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# Scaling laws for total reaction cross sections

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We analyze the neutron-nucleus total reaction cross sections for all the available experimental data, in the energy range from about 10 MeV to 400 GeV. We obtain simple scaling laws proved to be valid over all the energy ranges studied and for stable nuclei of the whole mass number region. Moreover, the proton-nucleus and nucleus-nucleus total reaction cross sections are studied for all the available experimental data in the energy range from 1 GeV to 400 GeV and from 1$A$ GeV to 20$A$ GeV, respectively. We show that the scaling laws found for proton(nucleus)-nucleus total reaction cross sections, for energies less than 1 GeV, in our previous works [B. Abu-Ibrahim and A. Kohama, Phys. Rev. C **81**, 057601 (2010); B. Abu-Ibrahim, *ibid.* **83**, 044615 (2011)], are still valid in this energy region. The uncertainty of the prediction of the scaling laws is about 10%. Our findings, which are consistent with previously obtained results, are useful to predict total reaction cross sections for reactions involving nuclei with mass numbers larger than 8, and over a large energy scale.

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## I. INTRODUCTION

The total reaction cross section in high energy collisions is believed to reflect the geometrical size. The experimental proton-nucleus and nucleus-nucleus total reaction cross sections and the elastic scattering angular distributions are used extensively to determine the matter rms radii of unstable nuclei.

In a previous paper, we studied the scaling properties of proton-nucleus total reaction cross sections for stable nuclei of the whole mass number region, in the energy range from 30 MeV to about 1 GeV [1]. In that work, using the available experimental data, we found that the total reaction cross section of a proton incident on a nucleus with mass number $A(= Z + N)$ is proportional to $(Z^{2/3}\sigma_{pp}^{\rm tot} + N^{2/3}\sigma_{np}^{\rm tot})$. This expression is different from the known one which is proportional to $A^{2/3}$ [2]. Also, we studied in Ref. [3] the nucleus-nucleus total reaction cross sections in the energy range from 30$A$ MeV to 1$A$ GeV, a scaling law has been also found.

The authors of Ref. [4] tried to extend our scaling law for proton-nucleus total reaction cross sections to neutron-nucleus total reaction cross sections. They commented on our scaling law, Eq. (1), of Ref. [1] as follows: "we find that the apparently obvious extension of this approximation expression for proton-nucleus reaction cross section to neutron-nucleus (n-N) reaction cross section as $\alpha(Z^{2/3}\sigma_{np}^{\rm tot} + N^{2/3}\sigma_{nn}^{\rm tot})$ does not lead to any reasonably acceptable results".

In this paper, we study the neutron-nucleus total reaction cross sections for stable nuclei for all the available experimental data, in the energy range from about 10 MeV to 400 GeV. We show here that the extension of our previous scaling formula for proton-nucleus total reaction cross sections to neutron-nucleus total reaction cross sections as $\alpha(Z^{2/3}\sigma_{np}^{\rm tot} + N^{2/3}\sigma_{nn}^{\rm tot})$ is successful. Also, we extend our previous work for proton-nucleus total reaction cross sections [1] to a higher energy range from 1 GeV to about 400 GeV, and our work for nucleus-nucleus total reaction cross sections [3] to energy up to 20$A$ GeV. Here, we adopt the same prescription as our previous work [1,3] for describing and calculating the total reaction cross sections of neutron-nucleus. The symbol $E$ represents the neutron(proton) kinetic energy in the case of neutron(proton)-nucleus collision, while it represents the energy per nucleon in the case of nucleus-nucleus collision.

The organization of the present paper is as follows. In Sec. II, we present the empirical scaling laws for estimating the neutron-nucleus total reaction cross sections and compare the numerical results of the scaling laws with the available experimental data. In Sec. III, we show that our formula for proton-nucleus total reaction cross sections published in Ref. [1] is also applicable in the energy range from 1 GeV to 400 GeV. The nucleus-nucleus total reaction cross sections are discussed in Sec. IV, where we show also that our scaling law of Ref. [3] is applicable in this energy domain. A comparison between the coefficient of the scaling laws for the three cases: neutron-nucleus, proton-nucleus, and nucleus-nucleus total reaction cross sections is given in Sec. V. A brief outlook is given in Sec. VI.

## II. NEUTRON-NUCLEUS TOTAL REACTION CROSS SECTIONS

In this section, we extend our scaling law for proton-nucleus total reaction cross sections, Eq. (1) of Ref. [1], to the case of neutron-nucleus total reaction cross sections. We also report our results in the energy range from about 10 MeV to 400 GeV.

### A. Scaling law for neutron-nucleus total reaction cross sections

The total reaction cross sections of a neutron incident on a nucleus with mass number $A(= N + Z)$ can be written as

$$\sigma_{\rm R}(Z,N,E) = \pi C(E,A)\left[Z^{2/3}\sigma_{np}^{\rm tot}(E) + N^{2/3}\sigma_{nn}^{\rm tot}(E)\right] \simeq \pi C(E)\left[Z^{2/3}\sigma_{np}^{\rm tot}(E) + N^{2/3}\sigma_{nn}^{\rm tot}(E)\right], \quad (1)$$

where $Z(N)$ is the number of protons (neutrons) in the target nucleus, $A$ is the mass number, $\sigma_{nn}^{\rm tot}$ ($\sigma_{np}^{\rm tot}$) is the neutron-neutron (neutron-proton) total cross section, and $C(E)(\equiv \langle C(E,A)\rangle_A)$

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is an energy-dependent coefficient to be deduced from experimental data. It contains the nuclear structure information such as surface effect. $\langle \cdots \rangle_A$ implies the average value of the values for different mass numbers at each energy. $E$ is the neutron incident energy. In Eq. (1), $C(E,A)$ is replaced by $C(E)$, because $C(E,A)$ depends weakly on $A$, as we will see later.

