Coulomb corrections to experimental temperatures and densities in Fermi-energy heavy-ion collisions

J. Mabiala,[1,*] H. Zheng,[1,2] A. Bonasera,[1,3] P. Cammarata,[1,4] K. Hagel,[1] L. Heilborn,[1,4] Z. Kohley,[1,4,†] L. W. May,[1,4] A. B. McIntosh,[1] M. D. Youngs,[1] A. Zarrella,[1,4] and S. J. Yennello[1,4]

[1]*Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA*
[2]*Physics Department, Texas A&M University, College Station, Texas 77843, USA*
[3]*Laboratori Nazionali del Sud, INFN, via Santa Sofia, 62, 95123 Catania, Italy*
[4]*Chemistry Department, Texas A&M University, College Station, Texas 77843, USA*

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The quantum-fluctuation method using Coulomb corrections is explored to extract temperatures and densities of hot nuclear systems produced in Fermi-energy heavy-ion collisions. The role of the neutron-proton asymmetry of the system was investigated. Temperature values were observed to not depend on the system neutron-proton asymmetry while a clear dependence is seen for density values. Coulomb corrections have been shown to lower temperature values by almost 2 MeV. On the other hand, density results exhibit a small variation.

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Exploring the properties of nuclear matter at various densities and temperatures is one of the main goals of the study of heavy-ion reactions [1–3]. The determination of nuclear parameters (temperature, density, pressure, free energy, etc.) that characterize the nuclear equation of state (EOS), essential in understanding a number of important issues in astrophysics, remains a difficult task despite a wide body of available experimental data. Various methods can be found in the literature that have been developed and applied to the study of thermodynamic properties of highly excited nuclear systems. These include the slope thermometer from kinetic energy distributions of emitted particles [4–7], the population of excited states thermometer [8–10] and the double isotopic yield ratio method [6,7,10–14] to extract the density and temperature of the system. Also in other studies [15–17], the temperature was determined by assuming a degenerate Fermi gas. All these methods were derived from a classical approach. A coalescence approach was also developed to estimate the density [12–14,18–20]. Despite the fact that this method was derived from a classical physics, the densities obtained were found to be higher than those from a double-ratio densitometer. This may be due to the coalescence parameter that can mimic important quantum effects [21] resulting in relatively high densities.

Another method for measuring temperatures was proposed by Wuenschel *et al.* [22] based on quadrupole momentum fluctuations of fragments using a classical Maxwell–Boltzmann distribution. However, within the same framework but for a Fermi–Dirac distribution or a Bose–Einstein distribution, a new method for extracting simultaneously both density and temperature of the system was suggested in Refs. [23–25]. A proper treatment of the quantum statistical nature of particles produced during heavy-ion reactions is taken into account. In such an approach, particle multiplicity fluctuation is used in addition to quadrupole momentum fluctuation to infer temperature and density of the system. Also, important quantum effects such as fermion quenching or Bose–Einstein condensation (BEC) [26–29] can be traced when fermions and bosons are treated differently. In subsequent works [3,30,31], this method was further modified by explicitly taking into account Coulomb corrections.

In our recent works [32–34], we reported on the experimental temperatures and densities of fragmenting systems produced in near-Fermi-energy heavy-ion collisions by means of a quantum-fluctuation method for protons. Since the protons represent the vapor phase, the derived densities and temperatures were shown to sample the vapor branch of the liquid-gas coexistence curve. These experimental quantities and the corresponding critical values have shown a dependence on the system neutron-proton asymmetry.

This paper extends our previous analysis [32–34] using protons as probe particles. We now provide additional results from the same experimental data set by Coulomb correcting temperature and density values.

The experiment was performed at the K-500 superconducting cyclotron facility at Texas A&M University. ^{64,70}Zn and ^{64}Ni beams were used to respectively irradiate ^{64,70}Zn and ^{64}Ni targets at 35 MeV/nucleon. Charged particles and free neutrons produced in the reactions were collected using the 4π NIMROD-ISiS array [35,36]. Further details of the experiment may be found in Refs. [37–39]. The excellent energy resolution enabled isotopic resolution of charged particles up to $Z = 17$ and elemental resolution up to the charge of the beam. The quasiprojectile (QP), the large, excited, primary fragment of the projectile following a noncentral collision with the target, was reconstructed from events in which all charged particles and free neutrons were isotopically identified. The neutron ball provided event-by-event experimental information on the free neutrons emitted during a reaction. The number of free neutrons emitted by the QP was deduced from the total measured number of neutrons, background, and efficiencies for measuring neutrons produced from QP and quasitarget sources [22]. The excitation energy was deduced using the

*jmabiala@comp.tamu.edu
†Present address: National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA.

transverse kinetic energy of the charged particles, the neutron multiplicity, the average neutron kinetic energy determined using the Coulomb-shifted proton energy distribution, and the energy needed for the breakup (Q value). This method of reconstruction has previously been fully described in Refs. [22,40–44]. Using the three reaction systems, we selected a QP mass range not too far from the projectile mass ($54 \leqslant A \leqslant 64$) and a span in neutron-proton asymmetry [$m_s = (N - Z)/A$] with sufficient statistics.

