Intermediate-energy deuteron scattering from α-cluster nuclei

Yu. A. Berezhnoy*
Karazin Kharkov National University, 61077 Kharkov, Ukraine

D. V. Fedorchenko† and V. V. Pilipenko‡
National Science Center "Kharkov Institute of Physics and Technology", 61108 Kharkov, Ukraine

V. P. Mikhailyuk§
Institute for Nuclear Research, 03680 Kiev, Ukraine
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The polarization observables for elastic 400 and 700 MeV deuteron scattering from ^{12}C and ^{16}O nuclei have been analyzed in the framework of the multiple diffraction scattering theory and the α-cluster model with dispersion. The rigid projectile approximation with "effective" d-α scattering amplitude as well as the three-body $n + p + A$ model, which uses nucleon-A scattering amplitudes and the deuteron ground-state wave function with both S and D waves, are applied for the calculations. Both approaches are compared with each other and with the available experimental data.

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I. INTRODUCTION

The theoretical description of light-nuclei scattering from various nuclei was a subject of numerous publications (see, for instance Ref. [1], and references therein). In the intermediate energy region the proper description of such processes requires considering both the incident nucleus internal structure and the structural features of the target nuclei. Moreover, in the consideration of the nucleus-nucleus scattering the influence of the nuclear environment, in which nucleons are placed, can play an important role [2]. In this case, solution of the many-body scattering problem is highly complicated, because the nucleons or clusters of the incident nuclei interact both with each other and with the target nucleus structural components. This interaction distorts the standard "elementary" free scattering amplitudes and leads to introduction of effective amplitudes [3–6].

From this point of view, the deuteron as the lightest complex nucleus is a unique object for theoretical treatment by itself and also as a test nucleus for validation of different theoretical models. So, on the one hand, its complex structure can imply the necessity of using "effective" amplitudes to describe deuteron-nucleus interaction. On the other hand, taking into account the small binding energy of the deuteron and the successful description of polarized proton scattering observables at intermediate energies on the basis of the multiple diffraction scattering theory (MDST) [7] variants using "free" nucleon-nucleon (or nucleon-cluster) amplitudes, one can expect that such MDST models should also be sufficiently accurate for calculations of deuteron scattering observables in this energy region.

The frequently used models for deuteron scattering at energies of about several tens of MeV are various versions of the folding model [8,9], the microscopic optical model with the effective Skyrme interaction [10,11], the three-body $n + p + A$ Faddeev-like scattering model with nonlocal and local optical potentials [12,13], and the diffraction model [14,15]. For the deuteron energy range $E_d = 100$–200 MeV the phenomenological optical model with the global deuteron optical potential [16,17] is adopted. We should also specially mention the continuum-discretized coupled channels (CDCC) method, which was successfully employed for studying the deuteron-nucleus scattering and reactions in a wide energy range from tens to hundreds of MeV during recent decades [18–22]. The CDCC formalism is a three-body approach, which describes the deuteron-nucleus interaction in terms of the nucleon-target optical potentials and explicitly includes effects of the deuteron virtual breakup channels, which can be important for such weakly bound projectiles as deuterons. The approaches suitable for use at the intermediate energies ($E_d =$ 200–700 MeV) are the distorted wave impulse approximation [23–26], relativistic and nonrelativistic optical models with various nucleon-nucleus optical potentials [27,28], as well as the multiple diffraction scattering models [29–34].

The main purpose of this paper is to develop an approach for describing the deuteron-nucleus scattering using the MDST variant that is based on the consideration of cluster degrees of freedom of the target nuclei. This approach seems to be rather attractive, as compared with the usual MDST variant making use of NN amplitudes and the single-nucleon description of the target nuclei, owing to the fact that it allows for nucleon-nucleon correlations of alpha-particle type and correlations between the alpha particles themselves in the used multicluster densities of target nuclei, and the nucleon-α amplitude as an elementary building block of the model effectively takes into account different processes of many-body interactions of incident particles with nucleons of the α cluster. It was shown in [35] that this cluster approach yielded a more accurate description of the polarization observables for p-^{12}C and p-^{16}O scattering.

*yuberezhnoy@pht.univer.kharkov.ua
†d.fedorchenko@gmail.com
‡vpilipenko@kipt.kharkov.ua
§mikhailyuk@kinr.kiev.ua

Recently [36,37] such an approach based on the use of the α-cluster model with dispersion [35,38] was employed to describe the elastic deuteron scattering from ^{12}C and ^{16}O nuclei. In these calculations two models were considered. The first of them is the rigid projectile (RP) approximation [39,40], which treats the incident nucleus as an elementary object. In this approximation the total deuteron-nucleus scattering amplitude was constructed in the MDST from "effective" deuteron-cluster amplitudes in the same way as the p-A scattering amplitude from proton-cluster amplitudes in Refs. [35,38]. The second approach is the three-body $n + p + A$ (TB) model [41], which incorporates the α-cluster structure of the target nuclei by using the elastic p-^{12}C and p-^{16}O scattering amplitudes calculated in Refs. [35,38].

The calculations revealed that, while the cross sections and analyzing powers A_y were in reasonable agreement with experimental data, the tensor analyzing powers A_{yy} had some discrepancies with experiment. A possible cause of such discrepancies are the approximations employed in these calculations: in the TB model we neglected the D wave in the deuteron ground state [36], and in the RP model the spin-spin terms in the "effective" deuteron-α amplitude were omitted [37].