The input quantities needed for the calculations of the reaction cross sections from Eq. (1) are the values of $\sigma_{np}^{\mathrm{tot}}$ and $\sigma_{nn}^{\mathrm{tot}}$. For computational purposes, we assumed that the $\sigma_{pp}^{\mathrm{tot}}$ values for each energy adequately represent the $\sigma_{nn}^{\mathrm{tot}}$ values, since only limited quantities of experimental data exist for neutron-neutron collision. They are taken from the fitting of Ref. [5] for the energy region from 10 MeV to 5 GeV and from the fitting of Ref. [6] for the energy region from 5 GeV to 500 GeV. We show this fitting in the Appendix.

Figure 1 shows the values of $\pi C(E,A)$ as a function of the mass number of the target nucleus ($A$) at six different neutron energies. The solid line in each panel represents the values of $\pi C(E)$ obtained by averaging $\pi C(E,A)$ over all the available experimental data for each energy. The dashed lines denote the standard deviations of $\pi C(E)$. One can see from the figure that the values of $C(E,A)$ depend strongly on the energy, while they depend weakly on the mass number of the target nucleus ($A$), over all the energy range from 14 MeV to about 350 GeV. These results validate the replacement of $\pi C(E,A)$ with $\pi C(E)$ in Eq. (1). The $C(E)$ value is determined at each energy as explained in Refs. [1,3]. The values of $\pi C(E)$ and their standard deviations, defined by $\Delta C(E) = \sqrt{\frac{1}{N_{\mathrm{data}}}\sum_{i=1}^{N_{\mathrm{data}}}(C_i - \bar{C})^2}$, as a function of energy are listed in Table I.

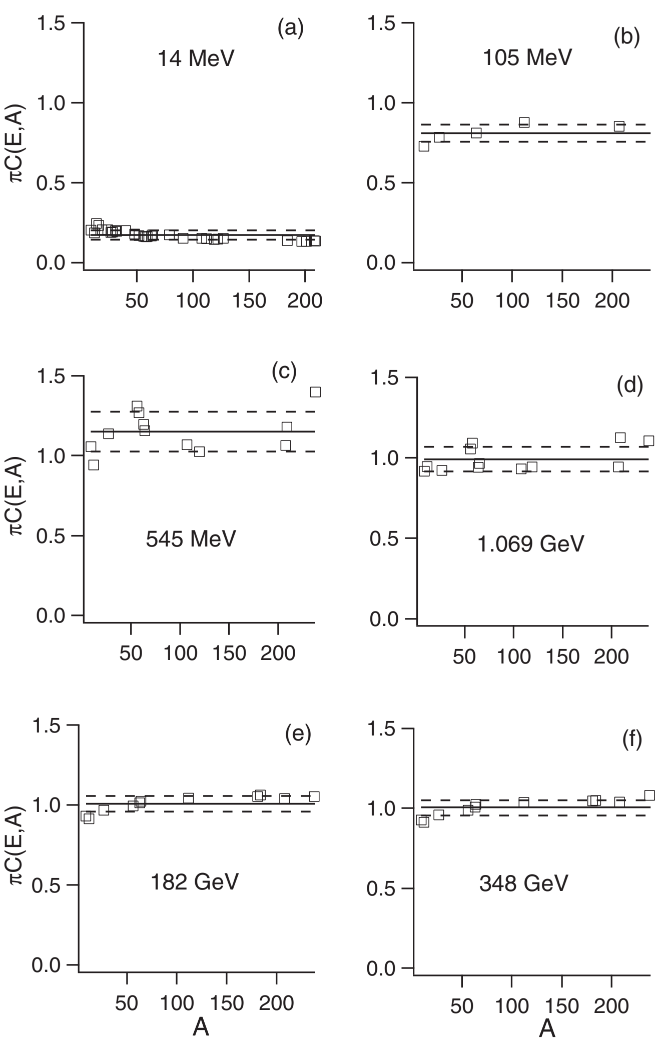


FIG. 1. Values of $\pi C(E,A)$ as a function of target mass number, $A$, for each energy, $E$, indicated by squares. The solid lines indicate the average values. The dashed lines indicate standard deviations around each average value. The experimental data are taken from Refs. [7–10].

TABLE I. Values of coefficient, $\pi C(E_n)$, defined in Eq. (1), and their standard deviations. The curves in Fig. 2 are drawn using these values.

| $E$ (MeV) [Ref.] | $\pi C(E)$ | $E$ (MeV) [Ref.] | $\pi C(E)$ |
|---|---|---|---|
| 14.0 [7] | $0.173 \pm 0.029$ | 250 [15] | $1.301 \pm 0.150$ |
| 21.0 [12] | $0.227 \pm 0.021$ | 300 [15] | $1.285 \pm 0.100$ |
| 25.5 [12] | $0.271 \pm 0.018$ | 350 [15] | $1.254 \pm 0.079$ |
| 29.2 [12] | $0.323 \pm 0.015$ | 400 [15] | $1.263 \pm 0.071$ |
| 40.3 [13] | $0.386 \pm 0.034$ | 451 [9] | $1.367 \pm 0.130$ |
| 45.0 [14] | $0.494 \pm 0.021$ | 488 [9] | $1.365 \pm 0.158$ |
| 50.0 [14] | $0.503 \pm 0.040$ | 545 [9] | $1.149 \pm 0.125$ |
| 60.0 [14] | $0.525 \pm 0.056$ | 624 [9] | $1.121 \pm 0.129$ |
| 80.0 [15] | $0.749 \pm 0.059$ | 705 [9] | $1.046 \pm 0.108$ |
| 105.0 [8] | $0.811 \pm 0.053$ | 788 [9] | $0.982 \pm 0.101$ |
| 140.0 [8] | $0.984 \pm 0.063$ | 873 [9] | $0.983 \pm 0.085$ |
| 200.0 [15] | $1.267 \pm 0.109$ | 959 [9] | $1.011 \pm 0.091$ |
| $E$ (GeV) [Ref.] | $\pi C(E)$ | $E$ (GeV) [Ref.] | $\pi C(E)$ |
| 1.069 [9] | $0.990 \pm 0.076$ | 5.0 [11] | $0.837 \pm 0.077$ |
| 1.202 [9] | $0.989 \pm 0.089$ | 8.3 [16] | $0.884 \pm 0.086$ |
| 1.545 [9] | $1.030 \pm 0.071$ | 182.0 [10] | $1.009 \pm 0.048$ |
| 1.361 [9] | $0.990 \pm 0.096$ | 262.0 [10] | $1.008 \pm 0.051$ |
| 1.731 [9] | $1.020 \pm 0.085$ | 348.0 [10] | $1.007 \pm 0.051$ |
| 3.000 [15] | $0.950 \pm 0.049$ | | |

