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Estimation of the electric conductivity of the quark gluon plasma via asymmetric heavy-ion collisions

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We show that in asymmetric heavy-ion collisions, especially off-central Cu + Au collisions, a sizable strength of electric field directed from Au nucleus to Cu nucleus is generated in the overlapping region, because of the difference in the number of electric charges between the two nuclei. This electric field would induce an electric current in the matter created after the collision, which results in a dipole deformation of the charge distribution. The directed flow parameters $v_1^{\pm}$ of charged particles turn out to be sensitive to the charge dipole and provide us with information about electric conductivity of the quark gluon plasma.

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Introduction. The quark gluon plasma (QGP), which consists of deconfined quarks and gluons, is expected to have filled the early universe [1]. We are now at the stage to study properties of the QGP experimentally through the relativistic heavy-ion collisions using the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN. One of the most interesting observations is the very strong elliptic flow in off-central collisions, which indicates a very small ratio of shear viscosity to entropy density, η/s [2–5]. We are now trying to learn more detailed properties of the QGP by constraining transport coefficients such as shear viscosity, bulk viscosity, and charge diffusion constants.

In this Rapid Communication, we propose a new way of estimating the electric conductivity σ of the QGP via asymmetric nucleus-nucleus collisions at ultrarelativistic energies. Theoretically, lattice QCD simulations [6–9] and perturbative QCD calculations [10] have been utilized to estimate electric conductivity of the QGP. So far, the estimated values of σ have differed significantly from each other, and experimental information is intently awaited. Very recently, asymmetric collisions between copper (Cu) and gold (Au) nuclei have been performed at RHIC, and the PHENIX Collaboration reported their first results [11]. We show that Cu + Au collisions can be useful for extracting the electric conductivity of the QGP. In off-central Cu + Au collisions, a substantial magnitude of electric field directed from a colliding Au nucleus to a Cu nucleus is generated in the overlapping region. This happens only when the two colliding nuclei carry different numbers of electric charge.[1]

This electric field would induce a current in the matter created after the collision, resulting in a dipole deformation of the charge distribution in the medium. Later, the time evolution of the system is dominated by a strong radial flow, which is an outward collective motion of the medium. Henceforth the charge asymmetry formed in the early stage is frozen. Thus, we argue that charge-dependent directed flow of the observed hadrons is sensitive to the charge dipole formed at the early stage, which reflects the electric conductivity of the QGP.

Conventionally the electric conductivity of the QGP is estimated from experiments via the Kubo formula [13]. The production rate of thermal dileptons is expressed by the electric current-current correlation function [14,15] and its small frequency region is governed by the transport peak [16]. Thus one can estimate the electric conductivity through comparison of theoretical results with dilepton invariant mass spectra [17]. Compared to this method, the present approach is a rather direct one, in which the response of the matter to an applied electric field is directly quantified.

The effects of transient strong electromagnetic fields have been under intensive discussions recently, especially in the context of the chiral magnetic effect [18–20]. So far, there has been no experimental evidence that strong fields actually exist. Observation of a charge-dependent directed flow would also provide evidence that a strong electromagnetic field is actually created in heavy-ion collisions.

Electric fields in Cu + Au collisions. Here, we show that, in off-central collisions between copper and gold nuclei, a sizable strength of electric field is generated in the overlapping regions of two nuclei. Because of the difference in the number of protons between the two nuclei, the generated electric field tends to the copper nucleus. The situation is different from the electromagnetic fields in the collisions of the same species of nuclei [12,21]. In symmetric collisions such as Au + Au or Cu + Cu, the event-averaged electric field does not have a specific direction, although the magnitude of the electric fields generated in each event is considerably large [$|e\vec{E}| \sim |e\vec{B}| \sim O(m_\pi^2)$]. We have performed event-by-event calculations of the electromagnetic fields in Cu + Au collisions to show that there should be a significantly large copper-directed electric field.

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[1]The possibility of estimating the electric conductivity of the QGP is suggested in Ref. [12].