In the present paper, to calculate the elastic d-^{12}C and d-^{16}O scattering observables, we use the RP approximation with an "effective" full-form spin-dependent d-α scattering amplitude (Sec. III) as well as the TB model, in which we include the deuteron ground-state wave function with both S and D waves (Sec. IV). The general formulas used in these calculations are given in Sec. II and in the Appendix, and the results of comparisons of both approaches with each other and with the available experimental data are presented in Sec. V.

II. THE SPIN-DEPENDENT MDST SCATTERING AMPLITUDE AND OBSERVABLES

In the general case of nucleus-nucleus collision, the MDST scattering amplitude is [7]

$$F(\mathbf{q}) = \frac{ik_{\rm in}}{2\pi}\int d^2b\, \exp(i\mathbf{qb})\langle\Psi_f,\varphi_f|\delta(\mathbf{R}_{\rm in})\delta(\mathbf{R}_t) \times \left\{1-\hat{Z}\prod_{i=1}^{N}\prod_{j=1}^{N_{\rm in}}[1-\omega_{ij}(\mathbf{b}-\mathbf{r}_i+\boldsymbol{\rho}_j)]\right\}|\Psi_i,\varphi_i\rangle, \tag{1}$$

where $N_{\rm in}$ and N are the numbers of subunits in the incident and target nuclei respectively, $k_{\rm in}$ is the wave vector of the incident particle, $\varphi_{i,f}(\{\boldsymbol{\rho}_j\})$ and $\Psi_{i,f}(\{\mathbf{r}_i\})$ are the wave functions of initial and final states of the incident particle and target nucleus, $\boldsymbol{\rho}_j$ and $\mathbf{r}_i$ are the radius vectors of the scattered subunits of the colliding nuclei, $\mathbf{b}$ is the impact parameter, $\mathbf{q}$ is the momentum transfer, $\delta(\mathbf{r})$ is the Dirac delta function, $\mathbf{R}_{\rm in}$ and $\mathbf{R}_t$ are the center-of-mass radius vectors

$$\mathbf{R}_{\rm in} = \frac{1}{N_{\rm in}}\sum_{j=1}^{N_{\rm in}}\boldsymbol{\rho}_j, \quad \mathbf{R}_t = \frac{1}{N}\sum_{i=1}^{N}\mathbf{r}_i, \tag{2}$$

$\omega_{ij}(\mathbf{b})$ are the profile functions, describing interaction between the jth structure component of the incident particle and the ith structure component of the target nucleus, and $\hat{Z}$ is the Z-ordering operator. The profile functions $\omega_{ij}(\mathbf{b})$ are given by the inverse Fourier transform of "elementary" scattering amplitudes $f_{ij}(\mathbf{q})$,

$$\omega_{ij}(\mathbf{b}) = \frac{1}{2\pi i k_j}\int d^2q\, \exp(-i\mathbf{qb}) f_{ij}(\mathbf{q}). \tag{3}$$

In the case of deuteron scattering on zero-spin nuclei, the scattering amplitude (1) becomes a matrix in the spin space and we can write its matrix elements as

$$[F(\mathbf{q})]_{M'M} = \langle\chi_{1M'}|\hat{\mathcal{F}}(\mathbf{q})|\chi_{1M}\rangle, \quad M,M' = -1,0,1, \tag{4}$$

where the elastic scattering amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$ is introduced, and χ_{1M} are the basis spin functions for the spin $S = 1$.

The expression for operator $\hat{\mathcal{F}}(\mathbf{q})$ formally coincides with the expression (1) where the profile functions now should be understood as spin operators $\hat{\omega}_{ij}$, and in the wave functions $\varphi_{i,f}$ only the coordinate parts should be taken into account.

For deuteron scattering from zero-spin nuclei, the scattering amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$, complying with the requirements of P and T invariance, has four independent terms with different spin structure, and its possible representation is

$$\hat{\mathcal{F}}(\mathbf{q}) = \mathcal{A}_1(\mathbf{q}) + \mathcal{A}_2(\mathbf{q})(\mathbf{Sn}) + \mathcal{A}_3(\mathbf{q})(\mathbf{Sn})^2 + \mathcal{A}_4(\mathbf{q})(\mathbf{S}\hat{\mathbf{k}}_d)(\mathbf{S}\hat{\mathbf{k}}'_d), \tag{5}$$

where $\mathbf{S}$ is the deuteron spin operator, $\mathbf{n} = (\mathbf{k}_d \times \mathbf{k}'_d)/|\mathbf{k}_d \times \mathbf{k}'_d|$, $\mathbf{k}_d$ and $\mathbf{k}'_d$ are the vectors of initial and final deuteron momenta, and $\mathcal{A}_i(\mathbf{q})$ are the invariant scalar amplitudes.

Besides the representation (5), there are also other convenient representations for the operator $\hat{\mathcal{F}}(\mathbf{q})$. In particular, experimental measurements of polarization observables are commonly performed using the individual spiral coordinate systems for incident and scattered particles. In this case, instead of the spin-matrix amplitude representation (5), it is more expedient to use the products $\chi_{i'}\chi_j^\dagger$ of the basis spin functions as basis matrices (here, $i' = x',y',z'$ and $j = x,y,z$ denote the Cartesian components of the spin functions in the spiral systems of the scattered and incident deuterons). Rewriting (5) in this representation, we obtain

$$\hat{\mathcal{F}}(\mathbf{q}) = a(\mathbf{q})\chi_{x'}\chi_x^\dagger + b(\mathbf{q})\chi_{y'}\chi_y^\dagger + c(\mathbf{q})\chi_{z'}\chi_z^\dagger + d(\mathbf{q})\chi_{z'}\chi_x^\dagger + e(\mathbf{q})\chi_{x'}\chi_z^\dagger. \tag{6}$$