To facilitate the numerical calculations of $\sigma_{\mathrm{R}}$ using Eq. (1), it would be useful to prepare the analytical form for $C(E)$ as a function of neutron incident energy, $E$. We adopt the same expression as in Ref. [1],

$$\pi C(E) = a_1 - a_2 \exp(-a_3 E^{a_4}) \cos(a_5 E^{a_6}), \tag{2}$$

where $E$ in units of MeV. The constants are given by $a_1 = 1.015827$, $a_2 = 1.540624$, $a_3 = 0.19911$, $a_4 = 0.355785$, $a_5 = 0.053$, and $a_6 = 0.6767812$. The numerical results of Eq. (2) are represented by the dashed curve in Fig. 7. It is valid in the energy range from 20 MeV to 1 GeV. The behavior of the constant $C(E)$ as a function of energy will be discussed in Sec. V. For energy range from 1 GeV to 10 GeV, the value of $\pi C(E)$ is $0.915 \pm 0.03$. For energies from 10 GeV to 500 GeV the value of $\pi C(E)$ is $1.0 \pm 0.01$, and it is constant over this energy range.

### B. Comparison with the experimental data

Here we report our numerical results of Eq. (1) for the neutron-nucleus total reaction cross sections. Each panel in

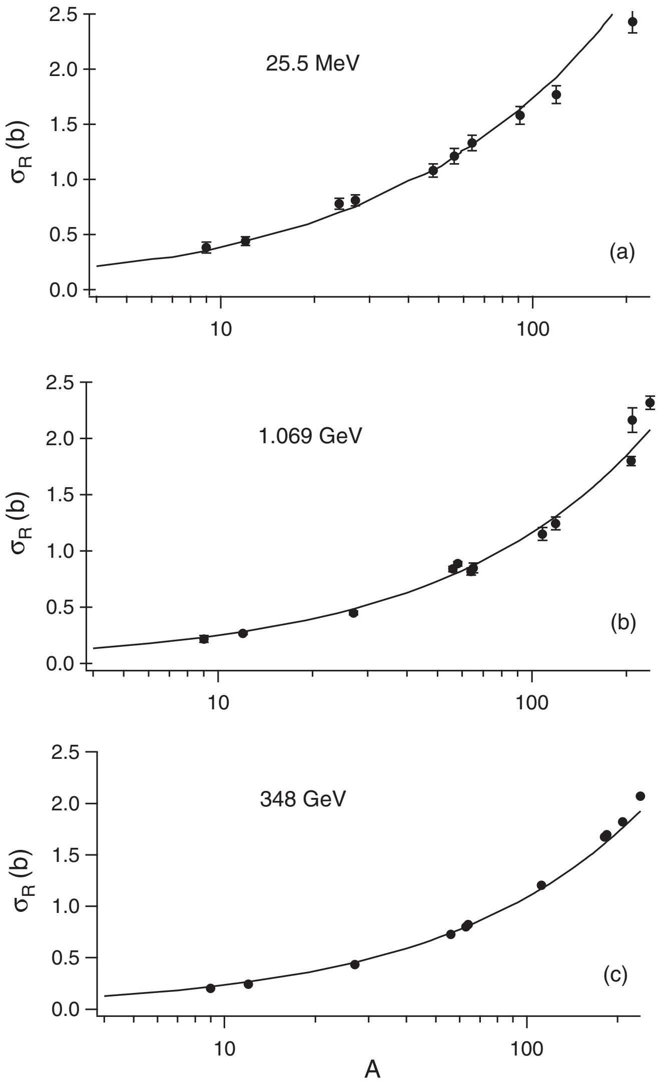


FIG. 2. Comparison of the numerical results obtained using Eq. (1) with the experimental data of total reaction cross sections for neutron-nucleus reactions as a function of mass number, $A$, for $E = 25.5$ MeV, 1.069 GeV, and 348 GeV. The solid curves are the numerical results. The values of $\pi C(E)$ are listed in Table I. The experimental data are shown by solid circles with error bars, and they are taken from Refs. [9,10,12].

Fig. 2 compares the numerical results obtained using Eq. (1) with experimental data of total reaction cross sections, $\sigma_R$, for neutron-nucleus reactions, at a given energy, as a function of target nucleus mass number $A$. The solid curves are the numerical results of Eq. (1). The values of the coefficient, $\pi C(E)$, which provide the solid curves, at different energies, are listed in Table I. The agreement of the numerical results with the empirical data is fairly good.

In Fig. 3, we compare the numerical results obtained using Eq. (1) with the experimental data of the total reaction cross sections for neutron incident on a target nucleus, as a function of neutron incident energy. The values of $\pi C(E)$ are obtained from Eq. (2) in the energy range from 10 MeV to 1 GeV and taken to be one in the energy range from 1 GeV to 400 GeV.