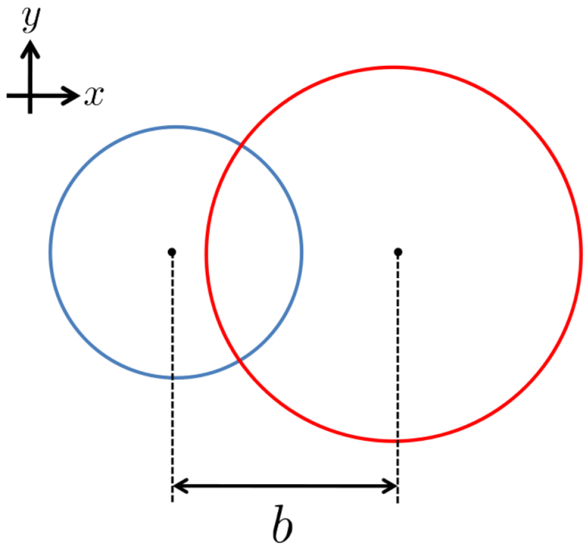

FIG. 1. (Color online) Transverse plane of off-central Cu + Au collisions with impact parameter b. The left (right) circle indicates the edge of the Cu (Au) nucleus.

The electromagnetic fields are generated by the protons in nuclei. If we regard protons as point particles, the electric and magnetic fields at a position $\vec{x}$ and time t are written by the Liénard-Wiechert potentials,

$$|e|\vec{E}(t,\vec{x}) = \alpha_{\rm EM}\sum_n \frac{1-v_n^2}{R_n^3\left[1-(\vec{R}_n\times\vec{v}_n)^2/R_n^2\right]^{3/2}}\vec{R}_n, \quad (1)$$

$$|e|\vec{B}(t,\vec{x}) = \alpha_{\rm EM}\sum_n \frac{1-v_n^2}{R_n^3\left[1-(\vec{R}_n\times\vec{v}_n)^2/R_n^2\right]^{3/2}}\vec{v}_n\times\vec{R}_n, \quad (2)$$

where $\vec{R}_n \equiv \vec{x} - \vec{x}_n(t)$ with $\vec{x}_n(t)$ the position vector of the nth proton, $\vec{v}_n$ is the velocity vector of nth proton, $|e|$ is the electric charge of a proton, and $\alpha_{\rm EM}$ is the fine structure constant. We define the origin of the spatial coordinate as the middle of the centers of the nuclei and x and y axes as in Fig. 1.[2] The summation is taken over all the protons in the colliding two nuclei. The positions of the protons inside a nucleus are sampled from the Woods-Saxon distribution with the standard parameters [22].

Figure 2 shows the event-averaged electric fields in Cu + Au collisions at impact parameter $b = 4$ fm. Each vector represents direction and magnitude of the electric field at that point. We find that the electric field in the central region of the overlapping area has a specific tendency to go from Au to Cu. Although the direction of electric fields fluctuates on an event-by-event basis because of the fluctuation in the proton positions inside colliding nuclei, the direction is correlated with the reaction plane for asymmetric collisions. The magnitude of the electric fields is as large $[|e\vec{E}| \sim O(m_\pi^2)]$ as the electric and magnetic fields in Au + Au collisions at the same collision energy.

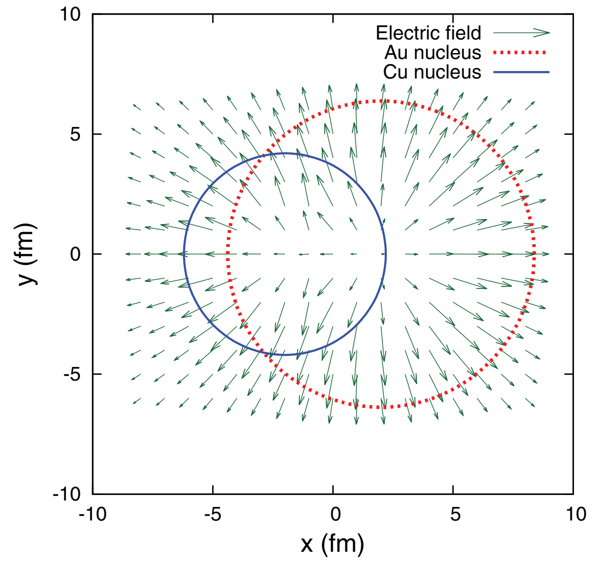

FIG. 2. (Color online) Event-averaged electric field in the transverse plane in off-central Cu + Au collisions at $t = 0$ (the collision time) with impact parameter $b = 4$ fm at $\sqrt{s_{NN}} = 200$ GeV. Vectors are shown only in $|y| < 6$ fm. The average is taken over 10^4 events.

We have also calculated the time dependence of the averaged electric fields as shown in Fig. 3. The strength of the fields decays as the spectators fly away. Nevertheless, it is notable that even at $t = 1$ fm/c the electric field is considerably larger than the so-called "critical field" for electrons, $|e|B_c = |e|E_c = m_e^2$ [23].