Here $d(\mathbf{q}) = -e(\mathbf{q})$ and, as in (5), we still have four independent invariant amplitudes

$$a(\mathbf{q}) = \mathcal{A}_0(\mathbf{q})\cos\theta + i\mathcal{A}_2(\mathbf{q})\sin\theta, \tag{7}$$

$$b(\mathbf{q}) = \mathcal{A}_1(\mathbf{q}) + \mathcal{A}_4(\mathbf{q})\cos\theta, \tag{8}$$

$$c(\mathbf{q}) = a(\mathbf{q}) - \mathcal{A}_4(\mathbf{q}), \tag{9}$$

$$d(\mathbf{q}) = -e(\mathbf{q}) = \mathcal{A}_0(\mathbf{q})\sin\theta - i\mathcal{A}_2(\mathbf{q})\cos\theta, \tag{10}$$

where

$$\mathcal{A}_0(\mathbf{q}) = \mathcal{A}_1(\mathbf{q}) + \mathcal{A}_3(\mathbf{q}) + \mathcal{A}_4(\mathbf{q})\cos\theta. \tag{11}$$

The full set of observables, completely determining the amplitude of deuteron scattering from zero-spin nuclei, includes the differential cross section $d\sigma(\theta)/d\Omega$ and, for example, four analyzing powers $A_y(\theta)$, $A_{yy}(\theta)$, $A_{xx}(\theta)$, $A_{xz}(\theta)$ and three polarization transfer coefficients $K_{zz}^{z'z'}(\theta)$, $K_{zz}^{x'z'}(\theta)$, $K_z^{y'z'}(\theta)$.

In the representation (6) these observables are determined by

$$\frac{d\sigma}{d\Omega} = \frac{1}{3}[|a|^2 + |b|^2 + |c|^2 + 2|d|^2], \quad (12)$$

$$3A_y \frac{d\sigma}{d\Omega} = 2\,\mathrm{Im}[(a+c)d^*], \quad (13)$$

$$3A_{yy} \frac{d\sigma}{d\Omega} = |a|^2 - 2|b|^2 + |c|^2 + 2|d|^2, \quad (14)$$

$$3A_{xx} \frac{d\sigma}{d\Omega} = -2|a|^2 + |b|^2 + |c|^2 - |d|^2, \quad (15)$$

$$3A_{xz} \frac{d\sigma}{d\Omega} = 3\mathrm{Re}[(a-c)d^*], \quad (16)$$

$$3K_{zz}^{z'z'} \frac{d\sigma}{d\Omega} = |a|^2 + |b|^2 + 4|c|^2 - 4|d|^2, \quad (17)$$

$$3K_{zz}^{x'z'} \frac{d\sigma}{d\Omega} = -3\,\mathrm{Re}[(a-2c)d^*], \quad (18)$$

$$3K_{z}^{y'z'} \frac{d\sigma}{d\Omega} = 3\,\mathrm{Im}[bd^*]. \quad (19)$$

III. RP APPROXIMATION

In the rigid projectile (RP) approximation [39,40] the incident nucleus is treated as an elementary object, and the MDST nucleus-nucleus scattering amplitude is constructed from "effective" amplitudes of the incident particle scattering on the components of the target nucleus. The purpose of introducing the RP approximation is to reduce the problem of nucleus-nucleus collision to the usual MDST approach, i.e., a pointlike particle scattering from multiple scattering centers.

For the case of deuteron scattering, this RP approximation coincides with the approach used for the proton scattering from ^{12}C and ^{16}O nuclei on the basis of the α-cluster model with dispersion [35,38]. Following the approach given in Refs. [35,38] and using the relation (1), in which we have to put $N_{\mathrm{in}} = 1$, the scattering amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$ can be presented in the form

$$\hat{\mathcal{F}}(\mathbf{q}) = \sum_{m=1}^{N} \alpha_m \hat{\mathcal{F}}_m(\mathbf{q}), \quad (20)$$

where N is the number of clusters constituting the target nucleus ($N = 3$ for ^{12}C and $N = 4$ for the ^{16}O nucleus), m is the scattering order, and α_m are the following coefficients: $\alpha = \{3,-3,1\}$ for the ^{12}C nucleus and $\alpha = \{4,-6,4,-1\}$ for the ^{16}O nucleus.

Within the framework of the α-cluster model with dispersion, the operators $\hat{\mathcal{F}}_m$ in (20) are

$$\hat{\mathcal{F}}_m(\mathbf{q}) = \frac{ik_d}{2\pi}\int d^2b \left(\prod_{j=1}^{N-1} d^3\xi_j\right) \exp(i\,\mathbf{q}_d\mathbf{b})\rho(\{\boldsymbol{\xi}_i\}) \prod_{j=1}^{m} \times \left[\frac{1}{2\pi i k_d}\int d^2q_j \exp[-i\,\mathbf{q}_j(\mathbf{b}-\mathbf{r}_j)]\hat{f}_{d\alpha}(\mathbf{q}_j)\right], \quad (21)$$

where k_d is the wave vector of the incident deuteron, $\hat{f}_{d\alpha}(\mathbf{q})$ is the deuteron-α scattering amplitude, $\boldsymbol{\xi}_i$ are the Jacobi coordinates of the α clusters, and $\rho(\{\boldsymbol{\xi}_i\})$ are the multicluster densities of the ^{12}C and ^{16}O nuclei [35,38]:

$$\rho(\{\boldsymbol{\xi}_i\}) = C\int \prod_{j\neq i}^{N-1} \delta(\boldsymbol{\xi}_i'\boldsymbol{\xi}_j') \prod_{l=1}^{N-1}\left\{\delta(\xi_l' - \lambda_l d) \times \exp\left[-\frac{\mu_l(\boldsymbol{\xi}_l - \boldsymbol{\xi}_l')^2}{2\Delta^2}\right] d^3\xi_l'\right\}. \quad (22)$$