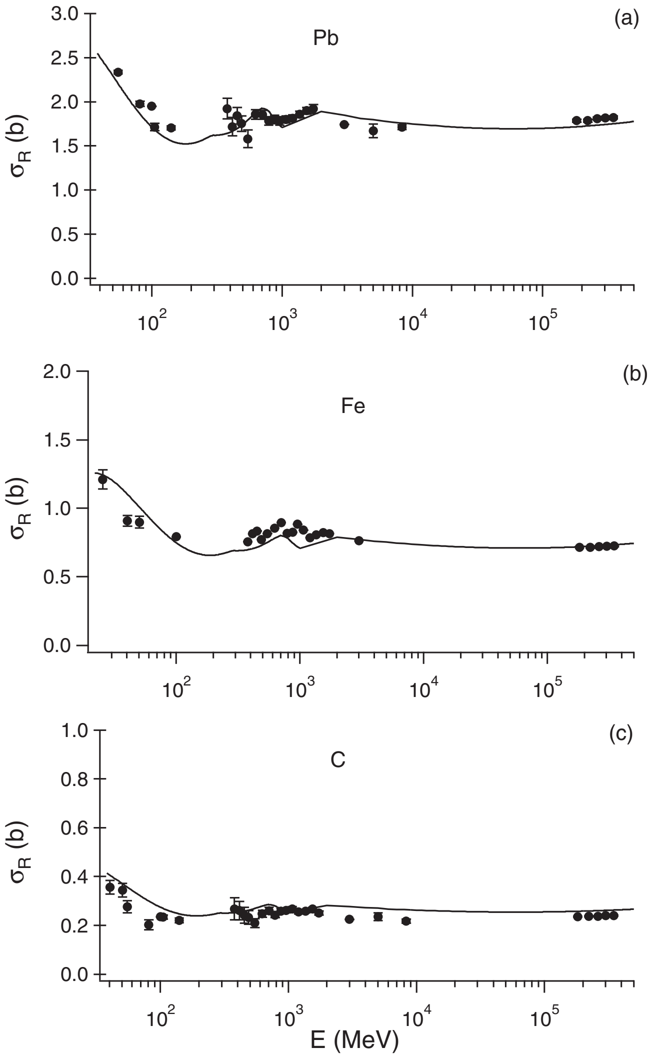


FIG. 3. Comparison of the numerical results obtained using Eqs. (1) and (2) with the experimental data of the total reaction cross sections for neutron-nucleus reactions as a function of energy, $E$. The solid curves are the numerical results of Eqs. (1) and (2). The experimental data (solid circles with error bars) are taken from Refs. [7–16].

The target nuclei are C, Fe, and Pb. We have performed the calculations for several target nuclei, here we show only three. The figure covers the energy range from about 10 MeV to 348 GeV. Over this large energy range the scaling law gives a good prediction to the total reaction cross sections.

### C. Parameter-free scaling law for $\sigma_R$

Here we extend the parameter-free formula for proton-nucleus total reaction cross sections Eq. (4) of Ref. [1] to the case of neutron-nucleus total reaction cross sections.

In the case of a neutron incident on a nucleus of mass number $A$ ($A = Z + N$), Eq. (4) of Ref. [1] becomes

$$\sigma_R(Z_1,N_1) = (1 \pm \Delta)\left(\frac{\sigma_{np}^{\text{tot}} Z_1^{2/3} + \sigma_{np}^{\text{tot}} N_1^{2/3}}{\sigma_{np}^{\text{tot}} Z_2^{2/3} + \sigma_{np}^{\text{tot}} N_2^{2/3}}\right) \sigma_R(Z_2,N_2), \tag{3}$$

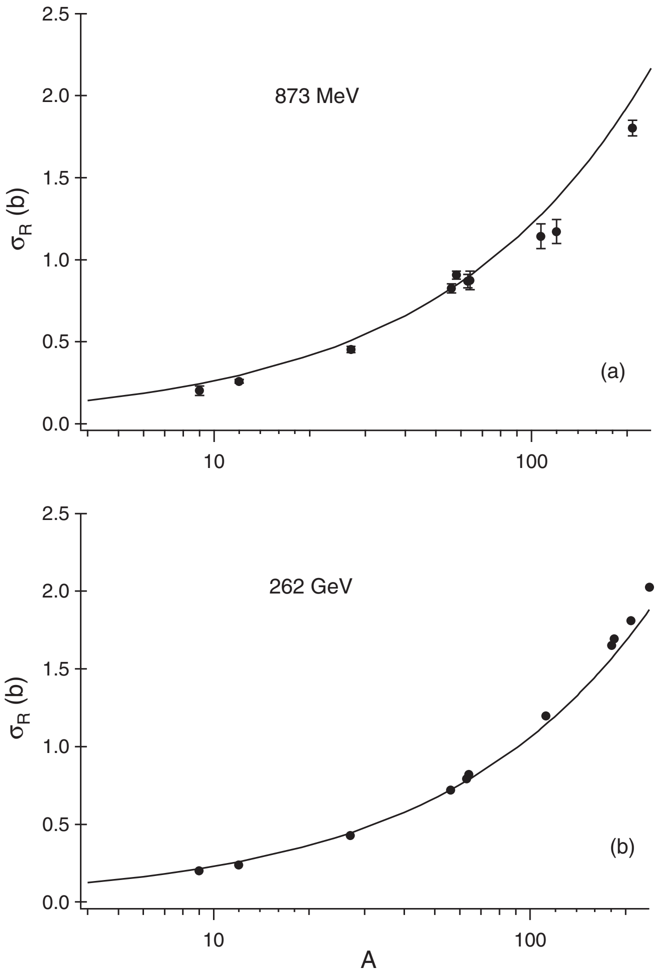


FIG. 4. Comparison of the numerical results of Eq. (3) with the experimental data of the total reaction cross sections for neutron-nucleus reactions as a function of target mass number, $A$. The solid curves represent the numerical results obtained using Eq. (3). The experimental data are taken from Refs. [9,10].

where $\Delta$ implies the uncertainty coming from that of $C(E)$. Here, $\Delta$ is defined by $\Delta \equiv \langle\delta(E)\rangle_E$ and $\delta(E) \equiv 2\Delta C(E)/C(E)$. This expression assumes that the uncertainties in the $C(E)$ values for the two nuclei being compared are correlated to yield the most conservative (i.e., largest) value of $\Delta$. $\langle\delta(E)\rangle_E$ implies the average of $\delta(E)$ values at various energies. We evaluate the value of $\Delta$ to be $\Delta \simeq 0.15$. As can be seen, no fitting parameter appears here, but, at most, 15% uncertainty should be taken into account.

