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Self-consistent Hartree-Fock and RPA Green's function method indicate no pygmy resonance in the monopole response of neutron-rich Ni isotopes

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The random phase approximation (RPA) monopole strength and the unperturbed particle-hole excitation strength are studied, in which the strength in the continuum is properly treated without discretizing unbound particle spectra. The model is the self-consistent Hartree-Fock calculation and the RPA Green's function method with Skyrme interactions. Numerical examples are the Ni isotopes, especially $^{68}_{28}\mathrm{Ni}_{40}$, in which an experimental observation of a low-lying peak with an appreciable amount of monopole strength at 12.9 ± 1.0 MeV was recently reported. In the present study it is concluded that sharp monopole peaks with the width of the order of 1 MeV can hardly be expected for ^{68}Ni in that energy region. Instead, a broad shoulder of monopole strength consisting of neutron excitations to nonresonant one-particle states (called "threshold strength") with relatively low angular momenta (ℓ, j) is obtained in the continuum energy region above the particle threshold, which is considerably lower than that of the isoscalar giant monopole resonance. In the case of monopole excitations of ^{68}Ni there are no unperturbed particle-hole states below 20 MeV, in which the particle expresses a neutron (or proton) resonant state. It is emphasized that in the theoretical estimate a proper treatment of the continuum is extremely important.

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Using inelastic α scattering at $50A$ MeV on the unstable nucleus ^{68}Ni in inverse kinematics Vandebrouck *et al.* recently observed [1] a soft monopole mode at 12.9 ± 1.0 MeV, in addition to the isoscalar giant monopole resonance (ISGMR), of which the centroid is placed at 21.1 ± 1.9 MeV. This observation is the first one in which the low-lying monopole peak so-called "pigmy resonance" is reported. The soft monopole mode reported in Ref. [1] was said to be approximately in agreement with the result of the calculation of Ref. [2], in which the random phase approximation (RPA) calculation is performed based on the discretized particle spectra. First of all, it is our impression that though the experimental result reported in Ref. [1] is very exciting, the observation of the low-energy mode in the monopole response function must be further confirmed by additional experiments in the future. This is because the angular distribution of the 12.9 MeV peak for the forward angle, $\theta_{c.m.} < 5^\circ$, is not reported, which is needed to identify exclusively the monopole strength. Second, in the case of monopole excitations of ^{68}Ni it is difficult for us to imagine the unperturbed discrete particle-hole (p-h) excitations below 20 MeV, which can be properly used as the basis for the RPA calculation. Consequently, it is hard to expect that some monopole peaks with the widths of the order of 1 MeV appear in the low-energy region. The RPA calculation, which is performed using positive-energy particle states obtained by discretizing particle spectra in the continuum, is not acceptable, because the energies as well as wave functions of such particle states have, in general, no correspondence to those of one-particle resonant states.

In halo nuclei, in which the wave function of very weakly bound $\ell = 0$ or 1 nucleon(s) extends largely to the outside of the core nucleus, the threshold strength that comes from the excitation of such bound nucleon(s) to nonresonant one-particle states may appear as a relatively sharp peak slightly above the particle threshold [3]. In contrast, in nuclei such as ^{68}Ni, of which the neutron separation energy is 7.8 MeV, the shoulder originating from the threshold strength is generally broad and the width may be the order of several MeV or larger. Consequently, the strength may be difficult to experimentally separate from the background strength.

It may sometimes occur especially for neutrons with small orbital angular-momenta ℓ that no one-particle resonant states are available, to which those neutrons are excited by a given low-multipole excitation operator. In such cases the entire p-h strength for the neutrons in a given hole orbit would appear as threshold strength [4]. One such example is the isoscalar (compression) dipole strength of the neutron-drip-line nucleus ^{22}C [5], in which the major strength should appear as the threshold strength with a low-energy broad peak instead of an easily recognizable giant resonance in a high-energy region. The only way to reliably calculate the response functions in such cases is to treat the continuum as it is.