Here d is the mean distance between the α clusters in the ^{12}C and ^{16}O nuclei, the parameter Δ characterizes the probability of α-cluster displacement from their most probable positions at the vertices of the equilateral triangle and tetrahedron, C are the normalization constants [$\rho(\{\boldsymbol{\xi}_i\})$ are normalized to unity], and coefficients λ_i, μ_i are the following:

$$\lambda_1 = 1, \quad \lambda_2 = \frac{\sqrt{3}}{2}, \quad \lambda_3 = \sqrt{\frac{2}{3}}, \quad (23)$$

$$\mu_1 = 1, \quad \mu_2 = \frac{4}{3}, \quad \mu_3 = \frac{3}{2}. \quad (24)$$

The values of the parameters d and Δ were determined in [35,38] from the fitting of the charge form factors calculated for the target nuclei by the α-cluster model with dispersion to the corresponding experimental data. The obtained values of these parameters are $d = 2.98$ fm, $\Delta = 0.346$ fm for the ^{12}C nucleus and $d = 3.16$ fm, $\Delta = 0.643$ fm for the ^{16}O nucleus.

It is convenient to represent the deuteron-α scattering amplitude $f_{d\alpha}(\mathbf{q})$ in the following form:

$$f_{d\alpha}(\mathbf{q}) = f_1(\mathbf{q}) + qf_2(\mathbf{q})(\mathbf{Sn}) + q^2 f_3(\mathbf{q})(\mathbf{Sn})^2 + q^2 f_4(\mathbf{q})(\mathbf{S}\hat{\mathbf{q}})^2. \quad (25)$$

By analogy with the approach given in Ref. [35], the invariant amplitudes $f_i(\mathbf{q})$ can be approximated as two-Gaussian functions:

$$f_i(\mathbf{q}) = k_d \sum_{j=1}^{2} G_j^{(i)} \exp\left(-\beta_j^{(i)} q^2\right). \quad (26)$$

According to this approach, here the parameters $G_1^{(i)}$ and $\beta_1^{(i)}$ of the effective deuteron-α amplitude $f_{d\alpha}(\mathbf{q})$ are considered as fitting ones, and the remaining parameters in (26) have been derived from the requirement that the terms with $G_2^{(i)}$ must coincide with the leading terms of the double scattering part of the deuteron-^{4}He amplitude, calculated in the MDST approach with the Gaussian deuteron-nucleon amplitude and the single-nucleon density of the ^{4}He nucleus taken in the oscillator form. This leads to the following relations:

$$G_2^{(i)} = \frac{3iG_1^{(1)}G_1^{(i)}\beta_1^{(1)}}{8\left(\beta_1^{(1)} + \beta_1^{(i)}\right)^2}, \quad \beta_2^{(i)} = \frac{\beta_1^{(1)}\beta_1^{(i)}}{\beta_1^{(1)} + \beta_1^{(i)}}. \quad (27)$$

Finally, executing the integration in (21), we obtain the relations for the elastic d-^{12}C and d-^{16}O scattering amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$ in the form (5). The corresponding analytical