The temperatures of reconstructed QPs are obtained with the quadrupole momentum fluctuation method reported in Refs. [23–25,30]. The variance of the transverse quadrupole momentum $Q_{xy} = p_x^2 - p_y^2$ is expressed in terms of the temperature T, using the correct quantum distribution for fermions, as

$$\langle \sigma_{xy}^2 \rangle = 4m^2 T^2 F_{QC}, \tag{1}$$

where m is the mass of the particle being used as the probe and F_{QC} is the quantum-correction factor. The quadrupole was defined in transverse direction in order to minimize nonequilibrium effects. In Ref. [24], Eq. (1) was solved numerically for a Fermi gas and F_{QC} was conveniently parametrized as

$$F_{QC} = 0.2\left(\frac{T}{\varepsilon_f}\right)^{-1.71} + 1, \tag{2}$$

where the temperature relative to the Fermi energy (T/ε_f) was also parametrized in terms of the normalized multiplicity fluctuation $\sigma_N^2/\langle N \rangle$ as

$$\frac{T}{\varepsilon_f} = -0.442 + \frac{0.442}{\left(1 - \frac{\sigma_N^2}{\langle N \rangle}\right)^{0.656}} + 0.345\frac{\sigma_N^2}{\langle N \rangle} - 0.12\left(\frac{\sigma_N^2}{\langle N \rangle}\right)^2. \tag{3}$$

The quantity F_{QC} should converge to one for classical systems where particle normalized multiplicity fluctuations are equal to one. The nucleon density ρ is determined from the Fermi-energy relation $\varepsilon_f = \varepsilon_{f0}(\rho/\rho_0)^{2/3}$, using ε_f derived from Eqs. (1)–(3) and the ground-state values of Fermi energy (ε_{f0}) and nucleon density (ρ_0).

As already mentioned above, the QP mass was required to be in the range $54 \leqslant A \leqslant 64$. The effect of the m_s bin width on the extracted temperature and density values was investigated. The dependence of these quantities on the m_s bin width is shown in Fig. 1. Figure 1(a) shows the temperature calculated by using the proton probes for the selected quasiprojectiles as a function of the range in QP asymmetry included in the calculation. These temperatures decrease from 9.15 MeV at an m_s bin width of 0.4 to 8.26 MeV at an m_s bin width of 0.025; a 10% change. The decrease in temperature is linear over the majority of the range; extrapolating to an m_s bin width of zero, the temperature would be 8.18 MeV. Figure 1(b) shows the calculated density probed by the free protons, for the same ranges in m_s bin width. The density increases from 0.0122 fm^{-3} at the largest m_s bin width to 0.015 fm^{-3} at the narrowest m_s bin width. This is nearly a 20% increase. Extrapolating to an m_s bin width of zero, the density would be 0.154 fm^{-3}. The reason for the evolution of the temperature and density is due to the width of the free-proton multiplicity distribution, which determines σ_N^2 and $\langle N \rangle$. The mean of the multiplicity distribution evolves with m_s, and when a range of m_s is summed over, the multiplicity distribution is artificially broadened. The extrapolation to an m_s bin width of zero provides a simple and intuitive way to correct for this. Thus, the previously published results using the classical momentum quadrupole fluctuation thermometer [34,35,45] are not impacted by this effect. It is found that the m_s bin width should be taken as small as possible (close to zero) to estimate true values of T and ρ as it should be by fixing the mass A and the charge Z of the QP. For an optimum balance between the m_s bin width as small as possible and statistics, the data were sorted in a bin width of 0.05.