More than 15 years ago we studied response functions of various types (isoscalar and isovector multipole modes, charge exchange modes, and spin-dependent modes) in unstable nuclei, using the Hartree-Fock (HF) plus the self-consistent RPA with Skyrme interactions, which is solved in coordinate space using the Green's function method. One of the characteristic features of the response functions in neutron-rich nuclei is the possible appearance of an appreciable amount of low-energy neutron threshold strength, which is not related to a resonant behavior of unbound single-particle states. For a given bound neutron with the binding energy ε_B the relevant threshold strength starts to appear at the excitation energy (E_x) equal to ε_B. The strength of excitations to neutron orbits with ℓ rises as the $\ell + \frac{1}{2}$ power of available energies, $(E_x - \varepsilon_B)^{\ell+1/2}$ [6], which rises more steeply for lower ℓ values just above the threshold energy [3,7]. The shape of the threshold strength is discussed in, for example, Ref. [8]. One of the well-known examples of the threshold strength is the sharp and strong low-energy peak of the dipole strength in halo nuclei, which is detected, for example, by Coulomb breakup reactions of halo nuclei [9].

Since the model used in the present paper is the same as that used in our paper in 1997 [10], here we write only the necessary points. Neglecting the pair correlation, the self-consistent HF + RPA calcurations with the Skyrme interactions are performed in the coordinate system, and the strength distributions $S(E)$ are obtained from the imaginary part of the RPA Green's function, G_{RPA}, as

$$S(E) = \sum_n |\langle n \mid Q \mid 0\rangle|^2 \, \delta(E - E_n)$$

$$= \frac{1}{\pi} \mathrm{Im}\,\mathrm{Tr}[Q^\dagger(\vec{r})\, G_{\mathrm{RPA}}(\vec{r};\vec{r}\,';E)\, Q(\vec{r}\,')]. \quad (1)$$

where Q expresses one-body operators

$$Q^{\lambda=0,\,\tau=0} = \frac{1}{\sqrt{4\pi}} \sum_i r_i^2 \text{ for isoscalar monopole strength.} \quad (2)$$

In Fig. 1 the monopole strength of the nucleus $^{68}_{28}\mathrm{Ni}_{40}$ is shown, which is calculated using the SLy4 interaction. The neutron separation energy S_n is experimentally 7.79 MeV, while 8.92 MeV for the SLy4 interaction, which is equal to the binding energy of the occupied least-bound neutrons in the $1f_{5/2}$ orbit. The calculated energies of neutrons in $2p_{1/2}$, $2p_{3/2}$, and $1f_{7/2}$ orbits are −9.25, −11.39, and −16.45 MeV, respectively. The proton separation energy S_p is experimentally 15.43 MeV, while 14.9 MeV for the SLy4 interaction, which is equal to the binding energy of the occupied least-bound protons in the $1f_{7/2}$ orbit. In Fig. 1(a) it is seen that the RPA strength increases monotonically from the particle threshold to the ISGMR peak around 21 MeV, in contrast to Fig. 1 of Ref. [2]. It is also seen that the low-energy threshold strength is basically of single-particle nature and obtains only a minor influence by particle-hole correlations. In Fig. 1(b) the neutron unperturbed monopole strengths, which contribute appreciably to the total unperturbed strength below 20 MeV shown in Fig. 1(a), are shown for respective occupied hole orbits. In the monopole case the (ℓ, j) value of particle states must be the same as that of respective hole orbits, and none of the neutron one-particle resonances with $3p_{1/2}$, $2f_{5/2}$, $3p_{3/2}$, and $2f_{7/2}$ exist. Consequently, the entire monopole strengths related to those neutrons in the hole orbits appear as respective threshold strengths in Fig. 1(b).