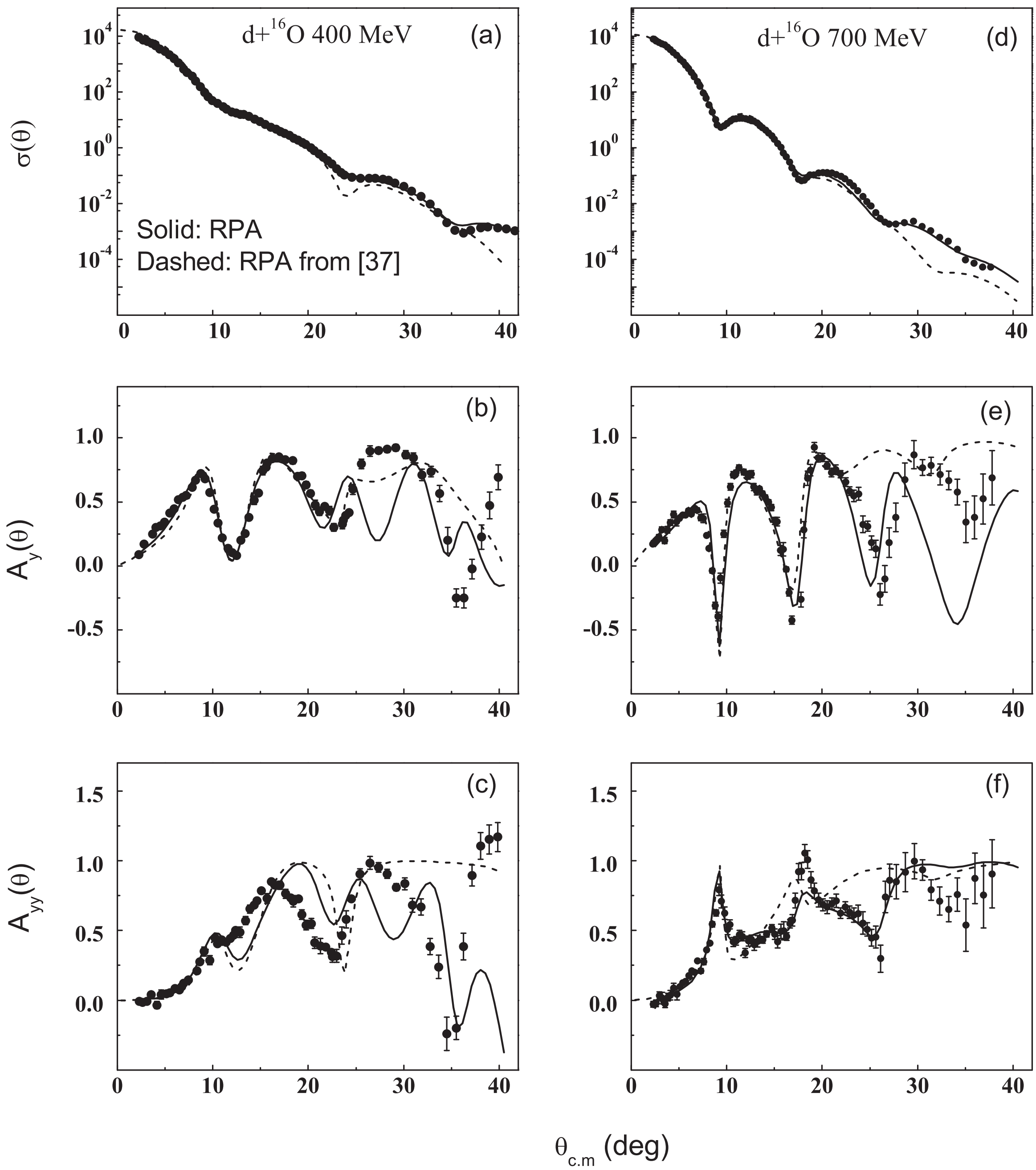

FIG. 1. Differential cross section $\sigma(\theta) \equiv d\sigma(\theta)/d\Omega$ (mb/sr) and analyzing powers $A_y(\theta)$, $A_{yy}(\theta)$ for the elastic deuteron scattering from ^{16}O nuclei at 400 and 700 MeV, calculated in the RP approximation. The solid curves have been calculated using the deuteron-α scattering amplitude $f_{d\alpha}(\mathbf{q})$ in the form (25), and the dashed curves were obtained in Ref. [37], allowing only for the first two terms in (25). The experimental data are from Ref. [43].

expressions for the invariant scalar amplitudes $\mathcal{A}_i(\mathbf{q})$ are given in the Appendix.

The results of such calculations are presented in Fig. 1 by solid curves. The dashed curves in this figure were calculated in Ref. [37], allowing only for the first two terms in (25).

The set of adjustable parameters $G_1^{(i)}$ and $\beta_1^{(i)}$, obtained from fitting the experimentally measured elastic d-^{16}O scattering observables $d\sigma(\theta)/d\Omega$, $A_y(\theta)$, and $A_{yy}(\theta)$, is given in Table I.

Figure 1 shows that using the full-form spin-dependent d-α scattering amplitude (25) in the RP-model we can significantly improve the agreement between the calculated and measured scattering observables as compared with the similar approach that takes into account only the terms linear in the deuteron spin operator $\mathbf{S}$ [37].

The observed differences between the observables calculated in this approach and the experimental data may be ascribed to the fact that in the region of momentum transfer $q \geqslant 3$ fm^{-1} ($\theta \geqslant 25^\circ$ for the deuteron energy of 700 MeV and $\theta \geqslant 30^\circ$ for 400 MeV), the conditions of applicability of the α-cluster model with dispersion are violated [35,38].

TABLE I. The parameters of d-α amplitude at 400 and 700 MeV.

Parameters	Energy (MeV)	
	400	700
$G_1^{(1)}$ (fm^2)	$-0.758+i2.162$	$-0.521+i1.619$
$\beta_1^{(1)}$ (fm^2)	$1.627+i0.899$	$1.015-i0.265$
$G_1^{(2)}$ (fm^3)	$0.289+i1.131$	$0.211+i0.795$
$\beta_1^{(2)}$ (fm^2)	$0.688+i0.316$	$0.604-i0.140$
$G_1^{(3)}$ (fm^4)	$0.211-i0.082$	$0.034-i0.003$
$\beta_1^{(3)}$ (fm^2)	$1.199-i0.152$	$0.184+i0.110$
$G_1^{(4)}$ (fm^4)	$0.011+i0.011$	$-0.145-i0.086$
$\beta_1^{(4)}$ (fm^2)	$0.163-i0.700$	$0.510+i0.132$

IV. TB APPROXIMATION

Within the three-body $n+p+A$ (TB) model, the d-A amplitude is constructed using known nucleon-nucleus scattering amplitudes. The advantage of this approach, as compared with the RP approximation, lies in the fact that the nucleon-nucleus scattering amplitudes and the deuteron ground-state wave function can be incorporated into the model from an independent analysis and, therefore, this TB approximation contains no free adjustable parameters.