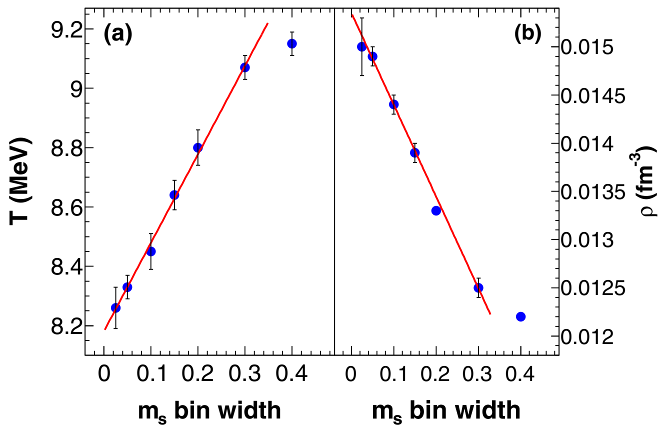

FIG. 1. (Color online) (a) Temperature and (b) density values as a function of the m_s bin width for an excitation energy of 5.5 MeV for the QP. All m_s bins are centered around 0.14. Protons are used as the probe particle. The solid line is a linear fit to the data. Error bars represent statistical errors and are not shown when smaller than the symbols.

In our previously reported studies [28,32–34], we were concerned that temperature and density results derived by means of this technique might be distorted by Coulomb effects. The dynamics of nucleons inside the nucleus is affected by Coulomb interactions. A charged particle that leaves an excited system will then experience a Coulomb acceleration. Therefore, the momentum distribution for particles is modified by the Coulomb field. In Refs. [30,31], Zheng *et al.* addressed this issue in the determination of densities and temperatures of hot sources produced in heavy-ion collisions. A method borrowed from electron scattering was adopted and applied to classical as well as to quantum systems. The Coulomb field is taken to be the Fourier transform of the Coulomb potential of the source. In this way, the equations of quadrupole momentum fluctuation, the average multiplicity, as well as the multiplicity fluctuation containing the Coulomb field term were numerically solved to derive the temperature T, the density ρ, and the volume V of the system. By using model calculations, the authors showed that derived temperatures of protons after Coulomb correction were very similar to those of neutrons. The Coulomb correction had a small effect on

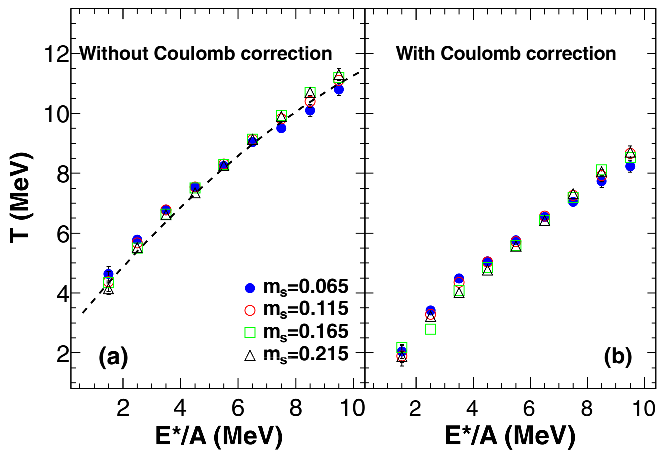

FIG. 2. (Color online) Temperatures of the gas phase for QPs that differ in neutron-proton asymmetry (m_s) as a function of the excitation energy per nucleon. Protons are used as the probe particle. Panels (a) and (b) correspond respectively to results without Coulomb and with Coulomb correction. Symbols correspond to data with m_s bin width of 0.05 while the dashed line represents the extrapolation of data to zero-m_s bin width (as seen in Fig. 1 for $\langle E^*/A \rangle = 5.5$ MeV). Statistical errors are indicated by the bars and are not shown when smaller than the symbols.

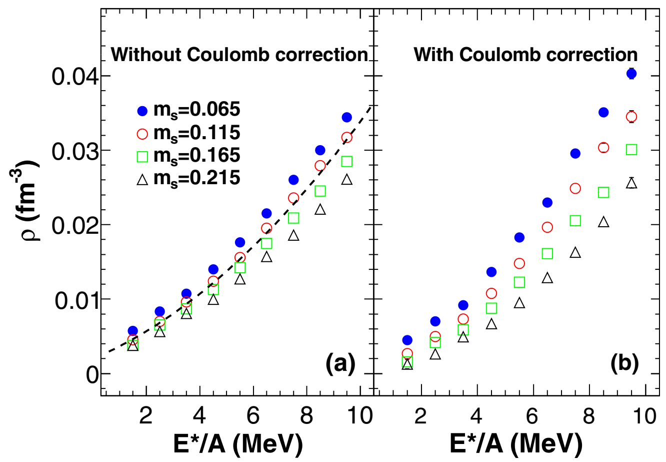

FIG. 3. (Color online) Density as a function of the QP excitation energy per nucleon for different asymmetries m_s, as probed by protons in the quantum momentum quadrupole fluctuation method. Panels (a) and (b) correspond respectively to results without Coulomb and with Coulomb correction. Symbols correspond to data with m_s bin width of 0.05 while the dashed line represents the extrapolation of data to zero-m_s bin width (as seen in Fig. 1 for $\langle E^*/A \rangle = 5.5$ MeV). Error bars corresponding to statistical errors are smaller than the points.

