Surface-integral formalism of deuteron stripping

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(Received 29 June 2014; published 8 September 2014)

The purpose of this paper is to develop an alternative theory of deuteron stripping to resonance states based on the surface-integral formalism of Kadyrov *et al.* [Ann. Phys. **324**, 1516 (2009)] and continuum-discretized coupled channels (CDCC). First we demonstrate how the surface-integral formalism works in the three-body model and then we consider a more realistic problem in which a composite structure of target nuclei is taken via optical potentials. We explore different choices of channel wave functions and transition operators and show that a conventional CDCC volume matrix element can be written in terms of a surface-integral matrix element, which is peripheral, and an auxiliary matrix element, which determines the contribution of the nuclear interior over the variable r_{nA}. This auxiliary matrix element appears because of the inconsistency in treating of the n-A potential: This potential should be real in the final state to support bound states or resonance scattering and complex in the initial state to describe n-A scattering. Our main result is formulation of the theory of the stripping to resonance states using the prior form of the surface-integral formalism and CDCC method. It is demonstrated that the conventional CDCC volume matrix element coincides with the surface matrix element, which converges for the stripping to the resonance state. Also the surface representation (over the variable $\mathbf{r}_{nA}$) of the stripping matrix element enhances the peripheral part of the amplitude although the internal contribution does not disappear and increases with an increase of the deuteron energy. We present calculations corroborating our findings for both stripping to the bound state and the resonance.

DOI: 10.1103/PhysRevC.90.034604 PACS number(s): 24.30.−v, 25.45.Hi, 24.10.−i

I. INTRODUCTION

Theory of the nucleon transfer reaction formulated in terms of the matrix element containing the potential transition operator is based on the perturbation approach over the potential transition operator. It can be formulated in two forms: post or prior. In the post (prior) form the initial (final) scattering wave function is approximated by a simpler channel wave function. The distorted-wave Born approximation (DWBA), which is the simplest approach, is the first-order perturbation theory over the potential transition operator (which is different in the post and prior forms) sandwiched by the initial and final-channel wave functions. These channel wave functions are given by a product of the bound-state wave function of the the initial (final nuclei) multiplied by a corresponding distorted wave. The DWBA is based on the assumption that the probability of direct reactions is so small that they can be treated as direct transitions from the initial to the final channel without any coupling to the other channels, which is not always true. A definite improvement is the continuum-discretized coupled channels (CDCC) method applied for the analysis of the deuteron stripping. In the CDCC method, in addition to $d + A$ channel, the three-body breakup channel $p + n + A$ is included. However, the CDCC method has its own limitations. The main one is related with the contribution of the rearrangement channels. For example, for deuteron stripping these rearrangement channels are the proton or neutron bound to the target. Because rearrangement channels are not orthogonal to the initial $d + A$ channel and to the breakup $p + n + A$ channel, their accurate inclusion makes the problem very complicated and the only legitimate solution is the Faddeev formalism [1], which allows one to treat consistently nonorthogonal channels without double counting. However, it is quite difficult to use the Faddeev formalism on a routine basis and its application, owing to the technical problems with the Coulomb interaction, is limited only to light nuclei. Hence, the CDCC method is still useful, but one needs to clearly understand the shortcomings of the CDCC method and one of them is the absence of the rearrangement channels in the asymptotic regions.

In practical calculations it is assumed that in a limited region near the target the CDCC wave function reproduces the three-body wave function reasonably well. To calculate the stripping matrix element the standard iteration procedure is used: The CDCC wave function does not have rearrangement channels in the asymptotic region but can be used to calculate the reaction matrix element contributed by the final volume around the target. Here the question of the uniqueness appears: How would the solution of the CDCC equation change if we add the rearrangement channel wave function to the original CDCC wave function. For example, if we consider the deuteron-stripping reaction $d + A \to p + F$, where $F = (n\,A)$, what would happen if we use $\Psi_i^{\mathrm{CDCC}(+)} + \varphi_F\,\chi_{pF}^{(+)}$ as the initial wave function rather than just the CDCC wave function $\Psi_i^{\mathrm{CDCC}(+)}$, where φ_F is the (nA) bound-state wave function and χ_{pF} is the p-F distorted wave. It was shown in Ref. [2] that the presence of two optical potentials U_{pA} and U_{nA} suppresses the contribution from two rearrangement channels, $p + (nA)$ and $n + (pA)$, resolving the uniqueness problem. However, if only one optical potential is present, then the issue of uniqueness should be checked. To suppress the rearrangement channels,

truncation over angular momentum is being used. Then a sensitivity to the maximal orbital angular momentum $l_{pn}^{\max}$ of the relative motion of p and n should be checked.

Despite the shortcomings of the CDCC approach, it remains the best option unless the Faddeev equations are solved. In this work we use the CDCC approach to develop the theory of the deuteron stripping to resonance states. However, instead of the standard formulation of the theory with the matrix element expressed in terms of the volume integral, we develop here the theory of the deuteron stripping based on the surface-integral formalism [3,4] and the CDCC approach. The first such attempt was done in a recent work [5], where both DWBA and CDCC methods were used to derive the deuteron-stripping reaction amplitude populating bound states and resonances. However, the CDCC part was not complete because the surface integral was extended to the region where the CDCC method fails. Here we present another formulation of the theory of the stripping to resonance states using the surface-integral formulation based on the CDCC approach in a finite region around the target, that is, in the region where the CDCC method should work. In Ref. [5] the surface integral in the post form for stripping to bound states was taken over variable $\mathbf{r}_{nA}$ while the volume integral over the second Jacobian variable $\boldsymbol{\rho}_{pF}$ was taken over the whole space. Now in our new formulation the matrix element is expressed in terms of the surface integral over $\boldsymbol{\rho}_{pF}$ at some finite ρ_{pF} determined by the transition operator, while the volume integral over the second Jacobian variable $\mathbf{r}_{nA}$ is taken over the limited volume space because of the presence of the bound-state wave function φ_F. We also use the prior form for the analysis of the stripping to resonance states. In this case the matrix element can be expressed in terms of surface integral over $\boldsymbol{\rho}_{dA}$ taken at some finite radius determined by the transition operator and the bound-state wave function φ_{pn} and the volume integral over the second Jacobian variable $\mathbf{r}_{pn}$, which is taken over the limited volume because of the presence of φ_{pn}. Thus, even for stripping to resonance the matrix element is taken over the limited space where the CDCC method works.

We explore different choices of the channel wave functions and, correspondingly, different transition operators. One of the main unsolved problems in the conventional theory for the deuteron-stripping reaction $d + A \to p + F$ is the inconsistency in the treatment of the n-A potential, which should be real to support the final bound or resonance state $(n\,A)$ but complex to describe the initial n-A scattering. We show how this inconsistency leads to the appearance of the auxiliary term when connecting the conventional volume matrix element with the surface-integral form. We also present calculations using the FRESCO code [6] for stripping to bound states and resonances. The main goal of this work is to present an advanced theory of the deuteron stripping to a resonance, which further leads to the three-body continuum in the final state. Such reactions can occur in broad interval of the deuteron incident energies. Note that the deuteron stripping to resonance requires 2.224 MeV, the deuteron binding energy, to break the deuteron and additional energy to excite a resonance state. Hence, the Q value of the reaction is negative. That is why we do not consider here deuteron stripping at sub-Coulomb energies with new interesting physics [7]. Such reactions can be studied using the Faddeev formalism. The theory, which we present here, is aimed to analyze the deuteron-stripping reactions from low energies near the Coulomb barrier up to the deuteron incident energies $E_d \sim 100$ MeV.

II. THREE-BODY THEORY OF DEUTERON STRIPPING POPULATING BOUND STATES IN THE SURFACE-INTEGRAL FORMALISM

Let us consider the deuteron stripping to bound states,

$$d + A \to p + F, \tag{1}$$

where $F = (A\,n)$ is the bound state.

The reaction amplitude can be calculated exactly in the three-body model using the Faddeev integral equations in the Alt-Grassberger-Sandhas (AGS) form [8–13], but it neglects internal degrees of freedom of the target or can only account for a few [14,15]. Moreover, the formalism is limited to targets with not-too-large charges. Nowadays, deuteron stripping on heavy nuclei with atomic number $A \sim 100$ is the most important and urgent because it can provide missing vital information about (n, γ) s or r processes in stellar evolution. The generalized Faddeev approach, which explicitly includes target excitations and the Coulomb interaction for arbitrary charges, was developed [16] but no computer codes based on the formalism are yet available. Besides, the Faddeev formalism is too complicated for use on an everyday basis, especially by experimental groups.

In the traditional approach the reaction amplitude is calculated using the iteration procedure, in which the volume matrix element containing the exact scattering wave function (in the initial state-post form or in the final state-prior form) is approximated by the one in which the exact scattering wave function is replaced with some model wave function. This approximation is used because nowadays there are no tools to calculate the many-body scattering wave function accurately, especially in the asymptotic regions with many open channels. Moreover, should this asymptotic behavior be available, there is no need to calculate the matrix elements because the amplitude of the asymptotic outgoing wave in the corresponding channel is the reaction amplitude for transition to this channel. The idea of the iteration procedure is that the matrix element containing the scattering wave function, which is not accurate asymptotically, is still suited to calculate the reaction amplitude, because this matrix element is contributed by a limited volume around the target where the model scattering wave function may be accurate enough.

First we consider the surface-integral formalism in a three-body model, in which all three particles are structureless and all the interaction potentials between them are real. After that, we specifically consider deuteron-stripping reactions extending the three-body model, which requires using optical potentials. Different options and ways in which they affect the reaction amplitude are discussed.

We start from the consideration of reaction (1) in the three-body model $p + n + A$. We introduce the Jacobian variables $\mathbf{r}_\alpha$ and $\boldsymbol{\rho}_\alpha$ commonly used to describe three-body systems, where $\mathbf{r}_\alpha$ is the radius vector connecting the center of masses of particles β and γ, while $\boldsymbol{\rho}_\alpha$ is the radius-vector

connecting the center of mass of particle α and the center of mass of the system $\beta+\gamma$ [17]. We also need a hyperradius in the six-dimensional configuration space defined according to $X_\alpha = (\mu_\alpha\, r_\alpha^2/m + M_\alpha\, \rho_\alpha^2/m)^{1/2}$, where m is the nucleon mass and μ_α is the reduced mass of particles β and γ, $M_\alpha = m_\alpha\, m_{\beta\gamma}/M$ is the reduced mass of particle α and the bound system $(\beta\,\gamma)$, and M is the total mass of the three-body system. Let us introduce the asymptotic region Ω_α corresponding to the case when two particles β and γ are close to each other while the third particle α is far away. In this region $r_\alpha/\rho_\alpha \to 0$ at $\rho_\alpha \to \infty$ [17]. Also, we denote by Ω_0 the asymptotic region where all three particles are far away (breakup channel), that is r_α, $\rho_\alpha \to \infty$ and $r_\alpha/\rho_\alpha \to$ const $\neq 0$. The asymptotic behavior of the three-body wave function of three charged particles in different asymptotic regions was discussed in [3,4,17–20]. For the asymptotic behavior of the three-body wave function we have

$$\Psi_\alpha^{(+)} = \Psi_\alpha^{(0)} - \sum_\nu \frac{M_\nu}{2\,\pi}\, \mathcal{M}_{\nu\,\alpha}^{(\mathrm{as})}\, u^{(+)}(\rho_\nu)\, \phi_\nu + \Psi_0^{(+)}. \qquad (2)$$

Here $\Psi_\alpha^{(+)}$ is the scattering wave function with the incident wave in the initial channel α. The two-cluster channel α is defined as the channel $\alpha + (\beta\,\gamma)$, where the free particle carries the name of the channel. For reaction (1) the incident channel α is $d + A$, that is $\alpha = A$ and $\Psi_\alpha^{(+)} \equiv \Psi_{\mathbf{k}_{dA}}^{(+)}$ is the $d + A$ scattering wave function calculated in the three-body model $p + n + A$. $\mathbf{k}_{ij}$ is the relative momentum of particles i and j; $\Psi_\alpha^{(0)}$ is the incident wave in the entry channel α. The sum over the final two-body channels ν contains the elastic and rearrangement channels, $\mathcal{M}_{\nu\,\alpha}^{(\mathrm{as})}$ is the reaction amplitude leading to the final two-body channel ν, $\phi_\alpha = \varphi_{\beta\,\gamma}$ is the bound-state wave function of the pair $(\beta\,\gamma)$ in the channel α; for example, for the channel $A + d$, $\phi_\alpha = \varphi_{pn}$ and for the channel $\beta = p + (n\,A)$, $\phi_\beta = \varphi_{nA}$. Also, $u^{(+)}(\rho_\nu)$ is the outgoing wave in the two-fragment channel ν. It should be understood that each νth asymptotic term dominates in its asymptotic region Ω_ν. In the case of reaction (1) under consideration, $\alpha = A$, $\beta = p$, $\gamma = n$. $\Psi_0^{(+)}$ is the asymptotic component of $\Psi_\alpha^{(+)}$ in the asymptotic region Ω_0. We remind the reader again that in the three-body model $p + n + A$ the nucleus A is a structureless constituent particle; that is, all the channels related to the target excitation and target breakup are neglected.

Equation (2) is of fundamental importance because it provides a model-independent definition of the reaction amplitude $\mathcal{M}_{\nu\,\alpha}^{(\mathrm{as})}$ as the amplitude of the outgoing spherical wave in the final channel ν formed from the initial channel α for an arbitrary collision of composite nuclei. However, its practical implementation in the many-body case, except for three- and four-body systems, is hardly yet possible, because contemporary microscopic methods fail to provide the correct asymptotic behavior. That is why, if we are not going to use the Faddeev or Faddeev-Yakubovski [21] coupled equations, the conventional methods for the determination of reaction amplitudes is to calculate the volume matrix elements in the post or prior forms.

Below we remind the reader how to derive these matrix elements in the three-body model, which can be extended to a many-body system. Let us consider the three-body wave function $\Psi_\alpha^{(+)}$ containing the incident wave in the channel $\alpha + (\beta\,\gamma)$. It satisfies the Schrödinger equation

$$(E - H_\alpha - \overline{K}_\alpha - \overline{V}_\alpha)\, \Psi_\alpha^{(+)} = 0. \qquad (3)$$

Here $H_\alpha = K_\alpha + V_\alpha$ is the Hamiltonian describing the relative motion of the system $\beta + \gamma$, K_α is the kinetic-energy operator of the relative motion of β and γ, $\overline{K}_\alpha$ is the kinetic-energy operator of the relative motion of α and the center of mass of $\beta + \gamma$, $\overline{V}_\alpha = V - V_\alpha$, $V = V_\alpha + V_\beta + V_\gamma$ is the total interaction potential in the three-body system, V_α is the interaction potential between β and γ, $E = \overline{E}_\alpha - \varepsilon_\alpha$ is the total energy of three-body system, $\overline{E}_\alpha$ is the relative kinetic energy of the particle α and the pair $(\beta\,\gamma)$, $\varepsilon_\alpha = m_\beta + m_\gamma - m_{\beta\,\gamma}$ is the binding energy of the bound state $(\beta\,\gamma)$, and m_α is the mass of particle α.

Equation (3) can be rewritten in the channel $\beta \neq \alpha$ representation as

$$(E - H_\beta - \overline{K}_\beta - \overline{V}_\beta)\, \Psi_\alpha^{(+)} = 0. \qquad (4)$$

Note that, according to Eq. (2), $\Psi_\alpha^{(+)}$ has the incident wave only in the channel α. Now neglecting the Coulomb interaction for a while (this does not affect the final result), we introduce the channel wave function in the channel β,

$$\Phi_\beta^{(0)} = e^{i\,\mathbf{q}_\beta\cdot\boldsymbol{\rho}_\beta}\, \phi_\beta, \qquad (5)$$

where $\mathbf{q}_\beta$ is the relative momentum of particle β and the bound state $(\alpha\,\gamma)$, that is, the momentum conjugated to the Jacobian coordinate $\boldsymbol{\rho}_\beta$. Multiplying Eq. (4) from the left by the channel wave function $\Phi_\beta^{(0)}$ we get

$$\langle \Phi_\beta^{(0)} | (E - \overrightarrow{H}_\beta - \overrightarrow{\overline{K}}_\beta) | \Psi_\alpha^{(+)} \rangle = \langle \Phi_\beta^{(0)} | \overline{V}_\beta | \Psi_\alpha^{(+)} \rangle. \qquad (6)$$

Taking into account that

$$(E - H_\beta - \overline{K}_\beta)\, \Phi_\beta^{(0)} = 0, \qquad (7)$$

we can rewrite

$$\langle \Phi_\beta^{(0)} | (\overleftarrow{\overline{K}}_\beta - \overrightarrow{\overline{K}}_\beta) | \Psi_\alpha^{(+)} \rangle = \langle \Phi_\beta^{(0)} | \overline{V}_\beta | \Psi_\alpha^{(+)} \rangle = \mathcal{M}_{\beta\,\alpha}. \qquad (8)$$

Here we took into account that the right-hand side is the conventional reaction amplitude $\mathcal{M}_{\beta\,\alpha} = \langle \Phi_\beta^{(0)} | \overline{V}_\beta | \Psi_\alpha^{(+)} \rangle$. The operator $\overrightarrow{\overline{K}}_\beta$ ($\overleftarrow{\overline{K}}_\beta$) acts to the right (left). When deriving this equation we took into account that H_β is Hermitian if $(\alpha\gamma)$ is a bound state, that is,

$$\langle \Phi_\beta^{(0)} | (\overleftarrow{H}_\beta - \overrightarrow{H}_\beta) | \Psi_\alpha^{(+)} \rangle = \langle \Phi_\beta^{(0)} | (\overrightarrow{H}_\beta - \overrightarrow{H}_\beta) | \Psi_\alpha^{(+)} \rangle = 0. \qquad (9)$$

It follows from the fact that V_β is a Hermitian operator. Because $\Phi_\beta^{(0)}$ contains the bound state $(\alpha\,\gamma)$ we can take the integral over $\mathbf{r}_\beta$ by parts twice transforming $\overleftarrow{K}_\beta$ into $\overrightarrow{K}_\beta$. Hence, $H_\beta = K_\beta + V_\beta$ is also the Hermitian operator. This validates Eq. (9).