In the TB approximation, the operator $\hat{\mathcal{F}}(\mathbf{q})$ can be represented in the following general form [41]:

$$\hat{\mathcal{F}}(\mathbf{q}) = \frac{ik_d}{2\pi}\int d^2b\,\exp(i\mathbf{q}\mathbf{b})\langle\Psi_0|\hat{\Omega}\times\left(\mathbf{b}+\frac{1}{2}\mathbf{r}_\perp,\mathbf{b}-\frac{1}{2}\mathbf{r}_\perp\right)|\Psi_0\rangle, \quad (28)$$

$$\hat{\Omega}(\mathbf{b}_p,\mathbf{b}_n) = \hat{\omega}_p(\mathbf{b}_p)+\hat{\omega}_n(\mathbf{b}_n)-\hat{\omega}_p(\mathbf{b}_p)\hat{\omega}_n(\mathbf{b}_n), \quad (29)$$

where $\mathbf{r}_\perp$ is the projection of the internal deuteron coordinate $\mathbf{r}=\mathbf{r}_n-\mathbf{r}_p$ onto the plane perpendicular to the incident beam, Ψ_0 is the deuteron ground-state wave function, and $\hat{\omega}_p(\mathbf{r})$ and $\hat{\omega}_n(\mathbf{r})$ are the profile functions for the proton and neutron scattering from the target nuclei.

The operator $\hat{\mathcal{F}}(\mathbf{q})$ can be presented as

$$\hat{\mathcal{F}}(\mathbf{q}) = \hat{\mathcal{F}}_1(\mathbf{q})+\hat{\mathcal{F}}_2(\mathbf{q}), \quad (30)$$

where the single and double scattering amplitude operators $\hat{\mathcal{F}}_i(\mathbf{q})$ are

$$\hat{\mathcal{F}}_1(\mathbf{q}) = \frac{k_d}{k}\langle\Psi_0|\hat{f}_p(\mathbf{q})+\hat{f}_n(\mathbf{q})|\Psi_0\rangle, \quad (31)$$

$$\hat{\mathcal{F}}_2(\mathbf{q}) = \frac{ik_d}{2\pi k^2}\int d^2q'\langle\Psi_0|\exp(i\mathbf{q}'\mathbf{r})\hat{f}_p(\mathbf{q}_1)\hat{f}_n(\mathbf{q}_2)|\Psi_0\rangle, \quad (32)$$

and $\mathbf{q}_{1,2}=\frac{1}{2}\mathbf{q}\mp\mathbf{q}'$.

The amplitudes $\hat{f}_{p,n}(\mathbf{q})$, describing the elastic proton (neutron) scattering from zero-spin target nuclei, can be presented in the form

$$\hat{f}_{p,n}(\mathbf{q}) = f_c^{(p,n)}(\mathbf{q})+qf_s^{(p,n)}(\mathbf{q})(\boldsymbol{\sigma}_{p,n}\mathbf{n}), \quad (33)$$

where $\mathbf{n}=(\mathbf{k}\times\mathbf{k}')/|\mathbf{k}\times\mathbf{k}'|$ and $\mathbf{k}$, $\mathbf{k}'$ are the initial and final nucleon momenta, and $\boldsymbol{\sigma}$ are the Pauli matrices.

TABLE II. The parameters of p-α amplitude at 200 and 350 MeV.

Parameters	Energy (MeV)	
	200	350
$G_1^{(1)}$ (fm^2)	$-0.188+i0.829$	$-0.092+i0.857$
$\beta_1^{(1)}$ (fm^2)	$0.258-i0.010$	$0.309-i0.116$
$G_1^{(2)}$ (fm^3)	$0.129+i0.705$	$0.206+i0.397$
$\beta_1^{(2)}$ (fm^2)	$0.705+i0.289$	$0.498+i0.098$

Within the α-cluster model with dispersion the scalar amplitudes $f_i^{(p,n)}(\mathbf{q})$ $(i=c,s)$ in (33) are determined from the MDST expressions for the incident nucleon scattering on the α clusters of the target nuclei. General formulas for such calculations are given by Eqs. (20) and (21), in which we have to replace the deuteron-α scattering operators $\hat{f}_{d\alpha}(\mathbf{q}_j)$ by the nucleon-α operators $\hat{f}_{N\alpha}(\mathbf{q}_j)$.

We take the operators $\hat{f}_{N\alpha}(\mathbf{q}_j)$ in the form

$$\hat{f}_{N\alpha}(\mathbf{q}) = f_1(\mathbf{q})+qf_2(\mathbf{q})(\boldsymbol{\sigma}_N\mathbf{n}), \quad (34)$$

where the amplitudes $f_i(\mathbf{q})$ are determined by Eqs. (26) and (27).

We have derived the values of the adjustable parameters $G_1^{(i)}$ and $\beta_1^{(i)}$ of the amplitudes $f_i(\mathbf{q})$ from an independent analysis of the p-^{4}He elastic scattering data, and the obtained numerical values of these parameters are given in Table II.

Finally, neglecting the differences between the proton- and neutron-nucleus scattering amplitudes, we use for the scalar amplitudes $f_i^{(p,n)}(\mathbf{q})$ in (33) the same relations as for the elastic p-^{12}C and p-^{16}O scattering amplitudes calculated in Refs. [35,38].

The deuteron ground state wave function Ψ_0 with allowance for both the S and D waves is

$$\Psi_0 = \frac{1}{\sqrt{4\pi}r}\left\{u(r)+\frac{1}{\sqrt{8}}w(r)S_{12}\right\}\chi_{1M}, \quad (35)$$

where $S_{12}=3(\boldsymbol{\sigma}_p\hat{\mathbf{r}})(\boldsymbol{\sigma}_n\hat{\mathbf{r}})-\boldsymbol{\sigma}_p\boldsymbol{\sigma}_n$.

