contains “a plasma” part $\tau n(\omega_{in,N})$ that is nearly identical for both mechanisms. Figure 5 shows the ζ function component that is defined by an atomic-nuclear part of the excitation process, namely, product of the cross section and the width of a working section of photon spectrum, that is, actually, the radiation width of the process. It is clear that, if detuning Δ is small, the IEB process will dominate over the direct photoexcitation.

It is interesting to note that the efficiency of excitation through IEB in the ^{84}Rb^{32+} ion can exceed even the efficiency of the photoexcitation for the $E1$ transition with energy $\omega_N = 3.4$ keV and reduced probability $B_{W.u.}(E1) = 1$ (see Sec. V for explanations), and there is nothing surprising in this. The mechanism of compensation of additional electron-photon vertexes in the diagram of the third order, which was considered at the beginning of Sec. II, is very efficient due to smallness of the $\omega_N r_0$ parameter. (At the same time the radiation width of even such $E1$ transition remains of course relatively small, 5.1×10^{-8} eV, since its energy is only 3.4 keV, and the radiation width of the $E1$ transition is proportional to ω_N^3.)

V. DISCUSSION OF RESULTS

The imposed condition $\Delta \leqslant \Gamma_n$ is rigid, but, as it was already mentioned above, is not contradictory. In hot dense unstable plasma, there is a permanent exchange of electrons between plasma and ions. The charge of ions change and, as a consequence, changes the Coulomb field affected the bound electrons. The electron binding energies in the ion shell E_i, E_n, E_f, and the energy of transition $\omega_{nf} = E_n - E_f$ change accordingly. The energy of transition ω_{nf} approaches to or moves away from the resonance value ω_N. According to Eqs. (8)–(10), the cross section of nuclear excitation under the mechanism of the inverse electronic bridge depends quadratically on the mismatch value Δ. In the resonance, the cross section $\sigma^{(3)}_{\rm IEB}$ possesses the maximum value, which is much larger than the cross section of photoexcitation $\sigma^{(1)}_{\gamma}$. Therefore, the reduction of Δ to the values comparable with Γ_n (even for short time) leads to effective excitation of the nucleus to the isomeric state. It is not unlikely that inclusion of the studied mechanism of excitation of nuclei in the theoretical analysis similar to that one made in [6], will allow making a more precise prediction of the yield of nuclear isomers in experiments with hot laser plasma. The required precision calculation of electronic transition energies in ions is difficult, but a problem for modern computer codes which can be overcome.

In conclusion, let us note two important circumstances. First, the value $P_{\rm NEET}$ is the relative probability of nucleus excitation in electron transition. It cannot exceed 1 by definition. Let us make sure that for the nuclear $E1$ transition, this condition is valid also in the case $\zeta^{(3)}_{\rm IEB}/\zeta^{(1)}_{\gamma} \approx 1/(\omega_N r_0)^6 \gg 1$. For this purpose, let us recollect that $\Gamma^{\rm rad}_N(E1;\omega) \sim \omega^3 R_N^2 B_{W.u.}(E1)$ and $\Gamma^{\rm rad}_A(E1;\omega) \sim \omega^3 r_0^2$. Substituting these relations into Eqs. (11)–(12), we shall obtain the following expression for the excitation efficiency ratio:

$$\frac{\zeta^{(3)}_{\rm IEB}}{\zeta^{(1)}_{\gamma}} \approx \left(\frac{r_0}{R_N}\right)^2 \frac{P_{\rm NEET}}{B_{W.u.}(E1)}.$$

Here $B_{W.u.}$ is a so-called reduced probability in Weisskopf units that depends on the properties of a particular nuclear transition. By definition, it is equal to the ratio of the measured experimentally reduced probability of the nuclear transition $B(E/ML, I_I \to I_F)$ to the reduced probability in Weisskopf model $B(W;E/ML)$. The latter value for the E/ML transition in a nucleus with atomic number A and radius $R_0 = 1.2A^{1/3}$ fm is calculated using the equations [38]

$$B(W;EL) = \frac{e^2}{4\pi}\left(\frac{3}{3+L}\right)^2 R_0^{2L},$$

$$B(W;ML) = B(W;EL)\frac{10}{(M_P R_0)^2},$$

where M_P is the proton mass.