Now using the Green's theorem

$$L = \langle f(\mathbf{r}) | \overleftarrow{K} - \overrightarrow{K} | g(\mathbf{r}) \rangle = -\frac{1}{2\mu^2} \lim_{r\to\infty} r^2 \int d\hat{\mathbf{r}} \left[g(\mathbf{r}) \frac{\partial f^*(\mathbf{r})}{\partial r} - f^*(\mathbf{r}) \frac{\partial g(\mathbf{r})}{\partial r} \right], \qquad (10)$$

we can transform the volume integral on the left-hand side of Eq. (8) into a surface one in the subspace over $\boldsymbol{\rho}_\beta$:

$$\langle\Phi_\beta^{(0)}|(\overleftarrow{K}_\beta - \overrightarrow{K}_\beta)|\Psi_\alpha^{(+)}\rangle = -\frac{1}{2M_\beta^2}\lim_{\rho_\beta\to\infty}\rho_\beta^2\int d\,\mathbf{r}_\beta\,\phi_\beta^*\int d\hat{\boldsymbol{\rho}}_\beta \times\left[\Psi_\alpha^{(+)}\,\frac{\partial\, e^{-i\,\mathbf{q}_\beta\cdot\boldsymbol{\rho}_\beta}}{\partial\rho_\beta} - e^{-i\,\mathbf{q}_\beta\cdot\boldsymbol{\rho}_\beta}\,\frac{\partial\Psi_\alpha^{(+)}}{\partial\rho_\beta}\right]. \quad (11)$$

Taking into account that the leading asymptotic term of $\Psi_\alpha^{(+)}$ in Ω_β is [see Eq. (2)]

$$\Psi_\alpha^{(+)} \overset{\Omega_\beta}{\approx} -\frac{M_\beta}{2\pi}\,\mathcal{M}_{\beta\alpha}^{(\mathrm{as})}\,u^{(+)}(\rho_\beta)\,\phi_\beta \quad (12)$$

and using the asymptotic equation [22]

$$e^{i\mathbf{q}_\beta\cdot\boldsymbol{\rho}_\beta}\xrightarrow{\rho_\beta\to\infty}\frac{1}{2\pi q_\beta\rho_\beta}[e^{iq_\beta\rho_\beta}\delta(\hat{\mathbf{q}}_\beta-\hat{\boldsymbol{\rho}}_\beta) - e^{-iq_\beta\rho_\beta}\delta(\hat{\mathbf{q}}_\beta+\hat{\boldsymbol{\rho}}_\beta)], \quad (13)$$

and the normalization integral

$$\int d\mathbf{r}_\beta\,|\phi_\beta|^2 = 1, \quad (14)$$

we get

$$\langle\Phi_\beta^{(0)}|(\overleftarrow{K}_\beta - \overrightarrow{K}_\beta)|\Psi_\alpha^{(+)}\rangle = \mathcal{M}_{\beta\alpha}^{(\mathrm{as})}. \quad (15)$$

Hence,

$$\mathcal{M}_{\beta\alpha}^{(\mathrm{as})} = \mathcal{M}_{\beta\alpha}. \quad (16)$$

Thus, we have proven that the conventional reaction amplitude $\mathcal{M}_{\beta\alpha}$ given by the volume matrix element coincides with the amplitude $\mathcal{M}_{\beta\alpha}^{(\mathrm{as})}$ of the outgoing scattered wave in the channel β with the incident wave in the channel α. In the standard applications to decrease the transition operator, one can subtract the final-channel potential U_β in the matrix element on the right-hand side of Eq. (8), which leads to the final-channel wave function $\Phi_\beta^{(-)} = \chi_\beta^{(-)}\phi_\beta$, where $\chi_\beta^{(-)}$ is the distorted wave generated by the channel potential U_β and describing the scattering of particle β and the bound state $(\alpha\,\gamma)$. The channel potential is arbitrary and can be real or complex. From the derivation it is clear that the matrix element does not depend on the choice of U_β if $\Psi_\alpha^{(+)}$ is the exact three-body wave function. Then we have

$$\mathcal{M}_{\beta\alpha} = \langle\Phi_\beta^{(-)}|(\overleftarrow{K}_\beta - \overrightarrow{K}_\beta)|\Psi_\alpha^{(+)}\rangle = \langle\Phi_\beta^{(-)}|\overline{V}_\beta - U_\beta|\,\Psi_\alpha^{(+)}\rangle. \quad (17)$$

After introducing the distorted wave in the channel β, we can turn on the Coulomb interaction.

Now let us discuss the lessons which we can learn from derivation of Eq. (17).

(i) This equation proves that, indeed, the volume matrix element, which is used in standard calculations of the reaction amplitude $\mathcal{M}_{\beta\alpha}$, is, in fact, the amplitude $\mathcal{M}_{\beta\alpha}^{(\mathrm{as})}$ of the leading asymptotic term of the exact three-body scattering wave function in the asymptotic domain Ω_β.

(ii) Equation (4) is important for deriving Eq. (17). The former shows that the exact scattering wave function Ψ_α also satisfies the Schrödinger equation in the channel β representation; that is, it has correct asymptotic behavior in the channel $\beta\neq\alpha$. The corresponding integral equation for Ψ_α will be homogeneous in the $\beta\neq\alpha$ channel.

(iii) There is a clear advantage of using the volume matrix element rather than to calculate the amplitude of the asymptotic scattering wave function in the corresponding asymptotic domain. Because $\phi_\beta = \varphi_{\alpha\,\gamma}$ is the bound-state wave function of the pair $(\alpha\,\gamma)$, the integration over the Jacobian coordinate r_β is limited. The transition operator $\overline{V}_\beta - U_\beta$, where $\overline{V}_\beta = V_\alpha + V_\gamma$, $V_\alpha \equiv V_{\beta\,\gamma}$, and $V_\gamma \equiv V_{\alpha\,\beta}$, cuts the integration over the second Jacobian variable ρ_β at some finite value. Hence, it is sufficient to know the scattering wave function $\Psi_\alpha^{(+)}$, developing from the initial state $\Psi_\alpha^{(0)}$, only in the constrained domain in the coordinate space $\{\mathbf{r}_\beta,\boldsymbol{\rho}_\beta\}$ around target nucleus α.

Let us introduce R_β as a quantity larger than the nuclear interaction radius R_β^N in the two-body subsystem $(\alpha\gamma)$ and $\overline{R}_\beta$ to be a quantity larger than nuclear interaction radius in the two-cluster channel β. These are the values which should be taken into account in the volume matrix element to achieve the required accuracy, which is typically $\sim$1% or better.

It is worth mentioning that R_β may be taken significantly larger than the nuclear interaction radius R_β^N of particles α and γ. Clearly, R_β should be larger than $1/\kappa_\beta$, where $\kappa_\beta \equiv \kappa_{\alpha\,\gamma}$ is the bound-state wave number of the bound state $(\alpha\gamma)$. We define a hyperradius corresponding to $\{R_\beta, \overline{R}_\beta\}$ as $X_0 = (\mu_\beta\,R_\beta^2/m + M_\beta\,\overline{R}_\beta^2/m)^{1/2}$. With this we can rewrite Eq. (17) as

$$\begin{aligned}\mathcal{M}_{\beta\alpha} &\approx \langle\Phi_\beta^{(-)}|\overline{V}_\beta - U_\beta|\,\Psi_\alpha^{(+)}\rangle|_{X_\beta\leqslant X_0}\\ &= \langle\Phi_\beta^{(-)}|(\overleftarrow{K}_\beta - \overrightarrow{K}_\beta)|\Psi_\alpha^{(+)}\rangle|_{X_\beta\leqslant X_0}\\ &= -\frac{\overline{R}_\beta^2}{2M_\beta}\int_{r_\beta\leqslant R_\beta} d\mathbf{r}_\beta\,\phi_\beta^*\int d\,\Omega_{\boldsymbol{\rho}_\beta}\\ &\quad\times\left[\Psi_\alpha^{(+)}\,\frac{\partial\chi_\beta^{(-)*}}{\partial\rho_\beta} - \chi_\beta^{(-)*}\,\frac{\partial\Psi_\alpha^{(+)}}{\partial\rho_\beta}\right]\Bigg|_{\rho_\beta=\overline{R}_\beta}. \end{aligned} \quad (18)$$

Equation (18) is our first main result in this section. It shows that in the three-body method the volume matrix element can be transformed into the peripheral matrix element. The surface integral over $\Omega_{\boldsymbol{\rho}_\beta}$ in Eq. (18) is taken along the sphere with the radius $\rho_\beta = \overline{R}_\beta$ encircling the finite volume inside of this sphere, while the integral over $\mathbf{r}_\beta$ is taken over the volume confined by the sphere with the radius $r_\beta = R_\beta$. If we take the limit $\overline{R}_\beta\to\infty$, we get identity, $\mathcal{M}_{\beta\alpha}\equiv\mathcal{M}_{\beta\alpha}^{(\mathrm{as})}$. However, in practical calculations we can constrain the integration region by a finite $\overline{R}_\beta$; that is, we need to know the wave function $\Psi_\alpha^{(+)}$ only in a limited volume around the target. The value of $\overline{R}_\beta$

can be determined by checking the convergence of the matrix element as function of $\overline{R}_\beta$. If $\overline{R}_\beta$ is not too large, then we do not need to know the asymptotic behavior of $\Psi_\alpha^{(+)}$. Although Eq. (18) has been derived in a three-body model, the derivation is valid also for a many-body case assuming that $\Psi_\alpha^{(+)}$ is the exact many-body scattering wave function with the incident wave in the channel $\alpha + (\beta\gamma)$.

Note that in the prior formalism the stripping reaction matrix element is given by

$$\begin{aligned}\mathcal{M}_{\beta\alpha} &\approx \langle\Psi_\beta^{(-)}|\overline{V}_\alpha - U_\alpha|\,\Phi_\alpha^{(+)}\rangle|_{X_\alpha\leqslant X_0}\\ &= \langle\Psi_\beta^{(-)}|(\overleftarrow{K}_\alpha - \overrightarrow{K}_\alpha)|\Phi_\alpha^{(+)}\rangle|_{X_\alpha\leqslant X_0}\\ &= -\frac{\overline{R}_\alpha^2}{2\,M_\alpha}\int_{r_\alpha\leqslant R_\alpha} d\mathbf{r}_\alpha\,\phi_\alpha\int d\,\Omega_{\boldsymbol{\rho}_\alpha}\\ &\quad\times\left[\chi_\alpha^{(+)}\frac{\partial\Psi_\beta^{(-)*}}{\partial\rho_\alpha} - \Psi_\beta^{(-)*}\frac{\partial\chi_\alpha^{(+)}}{\partial\rho_\alpha}\right]\Bigg|_{\rho_\alpha=\overline{R}_\alpha},\end{aligned}\tag{19}$$

where $\phi_\alpha = \varphi_{\beta\,\gamma}$. Though the post and prior forms are identical, there are computational advantages in using specific form depending on the reaction under consideration. We address it below.

The fact that the integration volume is constrained is quite important because it justifies the usage of the different approximations for the exact scattering wave function, which are valid in the limited space around nucleus even if these approximations do not provide wave functions with correct asymptotic behavior in the rearrangement channels. Such approximations are well known: DWBA, CDCC [2,23–25], and adiabatic method (ADWA) [26]. In the DWBA the initial scattering wave function contains only the contribution from the incident channel $\alpha + (\beta\,\gamma)$. In the CDCC method the initial wave function is contributed by the channel $\alpha + (\beta\,\gamma)$, in which the pair $(\beta\,\gamma)$ is taken in the bound state plus discretized states describing the three-body system $\alpha + \beta + \gamma$ in the continuum. The adiabatic approach, as does the CDCC method, also takes into account the continuum states of the $(\beta\,\gamma)$ system but in a more simplified way. All three methods fail to provide correct asymptotic behavior in the rearrangement channels. Nevertheless, all three methods, being not perfect, still give reasonable transfer reaction cross sections. The accuracy of the each method depends on the kinematics, energy, interacting nuclei, and purposes. When the energy increases, the contribution of the deuteron breakup channel also increases, making the ADWA and CDCC more adequate than the DWBA. In addition, this creates another problem to be dealt with: the increase of the contribution from the nuclear interior. In the internal region a strong coupling of different channels occurs and antisymmetrization effects are important. Meanwhile, the existing approaches, DWBA, ADWA, and CDCC, are based on the three-body model extended by adopting optical potentials and they are designed to treat mostly peripheral reactions. The surface-integral formalism developed here in the combination with the R-matrix method can provide a solution.

Finally, one important feature of Eq. (18) remains to be discussed. Assume that we use the CDCC wave function to calculate $\Psi_\alpha^{(+)}$. In the CDCC method particles β and γ are kept close to each other by using the projection operator, which truncates the number of the allowed β-γ partial waves. At the same time the surface integral over $\Omega_{\boldsymbol{\rho}_\beta}$ is calculated at $\rho_\beta = \overline{R}_\beta$. As $\overline{R}_\beta$ can be significantly larger than the nucleus radius, the dominant contribution to the volume integral over r_β should come from $R_\beta^N \leqslant r_\beta \leqslant R_\beta$. Hence, the reaction amplitude given by Eq. (18) is entirely peripheral in the subspace over $\mathbf{r}_\beta$ and $\boldsymbol{\rho}_\beta$ and can be rewritten as

$$\begin{aligned}\mathcal{M}_{\beta\alpha} = -\frac{\overline{R}_\beta^2}{2\,M_\beta}\int_{R_\beta^N\leqslant r_\beta\leqslant R_\beta} d\mathbf{r}_\beta\,\phi_\beta^*\int d\,\Omega_{\boldsymbol{\rho}_\beta}\\ \times\left[\Psi_\alpha^{\mathrm{CDCC}(+)}\frac{\partial\chi_\beta^{(-)*}}{\partial\rho_\beta} - \chi_\beta^{(-)*}\frac{\partial\Psi_\alpha^{\mathrm{CDCC}(+)}}{\partial\rho_\beta}\right]\Bigg|_{\rho_\beta=\overline{R}_\beta},\end{aligned}\tag{20}$$

where $\phi_\beta(r_\beta) \approx C_\beta\,W_{-\eta_\beta,l_\beta+1/2}(2\,\kappa_\beta\,r_\beta)/r_\beta$ is the radial part of the bound-state wave function, C_β is the asymptotic normalization coefficient (ANC) of the bound state $(\alpha\gamma)$, $W_{-\eta_\beta,l_\beta+1/2}(2\,\kappa_\beta\,r_\beta)$ is the Whittaker function, η_β is the Coulomb parameter, and $l_\beta \equiv l_{\alpha\gamma}$ is the orbital angular momentum of the bound state $(\alpha\gamma)$. In the many-body case ϕ_β should be replaced by the corresponding overlap function. Transition from the three-body model to the CDCC method requires using of the optical potentials, which effectively take into account the internal structure of the target.

III. DEUTERON STRIPPING TO A BOUND STATE: FROM MANY-BODY TO THREE-BODY MODEL

Post form

In the previous section we considered the deuteron-stripping reaction in the three-body problem; that is, all three particles, p, n, and A are structureless constituents. Hence, all the interaction potentials are real. Definitely internal degrees of freedom of the target should be taken into account. However, a rigorous practical many-body theory of transfer reactions is not yet available and contemporary nuclear reaction theory uses the three-body model in which the internal structure of the target is taken into account effectively by replacing N-A optical potentials.

Here we consider this reduction of the many-body problem to the three-body one and apply the surface-integral formalism developed in the previous section specifically for the deuteron-stripping reaction. We neglect the antisymmetrization between the existing proton and the rest of the nucleons in the target A. To derive an equation for the reaction amplitude, we start from the Schrödinger equation for the total scattering wave function $\Psi_i^{(+)}$ developing from the initial channel,

$$(E - K_{pF} - K_{nA} - V_{nA} - V_{pA} - V_{pn} - H_A)\,\Psi_i^{(+)} = 0,\tag{21}$$

where V_{nA} (V_{pA}) is the n-A (p-A) interaction potential given by the sum of NN potentials (three-body forces can also be included), H_A is the internal Hamiltonian of nucleus A. $\Psi_i^{(+)}$ has the incident wave in the initial channel $d + A$ and outgoing waves in both direct and rearrangement channels.

Standard choice of the exit-channel wave function

To proceed further, we need to adopt a suitable form of the final-channel wave function. Here we show how to derive and transform the stripping reaction amplitude in the case when the exit-channel wave function is taken in the standard form

$$\Phi_f^{(-)} = \chi_{pF}^{(-)}\,\varphi_F, \tag{22}$$

where $\chi_{pF}^{(-)}$ is the distorted wave of particles p and F in the final channel and φ_F is the bound-state wave function of nucleus F in the final channel. The wave function $\Phi_f^{(-)}$ is a solution of the Schrödinger equation

$$(E - K_{pF} - U_{pF} - K_{nA} - V_{nA} - H_A)\,\Phi_f^{(-)} = 0. \tag{23}$$

Multiplying Eq. (21) from the left by $\Phi_f^{(-)*}$ and taking into account Eq. (23), we get

$$\begin{aligned}
&\langle\Phi_f^{(-)}|E - \overrightarrow{K}_{pF} - \overrightarrow{K}_{nA} - V_{nA} - \overrightarrow{H}_A \\
&\quad -[V_{pA} + V_{pn} - U_{pF}] - U_{pF}|\Psi_i^{(+)}\rangle \\
&= \langle\Phi_f^{(-)}|\overleftarrow{K}_{pF} - \overrightarrow{K}_{pF} + \overleftarrow{K}_{nA} - \overrightarrow{K}_{nA} \\
&\quad + \overleftarrow{H}_A - \overrightarrow{H}_A - [V_{pA} + V_{pn} - U_{pF}]|\Psi_i^{(+)}\rangle \\
&= \langle\Phi_f^{(-)}|\overleftarrow{K}_{pF} - \overrightarrow{K}_{pF} - [V_{pA} + V_{pn} - U_{pF}]|\Psi_i^{(+)}\rangle = 0.
\end{aligned} \tag{24}$$

When deriving this equation we took into account that the operators H_A and K_{nA} are Hermitian because the final-channel wave function contains the bound state $F = (nA)$. Hence, $\langle\Phi_f^{(-)}|\overleftarrow{K}_{nA} + \overleftarrow{H}_A - \overrightarrow{K}_{nA} - \overrightarrow{H}_A|\Psi_i^{(+)}\rangle = 0$. We can rewrite Eq. (24) as

$$\mathcal{M}^{(\mathrm{as})} = \langle\Phi_f^{(-)}|\overleftarrow{K}_{pF} - \overrightarrow{K}_{pF}|\Psi_i^{(+)}\rangle \tag{25}$$

$$= \langle\Phi_f^{(-)}|V_{pA} + V_{pn} - U_{pF}|\Psi_i^{(+)}\rangle \equiv \mathcal{M}^{(\mathrm{post})}. \tag{26}$$

We can verify that the matrix element $\langle\Phi_f^{(-)}|\overleftarrow{K}_{pF} - \overrightarrow{K}_{pF}|\Psi_i^{(+)}\rangle$ is equal to the amplitude $\mathcal{M}^{(\mathrm{as})}$ of the leading asymptotic term of the exact $d + A$ scattering wave function $\Psi_i^{(+)}$ in the channel $p + F$. It can be proved by converting matrix element (25) into a surface integral in the subspace over ρ_{pF}. After taking the limit of the radius of the surface $\rho_{pF} \to \infty$ we get that the matrix element is nothing but the reaction amplitude $\mathcal{M}^{(\mathrm{as})}$ [3,4]. This amplitude is the model-independent definition of the reaction amplitude. Thus, it follows from Eq. (26) that the conventional reaction amplitude given by the volume matrix element $\mathcal{M}^{\mathrm{post}}$ is equal to $\mathcal{M}^{(\mathrm{as})}$. In Eq. (26) the internal degrees of freedom of the target A are taken into account properly. However, the exact many-body scattering wave function is not yet available and at this stage approximations are supposed to be used.

First we use the fact that, owing to the presence of the factor $\varphi_F\,[V_{pA} + V_{pn} - U_{pF}]$, the integration can be carried over a finite volume in the six-dimensional configuration space $\{\boldsymbol{\rho}_{pF}, \mathbf{r}_{nA}\}$, where we do not need to know the asymptotic behavior of the scattering wave function $\Psi_i^{(+)}$. The presence of the factor $\varphi_F\,[V_{pA} + V_{pn} - U_{pF}]$ in the matrix element constrains the integration over the Jacobian variables by a finite volume around the target nucleus. Clearly φ_F cuts the integration over the internal nucleon coordinates including the coordinates of the transferred neutron. We introduce $\mathcal{R}_{nA}$ as the maximal r_{nA}, which is required to achieve a desired accuracy for the integral over r_{nA}. We also introduce $\mathcal{R}_{pF}$ as the maximal ρ_{pF}, which is required to achieve a desired accuracy for the integral over ρ_{pF}. If R_{nA} is the channel radius for which we can use the radius of the strong n-A interaction, then $\mathcal{R}_{nA} > R_{nA}$ and may be significantly larger for loosely bound states. At some large-enough ρ_{pF} and finite $r_{nA} \leqslant \mathcal{R}_{nA}$ the nuclear part $V_{pA}^N + V_{pn} - U_{pF}^N$ of the transition operator becomes negligible.