Following Ref. [42], the radial parts $u(r)$ and $w(r)$ of the deuteron ground state wave function Ψ_0 we take in the form

$$u(r) = N\sum_{i=1}^{n}C_i\exp(-\alpha_i r), \quad (36)$$

$$w(r) = N\rho\sum_{i=1}^{n}D_i\exp(-\beta_i r)\left\{1+\frac{3}{\beta_i r}+\frac{3}{(\beta_i r)^2}\right\}, \quad (37)$$

TABLE III. Parameters of the deuteron ground-state wave function.

k	α_k (fm^{-1})	C_k	β_k (fm^{-1})	D_k
1	0.2316	1.0	0.2316	1.0
2	1.82007	−136.26749	1.07145	287.15485
3	1.83181	135.26749	1.03542	−288.15485
		$\rho=0.033$		

where

$$N = \left\{ \sum_{i,j=1}^{n} \left[\frac{C_i C_j}{\alpha_i + \alpha_j} + \rho^2 \frac{D_i D_j}{\beta_i + \beta_j} \right] \right\}^{-\frac{1}{2}}, \quad n = 3. \tag{38}$$

Note that in Ref. [42] such wave functions were obtained from analyzing the experimentally measured electromagnetic deuteron form factors, and the corresponding values of the deuteron ground-state wave function parameters are given in Table III.

Substituting (35)–(38) into (31) and (32), we obtain the following expressions for the single and double scattering amplitudes $\hat{\mathcal{F}}_i(\mathbf{q})$:

$$\hat{\mathcal{F}}_1(\mathbf{q}) = \frac{2k_d}{k} \left\{ f_c(\mathbf{q}) \left[S_1\left(\frac{q}{2}\right) + 8S_2\left(\frac{q}{2}\right) \right] + f_s(\mathbf{q}) \left[S_3\left(\frac{q}{2}\right) + S_4\left(\frac{q}{2}\right) \right] (\mathbf{Sn}) - 12 f_c(\mathbf{q}) S_2\left(\frac{q}{2}\right) (\mathbf{S\hat{q}})^2 \right\}, \tag{39}$$

$$\hat{\mathcal{F}}_2(\mathbf{q}) = \frac{ik_d}{2\pi k^2} \int d^2q' \left\{ f_c(\mathbf{q_1}) f_c(\mathbf{q_2}) \hat{\mathcal{F}}^{(cc)}(q,q') + \frac{2}{q_2} f_c(\mathbf{q_1}) f_s(\mathbf{q_2}) \hat{\mathcal{F}}^{(cs)}(q,q') + \frac{1}{q_1 q_2} f_s(\mathbf{q_1}) f_s(\mathbf{q_2}) \hat{\mathcal{F}}^{(ss)}(q,q') \right\}, \tag{40}$$

where

$$\hat{\mathcal{F}}^{(cc)}(q,q') = S_1(q') + 8S_2(q') - 12S_2(q')\sin^2\varphi'(\mathbf{Sn})^2 - 12S_2(q')\cos^2\varphi'(\mathbf{S\hat{q}})^2, \tag{41}$$

$$\hat{\mathcal{F}}^{(cs)}(q,q') = \{S_3(q') + S_4(q')\} \left[\frac{q}{2} + q'\cos\varphi' \right] (\mathbf{Sn}), \tag{42}$$

$$\begin{aligned}\hat{\mathcal{F}}^{(ss)}(q,q') = &\{S_1(q') + 8S_2(q')\} \left[q'^2 - \frac{q^2}{4} \right] + 2\left\{ S_1(q') \left(\frac{q^2}{4} - q'^2\cos^2\varphi' \right) + 2S_2(q') \left[q^2 - 4q'^2\cos^2\varphi' \right.\right. \\ &\left.\left. + 3\left(q'^2 - \frac{q^2}{4} \right)\sin^2\varphi' \right] \right\} (\mathbf{Sn})^2 - 2\left\{ S_1(q') q'^2 \sin^2\varphi' + 2S_2(q') \left[4q'^2\sin^2\varphi' - 3\left(q'^2 - \frac{q^2}{4} \right)\cos^2\varphi' \right] \right\} (\mathbf{S\hat{q}})^2.\end{aligned} \tag{43}$$

In (39)–(43) we introduce the structure form factors $S_i(q)$:

$$S_1(q) = \int dr \{u^2(r) + w^2(r)\} j_0(qr), \tag{44}$$

$$S_2(q) = \frac{1}{\sqrt{8}} \int dr \left\{ u(r)w(r) - \frac{1}{\sqrt{8}} w^2(r) \right\} j_2(qr), \tag{45}$$

$$S_3(q) = \int dr \left\{ u^2(r) + \frac{1}{4} w^2(r) \right\} j_0(qr), \tag{46}$$

$$S_4(q) = \frac{1}{\sqrt{8}} \int dr \left\{ 2u(r)w(r) + \frac{1}{\sqrt{8}} w^2(r) \right\} j_2(qr), \tag{47}$$

where $j_k(x)$ are the spherical Bessel functions.

After some algebraic transformations the above formulas allow us to obtain the elastic d-A scattering amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$ (30) in the form (5), but, in contrast to the RP approximation, in the TB-approach when performing final calculations it is necessary to use numerical integration.

V. RESULTS AND DISCUSSION

On the basis of the above TB and RP approximations, we have calculated the complete set of observables for the elastic deuteron scattering from ^{12}C and ^{16}O nuclei at 400 and 700 MeV. The results of such calculations are given in Figs. 2 and 3 by solid and dashed curves, respectively. The dot-dashed curves in these figures are calculated in the TB approximation with only the S wave included [36,37].