Electric dipole of the plasma and charge-dependent directed flow. The strong electric field toward the Cu nucleus at the early stage would induce an electric current in the medium that consists of the QGP after the thermalization

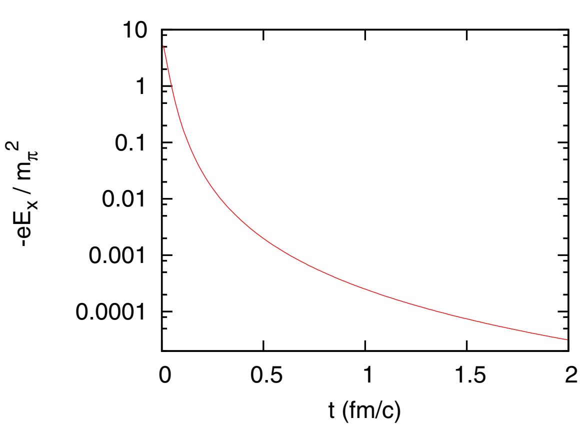

FIG. 3. (Color online) Event average of the time evolution of electric fields in off-central Cu + Au collisions ($b = 4$ fm) at $\vec{x} = \vec{0}$. The average is taken over 10^4 events.

[2]One may wonder whether we can take the origin of the azimuthal angle on the Au nucleus side in experiments. According to a report from the PHENIX group [11], it is indeed possible to experimentally determine on which side the Au or Cu nucleus is. By measuring the spectators, the origin of the azimuthal angle in the event plane of v_1 is determined and is always taken on the Au-going side.

time.[3] As a result, the charge distribution would be modified and a charge dipole would be formed. One can expect that the dipole-like deformation of the charge distribution at the early stage would also be present in the observed charge distribution. This is because the electromagnetic charge is an exactly conserved quantity and an inhomogeneity relaxation of a conserved charge density takes a long time. Once a radial flow starts, the medium expands rapidly and the charge dipole created at the early stage would be frozen. Thus, we can reasonably assume that the dipole-deformation in the plasma remains intact in the observed charge distribution.

The azimuthal angle distribution of the net charge, to the leading order in the multipole expansion, can be written as

$$\frac{d\,(N_+ - N_-)}{d\phi}(\phi) = (\bar{N}_+ - \bar{N}_-)\,(1 + 2d_e\cos\phi)\,, \tag{3}$$

where the azimuthal angle ϕ is measured from the x axis and $\bar{N}_\pm$ is defined as the angle average of the number distribution,

$$\bar{N}_\pm \equiv \int \frac{d\phi}{2\pi}\frac{dN_\pm}{d\phi}. \tag{4}$$

The dipole deformation of the medium is quantified by the value of d_e. We assume that the azimuthal distribution of the total number of particles is still written by v_1 without the effect of the electromagnetic fields as

$$\frac{d\,(N_+ + N_-)}{d\phi} = (\bar{N}_+ + \bar{N}_-)\,(1 + 2v_1\cos\phi)\,, \tag{5}$$

since electromagnetic fields are not expected to change the bulk flow significantly. From Eqs. (3) and (5), the distribution of charged particles can be written as

$$\begin{aligned}\frac{dN_+}{d\phi} &= \bar{N}_+\left[1 + \frac{\bar{N}_+ + \bar{N}_-}{2\bar{N}_+}2\,(v_1 + Ad_e)\cos\phi\right]\\ &= \bar{N}_+\{1 + 2[v_1 + A(d_e - v_1)]\cos\phi + O[(Ad_e)^2]\},\end{aligned} \tag{6}$$

where we have defined the charge asymmetry parameter $A \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$. Similarly,

$$\frac{dN_-}{d\phi} = \bar{N}_-\{1 + 2[v_1 - A(d_e - v_1)]\cos\phi + O[(Ad_e)^2]\}. \tag{7}$$

Thus, the directed-flow coefficients v_1 for positively and negatively charged particles are written as

$$v_1^\pm = v_1 \pm Ad_e', \tag{8}$$

where we have defined $d_e' \equiv d_e - v_1$. The values $v_1^\pm$ are linear functions of A and their slopes are given by the dipole-like deformation parameter d_e', which is written as

$$d_e' = \frac{1}{\bar{N}_+ - \bar{N}_-}\int r\,dr\,d\phi\,\cos\phi\big[j_e^0(r,\phi) - j^0_{e,\vec{E}=\vec{B}=0}(r,\phi)\big], \tag{9}$$

where $j_e^0(r,\phi)(j^0_{e,\vec{E}=\vec{B}=0}(r,\phi))$ is the transverse charge density in the presence (absence) of electromagnetic fields.