Now we consider the matrix element $\langle\chi_f^{(-)}\varphi_F|V_{pA}^C - U_{pF}^C|\Psi_i^{(+)}\rangle$ from the Coulomb part of the transition operator. At $r_{pA} \gg R_A$, where R_A is the radius of nucleus A, we can approximate in the leading order $V_{pA}^C(r_{pA}) \approx U_{pA}^C(r_{pA}) = Z_A\,e^2/r_{pA}$, while $U_{pF}^C(\rho_{pF}) = Z_A\,e^2/\rho_{pF}$, where Z_A is the charge of nucleus A. Taking into account that

$$\boldsymbol{\rho}_{pF} = \mathbf{r}_{pA} - \frac{1}{A+1}\,\mathbf{r}_{nA}, \tag{27}$$

we get for $r_{pA} \gg r_{nA}$

$$U_{pF}^C(\rho_{pF}) - U_{pA}(r_{pA}) \overset{r_{pA}\gg r_{nA}}{\approx} \frac{Z_A\,e^2}{r_{pA}}\,\frac{1}{A+1}\,\frac{\hat{\mathbf{r}}_{pA}\cdot\mathbf{r}_{nA}}{r_{pA}}, \tag{28}$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$ and A also represents the total number of nucleons in nucleus A. Hence, at large-enough r_{pA} the difference in the Coulomb potential becomes negligible; that is, the integration volume in the matrix element $\langle\chi_f^{(-)}\varphi_F|\,U_{pA}^C - U_{pF}^C\,|\Psi_i^{(+)}\rangle$ is also limited. Then we can rewrite

$$\mathcal{M}^{(\mathrm{post})} = \langle\Phi_f^{(-)}|V_{pA} + V_{pn} - U_{pF}|\Psi_i^{(+)}\rangle|_{X\leqslant X_0} \tag{29}$$

$$= \langle\Phi_f^{(-)}|\overleftarrow{K}_{pF} - \overrightarrow{K}_{pF}|\Psi_i^{(+)}\rangle|_{X\leqslant X_0}, \tag{30}$$

where the hyperradius is defined as

$$X = \sqrt{\frac{\mu_{nA}}{m}\,r_{nA}^2 + \frac{\mu_{pF}}{m}\,\rho_{pF}^2} \tag{31}$$

and

$$X_0 = \sqrt{\frac{\mu_{nA}}{m}\,\mathcal{R}_{nA}^2 + \frac{\mu_{pF}}{m}\,\mathcal{R}_{pF}^2}, \tag{32}$$

where m is the nucleon mass and μ_{ij} is the reduced mass of particles i and j.

Transforming now the matrix element containing the kinetic-energy operators into a surface integral in the subspace over ρ_{pF}, we get

$$\begin{aligned}
\mathcal{M}^{(\mathrm{post})} = &-\frac{\mathcal{R}_{pF}^2}{2\,\mu_{pF}}\int d\,\zeta_F\,\varphi_F^*(\zeta_F)\int d\,\Omega_{\boldsymbol{\rho}_{pF}} \\
&\times\left[\Psi_i^{(+)}\,\frac{\partial\,\chi_{pF}^{(-)*}(\boldsymbol{\rho}_{pF})}{\partial\,\rho_{pF}}\right. \\
&\left. - \chi_{pF}^{(-)*}(\boldsymbol{\rho}_{pF})\,\frac{\partial\,\Psi_i^{(+)}}{\partial\,\rho_{pF}}\right]\Bigg|_{\rho_{pF}=\mathcal{R}_{pF};\,r_{nA}\leqslant\mathcal{R}_{nA}}.
\end{aligned} \tag{33}$$

Here the surface integral is taken over the sphere with the radius $\rho_{pF} = \mathcal{R}_{pF}$, while the volume integral is taken over the set ζ_F of the internal coordinates of nucleus F subject to a condition that the coordinate r_{nA} is constrained by $r_{nA} \leqslant \mathcal{R}_{nA}$. Thus, the stripping matrix element is contributed by the finite volume in the space $\{\boldsymbol{\rho}_{pF}, \mathbf{r}_{nA}\}$. This important fact paves the way for different approximations used in the contemporary nuclear reaction theory, because within this finite volume the exact initial scattering wave function $\Psi_i^{(+)}$ can be approximated by wave functions, which do not have correct asymptotic behavior in the rearrangement channel $p + F$. Nevertheless, they approximate this wave function in the finite volume fairly enough, at least in the three-body approach. Such approximations are well known: the initial channel wave function $\chi_{dA}^{(+)} \varphi_{pn} \varphi_A$ used in the DWBA, the CDCC wave function $\Psi_i^{\mathrm{CDCC}(+)} \varphi_A$, or the adiabatic model wave function $\Psi_i^{\mathrm{AD}(+)} \varphi_A$. Note that all the three approaches are based on the three-body model, in which the target A is treated as structureless constituent particle. That is why in each approach the scattering wave function contains the target bound-state wave function φ_A in a factorized form. The composite structure of the target is taken into account effectively via the optical potentials.

Equation (29) is exact if the antisymmetrization effects are neglected. Assume now that in the integration region the wave function $\Psi_i^{(+)}$ can be approximated by the wave functions used in the DWBA, CDCC [2], or ADWA [26] methods. Usually such an approximation is done in the volume matrix element (29). Here we apply it after transforming the volume matrix element into the surface integral over ρ_{pF}, keeping the volume integral over ζ_F. This is the main difference between the standard approach and the one we use here. The replacement of $\Psi_i^{(+)}$ by the CDCC wave function, which is the most advanced among the three above-mentioned methods, leads to the following CDCC reaction amplitude in the surface approximation:

$$\mathcal{M}_{\mathrm{surf}}^{\mathrm{CDCC(post)}} = -\frac{\mathcal{R}_{pF}^2}{2\,\mu_{pF}} \int d\,\mathbf{r}_{nA}\, I_A^{F*}(\mathbf{r}_{nA}) \int d\,\Omega_{\boldsymbol{\rho}_{pF}} \times \left[\Psi_i^{\mathrm{CDCC}(+)} \frac{\partial\, \chi_{pF}^{(-)*}(\boldsymbol{\rho}_{pF})}{\partial\, \rho_{pF}} - \chi_{pF}^{(-)*}(\boldsymbol{\rho}_{pF}) \frac{\partial\, \Psi_i^{\mathrm{CDCC}(+)}}{\partial\, \rho_{pF}} \right]\Bigg|_{\rho_{pF}=\mathcal{R}_{pF};\, r_{nA}\leqslant\mathcal{R}_{nA}} . \tag{34}$$

Here $I_A^F(\mathbf{r}_{nA}) = \langle \varphi_A | \varphi_F \rangle$ is the overlap function of the bound-state wave functions of nuclei F and A. We remind the reader that here we neglected antisymmetrization effects. Thus, starting from the exact volume matrix element, we transformed it into the surface integral over ρ_{pF} leaving the volume integral over the second Jacobian variable $\mathbf{r}_{nA}$. After that, the exact scattering wave function was replaced by the CDCC one reducing the exact amplitude $\mathcal{M}^{(\mathrm{post})}$ in the surface-integral representation to the CDCC amplitude $\mathcal{M}_{\mathrm{surf}}^{\mathrm{CDCC(post)}}$ also in the surface-integral form.

Now the question is how this $\mathcal{M}_{\mathrm{surf}}^{\mathrm{CDCC(post)}}$ amplitude in the surface-integral form is related to the conventional CDCC amplitude given by the volume matrix element? Note that the conventional CDCC amplitude

$$\mathcal{M}_{\mathrm{conv}}^{\mathrm{CDCC(post)}} = \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| U_{pA} + V_{pn} - U_{pF} \middle| \Psi_i^{\mathrm{CDCC}(+)} \right\rangle\Big|_{X\leqslant X_0} \tag{35}$$

is also obtained from the exact matrix element (26) by approximating $\Psi_i^{(+)} \to \Psi_i^{\mathrm{CDCC}(+)}$ and $V_{PA} \to U_{pA}$. To answer this question, we transform Eq. (34) back to the volume integral. To do it we replace the surface integral by the volume integral in which the transition operator is given by the difference of the kinetic-energy operators $\overleftarrow{K} - \overrightarrow{K}$:

$$\mathcal{M}_{\mathrm{surf}}^{\mathrm{CDCC(post)}} = \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| \overleftarrow{K}_{pF} - \overrightarrow{K}_{pF} \middle| \Psi_i^{\mathrm{CDCC}(+)} \right\rangle\Big|_{X\leqslant X_0} \tag{36}$$

$$= \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| \overleftarrow{K} - \overrightarrow{K} \middle| \Psi_i^{\mathrm{CDCC}(+)} \right\rangle\Big|_{X\leqslant X_0} \tag{37}$$

$$= \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| U_{pA} + U_{nA} + V_{pn} - V_{nA}^{\mathrm{sp}} - U_{pF} \middle| \Psi_i^{\mathrm{CDCC}(+)} \right\rangle\Big|_{X\leqslant X_0} . \tag{38}$$

Here, to get Eq. (37) from Eq. (36), we took into account that the matrix element $\overleftarrow{K}_{nA} - \overrightarrow{K}_{nA}$ vanishes because, after two integrations by parts over r_{nA}, the surface integral at $r_{nA} \to \infty$ disappears owing to the presence of the overlap function I_A^F, and $\overleftarrow{K}_{nA}$ can be converted into $\overrightarrow{K}_{nA}$. Note that although the integration over r_{nA} is restricted by $r_{nA} \leqslant \mathcal{R}_{nA}$, we can extend it to infinity to make the matrix element from $\overleftarrow{K}_{nA} - \overrightarrow{K}_{nA}$ vanish. To get Eq. (38) we took into account that the CDCC wave function is the solution of the Schrödinger equation

$$(E - T - U_{pA} - U_{nA} - V_{pn})\Psi_i^{\mathrm{CDCC}(+)} = 0. \tag{39}$$

Note that often the truncation of the relative orbital angular momentum l_{pn} is used in the CDCC approach [2], which works as an additional suppression of the rearrangement channels [23] to the optical potentials U_{pA} and U_{nA}. This truncation is achieved by using the projector

$$\hat{P}_{pn} = \sum_{l_{pn}=0}^{l_{pn}^{\max}} \sum_{m_{l_{pn}}=-l_{pn}}^{l_{pn}} \int d\Omega_{\mathbf{r}_{pn}}\, Y_{l_{pn}\, m_{l_{pn}}}(\hat{\mathbf{r}}_{pn})\, Y_{l_{pn}\, m_{l_{pn}}}^*(\hat{\mathbf{r}}'_{pn}). \tag{40}$$

Suppression of the rearrangement channels is required to provide a unique solution of the CDCC Schrödinger equation (39). It has been shown in Ref. [23] that the suppression of the rearrangement channels by the optical potentials is stronger than by the projection operator $\hat{P}_{pn}$ and, *a priori*, there is no need to introduce the projector $\hat{P}_{pn}$ if two optical potentials U_{pA} and U_{nA} are being used. However, the constraint over l_{pn} can always be added if needed.

We have assumed also that the overlap function I_A^F is proportional to the single-particle bound-state wave function at all r_{nA}. Then $\chi_{pF}^{(-)}\, I_A^F$ satisfies the Schrödinger

equation

$$\left(E - K - V_{nA}^{\rm sp} - U_{pF}^{*}\right) \chi_{pF}^{(-)}\, I_A^F = 0, \tag{41}$$

where $V_{nA}^{\rm sp} = \langle \varphi_A | V_{nA} | \varphi_A \rangle$ is the single-particle n-A potential supporting the bound state.

There is an important point to be discussed here. The integration in Eq. (38) is taken at fixed $\rho_{pF} = \mathcal{R}_{pF}$ and $r_{nA} \leqslant \mathcal{R}_{nA}$, meaning that the integration over r_{pA} is also constrained. These constraints follow from the ones in the original matrix element (33). Replacing the exact scattering wave function $\Psi_i^{(+)}$ by $\Psi_i^{\rm CDCC(+)}\varphi_A$ in Eq. (33) we still keep the constraints of the integration region as in the original matrix element. This is because the CDCC method is valid only in the limited hypervolume with $X \leqslant X_0$, where the asymptotic regime of $\Psi_i^{(+)}$ has not yet been reached. Within this volume, the CDCC wave function is supposed to be a reasonable approximation to the exact one.

Note that the transition operator in Eq. (38) differs from the one in the conventional CDCC amplitude (35) and the difference is attributable to the additional transition operator $U_{nA} - V_{nA}^{\rm sp}$. The appearance of this additional transition operator is the price we pay for using energy-independent potentials. Because of the importance of this issue we would like to trace the appearance of this additional transition operator. First we should look back at the derivation of the exact matrix element (26). In this equation the potential describing n-A scattering in the initial state is real and coincides with the potential supporting $(n\,A)$ bound state. Hence, these potentials cancel each other out. However, after we replace the exact scattering wave function with the wave function $\Psi_i^{\rm CDCC(+)}\varphi_A$, the initial n-A potential becomes complex while the final-state n-A potential is the real mean-field neutron potential supporting the bound state. Replacing the n-A potential in the initial state with the energy-dependent one makes the problem of solving the CDCC equations difficult and impractical. That is why in practical applications the adopted initial n-A potential is a complex local energy-independent one. The conventional CDCC amplitude (35) can be derived from Eq. (26) by using the substitution $\Psi_i^{(+)} \to \Psi_i^{\rm CDCC(+)}\varphi_A$, where the CDCC wave function satisfies Eq. (39), and $V_{pA} + V_{pn} - U_{PF}$ by $U_{pA} + V_{pn} - U_{pF}$. However, a different expression for the CDCC amplitude can be obtained if we start its derivation from the equation

$$\mathcal{M}^{\rm (post)} = \langle \Phi_f^{(-)} | V_{pA} + V_{pn} + V_{nA} - V_{nA} - U_{pF} | \Psi_i^{(+)} \rangle |_{X \leqslant X_0}, \tag{42}$$

which is identical to Eq. (26) but in which we have not yet canceled out V_{nA} potentials. The potential $(+V_{nA})$ in the transition operator comes from the Schrödinger equation for $\Psi_i^{(+)}$ and $(-V_{nA})$ from the Schrödinger equation for $\Phi_f^{(-)}$. If we use the substitutions $\Psi_i^{(+)} \to \Psi_i^{\rm CDCC(+)}\varphi_A$, $V_{pA} \to U_{pA}$ and $(+V_{nA}) \to (+U_{nA})$ in Eq. (42), we get Eq. (38), which can be transformed to the surface integral over $\boldsymbol{\rho}_{pF}$ rather than the conventional one given by Eq. (35).

Now we can rewrite

$$\mathcal{M}_{\rm conv}^{\rm CDCC(post)} = \mathcal{M}_{\rm surf}^{\rm CDCC(post)} - \mathcal{M}_{\rm aux}^{\rm CDCC(post)}, \tag{43}$$

where $\mathcal{M}_{\rm conv}^{\rm CDCC(post)}$ is the conventional CDCC stripping amplitude given by Eq. (35) and

$$\mathcal{M}_{\rm aux}^{\rm CDCC(post)} = \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| U_{nA} - V_{nA}^{\rm sp} \middle| \Psi_i^{\rm CDCC(+)} \right\rangle \Big|_{\rho_{pF} \leqslant \mathcal{R}_{pF};\, r_{nA} \leqslant R_{nA}} \tag{44}$$

$$= i \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| \mathrm{Im}\, U_{nA} \middle| \Psi_i^{\rm CDCC(+)} \right\rangle \Big|_{\rho_{pF} \leqslant \mathcal{R}_{pF};\, r \leqslant R_{nA}} \tag{45}$$

is the auxiliary amplitude. Equation (45) follows from Eq. (44) assuming that $\mathrm{Re}U_{nA} = V_{nA}^{\rm sp}$. Thus, there is an ambiguity in the definition of the CDCC amplitude. If we replace the exact scattering wave function with the CDCC one in the volume matrix element (29) we obtain the conventional CDCC reaction amplitude (35). However, if we approximate the exact scattering wave function by the CDCC one in the surface-integral matrix element (34), we obtain the amplitude in the surface-integral formalism $\mathcal{M}_{\rm surf}^{\rm CDCC(post)}$, which differs from the conventional reaction amplitude $\mathcal{M}_{\rm conv}^{\rm CDCC(post)}$ by the auxiliary matrix element $\mathcal{M}_{\rm aux}^{\rm CDCC(post)}$; see Eq. (43).

The ambiguity in the definition of the CDCC amplitude is related with the matrix element taken from the transition operator $U_{nA} - V_{nA}^{\rm sp}$. The source of this ambiguity is the inconsistency in the treatment of the n-A potentials when the many-body problem is reduced to the three-body one: To describe the n-A interaction in the initial state, the optical U_{nA} is used while the real potential V_{nA} is adopted for describing the bound state $(n\,A)$ (see the Appendix, where we discuss how the inconsistency in the treatment of the n-A potential affects even the DWBA, which is more simpler than the CDCC). This inconsistency remains an open question in the contemporary nuclear reaction theory if we use energy-independent n-A potentials when reducing the many-body problem to the three-body one. A similar problem appears in the treatment of the deuteron-stripping reactions using the Faddeev formalism in the momentum space, in which the integration over the energy requires energy-dependent nucleon-target optical potentials. These potentials should provide scattering phase shifts at positive n-A relative energies and possible bound states at negative relative energies.

The replacement of the exact scattering wave function by the CDCC one is more accurate when it is done in the volume matrix element rather then in the surface one. The volume matrix element is contributed by the internal and peripheral (over the variable r_{nA}) parts. While at low energy the external part dominates, with energy increase the role of the internal part also increases. Meanwhile, the surface matrix element is mostly peripheral. It is evident from the following consideration. For large $\rho_{pF} \sim 30$ fm and small nonlocality $|\mathcal{R}_{pF} - \mathcal{R}_{dA}|$ of the post form (see calculations in Sec. V) ρ_{dA} is also large. Even if the initial CDCC wave function contains the p-n pair in the continuum, the constraint over l_{pn} constrains also the distance r_{pn}. Hence, large r_{nA} becomes dominant in the surface matrix element. Meanwhile, the auxiliary matrix element is entirely contributed by the internal region because of the presence of $\mathrm{Im}U_{nA}$. Thus, the conventional amplitude is contributed by the internal auxiliary

amplitude and mostly peripheral surface matrix element. Thus, we suggest to use Eq. (43) as the post CDCC amplitude, which can be expressed in terms of the predominantly peripheral surface matrix element and the auxiliary amplitude.