the extracted proton densities. The same behavior was also observed for composite fermions in the classical case. We have applied the same procedure to our experimental data.

In Fig. 2, we present QP temperatures as a function of the excitation energy per nucleon using protons as the probe particle. The curves represent different values of the source asymmetry m_s as indicated in the legend. These caloric curves show a monotonic rising behavior for both cases (without Coulomb and with Coulomb correction). A weak dependence on m_s is observed for temperatures extracted without as well as with Coulomb correction. In fact, data for the four curves with the m_s bin width of 0.05 are similar to those from the extrapolated zero-m_s bin width (dashed line). In addition, we observed that the Coulomb correction lowers the temperature value by almost 2 MeV.

The densities of QP regions probed by protons versus the excitation energy per nucleon are shown in Fig. 3. Figures 3(a) and 3(b) correspond to results without and with Coulomb correction, respectively. As protons refer to the gas component (low-density) region of the system in the liquid-to-gas-type phase transition, we observe that the density rises as the excitation energy increases. We interpret this as a result of higher excitation energy causing a larger number of protons to leave the liquid and enter the gas phase, resulting in a higher number density of protons in the gas phase. A clear dependence on m_s is seen in each panel for the four density curves: the larger the asymmetry, the lower the density. In our previous studies [34,44,45], a strong dependence of temperatures was shown within a classical treatment. However, in the present treatment where we extract simultaneously both temperature and density, the dependence on m_s is rather strongly exhibited in the density. Although the overall behavior is similar to model calculations reported in Refs. [30,31], the Coulomb correction amplifies the m_s dependence.

The correlation between the density and the temperature, as probed by protons, is presented in Fig. 4(a) for the four different source asymmetries. All curves display a rising behavior. It is also interesting to notice that, as the system temperature increases, the spacing between the proton density values for different asymmetries increases. These features may be attributed to the effects due to symmetry and Coulomb energies. From the values of density and excitation energy, we examine in Fig. 4(b) the energy density $\varepsilon = (E^*/A)\rho$ against the temperature. It is observed that ε monotonically increases as T increases and the differences between curves seen in Fig. 4(a) are less noticeable.

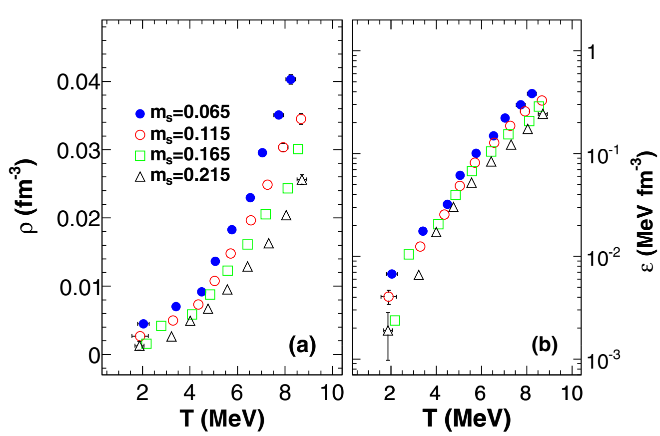

FIG. 4. (Color online) (a) Correlation between the density and the temperature of the system as probed by protons. (b) Energy density versus temperature. All quantities are corrected for Coulomb.

In summary, in this paper we presented and discussed temperatures and densities of hot sources produced in heavy-ion collisions near Fermi energies determined with the very recent quantum-fluctuation method. The evidence on the role of the source neutron-proton asymmetry on temperature and density results was provided. Coulomb corrections applied to derived temperatures and densities using protons as the probe particle have been shown to lower temperature values by almost 2 MeV while the effect on derived densities is small. The results of energy density versus temperature have shown a small dependence on the neutron-proton asymmetry of the system. We have seen in this work that the EOS can be studied with protons. However, a more complete study could include densities probed by neutrons in addition to protons.

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