No smearing is made in our calculated strength in Fig. 1, while smearing is already made in Fig. 1(a) of Ref. [2]. The reason why the "pigmy resonance" is obtained around 15 MeV in Ref. [2] is explained by the sentence in Ref. [2]: "Because of the neutron excess in ^{68}Ni, the $3p2f$ states are located close to the Fermi level and, moreover, the $2p1f$ shell is calculated just below the Fermi level." In order to make the above sentence meaningful, we would naturally interpret that their $3p2f$ states are supposed to approximately represent respective

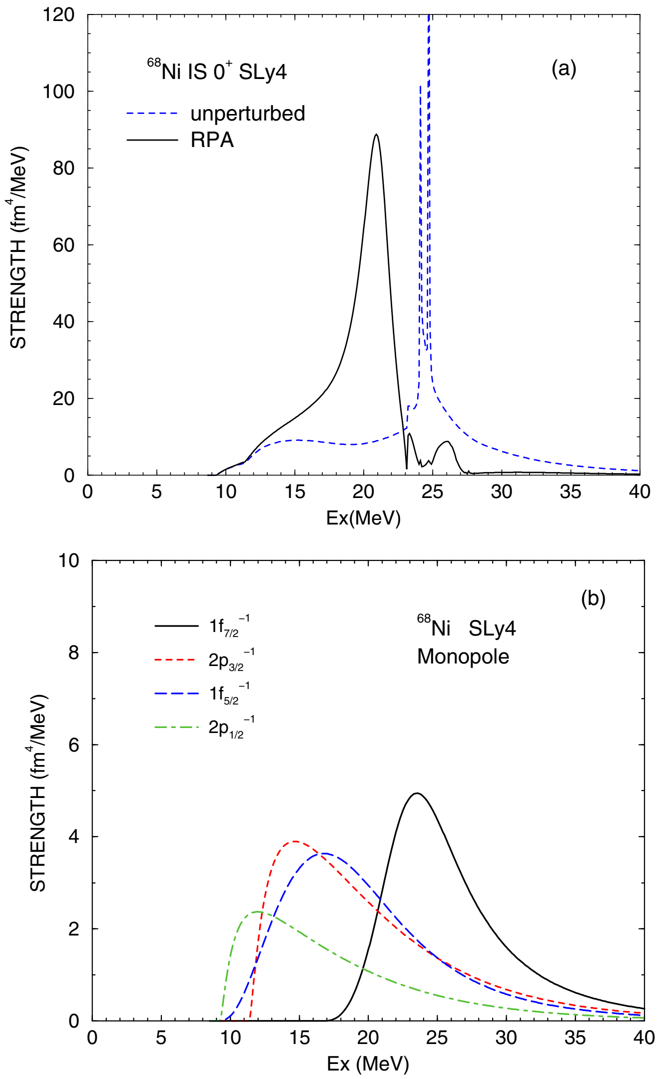

FIG. 1. (Color online) Monopole strength function (1) of ^{68}Ni as a function of excitation energy. The SLy4 interaction is used consistently both in the HF and the RPA calculations. (a) Unperturbed (neutron plus proton) monopole strength and isoscalar monopole RPA strength. The RPA strength denoted by the solid curve includes all possible strengths due to the coupling between bound and unbound states in RPA. On the other hand, in the unperturbed response denoted by the short-dashed curve the p-h strengths, in which both particle and hole orbits are bound, are not included because of the different dimensions of the strength. The energies of those unperturbed p-h excitations are the $1d_{5/2} \rightarrow 2d_{5/2}$ excitation at 27.60 MeV for neutrons and the excitations of $1p_{3/2} \rightarrow 2p_{3/2}$ at 27.58 MeV and $1p_{1/2} \rightarrow 2p_{1/2}$ at 27.46 MeV for protons. In addition, the proton excitation at 27.3 MeV from the bound $1d_{5/2}$ orbit to the one-particle resonant $2d_{5/2}$ orbit has such a narrow width that the strength is not plotted. The narrow peaks at 24.1 and 24.7 MeV in the unperturbed strength curve are the proton $2s_{1/2} \rightarrow s_{1/2}$ and $1d_{3/2} \rightarrow 2d_{3/2}$ excitations, respectively. ISGMR peak is obtained at slightly less than 21 MeV. (b) Some unperturbed neutron threshold strengths, which contribute appreciably to the total unperturbed strength below the energy of ISGMR in Fig. 1(a), are shown for respective occupied hole orbits, $(1f_{7/2})^{-1}$, $(2p_{3/2})^{-1}$, $(1f_{5/2})^{-1}$, and $(2p_{1/2})^{-1}$. Below 15 MeV the sum of the four curves shown is exactly equal to the short-dashed curve in Fig. 1(a).

one-particle resonant states. However, we find no such $3p2f$ one-particle resonant states in the continuum spectra.