As we have underscored, the constraint $X \leqslant X_0$ in the integration in the matrix elements in Eq. (43) comes from the constraint in the exact matrix element (42). The integrand in $\mathcal{M}_{\rm aux}^{\rm CDCC(post)}$, which contains the transition operator $\mathrm{Im}\, U_{nA}$, does not restrict the integration over ρ_{pF}, and the constraint $X \leqslant X_0$ comes only from the original matrix element (42). That is why the amplitude $\mathcal{M}_{\rm aux}^{\rm CDCC(post)}$ may depend on the choice of X_0. For peripheral reactions the internal contribution in the post form is small and $\mathcal{M}_{\rm aux}^{\rm CDCC(post)}$ is also small compared to $\mathcal{M}_{\rm conv}^{\rm CDCC(post)}$ because the depth of $\mathrm{Im}\, U_{nA}$ is significantly smaller than the depth of the real part of the transition operator in $\mathcal{M}_{\rm conv}^{\rm CDCC(post)}$, which is $\sim V_{pn}$. Then the conventional CDCC amplitude $\mathcal{M}_{\rm conv}^{\rm CDCC(post)}$ is close to the surface CDCC amplitude $\mathcal{M}_{\rm surf}^{\rm CDCC(post)}$.

Note that if we use the CDCC wave function satisfying the Schrödinger equation [27]

$$\left(E - T - U_{pA} - V_{nA}^{\rm sp} - V_{pn}\right) \Psi_i^{\rm CDCC(+)} = 0, \tag{46}$$

where the real $V_{nA}^{\rm sp}$ is being used rather than the optical potential U_{nA}, then

$$\mathcal{M}_{\rm surf}^{\rm CDCC(post)} = \mathcal{M}_{\rm conv}^{\rm CDCC(post)} = \left\langle \chi_{pF}^{(-)}\, I_A^F \middle| U_{pA} + V_{pn} - U_{pF} \times \left| \Psi_i^{\rm CDCC(+)} \right\rangle \right|_{X \leqslant X_0}; \tag{47}$$

that is, the CDCC surface-integral form and the conventional CDCC amplitudes coincide. However, in this case the rearrangement channel $p + (n\,A)$ is not suppressed and, hence, the solution of Eq. (47) is not unique. For example, one can consider $\Psi_i^{\rm CDCC(+)} + \varphi_{nA}\, \tilde{\chi}_{pF}^{(+)}$, where $\tilde{\chi}_{pF}^{(+)}$ is the p-F distorted wave. To decrease the contribution of the rearrangement channel the cutoff over l_{pn} was introduced in Ref. [27]; however, the suppression of the rearrangement channels by the angular momentum cutoff is weaker than by the optical potentials [23]. To achieve convergence, the integration radius over ρ_{pF} was extended up to 40 fm. In Ref. [27] it was also demonstrated that using of the CDCC wave function satisfying the Schrödinger equation with the U_{nA} optical potential rather than with $V_{nA}^{\rm sp}$ gives the angular distribution better agreeing with the experimental one.

We have expressed the conventional post CDCC amplitude $\mathcal{M}_{\rm conv}^{\rm CDCC(post)}$ given by the volume integral in terms of the surface-integral matrix element $\mathcal{M}_{\rm surf}^{\rm CDCC(post)}$ and the internal auxiliary amplitude $\mathcal{M}_{\rm aux}^{\rm CDCC(post)}$. There is no specific advantage in invoking the surface formalism when we use the final-channel wave function $\chi_{pF}^{(-)}\, I_A^F$ and the main goal here was to discuss the surface formalism just for better understanding of it. However, below we show another choice of the channel wave function, which clearly demonstrates the advantage of the surface formalism.

Greider-Goldberger-Watson-Johnson choice of the final-channel wave function

Here we consider a different choice of the exit-channel wave function. We choose it to be a solution of the Schrödinger equation

$$\left(E - K - V_{pA} - V_{nA}\right) \tilde{\Phi}_f^{(-)} = 0. \tag{48}$$

By comparing Eqs. (23) and (48) we can easily see the difference between the standard final-channel wave function $\Phi_f^{(-)}$ and the newly defined $\tilde{\Phi}_f^{(-)}$. Multiplying Eq. (21) from the left by $\tilde{\Phi}_f^{(-)*}$ and following a procedure similar to the one used for derivation of the exact reaction amplitude in the previous part, we get

$$\mathcal{M}^{\rm (post)} = \langle \tilde{\Phi}_f^{(-)} | V_{pA} + V_{nA} + V_{pn} - V_{pA} - V_{nA} | \Psi_i^{(+)} \rangle$$

$$= \langle \tilde{\Phi}_f^{(-)} | V_{pn} | \Psi_i^{(+)} \rangle |_{r_{pn} \leqslant R_{pn}} \tag{49}$$

$$= \langle \tilde{\Phi}_f^{(-)} | \overleftarrow{K} - \overrightarrow{K} | \Psi_i^{(+)} \rangle |_{r_{pn} \leqslant R_{pn}}. \tag{50}$$

The advantage of the new choice of the final-channel wave function is that the transition operator is just V_{np} and this keeps the nucleons of the deuteron within the range of their nuclear interaction. It allows us to simplify the initial scattering wave function. However, the new final-channel wave function, *a priori*, cannot be factorized into a product of the p-A distorted wave and the n-A bound-state wave function because now, owing to the presence of the V_{pA}, the recoil of the target can excite the system (nA) into any bound or continuum states. As a result, the final-channel wave function is contributed by the continuum component $p + n + A$ and integration over r_{dA} is not constrained. The asymptotic behavior of $\tilde{\Phi}_f^{(-)*}$ at large ρ_{pF} is given by the sum of the incident wave in the channel $p + F$ plus outgoing waves in all open two-body channels $p + F_n$, where n denotes bound or excited states of F plus three-body outgoing wave in the channel $p + n + A$. Converting the matrix element in Eq. (50) containing $\overleftarrow{K} - \overrightarrow{K}$, where $K = K_{pF} + K_{nA} + K_A$, into surface integrals we find that only the integral over ρ_{pF} survives giving the amplitude of the leading asymptotic term of the initial wave function in the rearrangement channel $p + F$. Thus, using the surface-integral formalism, it can be easily shown that the matrix element (49) coincides with the reaction amplitude for the stripping reaction $d + A \to p + F$. The first proof of Eq. (49) was provided by Greider [28]. Although the final result was correct, the proof contained an error. The first correct proof of Eq. (49) was presented by Goldberger and Watson [29] and extensively used by Johnson and co-workers in the formulation of the ADWA and its applications [26,30–33].

For practical application we consider the limit $A \to \infty$, in which the final-channel wave function can be factorized as

$$\tilde{\Phi}_f^{(-)} = \chi_{pA}^{(-)}\, \varphi_F. \tag{51}$$

In this limit we can choose $\mathbf{r}_{pA}$ and $\mathbf{r}_{nA}$ as two new independent Jacobian variables. Owing to the presence of the bound-state wave function φ_F and the potential V_{np} as the transition operator, the integration over both Jacobian variables is constrained. The reaction amplitude is reduced to

$$\begin{aligned}\mathcal{M}^{(\mathrm{post})} &= \langle \chi^{(-)}_{pF}\,\varphi_F|V_{pn}|\Psi^{(+)}_i\rangle|_{r_{nA}\leqslant\mathcal{R}_{nA};\,r_{pn}\leqslant R_{pn}} \qquad (52)\\ &= \langle \chi^{(-)}_{pF}\,\varphi_F|\overleftarrow{K}-\overrightarrow{K}|\Psi^{(+)}_i\rangle|_{r_{nA}\leqslant\mathcal{R}_{nA};\,r_{pn}\leqslant R_{pn}}\\ &= \langle \chi^{(-)}_{pF}\,\varphi_F|\overleftarrow{K}_{pA}-\overrightarrow{K}_{pA}|\Psi^{(+)}_i\rangle|_{r_{nA}\leqslant\mathcal{R}_{nA};\,r_{pn}\leqslant R_{pn}}. \qquad (53)\end{aligned}$$

Here we took into account that K_{nA} and K_A are Hermitian operators because of the presence of the bound-state wave function φ_F; that is, integrating twice by parts we can transform $\overleftarrow{K}_{nA}+\overleftarrow{K}_A$ to $\overrightarrow{K}_{nA}+\overrightarrow{K}_A$. Because $\mathbf{r}_{pA}=\mathbf{r}_{nA}+\mathbf{r}_{pn}$, the limitation of the integration over $\mathbf{r}_{pA}$ is $r_{pA}\leqslant\mathcal{R}_{pA}=\mathcal{R}_{nA}+R_{pn}$.

Now, as in the previous section, we approximate the exact scattering wave function $\Psi^{(+)}_i$ by the CDCC one $\Psi^{\mathrm{CDCC}(+)}_i\varphi_A$ and replace the potential V_{pA} in Eq. (48) by the optical potential U_{pA}. As discussed previously, it can be done in the volume matrix element (52) containing the transition operator V_{pn} or in the matrix element containing $\overleftarrow{K}_{pA}-\overrightarrow{K}_{pA}$. The obtained amplitudes differ by the term containing the transition operator $U_{nA}-V^{\mathrm{sp}}_{nA}$. Actually, if we do the approximation directly in the matrix element (52) we get the conventional post CDCC amplitude

$$\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{conv}}=\langle\chi^{(-)}_{pF}\,I_F|V_{pn}\big|\Psi^{\mathrm{CDCC}(+)}_i\big\rangle\big|_{r_{nA}\leqslant\mathcal{R}_{nA};\,r_{pn}\leqslant R_{pn}}. \qquad (54)$$

Owing to the presence of the short-range potential V_{pn}, we do not need to introduce an additional projector into the Schrödinger equation for the CDCC wave function, which constrains the distance between the proton and the neutron (see Eq. (40) and Ref. [2,27]). Another advantage of the presence of V_{pn} is a possibility to approximate the CDCC wave function by the first term of the Weinberg states expansion [33]. The Weinberg states φ^W_i are solutions of the equation with eigenvalues λ_i:

$$\big(-\varepsilon^d_{pn}-K_{pn}-\lambda_i\,V_{pn}\big)\varphi^W_i(\mathbf{r}_{pn})=0,\quad i=1,2,\ldots. \qquad (55)$$

This expansion significantly simplifies the calculation of the initial-state scattering wave function.

Now we approximate $\Psi^{(+)}_i$ by the CDCC wave function $\Psi^{\mathrm{CDCC}(+)}_i\varphi_A$ in the matrix element (53) to obtain the deuteron-stripping amplitude in the surface-integral formalism:

$$\begin{aligned}\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{surf}} &= \langle\chi^{(-)}_{pF}\,I_F|\overleftarrow{K}_{pA}-\overrightarrow{K}_{pA}\big|\Psi^{\mathrm{CDCC}(+)}_i\big\rangle\big|_{r_{nA}\leqslant\mathcal{R}_{nA};\,r_{pn}\leqslant R_{pn}} \qquad (56)\\ &= -\frac{\mathcal{R}^2_{pA}}{2\,\mu_{pA}}\int d\,\mathbf{r}_{nA}\,I^{F*}_A(\mathbf{r}_{nA})\int d\,\Omega_{\mathbf{r}_{pA}}\\ &\quad\times\left[\Psi^{\mathrm{CDCC}(+)}_i(\mathbf{r}_{dA},\mathbf{r}_{pn})\,\frac{\partial\,\chi^{(-)*}_{pA}(\mathbf{r}_{pA})}{\partial\,r_{pA}}-\chi^{(-)*}_{pA}(\mathbf{r}_{pA})\,\frac{\partial\,\Psi^{\mathrm{CDCC}(+)}_i(\mathbf{r}_{dA},\mathbf{r}_{pn})}{\partial\,r_{pA}}\right]\Bigg|_{r_{pA}=\mathcal{R}_{pA};r_{pn}\leqslant R_{pn}}. \qquad (57)\end{aligned}$$

In this representation the matrix element is actually the surface integral in the subspace over $\mathbf{r}_{pA}$ and the volume integral over $\mathbf{r}_{nA}$. The main advantage of the surface-integral form is that it is completely peripheral over r_{pA} and r_{nA}. We take into account that in the volume matrix element (52) the integration over r_{pA} is limited by $r_{pA}\leqslant\mathcal{R}_{pA}$, where $\mathcal{R}_{pA}=\mathcal{R}_{nA}+R_{pn}$. Actually we can take the surface integral at any $r_{pA}>\mathcal{R}_{pA}$ but we do not want to do it because with the increasing of the integration radius in the surface integral we risk being in the region where the CDCC wave function is not applicable. So it is better to use the minimally required integration radius, which is $\mathcal{R}_{pA}$. At $r_{pA}=\mathcal{R}_{pA}$ we make the integration over r_{nA} peripheral. From $\mathbf{r}_{nA}=\mathbf{r}_{pA}-\mathbf{r}_{pn}$ it follows that $\mathcal{R}_{pA}-R_{pn}\leqslant r_{nA}\leqslant\mathcal{R}_{pA}+R_{pn}$. Taking into account that $\mathcal{R}_{pA}\sim 25$–30 fm and that R_{pn} is small, we conclude that $R_{nA}\leqslant r_{nA}\leqslant\mathcal{R}_{nA}$, where $\mathcal{R}_{nA}=\mathcal{R}_{pA}-R_{pn}$ and R_{nA} is the n-A nuclear interaction radius.

At $r_{nA}\geqslant R_{nA}$ the radial overlap function can be replaced by its asymptotic term. We remind the reader that the overlap function can be written as

$$\begin{aligned}I^F_A(\mathbf{r}_{nA}) &= \sum_{j_{nA}\,m_{j_{nA}}\,m_{l_{nA}}}\big\langle J_A\,M_A\;\,j_{nA}\,m_{j_{nA}}\big|J_F\,M_F\big\rangle\\ &\quad\times\big\langle J_n\,M_n\;\,l_{nA}\,m_{l_{nA}}\big|j_{nA}\,m_{j_{nA}}\big\rangle Y_{l_{nA}\,m_{l_{nA}}}(\hat{\mathbf{r}}_{nA})\,I^F_{A\,j_{nA}\,l_{nA}}(r_{nA}). \qquad (58)\end{aligned}$$

Here l_{nA} ($m_{l_{nA}}$) is the relative orbital angular momentum (its projection) of n-A in the bound state $F=(n\,A)$, j_{nA} ($m_{j_{nA}}$) is the total angular momentum (its projection) of the neutron in the bound state, J_i (M_i) is the spin (its projection) of nucleus i. The radial overlap function at $r_{nA}>R_{nA}$ takes the form

$$I^F_{A\,j_{nA}\,l_{nA}}(r_{nA})\overset{r_{nA}\gtrapprox R_{nA}}{\approx}C^F_{A\,j_{nA}\,l_{nA}}\,i^{l_{nA}+1}\,\kappa_{nA}\,h^{(1)}_{l_{nA}}(i\,\kappa_{nA}\,r_{nA})\overset{r_{nA}\to\infty}{\approx}C^F_{A\,j_{nA}\,l_{nA}}\,\frac{e^{-\kappa_{nA}\,r_{nA}}}{r_{nA}}, \qquad (59)$$

where $h^{(1)}_{l_{nA}}(i\,\kappa_{nA}\,r_{nA})$ is the spherical Hankel function of the first order, $C^F_{A\,j_{nA}\,l_{nA}}$ is the ANC of the overlap function, $\kappa_{nA}=\sqrt{2\,\mu_{nA}\,\varepsilon^F_{nA}}$ is the bound-state wave number, and ε^F_{nA} is the binding energy of the ground state of F for the virtual decay $F\to n+A$. Taking into account Eqs (58) and (59), we get the final expression for the post-form CDCC deuteron-stripping

amplitude in the surface-integral formalism:

$$\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{surf}} = -\sum_{j_{nA}\, m_{j_{nA}}\, m_{l_{nA}}} \langle J_A\, M_A\ \ j_{nA}\, m_{j_{nA}} | J_F\, M_F\rangle \langle J_n\, M_n\ \ l_{nA}\, m_{l_{nA}} | j_{nA}\, m_{j_{nA}}\rangle$$
$$\times\, i^{-l_{nA}-1} C^{F}_{A\ j_{nA}\, l_{nA}}\ \kappa_{nA}\, \frac{\mathcal{R}^2_{pA}}{2\,\mu_{pA}} \int_{R_{nA}\leqslant r_{nA}\leqslant \mathcal{R}_{nA}} d\,\mathbf{r}_{nA}\, Y^{*}_{l_{nA}\, m_{l_{nA}}}(\hat{\mathbf{r}}_{nA})\, h^{(1)*}_{l_{nA}}(i\,\kappa_{nA}\, r_{nA})$$
$$\times \int d\Omega_{\mathbf{r}_{pA}} \left[\Psi^{\mathrm{CDCC(+)}}_{i}(\mathbf{r}_{dA}, \mathbf{r}_{pn})\, \frac{\partial\, \chi^{(-)*}_{pA}(\mathbf{r}_{pA})}{\partial\, r_{pA}} - \chi^{(-)*}_{pA}(\mathbf{r}_{pA})\, \frac{\partial\, \Psi^{\mathrm{CDCC(+)}}_{i}(\mathbf{r}_{dA}, \mathbf{r}_{pn})}{\partial\, r_{pA}} \right]\Bigg|_{r_{pA}=\mathcal{R}_{pA}; r_{pn}\leqslant R_{pn}} . \qquad (60)$$

Thus, the original volume matrix element can be converted into the surface integral over r_{pA}, which, owing to the constraint on the variable r_{pn}, leads to the dominant contributions for $r_{nA} \geqslant R_{nA}$. It allows us to parametrize the reaction amplitude in terms of the ANC. This peripheral character of the reaction amplitude is obtained because we used the modified final-channel wave function.

We can relate now the $\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{surf}}$ and the conventional CDCC amplitude $\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{conv}}$. To this end, we rewrite (54) as

$$\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{conv}} = \langle \chi^{(-)}_{pF}\, I_F | U_{pA} + U_{nA} + V_{pn} - U_{pA} - V^{\mathrm{sp}}_{nA} + \left[V^{\mathrm{sp}}_{nA} - U_{nA}\right] \left| \Psi^{\mathrm{CDCC(+)}}_{i} \right\rangle \Big|_{r_{nA}\leqslant \mathcal{R}_{nA};\, r_{pn}\leqslant R_{pn}}$$
$$= \mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{surf}} - \mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{aux}}. \qquad (61)$$

Thus, as before we can rewrite the conventional post CDCC volume matrix element in terms of two amplitudes: the entirely peripheral surface-integral matrix element and the internal auxiliary one. The matrix element in the surface-integral form is expressed in terms of the potential transition operator

$$\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{surf}} = \langle \chi^{(-)}_{pF}\, I_F | U_{pA} + U_{nA} + V_{pn} - U_{pA} - V^{\mathrm{sp}}_{nA} \left| \Psi^{\mathrm{CDCC(+)}}_{i} \right\rangle \Big|_{r_{nA}\leqslant \mathcal{R}_{nA};\, r_{pn}\leqslant R_{pn}} \qquad (62)$$
$$= \langle \chi^{(-)}_{pF}\, I_F | \overleftarrow{K} - \overrightarrow{K} \left| \Psi^{\mathrm{CDCC(+)}}_{i} \right\rangle \Big|_{r_{nA}\leqslant \mathcal{R}_{nA};\, r_{pn}\leqslant R_{pn}}. \qquad (63)$$

When deriving (63) we took into account that $\Psi^{\mathrm{CDCC(+)}}_{i}$ satisfies the Schrödinger equation with the potential $U_{pA} + U_{nA} + V_{pn}$ and the final-channel wave function is the solution of the Schrödinger equation with the potential $U_{pA} + V^{\mathrm{sp}}_{nA}$. It allows us to replace $U_{pA} + U_{nA} + V_{pn} - U_{pA} - V^{\mathrm{sp}}_{nA}$ in the matrix element (62) with $\overleftarrow{K} - \overrightarrow{K}$, which leads to the surface matrix element (60). The auxiliary matrix element, which is entirely contributed by the nuclear interior, is written as

$$\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{aux}} = \langle \chi^{(-)}_{pF}\, I_F | \left[U_{nA} - V^{\mathrm{sp}}_{nA}\right] \left| \Psi^{\mathrm{CDCC(+)}}_{i} \right\rangle \Big|_{r_{nA}\leqslant R_{nA};\, r_{pn}\leqslant R_{pn}}$$
$$= \langle \chi^{(-)}_{pF}\, I_F | \mathrm{Im}\, U_{nA} \left| \Psi^{\mathrm{CDCC(+)}}_{i} \right\rangle \Big|_{r_{nA}\leqslant R_{nA};\, r_{pn}\leqslant R_{pn}}. \qquad (64)$$

In Eq. (64) we adopted $\mathrm{Re}\, U_{nA} = V^{\mathrm{sp}}_{nA}$. We remind the reader that the auxiliary matrix element $\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{aux}}$ appears because of the inconsistency in treating the n-A potential. The auxiliary matrix element is contributed by the range of the imaginary part of the U_{nA} potential; that is, $r_{nA} \leqslant R_{nA}$. The depth of the imaginary part of U_{nA} is significantly smaller than that of V_{pn}. Also, the constraint $r_{pn} \leqslant R_{pn}$ keeps protons in the region with the strongest absorption. Hence, we expect that $|\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{aux}}|$ can be significantly smaller than $|\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{conv}}|$ at low energies and good matching of the initial and final momenta. In this case,

$$\mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{conv}} \approx \mathcal{M}^{\mathrm{CDCC(post)}}_{\mathrm{surf}}. \qquad (65)$$

Once again we repeat that adoption of the Greider-Goldberger-Watson-Johnson final-channel wave function allowed us to constrain the integration over r_{pn} by the range of the transition operator V_{pn} despite the fact that the CDCC wave function contains the components describing the p-n pair in the continuum. As we mentioned, it allows one to approximate the CDCC wave function by the first term of the Weinberg states expansion [33] and this significantly simplifies the calculation of the initial-state scattering wave function.