Estimate of the charge-dependent directed flow. Let us make an order-of-magnitude estimate of the value of the charge-dependent directed flow parameter Ad_e'. For that purpose, we first roughly evaluate the total charge that is transfered from the gold-side to copper-side in the presence of an electric field. The total charge Q transfered across a plane S from $t = 0$ to τ is written as

$$Q = \int_0^\tau dt \int_S \vec{J}\cdot d\vec{S} = \int_0^\tau dt \int_S \sigma\vec{E}\cdot d\vec{S}, \tag{10}$$

where we have used the constitutive relation $\vec{J} = \sigma\vec{E}$ with σ the electric conductivity. Let S be the plane which includes the origin and is perpendicular to the line which connects the centers of the two colliding nuclei at $t = 0$, the moment two nuclei contact. Neglecting the space-time dependence of σ, Q is rewritten as

$$Q \sim \sigma\tau\int_S \vec{E}\cdot d\vec{S}. \tag{11}$$

The integral in Eq. (11) is just the total electric flux that goes through the plane S. Hence, the total transfered charge Q is roughly given by

$$\int_S \vec{E}\cdot d\vec{S} \sim \frac{Z_{\rm Au} - Z_{\rm Cu}}{2}\frac{|e|}{\epsilon}, \tag{12}$$

where $Z_{\rm Au}$ and $Z_{\rm Cu}$ are the numbers of protons in the two nuclei, and ϵ is the dielectric constant of the QGP.

According to lattice QCD simulations, the electric conductivity of the QGP is estimated as

$$\sigma \sim B\,C_{\rm EM}T, \quad C_{\rm EM} \equiv \sum_f e_f^2, \tag{13}$$

where the sum in the electromagnetic vertex factor is taken over the flavors and B is a coefficient. If we consider u, d, and s quarks, $C_{\rm EM} = 8\pi\alpha_{\rm EM}/3$. The value of the coefficient B differs among calculations: $B \simeq 0.4$ in Refs. [7,8] and $B \simeq 7$ in Ref. [6]. On the other hand, perturbative QCD calculations predicts $\sigma \simeq 6T/e^2$ [10], which is much larger than the values from lattice QCD simulations.

As for τ, we take the time scale that the radial flow starts, $\tau \sim 1$ fm/c. If we take typical values for the other parameters, $T \sim 200$ MeV and $\epsilon \sim 1$, the total transfered charge is estimated as

$$\begin{aligned}Q &\sim BC_{\rm EM}T\tau\,\frac{Z_{\rm Au} - Z_{\rm Cu}}{2}\frac{|e|}{\epsilon}\\ &\sim B\frac{8\pi}{3}\alpha_{\rm EM}\times\ 200\ {\rm MeV}\times\ 1\ {\rm fm}/c\times\ 25|e|\\ &\sim 1.7\,|e|\times B.\end{aligned} \tag{14}$$

[3]The matter would be in the state of glasma before thermalized quark gluon plasma is formed. It is possible that the measured charge-dependent directed flow also reflects the conducting property of such matter. However, we expect that the conductivity of glasma is far smaller compared to that of quark gluon plasma. That is because glasma basically does not have charged particles as a constituent, while the QGP does. The quarks and antiquarks in the QGP are deconfined, which makes QGP have a high conductivity. That is why we assume that the charge asymmetry created in the evolution originates mostly from the property of QGP.

Now we can roughly evaluate Ad'_e. Let us choose the events in which the numbers of positive and negative hadrons are equal, $\bar{N}_+ = \bar{N}_-$, and assume that n charges have been transfered by the electric field. Then, the number n can be written as[4]

$$n = -\frac{1}{2}\int_{-\pi/2}^{\pi/2} d\phi \, \frac{d(N_+ - N_-)}{d\phi} = -2Ad'_e(\bar{N}_+ + \bar{N}_-). \tag{15}$$

Therefore, the directed flow parameter Ad'_e is written by n as

$$Ad'_e = -\frac{\pi n}{N_{\rm tot}}, \tag{16}$$

where $N_{\rm tot} \equiv 2\pi\left(\bar{N}_+ + \bar{N}_-\right)$ is the total number of charged particles. The number n is related to the total transferred charge roughly as $n \sim Q/|e|$. Therefore, Ad'_e, the charge-dependent part of the directed flow parameter, and the electric conductivity of the plasma are parametrically related as

$$Ad'_e \sim -\frac{\pi\sigma\tau}{N_{\rm tot}|e|}\int_S \vec{E}\cdot d\vec{S}. \tag{17}$$