In Fig. 4, we show the numerical results of Eq. (3) with the experimental data as a function of mass number, $A$. The top panel represents the calculation at neutron energy of 873 MeV, while the bottom one represents the calculation at neutron energy of 262 GeV. For the reference nucleus, which corresponds to $A_2$ in Eq. (3), we here choose a nucleus of a medium mass number, such as $^{56}$Fe. For numerical calculations, we choose a nucleus and the reaction cross section at each energy as ($^{56}$Fe, 825 $\pm$ 28 mb and 721 $\pm$ 7.2 mb), for 873 MeV and 262 GeV, respectively [9,10]. Here we neglect $\Delta$. These results show the validity of Eq. (3). The expression of Eq. (3) indicates that if we have the empirical value of the reaction cross section of a given nucleus, we can predict the reaction cross section of other nuclei, confirming what was done in [1]. The numerical results are consistent with the empirical data of stable nuclei. A possible way of estimating an unknown reaction cross section of a given nucleus at a given energy is either to use Eqs. (1) and (2) or to use Eq. (3). We do not need any parameter for the latter.

## III. PROTON-NUCLEUS TOTAL REACTION CROSS SECTIONS IN THE ENERGY RANGE FROM 1 GeV TO 400 GeV

Here we show that our scaling law for the proton nucleus total reaction cross sections published in Ref. [1] is not limited to the energy range from 30 MeV to 1 GeV, but it is also applicable up to proton energy of 400 GeV. At energies larger than 400 GeV, there is no experimental data to check our formula. Here, we summarize our results for proton-nucleus case in the energy range from 1 GeV to about 400 GeV.

For 16 energies covered the energy range from 1 GeV to 400 GeV, we analyzed all the available experimental data for proton-nucleus total reaction cross sections, at each energy, using Eq. (1) of Ref. [1], by the same procedure explained in the previous section and in Ref. [1]. The values of the coefficient $\pi C(E)$ obtained from the analysis are listed in Table II. We confirm that the values of $C(E,A)$ at each energy depend weakly on the target nucleus. We do not show a figure similar to Fig. 1 to avoid the repetition. Please, note that the small uncertainty values for each $\pi C(E)$ indicate that the values of $\pi C(E,A)$, at each energy, depend weakly on the target nucleus. The behavior of the values of $\pi C(E)$ and the uncertainty will be discussed in Sec. V.

We show in Fig. 5 a sample from our results, at 8 GeV and 400 GeV, for proton-nucleus case. Each panel compares the numerical results of Eq. (1) of Ref. [1] (solid curve) with the experimental data (solid circles with error bars) as a function of mass number, $A$. The values of $C(E)$ used in the calculation are listed in Table II. As can be seen from the figure, the agreement between the calculation and the experimental data is reasonable.

Moreover, we confirm the validity of Eq. (4) of Ref. [1] for protons at energies from 1 GeV to 400 GeV. We calculate the value of $\Delta$; we obtain 0.08, or uncertainty about 8%.

TABLE II. Values of coefficient, $\pi C(E)$, and their standard deviations, for proton-nucleus reactions.

| $E$ (GeV) [Ref.] | $\pi C(E)$ | $E$ (GeV) [Ref.] | $\pi C(E)$ |
|---|---|---|---|
| 1.00 [15] | $0.934 \pm 0.044$ | 2.00 [15] | $0.943 \pm 0.042$ |
| 3.00 [15] | $0.957 \pm 0.046$ | 4.15 [17] | $0.988 \pm 0.030$ |
| 5.13 [17] | $1.006 \pm 0.030$ | 6.12 [17] | $1.011 \pm 0.030$ |
| 7.12 [17] | $1.022 \pm 0.034$ | 8.11 [17] | $1.023 \pm 0.028$ |
| 19.08 [18] | $1.011 \pm 0.045$ | 29.07 [18] | $1.042 \pm 0.045$ |
| 39.07 [18] | $1.033 \pm 0.040$ | 49.07 [18] | $1.036 \pm 0.038$ |
| 59.07 [18] | $1.063 \pm 0.047$ | 199.06 [19] | $0.965 \pm 0.056$ |
| 279.06 [19] | $0.951 \pm 0.053$ | 400.00 [20] | $1.087 \pm 0.010$ |