The presence of the overlap function I^F_A constrains the integration over r_{nA}. As the result of these two constraints the surface matrix element taken at $r_{pA} = \mathcal{R}_{pA}$ leads to the dominant contribution at $r_{nA} \geqslant R_{nA}$. In other words, the surface matrix element is peripheral, allowing us to parametrize it in terms of the ANC for the bound state $F = (n\,A)$, which is the only model-independent spectroscopic information extractable from experiment [34]. The auxiliary term determines the contribution from the nuclear interior. Although here equations were obtained assuming an infinitely heavy target A, they should work also for a heavy target with a finite mass. Necessary corrections may be introduced using expansion over a small parameter $1/A$.

IV. DEUTERON STRIPPING TO A RESONANCE STATE

Now we proceed to the main goal of the present paper and apply the surface formalism used in the previous sections for stripping to bound states to describe the deuteron stripping populating resonance states.

Prior form

To treat the stripping to resonance states we use the prior formalism, in which the exact scattering wave function $\Psi^{(-)}_{f}$ is taken in the final state. We consider the deuteron-stripping reaction

$$d + A \to p + n + A, \qquad (66)$$

proceeding through the resonant subreaction $n + A \to F^* \to n + A$. The results can easily be extended for the deuteron-stripping reaction

$$d + A \to p + b + B, \tag{67}$$

which proceeds through the resonant subreaction $n + A \to F^* \to b + B$, where the channel $b + B$ differs from $n + A$.

The wave function $\Psi_f^{(-)}$ satisfies the Schrödinger equation

$$\Psi_f^{(-)*}(E - \overleftarrow{K} - V_{pA} - V_{nA} - V_{pn} - H_A) = 0 \tag{68}$$

and has the $p + n + A$ incident three-body wave in the continuum with the outgoing waves in all the open channels. Let $\Phi_i^{(+)} = \varphi_{pn}\,\chi_{dA}^{(+)}$ be the wave function of the entry channel and $\chi_{dA}^{(+)}$ be the $d + A$ distorted wave. We adopt the initial-channel wave function as the solution of the Schrödinger equation

$$(E - K - V_{pn} - U_{dA} - H_A)\,\Phi_i^{(+)} = 0, \tag{69}$$

with the $d + A$ incident wave. Because we do not consider excitation of the target nucleus A, in what follows we disregard the Hamiltonian H_A.

Multiplying Eq. (68) from the right by $\Phi_i^{(+)}$ and taking into account Eq. (69), we get

$$\mathcal{M}^{(\mathrm{as})} = \langle \Psi_f^{(-)} | \overleftarrow{K} - \overrightarrow{K} | \Phi_i^{(+)} \rangle \tag{70}$$

$$= \langle \Psi_f^{(-)} | V_{pA} + V_{nA} + V_{pn} - V_{pn} - U_{dA} \big| \Phi_i^{(+)} \rangle$$

$$= \langle \Psi_f^{(-)} | V_{pA} + V_{nA} - U_{dA} | \Phi_i^{(+)} \rangle = \mathcal{M}^{(\mathrm{prior})}. \tag{71}$$

Equation (71) is the standard prior form of the volume matrix element, while Eq. (70) is the matrix element, which can be written in a surface-integral form. This matrix element can be easily reduced to the amplitude of the leading asymptotic term of the $\Psi_f^{(-)*}$ in the channel $d + A$. This amplitude, by definition, is the deuteron-stripping amplitude $\mathcal{M}^{(\mathrm{as})}$. To show it we rewrite

$$\langle \Psi_f^{(-)} | \overleftarrow{K} - \overrightarrow{K} | \Phi_i^{(+)} \rangle = \langle \Psi_f^{(-)} | \overleftarrow{K}_{dA} - \overrightarrow{K}_{dA} | \Phi_i^{(+)} \rangle + \langle \Psi_f^{(-)} | \overleftarrow{K}_{pn} - \overrightarrow{K}_{pn} | \Phi_i^{(+)} \rangle. \tag{72}$$

The matrix element containing $\overleftarrow{K}_{pn} - \overrightarrow{K}_{pn}$ vanishes because it contains the deuteron bound-state wave function φ_{pn}. Taking the limit $R_{pn} \to \infty$ we get

$$\langle \Psi_f^{(-)} | \overleftarrow{K}_{pn} - \overrightarrow{K}_{pn} | \Phi_i^{(+)} \rangle = -\lim_{R_{pn}\to\infty} \frac{R_{pn}^2}{2\,\mu_{pn}} \int d\boldsymbol{\rho}_{dA}\,\chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA}) \int d\Omega_{\mathbf{r}_{pn}} \times \left[\Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn}) \frac{\partial \varphi_{pn}(r_{pn})}{\partial r_{pn}} - \varphi_{pn}(r_{pn}) \frac{\partial \Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn})}{\partial r_{pn}} \right] \Bigg|_{r_{pn}=R_{pn}} = 0. \tag{73}$$

Hence,

$$\mathcal{M}^{(\mathrm{as})} = \langle \Psi_f^{(-)} \big| \overleftarrow{K} - \overrightarrow{K} \big| \Phi_i^{(+)} \rangle = \langle \Psi_f^{(-)} \big| \overleftarrow{K}_{dA} - \overrightarrow{K}_{dA} \big| \Phi_i^{(+)} \rangle = -\lim_{R_{dA}\to\infty} \frac{R_{dA}^2}{2\,\mu_{dA}} \int d\mathbf{r}_{pn}\,\varphi_{pn}(\mathbf{r}_{pn}) \int d\Omega_{\boldsymbol{\rho}_{dA}} \times \left[\Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn}) \frac{\partial \chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA})}{\partial \rho_{dA}} - \chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA}) \frac{\partial \Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn})}{\partial \rho_{dA}} \right] \Bigg|_{\rho_{dA}=R_{dA}} = \mathcal{M}^{\mathrm{prior}}. \tag{74}$$

To prove that this equation reduces to $\mathcal{M}^{(\mathrm{as})}$, we have taken into account that at $\rho_{dA} \to \infty$ only the leading asymptotic term

$$\Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn}) \sim -\frac{\mu_{dA}}{2\,\pi}\,\mathcal{M}^{(\mathrm{as})}\,u^+(\rho_{dA})\,\varphi_{pn}, \tag{75}$$

where $u^+(\rho_{dA})$ is outgoing scattered wave in the $d + A$ two-body channel, will give nonvanishing contribution to the integral over $\boldsymbol{\rho}_{dA}$. Thus, in the prior form the conventional CDCC amplitude given by the volume matrix element is equal to the amplitude in the surface-integral formalism. It is because only one potential, V_{nA}^{sp}, is used in the prior formalism.

After proving that the volume matrix element (71) is equal to the amplitude of the total scattering wave function in the asymptotic final $d + A$ channel $\mathcal{M}^{(\mathrm{as})}$, we consider now the constraints on the integration volume in the matrix element (71). Owing to the presence of the deuteron bound-state wave function in the initial channel, the integration over r_{pn} is limited. At large r_{pA}, $V_{pA} \to U_{pA}^C$, where $U_{pA}^C = Z_A\,e^2/r_{pA}$ is the Coulomb potential between the proton and the center of mass of nucleus A; also at large r_{pA}, $U_{dA} \to U_{dA}^C$ because at large r_{pA} also r_{dA} is large because of the constrain of r_{pn}. For the same reason, when r_{pA} increases, also r_{nA} increases. Then V_{nA} vanishes when r_{pA} increases. As r_{dA} increases, the matrix element from the difference $U_{pA}^C - U_{dA}^C$ goes to zero as $d_0\,Z_A\,e^2/(2\,r_{dA}^2)$, where d_0 is the deuteron size [35]. Hence, the integration over r_{dA} is also constrained. Thus, the volume integral in Eq. (71) can be taken over the constrained volume

in the six-dimensional space $\{\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn}\}$ with the hyperradius $Y \leqslant Y_0$, where $Y_0 = (\mu_{pn}\,\mathcal{R}_{pn}^2/m + \mu_{dA}\,\mathcal{R}_{dA}^2/m)^{1/2}$. Also $\mathcal{R}_{pn}$ is the maximal r_{pn}, which is required to achieve a desired accuracy for the integral over r_{pn} and $\mathcal{R}_{dA}$ is the maximal ρ_{dA}, which is required to achieve a desired accuracy for the integral over ρ_{dA}.

Hence, we can rewrite (71) in form of the conventional volume and the surface-integral forms:

$$\mathcal{M}^{(\mathrm{prior})} = \langle \Psi_f^{(-)} | V_{pA} + V_{nA} - U_{dA} | \Phi_i^{(+)} \rangle |_{\rho_{dA} \leqslant \mathcal{R}_{dA}; r_{pn} \leqslant \mathcal{R}_{pn}} \tag{76}$$

$$= -\frac{R_{dA}^2}{2\,\mu_{dA}} \int d\mathbf{r}_{pn}\, \varphi_{pn}(\mathbf{r}_{pn}) \int d\Omega_{\boldsymbol{\rho}_{dA}} \times \left[\Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn}) \frac{\partial \chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA})}{\partial \rho_{dA}} - \chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA}) \times \frac{\partial \Psi_f^{(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn})}{\partial \rho_{dA}} \right] \Bigg|_{\rho_{dA} = \mathcal{R}_{dA}; r_{pn} \leqslant \mathcal{R}_{pn}} \tag{77}$$

$$= \mathcal{M}^{(\mathrm{as})}. \tag{78}$$

Thus, the integration in both forms, volume and surface, is constrained.

As in the previous sections, now we can get the prior form of the CDCC amplitude for the stripping to the resonance state in the conventional volume integral form and the surface formalism. To this end we replace $\Psi_f^{(-)}$ with the CDCC wave function $\Psi_f^{\mathrm{CDCC}(-)}$. If we do it in the matrix element (76) containing the volume integral we get the conventional CDCC prior form amplitude:

$$\mathcal{M}_{\mathrm{conv}}^{\mathrm{CDCC(prior)}} = \left\langle \Psi_f^{\mathrm{CDCC}(-)} \middle| U_{pA} + V_{nA}^{\mathrm{sp}} - U_{dA} | \varphi_{pn}\, \chi_{dA}^{(+)} \right\rangle |_{\rho_{dA} \leqslant \mathcal{R}_{dA}; r_{pn} \leqslant \mathcal{R}_{pn}}. \tag{79}$$

To obtain the prior form of the CDCC matrix element we replaced V_{pA} by the optical potential U_{pA}. Correspondingly, from Eq. (78) we get the CDCC prior form amplitude in the surface-integral representation:

$$\mathcal{M}_{\mathrm{surf}}^{\mathrm{CDCC(prior)}} = -\frac{R_{dA}^2}{2\,\mu_{dA}} \int d\mathbf{r}_{pn}\, \varphi_{pn}(\mathbf{r}_{pn}) \int d\Omega_{\boldsymbol{\rho}_{dA}} \times \left[\Psi_f^{\mathrm{CDCC}(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn}) \frac{\partial \chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA})}{\partial \rho_{dA}} - \chi_{dA}^{(+)}(\boldsymbol{\rho}_{dA}) \frac{\partial \Psi_f^{\mathrm{CDCC}(-)*}(\boldsymbol{\rho}_{dA}, \mathbf{r}_{pn})}{\partial \rho_{dA}} \right] \Bigg|_{\rho_{dA} = \mathcal{R}_{dA}; r_{pn} \leqslant \mathcal{R}_{pn}}. \tag{80}$$

Because the potential V_{nA}^{sp} is real, both conventional and surface-integral forms coincide. It is straightforward to see, but before showing it we discuss the CDCC wave function $\Psi_f^{\mathrm{CDCC}(-)}$. We consider the deuteron-stripping reaction populating a resonance state, which decays into the channel $n + A$.

Thus, we have the three-body system $p + n + A$ in the final state, in which we need to take into account explicitly the $n + A$ rescattering in the final state to describe the resonance in the n-A system. To this end in the finite volume around the target A we approximate the exact final-state scattering wave function by the CDCC wave function $\Psi_f^{\mathrm{CDCC}(-)}$, which satisfies the three-body Schrödinger equation

$$\Psi_f^{\mathrm{CDCC}(-)*} \left(E - \overleftarrow{K} - U_{pA} - V_{nA}^{\mathrm{sp}} - V_{pn} \right) = 0. \tag{81}$$

The CDCC method simplifies the problem by considering only one equation (81) with the incident wave describing the three-body system $p + n + A$ in the continuum. The simplest mechanism of the deuteron stripping populating a resonance state can be described as a virtual breakup of the deuteron with subsequent $n + A$ resonance scattering, in which the proton is a spectator. An effective way to describe the three-body system in the continuum, which takes into account the resonance scattering in the subsystem $n + A$, is to use the CDCC wave function which is expressed in terms of the product of the n-A scattering wave function times the scattering wave function of the proton off the center of mass of the system $n + A$. Then the only channel coupled to the three-body continuum that can be included in the CDCC method is the two-fragment channel $p + F$, where $F = (n\,A)$ is the bound state. A few bound states of the system $(n\,A)$ can be taken into account. Then we can write the CDCC wave function in the form

$$\Psi_f^{\mathrm{CDCC}(-)}(\boldsymbol{\rho}_{pF}, \mathbf{r}_{nA}) = \sum_{i=0}^{i_{\max}} \varphi_{nA}^{(i)}(\mathbf{r}_{nA})\, \chi_{\mathbf{q}_{pF}}^{(i)(-)}(\boldsymbol{\rho}_{pF}) + \sum_{j=1}^{j_{\max}} \overline{\psi}_{\mathbf{k}_{nA}}^{(j)(-)}(\mathbf{r}_{nA})\, \overline{\chi}_{\mathbf{q}_{pF}(\mathbf{k}_{nA})}^{(j)(-)}(\boldsymbol{\rho}_{pF}), \tag{82}$$

Here $\varphi_{nA}^{(i)}(\mathbf{r}_{nA})$ is the ith bound-state wave function of the system $F = (nA)$ with $i = 0$ corresponding to the ground state and $\chi_{\mathbf{q}_{pF}}^{(i)(-)}(\boldsymbol{\rho}_{pF})$ are the functions, which describe the relative motion of the center of mass of p and the $(n\,A)$ pair in the ith bound state. $\overline{\psi}_{\mathbf{k}_{nA}}^{(j)(-)}(\mathbf{r}_{nA})$ is the n-A scattering wave function obtained by averaging continuous breakup states in the jth bin and $\overline{\chi}_{\mathbf{q}_{pF}(\mathbf{k}_{nA})}^{(j)(-)}(\boldsymbol{\rho}_{pF})$ is the wave function describing the relative motion of the proton and the center of mass of the system $n + A$ in the continuum in the jth bin. In Eq. (82) the relative momentum $\mathbf{q}_{pF}(\mathbf{k}_{nA})$ of the particles p and F is related to the n-A relative momentum $\mathbf{k}_{nA}$ via the energy conservation law:

$$E = E_{dA} - \varepsilon_{pn}^{d} = E_{pF} - \varepsilon_{nA}^{F} = \frac{q_{pF}^2}{2\,\mu_{pF}} + \frac{k_{nA}^2}{2\,\mu_{nA}}. \tag{83}$$

The n-A interaction is taken as a real single-particle potential V_{nA}^{sp}, which can support the resonance in the n-A system. The corresponding scattering wave function is orthogonal to the bound states generated by this potential.

To be sure that Eq. (82) provides a unique solution of Eq. (81) we need to suppress the two-fragment rearrangement channels, $n + (pA)$ and $d + A$. Unfortunately, there is only one optical potential, U_{pA}, in Eq. (81). This potential to some extent suppresses the rearrangement channel $n + (p\,A)$

because it generates a substantial positive imaginary part to the potential $n + (p\,A)$ damping the outgoing neutron wave. However, the other rearrangement channel $d + A$ is not suppressed because the potential V_{pn} is real. To provide a unique solution of Eq. (81) a model space is introduced in which the CDCC solution becomes unique. This model space is achieved by cutting the n-A relative orbital angular momenta by some finite $l_{nA}^{\max}$ [36]. Although the solution is unique in such a model space because the rearrangement channels are absent in the asymptotic regions, the nonuniqueness is disguised in the dependence of the CDCC solution on the adopted model space [36]. Fortunately, in the case of the stripping to resonance the number of the resonant partial waves l_{nA} is limited by one or a few at most. To ensure the uniqueness of the CDCC solution only the number of the nonresonant partial waves (nonresonant background) in the subsystem n-A requires a cutoff that can create a model dependence on $l_{nA}^{\max}$. Note that a constraint on l_{nA} keeps n close to A, suppressing the contribution of the rearrangement channel $d + A$.

We write now the n-A scattering wave function taking into account the spins in the representation with given channel spin and its projections:

$$\psi^{(j)(-)}_{\mathbf{k}_{nA}\, s\, m_s\, m''_s}(\mathbf{r}_{nA}) = i\,\frac{2\,\pi}{k_{nA}\,r_{nA}} \sum_{J_F\, M_F\, l_{nA}\, m_{l_{nA}}\, m''_{l_{nA}}} \times \left\langle s\, m_s\ l_{nA}\, m_{l_{nA}} \middle| J_F\, M_F \right\rangle \times \left\langle s\, m''_s\ l_{nA}\, m''_{l_{nA}} \middle| J_F\, M_F \right\rangle i^{-l_{nA}}\, Y^*_{l_{nA}\, m_{l_{nA}}} \times (\hat{\mathbf{k}}_{nA}) Y_{l_{nA}\, m''_{l_{nA}}}(\hat{\mathbf{r}}_{nA}) \phi_{nA s m''_s} u^{(j)(+)*}_{k_{nA} s l_{nA} J_F}(r_{nA}). \tag{84}$$

Here s is the channel spin (m_s and m''_s are its projections before and after scattering) and l_{nA} is the n-A orbital angular momentum ($m_{l_{nA}}$ and $m''_{l_{nA}}$ are its projections before and after scattering), J_F (M_F) is the spin (its projection) of nucleus F, and $\phi_{nA\, s\, m''_s}$ is the spin function of the system $n + A$ with the channel spin s. We presented here only the diagonal components (over the channel spin and the orbital angular momenta) of the scattering wave function. General cases of the scattering wave function with different channel spins in the initial and final states and even including reaction channels are given in Ref. [5].