If pair correlation is negligible and one discretizes the unbound particle spectra by using an available discretization method, such as confining the system in a finite box or expanding the wave functions in terms of a finite number of harmonic-oscillator bases [11,12], one may obtain, for example, only one discrete state with a given (ℓj) in the energy range of 5 MeV above the threshold of respective (ℓj) particles. When the size of the finite box increases or the number of bases increases, the number of the given (ℓj) particle states in the same energy range increases and the energy of the lowest-lying discretized state decreases. As the number of bases approaches infinite, the shape of the total strength coming from many discretized particle states with the given (ℓj) may approach the shape of the curve shown in Fig. 1(b). The consideration above explains that the lowest-lying unbound particle states obtained by discretizing the continuum with an easily manageable finite number of bases can be neither similar to one-particle resonant states nor the proper representatives of threshold strength. In short, great caution must be taken against using discretized particle states as the constituents of continuum particle states in RPA.

Using the definition of one-particle resonant energy, at which the phase shift increases through $\pi/2$ as the energy increases, for the present HF potential of ^{68}Ni with the SLy4 interaction we obtain one-particle energy of the $2d_{5/2}$ neutron at -0.71 MeV and one-particle resonant energy of the $2d_{3/2}$ neutron at $+1.13$ MeV (with the width of 0.54 MeV). Thus, the calculated spin-orbit splitting for $2d$ neutrons is $1.13-(-0.71) = 1.84$ MeV. Similarly, one-particle resonant energies of the $2d_{5/2}$ and $2d_{3/2}$ protons are obtained at $+1.27$ and $+3.23$ MeV, respectively, with negligible widths. Thus, the calculated spin-orbit splitting for $2d$ protons is $3.23-1.27 = 1.96$ MeV. As seen from these numerical examples, spin-orbit splittings of one-particle resonant levels do not approach zero, though they are reduced compared with spin-orbit splittings of deeply bound one-particle states. In contrast, the following sentence in Ref. [11], "This expected vanishing of the spin-orbit splitting for unbound states," was made based on the discretized continuum particle spectra. We are afraid that "unbound states" in the sentence correspond to some of the infinite number of continuum states. Furthermore, the statement in Ref. [1] "the observation of the soft monopole mode could bring valuable information on spin-orbit splitting" was presumably made based on the numerical results in Ref. [11]. It is difficult for us to find a sensible meaning in the statement because of the nonresonant nature of the monopole strength below the ISGMR.

In Fig. 2 the monopole strength of the nucleus $^{78}_{28}$Ni$_{50}$ is shown, which is calculated using the SLy4 interaction. The neutron separation energy is not known experimentally, while 5.88 MeV for the SLy4 interaction, which is equal to the binding energy of the occupied least-bound neutrons in the $1g_{9/2}$ orbit. The calculated energies of neutrons in $2p_{1/2}$, $1f_{5/2}$, $2p_{3/2}$, and $1f_{7/2}$ orbits are -10.11, -10.71, -12.04, and -17.48 MeV, respectively. The proton separation energy is not known experimentally, while 20.8 MeV for the SLy4 interaction, which is equal to the binding energy of the