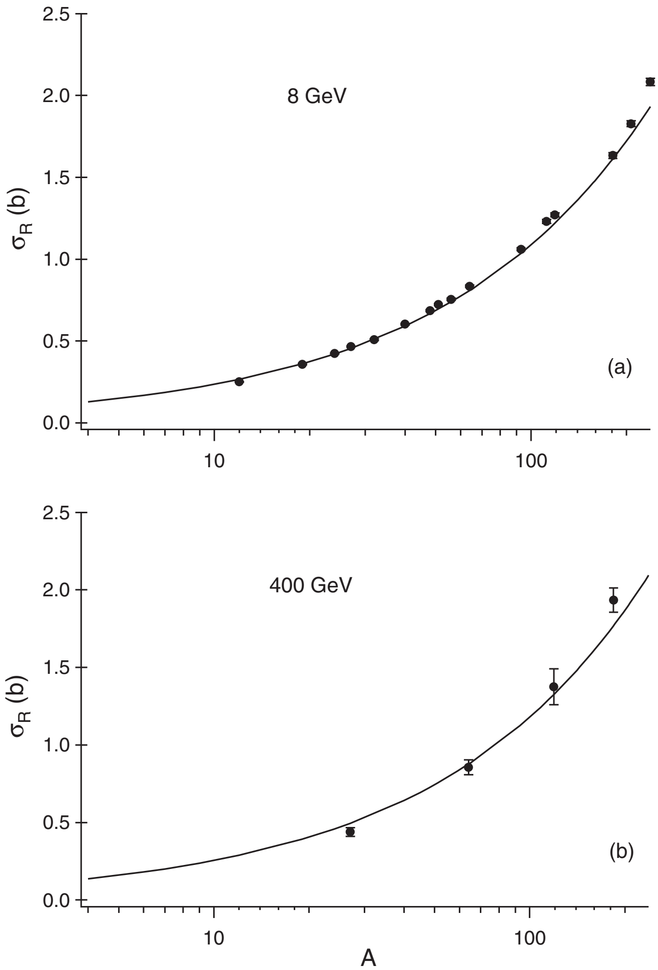


FIG. 5. Comparison of the numerical results with the experimental data of the total reaction cross sections for proton-nucleus reactions as a function of target mass number, $A$. The solid curves represent the numerical results obtained using Eq. (1) of Ref. [1]. The experimental data are taken from Refs. [17,20].

## IV. NUCLEUS-NUCLEUS TOTAL REACTION CROSS SECTIONS ABOVE 1$A$ GeV

In this section we show that our formula for the nucleus-nucleus total reaction cross sections [3] is also applicable to energies above 1$A$ GeV. We have collected all the available experimental data for nucleus-nucleus reaction cross sections above 1$A$ GeV. Due to the limited experimental data specially above 5$A$ GeV, we have used the theoretical calculation of Ref. [21], which is limited up to an energy of 20$A$ GeV.

We have analyzed all the nucleus-nucleus total reaction cross sections data at each energy using Eq. (1) of Ref. [3] by the same procedure explained in Sec. II and in Ref. [3]. The values of the coefficient $\pi C(E)$ obtained from the analysis are listed in Table III. We confirm that the values of $C(E, A_1, A_2)$ at each energy depend weakly on both the projectile and target mass numbers ($A_1$ and $A_2$).

A sample from our results, at 3.7$A$ GeV, is shown in Fig. 6 for a nucleus incident on a carbon target, as a function of mass number, $A$. Solid line is the numerical results of Eq. (1) of Ref. [3]. The values of $C(E)$ used in the calculation are listed in Table III. As one can see from the figure the agreement between the calculation and the experimental data is reasonable.

TABLE III. Values of coefficient, $\pi C(E)$, and their standard deviations, for nucleus-nucleus reactions.

| $E$ (GeV) [Ref.] | $\pi C(E)$ | $E$ (GeV) [Ref.] | $\pi C(E)$ |
|---|---|---|---|
| 2.1 [22,23] | $0.833 \pm 0.060$ | 3.3 [22,23] | $0.924 \pm 0.130$ |
| 3.7 [22,23] | $0.792 \pm 0.070$ | 5.0 [21] | $0.892 \pm 0.036$ |
| 10.0 [21] | $0.920 \pm 0.038$ | 15.0[21] | $0.930 \pm 0.039$ |
| 20.0 [21] | $0.930 \pm 0.040$ | | |

Also we have confirmed the validity of Eq. (3) of Ref. [3] for nucleus-nucleus reactions at energies up to 20$A$ GeV. We calculate, in the current energy range, the value of $\Delta$; we obtain 0.15, or uncertainty about 15%.

## V. COMPARISON BETWEEN THE COEFFICIENTS OF THE SCALING LAWS FOR NEUTRON-NUCLEUS, PROTON-NUCLEUS, AND NUCLEUS-NUCLEUS INTERACTIONS

In the present article and in two previous articles [1,3], we have done the analysis of the neutron-nucleus, proton-nucleus, and nucleus-nucleus total reaction cross sections in the energy range from about 10 MeV to 400 GeV, and over all the charts of nuclei. We find that they follow a few scaling laws, within 10% uncertainty (Eq. (1) in Ref. [1] for proton-nucleus reactions, extended to Eq. (1) for neutron-nucleus reactions in the present work, for nucleus-nucleus reactions Eq. (1) of Ref. [3]) they depend on one coefficient $C(E)$. In this section we focus on the values of $C(E)$ for the three reactions: neutron-nucleus, proton-nucleus, and nucleus-nucleus. We compare between their values at each energy and their energy dependence. We also focus on the uncertainty in the prediction of our scaling formulas as well.

Figure 7 compares the values of $\pi C(E)$ for the three reactions. The top panel shows the values of $\pi C(E)$ in the

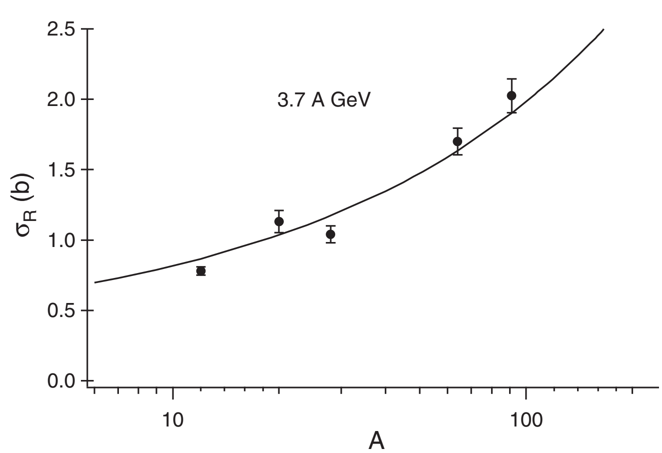


FIG. 6. Comparison of the numerical results with the experimental data of the total reaction cross sections for nucleus-nucleus reactions as a function of projectile mass number, $A$. The target is $^{12}$C. The solid curves represent the numerical results obtained using Eq. (1) of Ref. [3]. The experimental data are taken from Ref. [22].