Note that in practical application we need to use $\overline{\psi}^{(j)(-)*}_{\mathbf{k}_{nA} s\, m_s\, m''_s}(\mathbf{r}_{nA})$, which is expressed in terms of the binned radial wave function $\overline{u}^{(j)(+)}_{k_{nA}\, s\, l_{nA}\, J_F}(r_{nA})$ given by [37]

$$\overline{u}^{(j)(+)}_{k_{nA}\, s\, l_{nA}\, J_F}(r_{nA}) = \sqrt{\frac{2}{\pi\, N^{(j)}_{s\, l_{nA}\, J_F}}} \times \int_{k_{nA}^{(j-1)}}^{k_{nA}^{(j)}} dk_{nA}\, g^{(j)}_{s\, l_{nA}\, J_F}(k_{nA})\, u^{(+)}_{k_{nA}\, s\, l_{nA}\, J_F}(r_{nA}), \tag{85}$$

where $g^{(j)}_{s\, l_{nA}\, J_F}(k_{nA})$ is the weight function. The normalization constant is

$$N^{(j)}_{s\, l_{nA}\, J_F} = \int_{k_{nA}^{(j-1)}}^{k_{nA}^{(j)}} d\,k_{nA}\, \left| g^{(j)}_{s\, l_{nA}\, J_F}(k_{nA}) \right|^2. \tag{86}$$

The adopted normalization constant $N^{(j)}_{s\, l_{nA}\, J_F}$ makes an orthonormal set $\overline{u}^{(j)(+)*}_{k_{nA}\, s\, l_{nA}\, J_F}(r_{nA})$ when all the intervals $(k_{nA}^{j-1}, k_{nA}^{(j)})$ are nonoverlapping.

The next important step is adoption of the weight function $g^{(j)}_{s\, l_{nA}\, J_F}(k_{nA})$. In Ref. [37] two different prescriptions were used for the weight function for resonant and nonresonant bins. We use for the nonresonant bins

$$g^{(j)}_{s\, l_{nA}\, J_F}(k_{nA}) = e^{-i\,\delta_{s\, l_{nA}\, J_F}(k_{nA})} \tag{87}$$

and for the resonance bin

$$g_{s\, l_{nA}\, J_F}(k) = e^{i\delta_{s\, l_{nA}\, J_F}(k)} \sin[\delta_{s\, l_{nA}\, J_F}(k)], \tag{88}$$

where $\delta_{s\, l_{nA}\, J_F}(k_{nA})$ is the n-A scattering phase shift.

The radial scattering wave function $u^{(j)(+)}_{k_{nA} s\, l_{nA}, J_F}(r_{nA})$ should describe the resonance scattering in the bin covering the resonant region. In the R-matrix approach the coordinate space over r_{nA} is divided into the internal, $r_{nA} \leqslant R_{nA}$, and external, $r_{nA} > R_{nA}$, regions. In the internal region in the one-level approximation

$$u^{(\mathrm{int})}_{k_{nA}\, s\, l_{nA}\, J_F} = -i \sqrt{\frac{k_{nA}}{\mu_{nA}}}\, e^{-i\,\delta^{\mathrm{hs}}_{l_{nA}}} \times \frac{\left[\Gamma_{nA\, s\, l_{nA}\, J_F}(E_{nA})\right]^{1/2}}{E_R - E_{nA} - i\,\Gamma_{nA\, s\, l_{nA}\, J_F}(E_{nA})/2}\, X_{\mathrm{int}}. \tag{89}$$

Here $\Gamma_{nA\, s\, l_{nA}\, J_F}(E_{nA})$ is the partial resonance width in the channel $n + A$, $\delta^{\mathrm{hs}}_{l_{nA}}$ is the hard-sphere scattering phase shift, and X_{int} is an eigenfunction of the Hamiltonian describing the compound system $F = n + A$. At the channel radius $r_{nA} = R_{nA}$,

$$X_{\mathrm{int}} = \frac{1}{R_{nA}} \sqrt{2\,\mu_{nA}\,R_{nA}}\,\gamma_{s\, l_{nA}\, J_F}, \tag{90}$$

where $\gamma_{s\, l_{nA}\, J_F}$ is the reduced width amplitude in the channel with quantum numbers s, l_{nA}, and J^F. In the external region ($r_{nA} > R_{nA}$) in the representation with a given channel spin s and orbital angular momentum l_{nA} wave function, $u^{(j)(+)}_{k_{nA} s\, l_{nA}\, J_F}(r_{nA})$ takes the standard form

$$u^{(\mathrm{ext})(+)}_{k_{nA}\, s\, l_{nA}\, J_F} = \left[I_{l_{nA}}(k_{nA}, r_{nA}) - S^{J_F}_{nA\, s\, l_{nA}; nA\, s\, l_{nA}}\, O_{l_{nA}}(k_{nA}, r_{nA}) \right], \tag{91}$$

where $I_{l_{nA}}(k_{nA}, r_{nA})$ and $O_{l_{nA}}(k_{nA}, r_{nA})$ are incoming and outgoing spherical waves, respectively. By equating the internal $u^{(\mathrm{int})}_{k_{nA}\, s\, l_{nA}\, J_F}$ and external $u^{(\mathrm{ext})(+)}_{k_{nA}\, s\, l_{nA}\, J_F}$ wave functions at the channel radius $r_{nA} = R_{nA}$, we get an expression for the resonant S matrix elastic scattering element $S^{J_F}_{nA\, s\, l_{nA}; nA\, s\, l_{nA}}$, which at energies near the resonances takes the form

$$S^{J_F}_{nA\, s\, l_{nA}; nA\, s\, l_{nA}} = e^{-2\,i\,\delta^{\mathrm{hs}}_{l_{nA}}} \left[1 + i\,\frac{\Gamma_{nA\, s\, l_{nA}\, J_F}(E_{nA})}{E_R - E_{nA} - i\,\Gamma_{nA\, s\, l_{nA}\, J_F}(E_{nA})/2} \right], \tag{92}$$

where E_R is the real part of the resonance energy. $\delta^{\rm hs}_{l_{nA}}$ is the hard-sphere scattering phase shift in the channel $n+A$ determined by equation

$$e^{-2i\,\delta^{\rm hs}_{l_{nA}}} = \frac{I_{l_{nA}}(k_{nA}, R_{nA})}{O_{l_{nA}}(k_{nA}, R_{nA})}. \tag{93}$$

Thus, in the external region $u^{({\rm ext})(+)}_{k_{nA}\,s\,l_{nA}\,J_F}$ can be expressed in terms of the observable partial resonance widths and resonance energies.

Another possible approach is the potential one. In the potential approach first we introduce the overlap function $I^{F(-)*}_{A\,\mathbf{k}_{nA}} = \langle \psi^{(-)}_{F\,\mathbf{k}_{nA}} | \varphi_A \rangle$, where φ_A is the bound-state wave function of nucleus A and $\psi^{(-)}_F$ is the eigenfunction of the continuum spectrum of the Hamiltonian $H = K_{nA} + V_{nA} + H_A$ of the system $F = n + A$. This overlap function is approximated as [35]

$$I^{F(-)*}_{A\,\mathbf{k}_{nA}} = S^F_A\, u^{(-)*}_{\mathbf{k}_{nA}}, \tag{94}$$

where S^F_A is the spectroscopic factor of the configuration $n+A$ in F and $u^{(-)*}_{\mathbf{k}_{nA}}$ is a solution of the Schrödinger equation

$$\left(E_{nA} - K_{nA} - V^{\rm sp}_{nA}\right) u^{(-)*}_{\mathbf{k}_{nA}}(\mathbf{r}_{nA}) = 0. \tag{95}$$

The external part of the single-particle wave function $u^{({\rm ext})(+)}_{k_{nA}\,s\,l_{nA}\,J_F}$ (we recovered here the spins) is given by Eq. (91) where the elastic scattering S matrix is generated by the potential $V^{\rm sp}_{nA}$. This S matrix element in the single-particle model is given by

$$S^{({\rm sp})\,J_F}_{nA\,s\,l_{nA};nA\,s\,l_{nA}} = e^{-2i\,\delta^{\rm sp}_{s\,l_{nA}\,J_F}} \left(1 + i\,\frac{\Gamma^{\rm sp}_{nA\,s\,l_{nA}\,J_F}}{E_R - E_{nA} - i\,\Gamma^{\rm sp}_{nA\,s\,l_{nA}\,J_F}/2}\right), \tag{96}$$

where $\delta^{\rm sp}_{s\,l_{nA}\,J_F}$ is the potential nonresonance scattering phase shift and $\Gamma^{\rm sp}_{nA\,s\,l_{nA}\,J_F}$ is the single-particle neutron resonance width. Then the observable resonance width is written as

$$\Gamma_{nA\,s\,l_{nA}\,J_F} = S^F_A\,\Gamma^{\rm sp}_{nA\,s\,l_{nA}\,J_F}. \tag{97}$$

Now we return to the prior form of the stripping amplitude. After defining the CDCC wave function in the final state it is clear from Eqs. (81) and

$$(E - K - V_{pn} - U_{dA})\,\varphi_{pn}\,\chi^{(+)}_{dA} = 0 \tag{98}$$

that amplitudes (79) and (80) coincide. The main advantage of the surface amplitude (80) is that the convergence problem for the stripping to resonance is solved because the integration over ρ_{dA} is taken at the finite $\rho_{dA} = \mathcal{R}_{dA}$ and the integration over r_{pn} is constrained owing to the presence of φ_{pn}. Because of these two integrations the contribution of the peripheral region over r_{nA} in the surface matrix element is enhanced compared to the conventional volume matrix element. However, the surface matrix element is not fully peripheral over r_{nA} because of the large nonlocality of the prior amplitude (typically 20–25 fm). It means that small ρ_{pF}, and, correspondingly, small r_{nA} can contribute making nonperipheral contribution also possible, especially when the energy increases. Equation (80) is the main result of our paper.

There is one more point about CDCC to discuss. We have assumed that the CDCC wave function given by Eq. (82) is a solution of Eq. (81). As we have discussed, the constraint imposed on $l^{\rm max}_{nA}$ allows us to diminish the role of the rearrangement channels. However, it may not be enough and a more sophisticated truncation procedure is achieved by using the projector

$$\hat{P}_{nA} = \sum_{l_{nA}=0}^{l^{\rm max}_{nA}} \sum_{m_{l_{nA}}=-l_{nA}}^{l_{nA}} \int d\Omega_{\mathbf{r}_{nA}}\, Y_{l_{nA}\,m_{l_{nA}}}(\hat{\mathbf{r}}_{nA})\, Y^*_{l_{nA}\,m_{l_{nA}}}(\hat{\mathbf{r}}'_{nA}). \tag{99}$$

Applying the projector P_{nA} to Eq. (81) from the right we get the Schrödinger equation for the CDCC wave function in the final state in the projected model space,

$$\Psi^{{\rm CDCC}(-)*}_{(P_{nA})\,f}\left(E - \overleftarrow{K} - U^{P_{nA}}_{pA} - V^{P_{nA}}_{pn} - V_{nA}\right) = 0, \tag{100}$$

where $\Psi^{{\rm CDCC}(-)*}_{(P_{nA})\,f} = \Psi^{{\rm CDCC}(-)*}_f\,P_{nA}$, $U^{P_{nA}}_{pA} = P_{nA}\,U_{pA}\,P_{nA}$, and $V^{P_{nA}}_{pn} = P_{nA}\,V_{pn}\,P_{nA}$. Note that the projector P_{nA} acts on $\mathbf{r}_{nA}$; hence, it modifies U_{pA} and V_{pn}, which can be expressed in terms of the radii $\mathbf{r}_{nA}$ and $\boldsymbol{\rho}_{pF}$. The potential V_{nA} remains intact to the action of P_{nA} because it depends only on r_{nA} rather than on $\mathbf{r}_{nA}$.

In the projected model space the rearrangement channels are suppressed. For example, if we add to the CDCC wave function the component of the rearrangement channel $\varphi_{pn}\,\chi^{(+)}_{dA}$, application of the projector P_{nA} at $\rho_{dA} \gg r_{pn}$ brings an additional suppression factor ρ^{-2}_{dA} [2]. In the projected model space the conventional volume matrix element and the matrix element in the surface-integral formalism do not coincide. To show it we go back to the conventional volume matrix element (79), in which we replace $\Psi^{{\rm CDCC}(-)}_f$ with $\Psi^{(-)}_{(P_{nA})f}$ without changing potentials. Then we get

$$\mathcal{M}^{{\rm CDCC(prior)}}_{\rm conv} = \left\langle \Psi^{{\rm CDCC}(-)}_{(P_{nA})\,f} \middle| U_{pA} + V^{\rm sp}_{nA} - U_{dA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle\Big|_{\rho_{dA}\leqslant\mathcal{R}_{dA};\,r_{pn}\leqslant\mathcal{R}_{pn}}. \tag{101}$$

To transform this matrix element to the surface-integral form, we rewrite

$$U_{pA} + V^{\rm sp}_{nA} - U_{dA} = \left(U_{pA} - U^{P_{nA}}_{pA}\right) + \left[U^{P_{nA}}_{pA} + V^{P_{nA}}_{pn} + V^{\rm sp}_{nA}\right] - \left[V_{pn} + U_{dA}\right] + \left(V_{pn} - V^{P_{nA}}_{pn}\right). \tag{102}$$

Taking into account the Schrödinger equations for $\Psi^{\mathrm{CDCC}(-)*}_{(P_{nA})f}$ and $\varphi_{pn}\,\chi^{(+)}_{dA}$, we can replace the bracketed operator $[U^{P_{nA}}_{pA} + V^{P_{nA}}_{pn} + V^{\mathrm{sp}}_{nA}]$ with $E - \overleftarrow{K}$ and $[V_{pn} + U_{dA}]$ with $E - \overrightarrow{K}$. Then Eq. (101) can be reduced to

$$\mathcal{M}^{\mathrm{CDCC(prior)}}_{\mathrm{conv}} = \left\langle \Psi^{\mathrm{CDCC}(-)}_{(P_{nA})f} \middle| U_{pA} + V^{\mathrm{sp}}_{nA} - U_{dA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle \Big|_{\rho_{dA}\leqslant\mathcal{R}_{dA};\, r_{pn}\leqslant\mathcal{R}_{pn}} \tag{103}$$

$$= \mathcal{M}^{\mathrm{CDCC(prior)}}_{\mathrm{surf}} + M^{\mathrm{CDCC(prior)}}_{\mathrm{aux}}. \tag{104}$$

Here the matrix element in the surface-integral representation is

$$\begin{aligned}\mathcal{M}^{\mathrm{CDCC(prior)}}_{\mathrm{surf}} &= \left\langle \Psi^{\mathrm{CDCC}(-)}_{(P_{nA})f} \middle| \overrightarrow{K} - \overleftarrow{K} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle \Big|_{\rho_{dA}\leqslant\mathcal{R}_{dA};\, r_{pn}\leqslant\mathcal{R}_{pn}} \\ &= \left\langle \Psi^{\mathrm{CDCC}(-)}_{(P_{nA})f} \middle| \overrightarrow{K}_{dA} - \overleftarrow{K}_{dA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle \Big|_{\rho_{dA}\leqslant\mathcal{R}_{dA};\, r_{pn}\leqslant\mathcal{R}_{pn}} \\ &= -\frac{R^2_{dA}}{2\,\mu_{dA}} \int d\mathbf{r}_{pn}\,\varphi_{pn}(\mathbf{r}_{pn}) \int d\Omega_{\boldsymbol{\rho}_{dA}} \left[\Psi^{\mathrm{CDCC}(-)*}_{(P_{nA})f}(\boldsymbol{\rho}_{dA},\mathbf{r}_{pn}) \frac{\partial\,\chi^{(+)}_{dA}(\boldsymbol{\rho}_{dA})}{\partial\,\rho_{dA}} \right. \\ &\quad \left. - \chi^{(+)}_{dA}(\boldsymbol{\rho}_{dA}) \frac{\partial\,\Psi^{\mathrm{CDCC}(-)*}_{(P_{nA})f}(\boldsymbol{\rho}_{dA},\mathbf{r}_{pn})}{\partial\,\rho_{dA}} \right] \Bigg|_{\rho_{dA}=\mathcal{R}_{dA};\, r_{pn}\leqslant\mathcal{R}_{pn}}, \end{aligned} \tag{105}$$

where we took into account that the matrix element from $\overleftarrow{K}_{pn} - \overrightarrow{K}_{pn}$ vanishes. The auxiliary matrix element is given by

$$\begin{aligned}\mathcal{M}^{\mathrm{CDCC(prior)}}_{\mathrm{aux}} &= \left\langle \Psi^{\mathrm{CDCC}(-)}_{(P_{nA})f} \middle| U_{pA} - U^{P_{nA}}_{pA} + V_{pn} - V^{P_{nA}}_{pn} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle \Big|_{\rho_{dA}\leqslant\mathcal{R}_{dA};\, r_{pn}\leqslant\mathcal{R}_{pn}} \\ &= \left\langle \Psi^{\mathrm{CDCC}(-)}_{(P_{nA})f} \middle| P_{nA}(U_{pA} + V_{pn}) Q_{nA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle \Big|_{\rho_{dA}\leqslant\mathcal{R}_{dA};\, r_{pn}\leqslant\mathcal{R}_{pn}}. \end{aligned} \tag{106}$$

To obtain Eq. (106) we took into account that $P_{nA} + Q_{nA} = 1$, $P^2_{nA} = P_{nA}$, $\Psi^{\mathrm{CDCC}(-)*}_{(P_{nA})f}(U_{pA} - U^{P_{nA}}_{pA} + V_{pn} - V^{P_{nA}}_{pn}) = \Psi^{\mathrm{CDCC}(-)*}_{(P_{nA})f} P_{nA}(U_{pA} - U^{P_{nA}}_{pA} + V_{pn} - V^{P_{nA}}_{pn}) = \Psi^{\mathrm{CDCC}(-)}_{(P_{nA})f} P_{nA}(U_{pA} + V_{pn})\,Q_{nA}$. The potential $P_{nA}(U_{pA} + V_{pn})\,Q_{nA}$ couples low orbital angular momenta l_{nA} with the large l_{nA} from the subspace Q_{nA}. Thus, the auxiliary term adds a model dependence because by taking into account this term we go beyond the limits of the model space constrained by the projector P_{nA}.

V. NUMERICAL RESULTS

In this section we present some calculations corroborating our theoretical findings, although the code for the surface integral formalism in the CDCC approach is not yet available and the work on it is in progress.