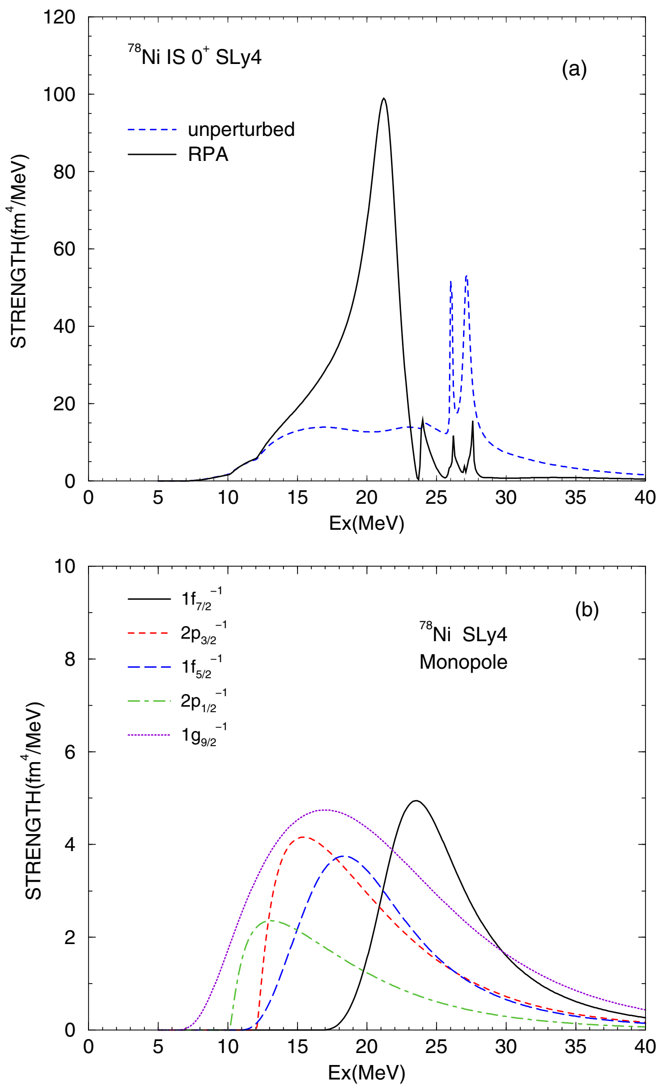

FIG. 2. (Color online) Monopole strength function of ^{78}Ni as a function of excitation energy. The SLy4 interaction is consistently used both in the HF and the RPA calculations. (a) Unperturbed (neutron plus proton) monopole strength and isoscalar monopole RPA strength. The RPA strength includes all possible strengths. On the other hand, in the unperturbed strength that is denoted by the short-dashed curve the p-h strengths, in which both particle and hole orbits are bound, are not included because of the different dimensions of the strength. (b) Some unperturbed neutron threshold strengths, which contribute appreciably to the total unperturbed strength below the energy of ISGMR in Fig. 2(a), are shown for respective occupied hole orbits.

occupied least-bound protons in the $1f_{7/2}$ orbit. In Fig. 2(a) one sees that the properties of ISGMR of ^{78}Ni are similar to those of ^{68}Ni. The main differences between the monopole strengths of ^{68}Ni and ^{78}Ni are that, due to the extra filling of the neutron $1g_{9/2}$ orbit in ^{78}Ni, S_n is considerably smaller, S_p is larger, and the height of the broad flat shoulder of the neutron threshold strength is appreciably higher. The RPA strength increases monotonically from the particle threshold to the ISGMR peak around 21 MeV. The strength function in Fig. 2(a), which is not smeared out, may be compared with Fig. 5(a) in Ref. [2], which is the result of smearing out. In Fig. 2(b) the neutron unperturbed monopole strengths, which contribute appreciably to the total unperturbed strength below 20 MeV in Fig. 2(a), are shown for respective occupied hole orbits. Since none of the neutron one-particle resonances exist for the $2g_{9/2}$, $3p_{1/2}$, $2f_{5/2}$, $3p_{3/2}$, and $2f_{7/2}$ orbits the entire monopole strengths related to those neutrons appear as the respective threshold strengths.

In conclusion, based on our continuum calculation without discretizing continuum particle spectra, we state that it is very unlikely to have some isoscalar monopole peaks with the width of the order of 1 MeV below the excitation energy of 20 MeV in ^{68}Ni. In nuclei other than halo nuclei the low-lying isoscalar monopole threshold strength may appear as a broad shoulder as in the present neutron-rich Ni isotopes. The broad shoulder may sometimes look like a broad bump. An example is the calculated isoscalar monopole strength in ^{60}Ca shown in Fig. 12 of Ref. [10]. However, the "width" of the bump in such cases is several MeV or larger. In order to compare our results with the observed strength, we may further smear out our RPA curve so as to approximately include the effect of 2p-2h components that are not taken care of in RPA. Then, the "width" of the bump would become even larger.

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