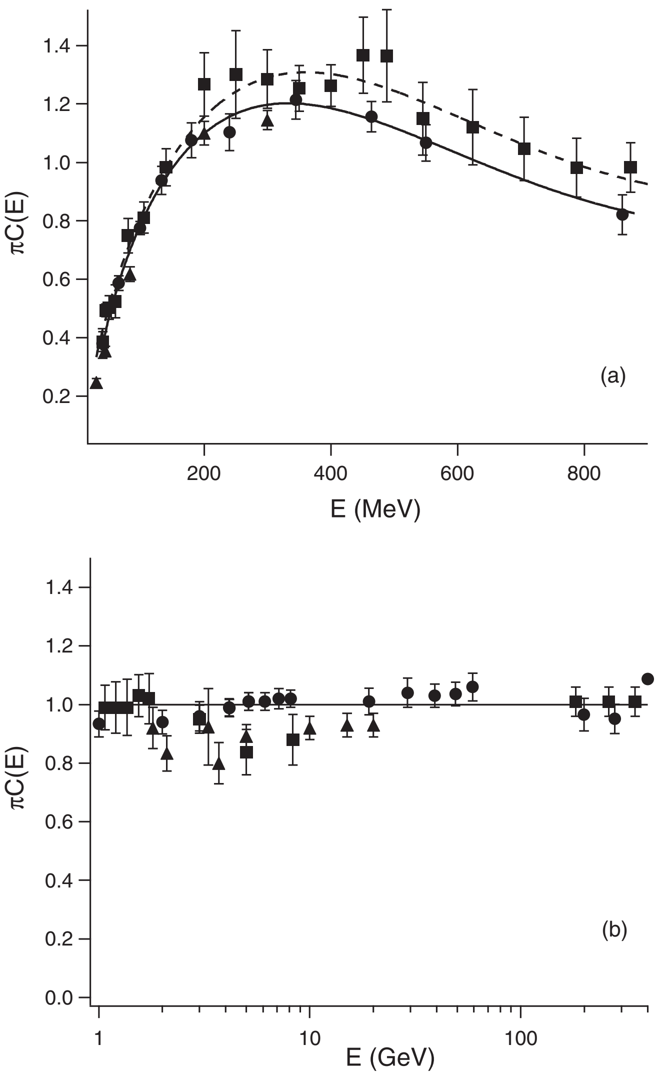


FIG. 7. Values of coefficient, $\pi C(E)$, as a function of the energy per nucleon, $E$. The top panel represent the values in the energy range from 30$A$ MeV to about 1$A$ GeV, while the bottom panel represent the values in the energy range from 1$A$ GeV to 400$A$ GeV. Solid circles with error bars represent the values for proton-nucleus reactions, solid squares with error bars represent the values for neutron-nucleus reactions, and solid triangles with error bars represent the values for nucleus-nucleus reactions. The dashed curve is the prediction of Eq. (2), while the solid curve is the prediction of Eq. (3) of Ref. [1].

energy range from 30 MeV to about 1 GeV. The bottom panel shows their values in the energy range from 1 GeV to about 400 GeV. Solid circles with error bars are for proton-nucleus reactions. Solid squares with error bars are those of neutron-nucleus reactions. Solid triangles with error bars are those of nucleus-nucleus reactions. The solid line in the top panel is the prediction of Eq. (3) of Ref. [1] while the dashed curve is the prediction of Eq. (2) of the present work. In the bottom panel the solid line represents the unity value to guide the eye. It is clear from Fig. 7, the values of $\pi C(E)$ for the three reactions have the same energy dependence. In the energy range from about 200 MeV to about 1 GeV, the difference between those of neutron-nucleus reactions and those of proton-nucleus reactions is about 10%. Those values for nucleus-nucleus reactions are close to those of proton-nucleus reactions. The difference in the values of $\pi C(E)$ for the neutron-nucleus, proton-nucleus, and nucleus-nucleus reactions may arise from the uncertainty of the experimental data. In all our analysis we have neglected the error bars of the experimental data. We mention here that the values of the total reaction cross sections of the neutron-nucleus, in this energy region, are taken from theoretical calculations [15], because there is no experimental data for neutron-nucleus in this energy region.

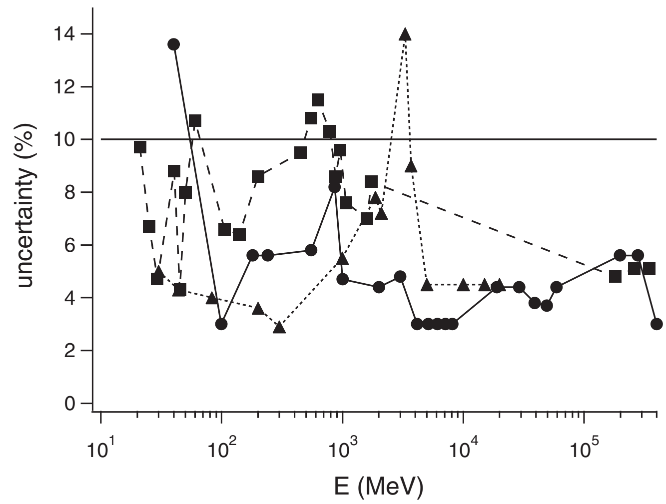


FIG. 8. The uncertainties in numerical results of the total reaction cross sections as a function of the incident energy per nucleon, $E$. Solid circles represent the values for proton-nucleus reactions, solid squares represent the values for neutron-nucleus reactions, and solid triangles represent the values for nucleus-nucleus reactions. The dashed curves are guide for the eye. Solid line represents uncertainty of 10%.

In the energy range from 1 GeV to about 400 GeV the values of $\pi C(E)$ are around one except in the energy region from about 2 GeV to about 10 GeV in which the values of $\pi C(E)$ fluctuate about 0.9. This fluctuation in this energy region may be due to the fluctuation of the neutron-proton total cross section $\sigma_{np}^{\text{tot}}$ in the same energy region, see Fig. 9, or due to the reasons discussed in the previous paragraph. It is worth noting that the constant value of $C(E)$ in the energy range from 10 GeV to 400 GeV is a reflection to the fact that no energy dependence of the total reaction cross sections in this energy region.