Let us evaluate the value $P_{\rm NEET}$ in the case, when the formula (15) is valid for $\zeta^{(3)}_{\rm IEB}/\zeta^{(1)}_{\gamma}$. From the relation $P_{\rm NEET} \approx (R_N/r_0)^2 B_{W.u.}(E1)/(\omega_N r_0)^6$ we obtain that $P_{\rm NEET} \lesssim 10^{-6}$ at the characteristic value $B_{W.u.}(E1) \simeq 10^{-6}$. Thus, the deduced above relation (15) does not really contradict the condition $P_{\rm NEET} \leqslant 1$.

Second, in the resonance mode, which was considered in the present work, time of the excitation process in a nucleus is defined by the lifetime of vacancy t_v in the ion shell. In the case of an elastic electron bridge, when initial $|i\rangle$ and final $|f\rangle$ electron states coincide, this is the vacancy lifetime in a state with E_i energy (see Fig. 1). If the process goes through inelastic electron bridge, as shown in the right part of Fig. 1, its duration is defined by the hole lifetime in a state with energy E_f. When the vacancy decay in the ion shell is slower than plasma cools down, an additional multiplier τ/t_v is to be added into the equation for the efficiency of the IEB process (11). For transitions in the optical part of the spectrum (with energies of some electronvolts), the characteristic time $t_v \simeq 10^{-8}$ s and the factor τ/t_v could reduce essentially the efficiency of the excitation through the IEB mechanism. However, for the discussed energies $\omega \simeq 1$–10 keV, the characteristic lifetime of transitions in the atomic or ionic shell is much less than in the optical area since the probability of electric dipole transitions is proportional to ω^3. Accordingly, even for the high-temperature laser plasma, lifetime of which is relatively low (an order of laser pulse duration), the condition $t_v < \tau$ will be always true. In particular, the estimations of $E1$ transition widths in the Rb ion, given above, indicate this. Thus, Eq. (11) quite adequately allows for physical conditions of the nuclear excitation process in plasma and can be used for qualitative assessment of the IEB mechanism efficiency.

VI. CONCLUSION

In conclusion, the results of the work can be summarized as follows. It was shown that in the case of resonance coincidence of the energies of nuclear and one of atomic transitions, the process of nuclei excitation by the mechanism of the inverse electronic bridge can provide a very high efficiency of excitation of the nuclei, including the excitation to the short-living state located closely to the isomer state. Adding such mechanism into corresponding computer codes is of great interest to the study of excitation of nuclei in high-temperature dense nonstationary plasma.

[1] Y. Izawa and C. Yamanaka, Phys. Lett. B **88**, 59 (1979).

[2] R. V. Arutyunyan, L. A. Bol'shov, V. D. Vikharev, S. Dorshakov, V. A. Kornilo, A. A. Krivolapov, V. P. Smirnov, and E. V. Tkalya, Soviet J. Nucl. Phys. **53**, 23 (1991).

[3] A. V. Andreev, R. V. Volkov, A. M. Dykhne, V. M. Gordienko, M. P. Kalashnikov, P. M. Mikheev, P. V. Nikles, A. B. Savel'ev, E. V. Tkalya, R. A. Chalykh *et al.*, J. Exp. Theor. Phys. **91**, 1163 (2000).

[4] K. W. D. Ledingham, P. McKenna, and R. P. Singhal, Science **300**, 1107 (2003).

[5] C. Granja, J. Jakubek, V. Linhart, S. Pospisil, T. Slavicek, J. Uher, Z. Vykydal, J. Kuba, M. Sinor, L. Drska *et al.*, Czech. J. Phys. **56**(Suppl. B), 478 (2006).

[6] F. Gobet, F. Hannachi, M. M. Aléonard, M. Gerbaux, G. Malka, J. N. Scheurer, M. Tarisien, G. Claverie, D. Descamps, F. Dorchies *et al.*, J. Phys. B: At. Mol. Opt. Phys. **41**, 145701 (2008).

[7] R. C. Haight and G. C. Baldwin, AIP Conf. Proc. **146**, 58 (1986).

[8] R. V. Arutyunyan and L. A. Bolshov, Sov. Phys. Dokl. **34**, 338 (1989).