A. Stripping to bound state: Reaction ^{14}C$(d, p)^{15}$C$(2s_{1/2}, E_x = 0.0$ MeV$)$

First we present the effect of the auxiliary matrix element (45). To this end we performed calculations using the prior DWBA amplitude,

$$\mathcal{M}^{\mathrm{DW(prior)}} = \left\langle \chi^{(-)}_{pF}\,\varphi^{F}_{nA} \middle| U_{pA} + U_{nA} - U_{dA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle, \tag{107}$$

and the prior CDCC amplitude,

$$\mathcal{M}^{\mathrm{CDCC(prior)}} = \left\langle \Psi^{\mathrm{CDCC}(-)}_{f} \middle| U_{pA} + U_{nA} - U_{dA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle. \tag{108}$$

In both amplitudes, to calculate the initial distorted wave $\chi^{(+)}_{dA}$ we use the optical potential U_{dA} prescribed by the ADWA using the zero-range Johnson-Sopper prescription [26] in which the d-A optical potential U_{dA} is given by the sum $U_{pA} + U_{nA}$ taken at $r_{pn} = 0$ and at half of the deuteron incident energy. In all the calculations we use Koning-Delaroche potential [38] for the N-A optical potentials. We use the spectroscopic factor $S^F_A = 1$ for $n + {}^{14}$C configuration in the ground state of ^{15}C. By comparing the differential cross sections obtained using the complex U_{nA} and the real $U_{nA} = V^{\mathrm{sp}}_{nA}$ we can estimate the effect of the auxiliary terms

$$\mathcal{M}^{\mathrm{DW(prior)}}_{\mathrm{aux}} = \langle \chi^{(-)}_{pF}\,\varphi_{nA} | \mathrm{Im}U_{nA} | \varphi_{pn}\,\chi^{(+)}_{dA} \rangle \tag{109}$$

and

$$\mathcal{M}^{\mathrm{CDCC(prior)}}_{\mathrm{aux}} = \left\langle \Psi^{\mathrm{CDCC}(-)}_{f} \middle| \mathrm{Im}U_{nA} | \varphi_{pn}\,\chi^{(+)}_{dA} \right\rangle. \tag{110}$$

Clearly, our calculations give a rather qualitative estimation of the auxiliary term effect because when we change the U_{nA} in the transition operator we simultaneously change the distorted wave $\chi^{(+)}_{dA}$ in the initial state, while in the auxiliary amplitudes (109) and (110), as U_{nA} changes only the transition operator should change. Hence, our calculations overestimate the effect of the auxiliary term. The calculations are done for the ^{14}C$(d,p)^{15}$C$(2s_{1/2}, E_x = 0.0$ MeV$)$ at the deuteron energy of $E_d = 23.4$ MeV. The results are shown in Figs. 1 and 2.

The replacement in U_{dA} of the real potential V^{sp}_{nA} by the complex optical potential U_{nA} changes the differential cross section at forward angles by 10% for the DWBA and by 11% for the CDCC.

As we can see in Figs. 1 and 2 the replacement of U_{nA} with V^{sp}_{nA} makes very little effect on the differential cross section in the region of the first stripping peak, confirming that at low energies the contribution from the nuclear interior is small at forward angles but increases with angle increasing. Hence, at low energies the replacement of U_{nA} with V^{sp}_{nA} does not affect

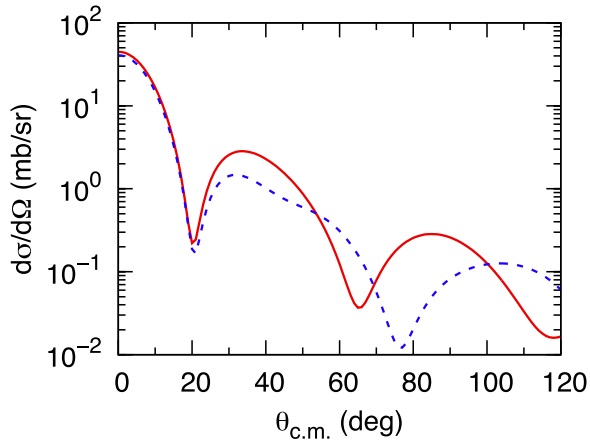

FIG. 1. (Color online) Prior DWBA differential cross sections for the ^{14}C(d, p)^{15}C($2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 23.4$ MeV. The solid red line is obtained using the optical potential U_{nA} when calculating U_{dA}; the blue dotted line is obtained with $U_{nA} = V_{nA}^{\rm sp}$ in U_{dA}.

the spectroscopic information, like ANCs or spectroscopic factors, which is extracted from the normalization of the calculated differential cross section to the experimental one in the first stripping peak.

Similar calculations for 60-MeV deuterons give quite different results. In Fig. 3 we present the prior DWBA differential cross sections for two different choices of the n-A potential used to calculate U_{dA}. As we see, the difference is quite significant but it comes mainly owing to the different initial distorted waves $\chi_{dA}^{(+)}$ generated by different U_{dA}. If for $E_d = 23.4$ MeV this difference was not important because the reaction was peripheral, it is not the case for 60 MeV, when the deuteron-stripping reaction is contributed also by the nuclear interior [39]. Unfortunately, we are not able to calculate the matrix element from ImU_{nA} without changing the initial distorted wave.

In the second type of calculations we compared the post and prior CDCC amplitudes for the ^{14}C$(d,p)^{15}$C$(2s_{1/2}, E_x =$ 0.0 MeV) reaction at the deuteron energy of $E_d = 23.4$ MeV.

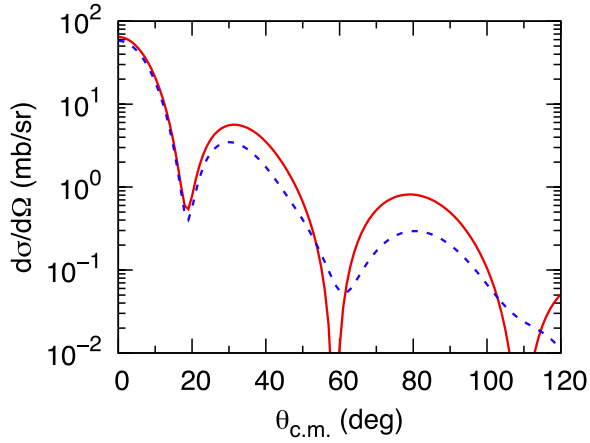

FIG. 2. (Color online) Prior CDCC differential cross sections for the ^{14}C(d, p)^{15}C($2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 23.4$ MeV. Notations are the same as in Fig. 1.

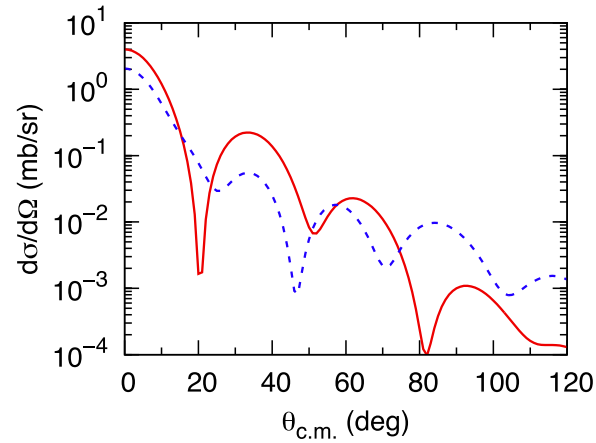

FIG. 3. (Color online) Prior DWBA differential cross sections for the ^{14}C(d, p)^{15}C($2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 60$ MeV. Notations are the same as in Fig. 1.

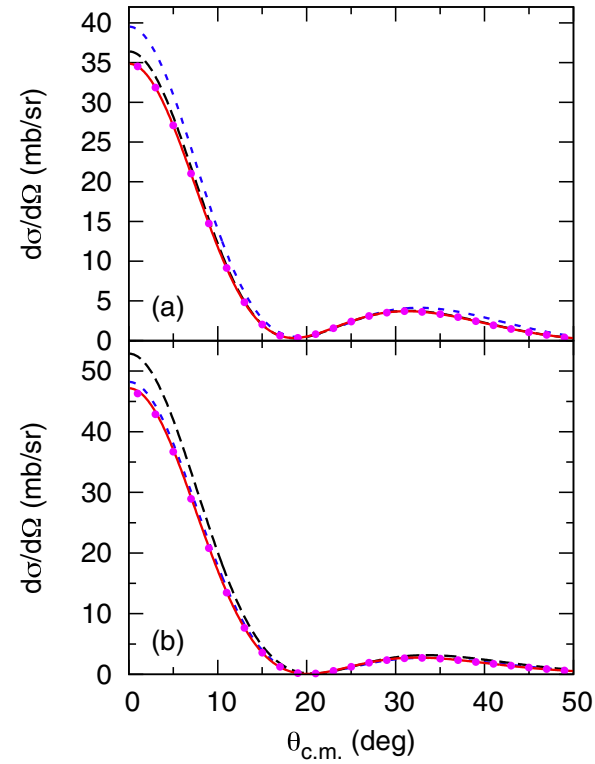

FIG. 4. (Color online) Post (a) and prior (b) CDCC differential cross sections for the ^{14}C(d, p)^{15}C($2s_{1/2}, E_x = 0.0$ MeV) at $E_d =$ 23.4 MeV. In the post form (a) the cutoff is introduced over the p-n partial waves in the continuum component of the initial CDCC scattering wave function: $l_{pn}^{\rm max} = 0$, red solid line; $l_{pn}^{\rm max} = 2$, black dashed line; $l_{pn}^{\rm max} = 4$, blue short dashed line; $l_{pn} = 6$, dots. In the prior form (b) the cutoff is introduced over the n-A partial waves in the continuum component of the final CDCC scattering wave function: $l_{nA}^{\rm max} = 1$, red solid line; $l_{nA}^{\rm max} = 2$, black dashed line; $l_{nA}^{\rm max} = 3$, blue short dashed line; $l_{nA} = 4$, dots.

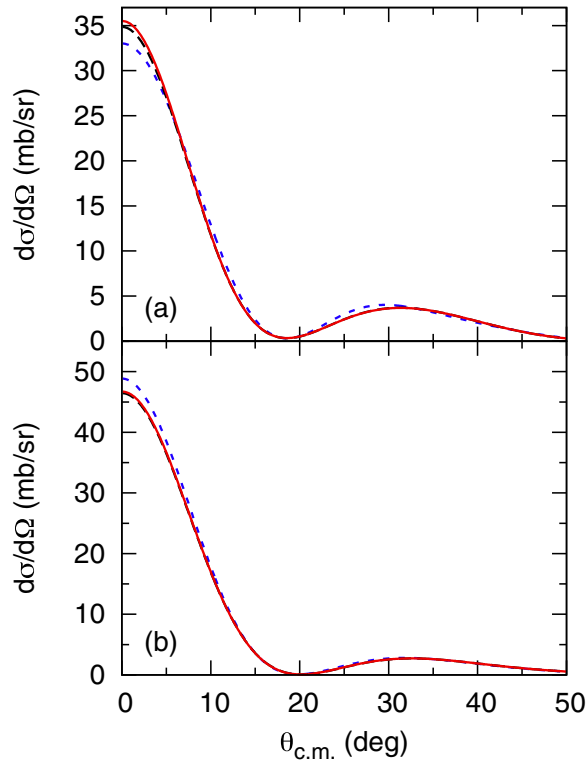

FIG. 5. (Color online) Convergence of the post (a) and prior (b) CDCC differential cross sections for the ^{14}C$(d, p)^{15}$C$(2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 23.4$ MeV. $R_{\rm match} = 20$ fm, red solid line; $R_{\rm match} = 30$ fm, black dashed line; $R_{\rm match} = 40$ fm, blue short dashed line.

In Fig. 4 we compare the dependence of the CDCC amplitudes on the maximum l_{pn} of the continuum p-n states in the post form and maximum l_{nA} of the continuum n-A states in the prior form. For both post and prior forms $l_{pn} = 4$ and $l_{nA} = 4$, correspondingly, are enough to achieve convergence.

Now in Fig. 5 we demonstrate the convergence of the post and prior CDCC differential cross sections for the ^{14}C$(d, p)^{15}$C$(2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 23.4$ MeV as functions of $\mathcal{R}_{dA}$ and $\mathcal{R}_{pF}$. In the FRESCO code this corresponds to parameter $R_{\rm match}$. The post form converges at $R_{\rm match} = 40$ fm, while the prior form converges at $R_{\rm match} = 30$ fm, although the post form has nonlocality range in the matrix element 9 fm versus 24 fm in the prior form. These calculations demonstrate that the integration volumes over $\boldsymbol{\rho}_{dA}$ and $\boldsymbol{\rho}_{pF}$ in the CDCC matrix elements are constrained.

In Fig. 6 we show the convergence of the post and prior CDCC differential cross sections as functions of r_{nA} for the ^{14}C$(d, p)^{15}$C$(2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 23.4$ MeV. To this end we calculated the post and prior CDCC differential cross sections in which the integration over r_{nA} was cut at the upper limit $r_{nA}^{\rm max}$. By increasing $r_{nA}^{\rm max}$ we can determine the convergence of the CDCC differential cross sections as functions of $r_{nA}^{\rm max}$. The convergence over r_{nA} is important because depending on $\mathbf{r}_{nA}$ the overlap function I_A^F is the only source of the spectroscopic information, which can be extracted from the deuteron-stripping reactions. In the case under consideration, owing to the small neutron binding energy

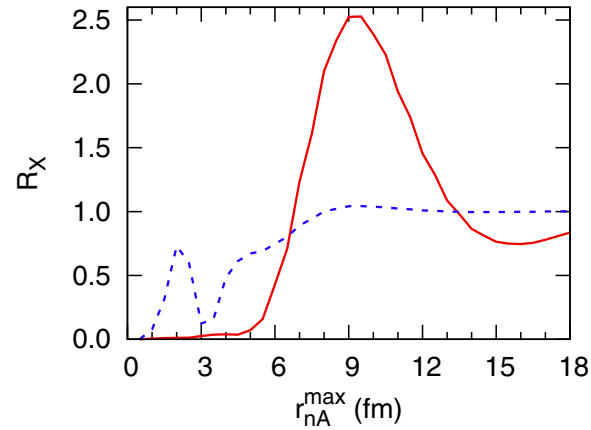

FIG. 6. (Color online) Dependence of the normalized post CDCC differential cross sections R_x on $r_{nA}^{\rm max}$ for the ^{14}C$(d, p)^{15}$C$(2s_{1/2}, E_x = 0.0$ MeV) at $E_d = 23.4$ MeV. R_x is the ratio of the peak CDCC differential cross section, in which the integral over r_{nA} is calculated up to $r_{nA}^{\rm max}$, to the full peak CDCC differential cross section calculated at $r_{nA}^{\rm max} \to \infty$. The solid red line is the normalized post CDCC form; the blue dotted line is the normalized prior CDCC fiorm.

$\varepsilon_{n\,^{14}{\rm C}}^{^{15}{\rm C}} = 1.218$ MeV in ^{15}C, we expect a very slow convergence of the CDCC matrix elements. Nevertheless, our calculations demonstrate that the prior form converges at $r_{nA} \approx 9$ fm, while the convergence of the post form is achieved at $r_{nA} > 20$ fm. This advantage of the prior form may be not decisive for the stripping to bound states but could be important for stripping to resonance states.

B. Stripping to resonance state: Reaction ^{16}O$(d, p)^{17}$O$(1d_{3/2})$

Now we proceed to the calculation of the stripping to a resonance state. We select the reaction ^{16}O$(d, p)^{17}$O$(1d_{3/2})$ at $E_d = 36$ MeV populating a resonance state of energy $E_x = 5.085$ MeV, which corresponds to the resonance level at 0.94 MeV. In all the calculations shown below we use the single-particle approach for the n-A resonant scattering wave function calculated in the Woods-Saxon potential with the radial parameter $r_0 = 1.25$ fm and diffuseness $a = 0.65$ fm.

In the first calculation we compare the post and prior calculations following the procedure developed in Ref. [5]. The post and prior ADWA and prior CCBA (coupled-channel Born approximation) are used for comparison. The prior ADWA is the standard prior DWBA in which the initial deuteron potential is given by the sum of the optical U_{PA} and U_{nA} potentials calculated at half of the deuteron energy using the zero-range Johnson-Sopper prescription [26]. In the prior CCBA the final-state wave function can be derived from Eq. (82). To do it we use the partial wave expansion of the binned n-A continuum scattering wave function leaving only the resonance partial wave $l_{nA} = 2$. The adopted bin covers the resonance region and $\chi^{({\rm res})(-)}_{\mathbf{q}_{pF}(\mathbf{k}_{nA})}(\boldsymbol{\rho}_{pF})$ corresponding to the resonance bin has asymptotically both incident and outgoing waves. The continuum resonance wave function component is coupled with two bound states in ^{17}O: the ground state $1d_{5/2}$

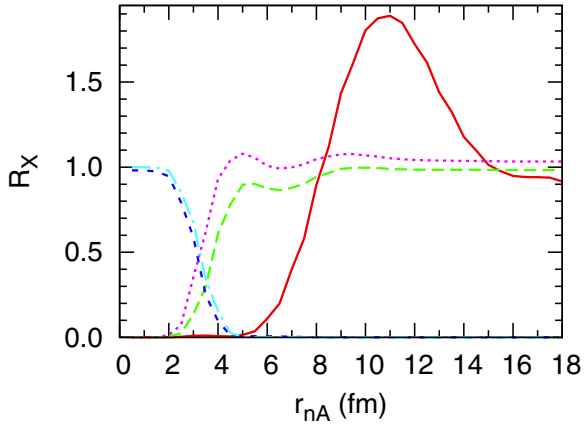

FIG. 7. (Color online) Dependence of the normalized ADWA and CCBA differential cross sections R_x on r_{nA} for the deuteron stripping to resonance $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}(1d_{3/2})$ at $E_d = 36$ MeV. Blue short and long dash-dotted lines, the ratios R_X of the peak prior ADWA and CCBA differential cross sections, correspondingly, in which the radial integral over r_{nA} is calculated for $r_{nA} \geqslant r_{nA}^{\min}$, to the full differential cross section. Similarly, magenta dotted and green dashed lines are the ratios R_X of the peak prior ADWA and CCBA differential cross sections, correspondingly, in which the radial integral over r_{nA} is calculated in the interval $0 \leqslant r_{nA} \leqslant r_{nA}^{\max}$, to the full differential cross section. The red solid line is the R_X dependence on $r_{nA}^{\max}$ calculated for the post ADWA form. Hence, r_{nA} on the abscissa is $r_{nA}^{\min}$ for the blue short and long dashed lines and $r_{nA}^{\max}$ for the dotted magenta, dashed green, and solid red lines.