In Fig. 8, we compare the precision of our three formulas for proton-nucleus, neutron-nucleus, and nucleus-nucleus. We estimate the fluctuations of the data around the numerical results at each energy. We introduce the fluctuation defined by the standard deviations of $C(E)$ divided by its mean values, that is, $\Delta C(E)/C(E)$. As we can see the uncertainty in our numerical results is less than 10% in general.

## VI. SUMMARY

In summary, we have analyzed the neutron-nucleus total reaction cross sections in the energy region from about 10 MeV to 400 GeV. Furthermore, both proton-nucleus and nucleus-nucleus total reaction cross sections have been studied in the energy region from 1 GeV to about 400 GeV, and from 1 $A$ GeV to 20 $A$ GeV, respectively.

For neutron-nucleus, we have obtained a new scaling law for neutron-nucleus total reaction cross sections in terms of the number of protons and neutrons and the total cross sections of proton-proton and proton-neutron reactions. This law are applicable to the energy range from about 10 MeV up to 400 GeV. Also, we have derived a simple relation for predicting the total reaction cross sections of a neutron with any nucleus within 15% uncertainty, using the empirical values of the total reaction cross section of a given nucleus at a given energy. A better result is obtained if a neighboring nucleus is adopted.

For proton-nucleus and nucleus-nucleus reactions, we have confirmed that the scaling laws found in our previous publication in the energy region from 30 MeV to 1 GeV is also valid in this energy region studied here.

By comparing the values of $C(E)$ (the scaling law coefficient) for neutron-nucleus, proton-nucleus and nucleus-nucleus reactions, we have obtained the followings: the values of the coefficients have the same behavior as a function of energy for the three reactions. At a given energy, they have almost the same values within 10% uncertainty in the energy region from about 10 MeV to 10 GeV, while in the energy region from about 10 GeV to about 400 GeV they are constant and equal to one for the three reactions.

The scaling laws provide a simple tool to compute the total reaction cross sections for nuclei with mass number larger than 8, and over energy range from 10 MeV to 400 GeV.

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## APPENDIX

We have calculated the proton-proton and neutron-proton total reaction cross sections in the energy range from 10 GeV to 400 GeV from the following parametrizations of Ref. [6]:

$$\begin{aligned}\sigma_{pp}^{\text{tot}} &= 35.45 + 0.308 \times (\log(s/28.94))^2 + 42.53 \\ &\quad \times (1.0/s)^{(0.458)} - 33.34 \times (1.0/s)^{(0.545)}, \\ \sigma_{np}^{\text{tot}} &= 35.8 + 0.308 \times (\log(s/28.94))^2 + 40.15 \\ &\quad \times (1.0/s)^{(0.458)} - 30.0 \times (1.0/s)^{(0.545)},\end{aligned} \tag{A1}$$

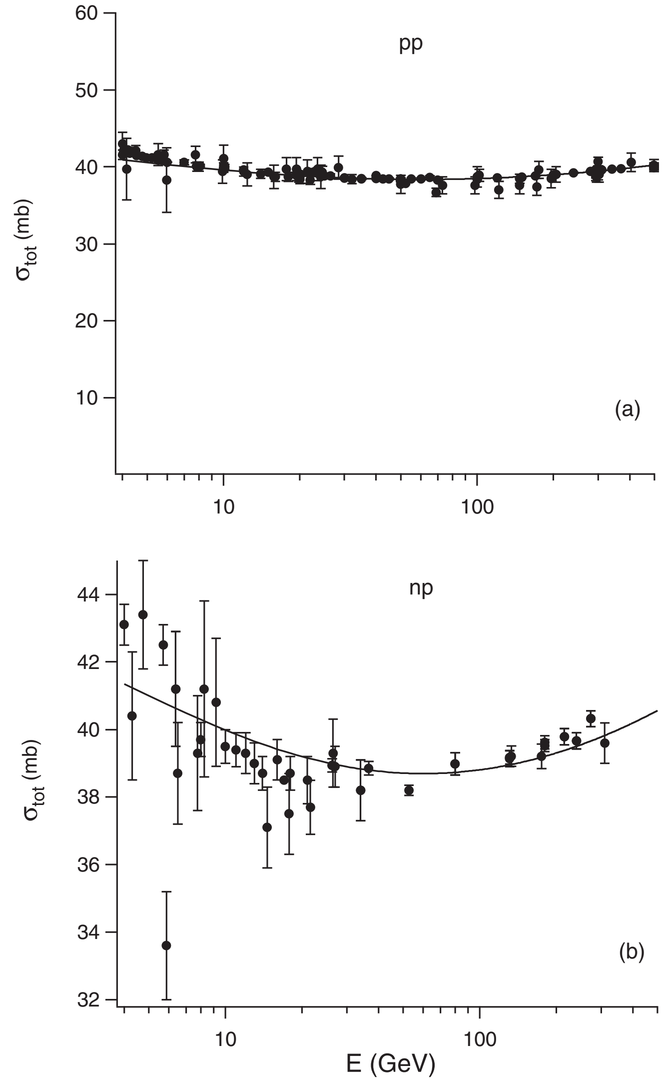


FIG. 9. Total cross sections for proton-proton (top) and neutron-proton (bottom). Solid curves are the fittings of Eq. (A1). The experimental data are taken from Ref. [24].

where $s = 2m(m + \sqrt{p^2 + m^2})$, $p = \sqrt{T(T + 2m)}$, $T$ is the kinetic energy, and $m$ is the nucleon mass.

Figure 9 compares the numerical results of the above equation, which have been used in our analysis, and the experimental data, in the energy region from 1 GeV to 500 GeV. The experimental data are taken from Ref. [24].

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