[9] E. V. Tkalya, E. V. Akhrameev, R. V. Arutyunayn, L. A. Bol'shov, and P. S. Kondratenko, Phys. Rev. C **85**, 044612 (2012).

[10] M. Morita, Progr. Theor. Phys. **49**, 1574 (1973).

[11] E. V. Tkalya, Nucl. Phys. A **539**, 209 (1992).

[12] V. I. Gol'danskii and V. A. Namiot, Phys. Lett. B **62**, 393 (1976).

[13] M. R. Harston and J. F. Chemin, Phys. Rev. C **59**, 2462 (1999).
[14] E. V. Tkalya, Sov. Phys. Dokl. **36**, 467 (1991).
[15] E. V. Tkalya, Sov. Phys. Dokl. **35**, 1069 (1990).
[16] E. V. Tkalya, Laser Phys. **14**, 360 (2004).
[17] O. V. Chutko, A. V. Andreev, V. M. Gordienko, M. A. Joukov, E. V. Petrova, A. A. Rusanov, A. B. Savel'ev, and E. V. Tkalya, Laser Phys. **13**, 190 (2003).
[18] P. Morel, J. M. Daugas, G. Gosselin, V. Méot, and D. Gogny, Nucl. Phys. A **746**, 608c (2004).
[19] P. Morel, V. Méot, G. Gosselin, D. Gogny, and W. Younes, Phys. Rev. A **69**, 063414 (2004).
[20] G. Gosselin and P. Morel, Phys. Rev. C **70**, 064603 (2004).
[21] G. Gosselin, V. Méot, and P. Morel, Phys. Rev. C **76**, 044611 (2007).
[22] O. Renner, L. Juha, J. Krasa, E. Krousky, M. Pfeifer, A. Velyhan, C. Granja, J. Jakubek, V. Linhart, T. Slavicek *et al.*, Laser and Particle Beams **26**, 249 (2008).
[23] G. Gosselin, N. Pillet, V. Méot, P. Morel, and A. Y. Dzyublik, Phys. Rev. C **79**, 014604 (2009).
[24] P. Morel, V. Méot, G. Gosselin, G. Faussurier, and C. Blancard, Phys. Rev. C **81**, 034609 (2010).
[25] S. K. Arigapudi and A. Pálffy, Phys. Rev. A **85**, 012710 (2012).
[26] J. Gunst, Y. A. Litvinov, C. H. Keitel, and A. Pálffy, Phys. Rev. Lett. **112**, 082501 (2014).
[27] J. J. Carroll Laser Phys. Lett. **1**, 275 (2004).
[28] R. V. Arutyunyan, E. V. Akhrameev, L. A. Bol'shov, P. S. Kondratenko, and E. V. Tkalya, JETP Lett. **98**, 772 (2013).
[29] R. V. Arutyunyan, L. A. Bolshov, and E. V. Tkalya, JETP Lett. **46**, 446 (1987).
[30] V. A. Krutov and V. N. Fomenko, Ann. Phys. (NY) **21**, 291 (1968).
[31] J. M. Eisenberg and W. Greiner, *Nuclear Theory. Excitation Mechanisms of the Nucleus. Electromagnetic and Weak Interactions* (North-Holland Publishing Co., London, 1970), Vol. II.
[32] *Handbook of Mathematical Functions*, edited by M. Abramovitz and I. A. Stegun, Applied Mathematics Series 55 (National Bureau of Standards, Washington, DC, 1964).
[33] E. V. Tkalya, Phys. Rev. A **75**, 022509 (2007).
[34] V. B. Berestetskii, E. M. Lifschitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, Oxford, 1982).
[35] G. A. Korn and T. M. Korn, *Mathematical Handbook* (McGraw-Hill Book Company, New York, 1968).
[36] F. G. Kondev, Nucl. Data Sheets **110**, 2815 (2009).
[37] I. M. Band and V. I. Fomichev, At. Data Nucl. Data Tabl. **23**, 295 (1979).
[38] A. Bohr and B. R. Mottelson, *Nuclear Structure. Single-Particle Motion* (W. A. Benjamin, Inc., New York/Amsterdam, 1969), Vol. I.