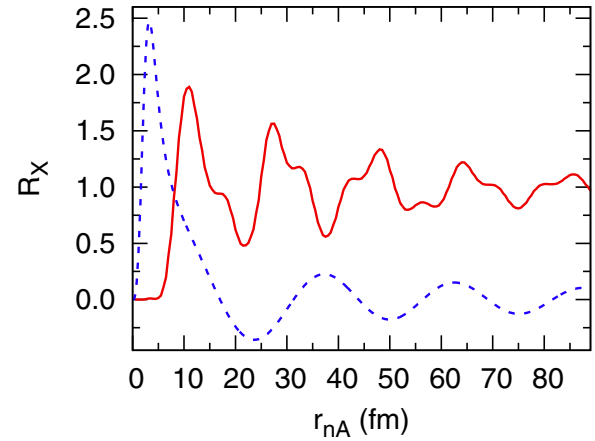

FIG. 8. (Color online) Solid red line, dependence on $r_{nA}^{\max}$ of the normalized post ADWA differential cross section R_x for the stripping to resonance $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}(1d_{3/2})$ at $E_d = 36$ MeV. R_X is calculated as the ratio of the ADWA differential cross section, in which the radial integral over r_{nA} is calculated for $0 \leqslant r_{nA} \leqslant r_{nA}^{\max}$, to the full differential cross section. At each $r_{nA}^{\max}$ the peak value of the differential cross section is used. Blue dotted line, dependence on r_{nA} of the binned radial resonant scattering wave function $\overline{\psi}^{(\mathrm{res})}_{k_{nA}\, s=1/2\, l_{nA}=2\, J_F=3/2}(r_{nA})$.

and the first excited state $2s_{1/2}$. These terms are given by the sum over $i = 0, 1$ in Eq. (82). Thus, schematically we can write the final-state wave function in CCBA as

$$\Psi_f^{\mathrm{CDCC}(-)}(\boldsymbol{\rho}_{pF}, \mathbf{r}_{nA}) = \varphi_{nA}^{(0)}(\mathbf{r}_{nA})\, \chi_{\mathbf{q}_{pF}}^{(0)(-)}(\boldsymbol{\rho}_{pF}) + \varphi_{nA}^{(1)}(\mathbf{r}_{nA})\, \chi_{\mathbf{q}_{pF}}^{(1)(-)}(\boldsymbol{\rho}_{pF}) + \overline{\psi}^{(\mathrm{res})(-)}_{\mathbf{k}_{nA},\, l_{nA}=3}(\mathbf{r}_{nA})\, \chi^{(\mathrm{res})(-)}_{\mathbf{q}_{pF}(\mathbf{k}_{nA})}(\boldsymbol{\rho}_{pF}). \quad (111)$$

Here, for simplicity, we omitted spins. The radial and momentum spherical harmonics are absorbed into $\overline{\psi}^{(\mathrm{res})(-)}_{\mathbf{k}_{nA}}(\mathbf{r}_{nA})$. The distorted waves $\chi^{(0)(-)}_{\mathbf{q}_{pF}}(\boldsymbol{\rho}_{pF})$ and $\chi^{(1)(-)}_{\mathbf{q}_{pF}}(\boldsymbol{\rho}_{pF})$ have only outgoing waves.

The results of the calculations are shown in Fig. 7. Dependence of the peak value of the normalized differential cross section R_X on the $r_{nA}^{\min}$ [blue short (ADWA) and long dashed-dotted (CCBA) lines] shows that the prior form converges pretty fast, being dominantly contributed by the region $r_{nA} \lesssim 5$ fm with following up small oscillations at larger $r_{nA}^{\min}$. These small oscillations are better exposed on the ADWA and CCBA lines, which show the dependence of the corresponding normalized cross section on $r_{nA}^{\max}$. These oscillations practically disappear for $r_{nA}^{\max} > 10$ fm; that is, the prior form converges at $r_{nA}^{\max} = \mathcal{R}_{nA} = 10$ fm. Because in both ADWA and CCBA calculations the ADWA prescriptions were used, the difference between both methods determines the effect of the coupling of the continuum resonant wave function in the final state with two bound states. As we see, this effect is not significant.

Meanwhile, the post form (solid red line) does not converge at much larger $r_{nA}^{\max}$ sustaining significant oscillations even at $r_{nA}^{\max} > 20$ fm. To demonstrate a poor convergence of the post form in Fig. 8, we show the oscillation of the post ADWA normalized differential cross section R_X as a function of $r_{nA}^{\max}$ (red solid line). For comparison we show also the oscillation of the binned (the bin size is 1 MeV) resonant scattering waver function. As we see, the oscillation of R_X is caused by the oscillation of the resonant scattering wave function. Hence, the prior form has an evident advantage over the post one when dealing with the stripping to resonance.

In Fig. 9 the angular distributions for the reaction $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}(1d_{3/2})$ at $E_d = 36$ using prior DWBA, ADWA, and CCBA are shown. The CCBA, as explained above, takes into account the coupling of the final resonant scattering wave function with the ground and first excited states in ^{17}O. As we can see the effect of coupling with the bound states has little effect on the angular distributions. In the single-particle potential approach for the resonant scattering wave function the normalization of the theoretical cross section to the experimental one determines the spectroscopic factor; see Eq. (94). From the normalization of the calculated differential cross sections we determined the spectroscopic factors: SF = 0.89 for the DWBA, SF = 0.66 for the ADWA, and SF = 0.73 for the CCBA. Using the single-particle neutron partial resonance width $\Gamma_{\mathrm{sp}} = 128$ keV, we get for the observable neutron widths $\Gamma_n = 113.9$ keV for the DWBA, $\Gamma_n = 84.5$ keV for the ADWA, and $\Gamma_n = 93.4$ keV for the CCBA. The observed experimental value is $\Gamma_n = 96 \pm 5$ keV. Thus, the prior CCBA and ADWA can be used to determine the observable partial resonance widths.

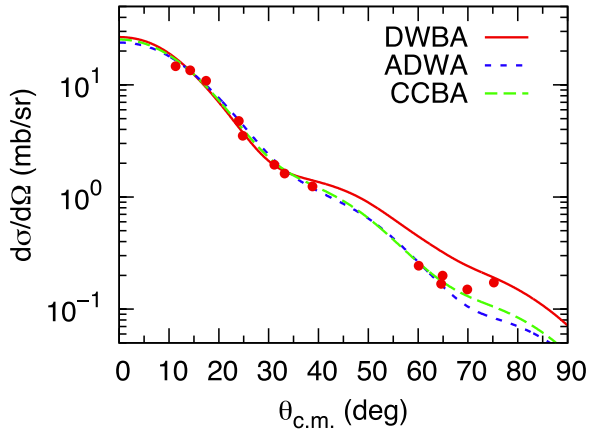

FIG. 9. (Color online) Angular distributions for the deuteron stripping to resonance ^{16}O$(d, p)^{17}$O$(1d_{3/2})$ at $E_d = 36$ MeV. The red solid line is the DWBA, the blue short dashed line is the ADWA, and the green dashed line is the CCBA. All the angular distributions are normalized in the region of the forward peak to the experimental one, red dots [40].

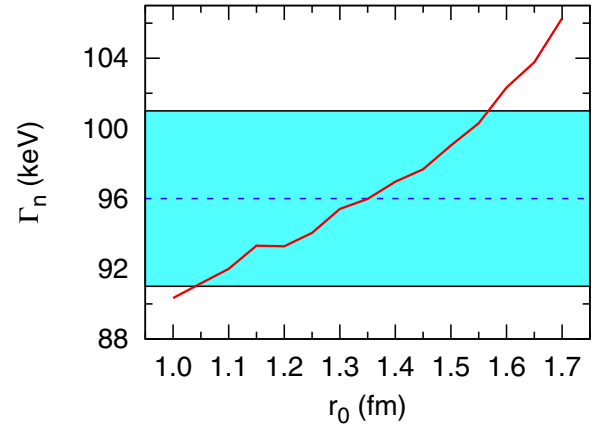

FIG. 11. (Color online) Solid red line- dependence on r_0 of the neutron resonance width extracted from the CCBA calculations of the ^{16}O$(d, p)^{17}$O$(1d_{3/2})$ reaction at $E_d = 36$ MeV. The blue dashed line is the experimental neutron resonance width of the $1d_{3/2}$ resonance in ^{17}O and the blue strip is the resonance width's experimental uncertainty.

Until now we have not discussed the impact of the resonant bin width. In all the calculations shown above we used the bin width of 1 MeV. To check the impact of the bin width, we performed prior CCBA calculations with three different bin widths. The results are shown in Fig. 10. The difference in the normalization of the CCBA calculated differential cross sections at 1 and 0.8 MeV is only 3.7%.

In our final calculations presented in Fig. 11 we check the dependence of the extracted neutron resonance width on the radius r_0 of the n-A Woods-Saxon potential, which supports the resonance state $1d_{3/2}$. This test is important for corroboration of our theoretical findings and shows how peripheral the deuteron stripping to resonance is. At each $1.0 \leqslant r_0 \leqslant 1.7$ we calculated the CCBA differential cross section, normalized it to the experimental one in the stripping peak in the angular distribution and determined the spectroscopic factor, which is the normalization factor. For each r_0 from the derivative of the calculated scattering phase shift we determine the single-particle neutron resonance width and, multiplying it by the determined spectroscopic factor, we find the observable resonance width shown in Fig. 11. As we can see the determined neutron resonance width Γ_n varies with variation of r_0 in the realistic interval 1.0–1.6 fm by $\pm 7\%$ from the experimental value of 96 keV.

The reaction is not peripheral and this is clearly demonstrated by the r_0 dependence of Γ_n. In the case of the completely peripheral reaction the extracted Γ_n should show none or a very little dependence on r_0. From Fig. 11 we can determine the radial parameter $r_0 = 1.35$ fm at which the extracted width coincides with the experimental one. In Fig. 12 we show the r_0 dependence of the spectroscopic factor. Clearly, the dependence on r_0 of the spectroscopic factor is much stronger than for Γ_n. Taking into account that at $r_0 = 1.35$ fm

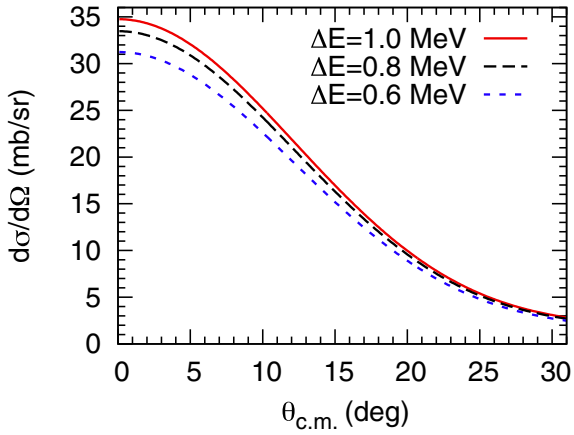

FIG. 10. (Color online) Angular distributions for the deuteron stripping to resonance ^{16}O$(d, p)^{17}$O$(1d_{3/2})$ at $E_d = 36$ MeV calculated using prior CCBA for three different bins: 1 MeV, red solid line; 0.8 MeV, dashed black line; 0.6 MeV, short dashed blue line.

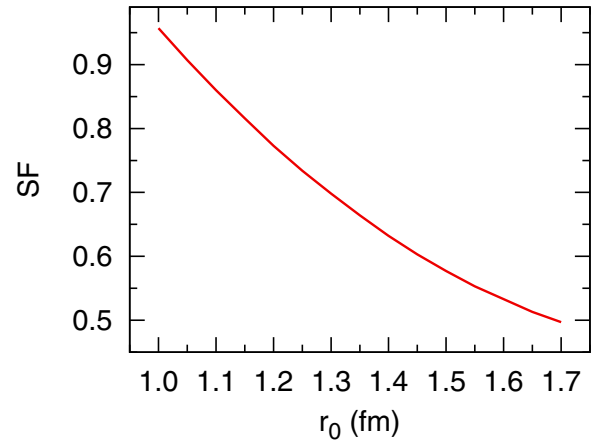

FIG. 12. (Color online) Solid red line- dependence on r_0 of the spectroscopic factor extracted from the CCBA calculations of the ^{16}O$(d, p)^{17}$O$(1d_{3/2})$ reaction at $E_d = 36$ MeV.

the calculated Γ_n coincides with the experimental one we can determine the spectroscopic factor to be $S_A^F = 0.66^{+0.25}_{-0.1}$.

VI. SUMMARY

The goal of this paper was to develop a theory of the deuteron stripping to resonances based on the surface-integral formalism. First we demonstrated how the surface-integral formalism worked for the deuteron stripping to bound states in the three-body model and then we considered a more realistic problem in which a composite structure of target nuclei is taken into account via optical potentials. We explored different choices of channel wave functions and transition operators and showed that the conventional CDCC volume matrix element can be written in terms of the surface-integral matrix element, which is peripheral, and the auxiliary matrix element, which determines the contribution of the nuclear interior over the variable r_{nA}. This auxiliary matrix element appears owing to the inconsistency in treating of the n-A potential: This potential should be real in the final state to support bound states or resonance scattering and complex in the initial state to describe n-A scattering.

Our main result is a formulation of the theory of the stripping to resonance states using the prior form of the surface-integral formalism and the CDCC method. It is demonstrated that the conventional CDCC volume matrix element coincides with the surface matrix element, which converges for the stripping to the resonance state. Also the surface representation (over the variable $\mathbf{r}_{nA}$) of the stripping matrix element enhances the peripheral part of the amplitude although the internal contribution does not disappear and increases with increasing deuteron energy.

Although the code for the surface-integral formalism in the CDCC approach is not yet available, we presented many calculations corroborating our findings both for the stripping to the bound state and the resonance. For the stripping to the bound state we use ^{14}C$(d, p)^{15}$C at 23.4 and 60 MeV of the deuteron incident energy. It is shown how the contribution of the auxiliary term changes with energy. For the stripping to resonance state we explore the ^{16}O$(d, p)^{17}$O$(1d_{3/2})$ reaction at $E_d = 36$ MeV. Because the CDCC code for stripping to resonance is not yet available we use the CCBA and demonstrate that the prior form converges, while the post form oscillates even at large distances. We demonstrate how the resonance width can be extracted from the analysis of the deuteron stripping to the resonance state.

ACKNOWLEDGMENTS

C.A.B. and A.M.M. acknowledge that this material is based upon their work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Science. C.B.A. is supported by Awards No. DE-SC004971 and No. DE-FG02-08ER41533. A.M.M. is supported by Awards No. DE-FG02-93ER40773 and No. DE-SC0004958. A.M.M. also acknowledges the support by the DOE-NNSA under Award No. DE-FG52-09NA29467. C.A.B. and A.M.M. also acknowledge the support by the U.S. National Science Foundation under Award No. PHY- 1415656. A.S.K. acknowledges the support by the Australian Research Council. D.Y.P. appreciates the warm hospitality he received during his visits to the Cyclotron Institute, Texas A&M University. D.Y.P. gratefully acknowledges the supports of the National Natural Science Foundation of China (Grants No. 11275018 and No. 11035001) and the Chinese Scholarship Council (Grant No. 201303070253).

APPENDIX: POST-PRIOR DWBA DISCREPANCY OWING TO THE n-A POTENTIAL INCONSISTENCY

Here we show how the inconsistency in the treatment of the n-A potential leads to the post-prior discrepancy of the DWBA amplitude. To this end we start from the post DWBA amplitude,

$$\mathcal{M}^{\mathrm{DW(post)}} = \langle \chi^{(-)*}_{pF}\, \varphi_{nA} | \Delta\, V_{pF} | \varphi_{pn}\, \chi^{(+)}_{dA} \rangle, \tag{A1}$$

and derive from it the prior DWBA form. Here

$$\Delta V_{pF} = U_{pA} + V_{pn} - U_{pF} \tag{A2}$$

is the potential transition operator in the post form. Let us take into account Schrödinger equations for the initial- and final-channel wave functions,

$$(E - K - V_{pn} - U_{dA})\, \varphi_{pn}\, \chi^{(+)}_{dA} = 0 \tag{A3}$$

and

$$\left(E - K - V^{\mathrm{sp}}_{nA} - U_{pF}\right) \varphi_{nA}\, \chi^{(-)*}_{pF} = 0. \tag{A4}$$

Then Eq. (A1) can be transformed into

$$\begin{aligned}
\mathcal{M}^{\mathrm{DW(post)}} &= \left\langle \chi^{(-)*}_{pF}\, I^F_A \middle| \Delta\, V_{pF} | \varphi_{pn}\, \chi^{(+)}_{dA} \right\rangle \\
&= \left\langle \chi^{(-)*}_{pF}\, I^F_A \middle| U_{pA} + V^{\mathrm{sp}}_{nA} - U_{dA} - \left[V^{\mathrm{sp}}_{nA} + U_{pF}\right] + [V_{pn} + U_{dA}] | \varphi_{pn}\, \chi^{(+)}_{dA} \right\rangle \\
&= \left\langle \chi^{(-)*}_{pF}\, I^F_A \middle| U_{pA} + V^{\mathrm{sp}}_{nA} - U_{dA} + \left[E - \overrightarrow{K} - V^{\mathrm{sp}}_{nA} - U_{pF}\right] | \varphi_{pn}\, \chi^{(+)}_{dA} \right\rangle \\
&= \left\langle \chi^{(-)*}_{pF}\, I^F_A \middle| U_{pA} + V^{\mathrm{sp}}_{nA} - U_{dA} + \left[E - \overleftarrow{K} - V^{\mathrm{sp}}_{nA} - U_{pF}\right] | \varphi_{pn}\, \chi^{(+)}_{dA} \right\rangle \\
&= \left\langle \chi^{(-)*}_{pF}\, I^F_A \middle| U_{pA} + V^{\mathrm{sp}}_{nA} - U_{dA} | \varphi_{pn}\, \chi^{(+)}_{dA} \right\rangle \\
&= \mathcal{M}^{\mathrm{DW(prior)}}.
\end{aligned} \tag{A5}$$

Here we took into account that the bracketed operators are the potentials in Eqs. (A3) and (A4). Also because the matrix element contains φ_{pn} and φ_{nA} the kinetic-energy operator $\overrightarrow{K}$ can be transformed into $\overleftarrow{K}$. Thus, if we use the real $V^{\rm sp}_{nA}$ potential, which generates the final bound state ($n\,A$), as the n-A potential in the transition operator in the prior DWBA amplitude, the post and prior DWBA amplitudes coincide. We note that in the proof of the equality of the post and prior forms we used the same $V^{\rm sp}_{nA}$ potential both in the Schrödinger equation for the final-state bound-state wave function and in the transition operator of the prior form. However, the often-used global optical potential U_{dA} is contributed by both U_{pA} and U_{nA} optical potentials. Similarly, in the ADWA U_{dA} is given by the sum of $U_{pA} + U_{nA}$ with the n-A optical potentials taken at half deuteron energy. If we adopt U_{nA} in the prior form transition operator rather than $V^{\rm sp}_{nA}$, then the post and prior form DWBA amplitudes differ by the auxiliary amplitude

$$\mathcal{M}^{\rm DW(post)} = \mathcal{M}'^{\rm DW(prior)} - \mathcal{M}^{\rm DW}_{\rm aux}, \tag{A6}$$

where the prior DWBA amplitude is given now by

$$\mathcal{M}'^{\rm DW(prior)} = \langle \chi^{(-)}_{pF}\, I^F_A \big| U_{pA} + U_{nA} - U_{dA} | \varphi_{pn}\, \chi^{(+)}_{dA} \rangle \tag{A7}$$

and

$$\mathcal{M}^{\rm DW(prior)}_{\rm aux} = \langle \chi^{(-)}_{pF}\, I^F_A \big| U_{nA} - V^{\rm sp}_{nA} | \varphi_{pn}\, \chi^{(+)}_{dA} \rangle. \tag{A8}$$

In a modified prior DWBA amplitude the transition operator contains the optical potential U_{nA} rather than the real potential $V^{\rm sp}_{nA}$ in the conventional prior form (A5). Thus, the post and prior DWBA amplitudes differ if we replace $V^{\rm sp}_{nA}$ with U_{nA} in the transition operator of the prior form, meaning that the inconsistency in the treatment of the n-A potential leads to the post-prior discrepancy. If we adopt $\mathrm{Re}\, U_{nA} = V^{\rm sp}_{nA}$, then

$$\mathcal{M}^{\rm DW(prior)}_{\rm aux} = \langle \chi^{(-)}_{pF}\, I^F_A \big| \mathrm{Im}\, U_{nA} | \varphi_{pn}\, \chi^{(+)}_{dA} \rangle. \tag{A9}$$

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