If one takes $N_{\rm tot} \sim 10^3$ and $n \sim 1$ [Eq. (14)], the order of magnitude of the directed-flow parameter is estimated as

$$Ad'_e \sim -B \times 10^{-3}. \tag{18}$$

This value would be within experimental reach if the parameter B is larger than of order unity. Note that the value (18) is negative since the electric field tends toward the Cu nucleus. Although the estimate above is a crude one, we can distinguish at least whether the created matter is in the perturbative or nonperturbative regime by looking at the order of magnitude of deference between $v_1^{\pm}$. This is because perturbative calculations indicate significantly larger values of B ($\sim 10^2$) compared to lattice calculations ($B \sim 1$). This would indicate much progress compared to the current situation where little is known about the actual conductivity of the matter created in heavy-ion collisions.

As seen in Eq. (17), the magnitude of the electric flux which goes through the QGP is an important quantity. We calculated the impact parameter dependence of the event-averaged electric flux that goes through the overlapping region, $\Phi = \int_S \vec{E}\cdot d\vec{S}$, which is shown in Fig. 4. The plane S is chosen so that it is perpendicular to the line connecting the two centers of the two nuclei at $t = 0$, and it crosses the thickest part of the almond (dotted line in Fig. 1). In most central collisions, the electric flux is zero, and it gets larger when one increases b. For $1 \lesssim b \lesssim 5$ fm, the flux is positive, which means that the field directs toward the Au nucleus. At larger b the electric flux changes its sign and the direction of the fields is flipped. This result can be understood in the following way. At very peripheral collisions, the plane S is closer to the center of the Cu nucleus than to that of the Au nucleus. As a result, the flux that comes from the Cu nucleus becomes denser, because of its smaller radius. The behavior of the electric flux as a function of the impact parameter would be reflected in the centrality dependence of charge-dependent v_1.

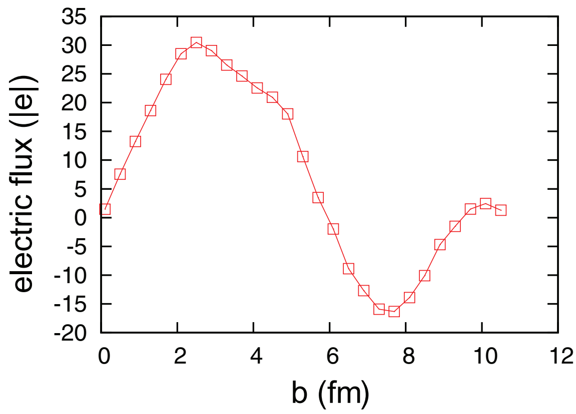

FIG. 4. (Color online) Event-averaged electric flux that goes through the thickest part of the almond-like shape of two overlapping nuclei, as a function of the impact parameter b. The value of the flux is the average over 2000 events for each b.

Let us comment on potential uncertainties in the estimate above. It is possible that the charge dipole formed at the early stage can be obscured in the later stages, namely the hydrodynamic evolution and hadronic collisions. In order to quantify these effects, we have to calculate the time evolution of the charge density under an electric field. The back reaction of matter to electromagnetic fields may also have to be taken into account [21,24]. Hence, it would be desirable to use a magnetohydrodynamic model combined with a hadronic afterburner. One should also consider the effects of fluctuations of the generated electric fields on an event-by-event basis, although the fields have a tendency to direct from Au to Cu nucleus on average. The charge dipole could be weakened by the fluctuations. Event-by-event simulations are necessary to consider the effect of such fluctuations. Finally, although we have assumed the dielectric constant is a constant, it can in general depend on frequency and wave length. Consideration of such effects is left as a future work.

Summary. We have pointed out that, in Cu + Au collisions, a sizable strength of electric field directed from Au to Cu nucleus is generated in the overlapping region. We have shown this by performing event-by-event numerical calculation of the produced electromagnetic fields. We have also pointed out that the electric field would induce an electric current in the matter created after the collision and it would result in a dipole deformation of the charge distribution in the medium. We have shown that the charge-dependent directed flow of hadrons is sensitive to the charge dipole in the medium and is useful in estimating the electric conductivity of the QGP.

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[4]Note that $Ad'_e = Ad_e$ for $\bar{N}_+ = \bar{N}_-$.

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