As is seen from Figs. 2 and 3, the results obtained are in reasonable agreement with the existing experimental data, especially for the differential cross sections and analyzing powers A_y. The using of the RP approach provides a somewhat better description of the behavior of analyzing powers A_{yy} as compared to the approach that uses the TB approximation. In addition, the agreement between the calculated and measured observables at an incident deuteron energy of 700 MeV is somewhat better than that at 400 MeV.

For the deuteron scattering at 400 MeV the discrepancies between the calculation results and experimental data can be explained by the fact that for lower energies the conditions of validity of the MDST are violated, and the transition from a predominantly diffraction scattering mechanism for essential refractive ones is expected [43].

Considering the TB approximation, we can point out that introduction of the D wave appears to be most essential for small scattering angles ($\theta \lesssim 10^\circ$). In this case, incorporation of the D wave into the model provides a more correct behavior of the analyzing powers $A_{yy}(\theta)$ in this region of scattering angles (Figs. 2 and 3, solid curves) in comparison with the TB approach which uses only the S wave [36,37] (Figs. 2 and 3, dot-dashed curves).

Figures 2 and 3 also show noticeable differences in the behavior of other polarization observables calculated in the TB and RP approaches. These differences can be explained by the existing ambiguities in determining the "effective" d-α amplitude parameters, which have been found in the RP approach from the fitting of the d-^{16}O elastic scattering data. Such ambiguities arise owing to the lack of a complete set of measured d-A scattering observables.

In the present paper we also compare the results obtained in the RP and TB approaches for the elastic d-^{4}He scattering observables. When performing the calculations in the TB

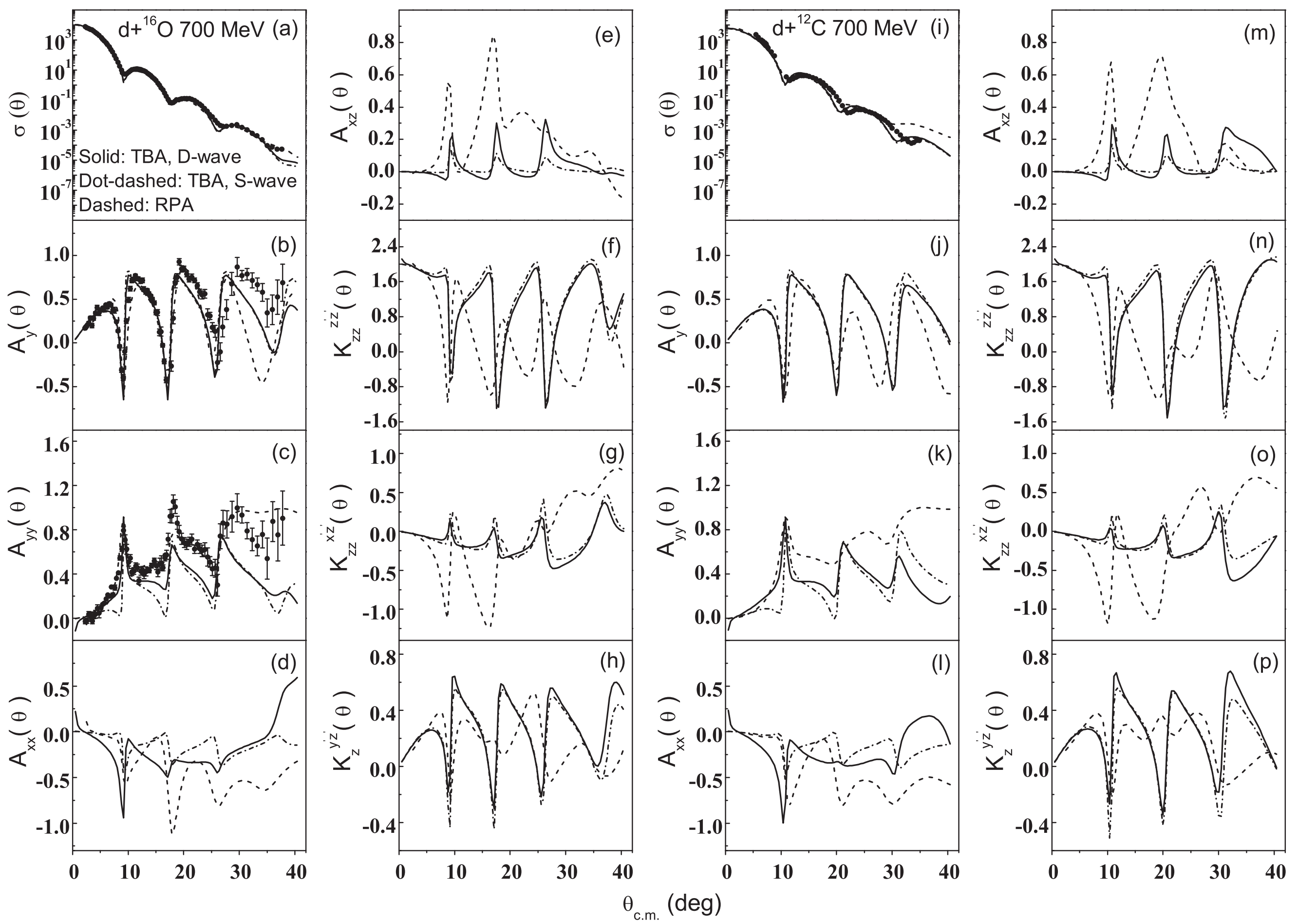

FIG. 2. Differential cross section $\sigma(\theta) \equiv d\sigma(\theta)/d\Omega$, analyzing powers $A_y(\theta)$, $A_{yy}(\theta)$, $A_{xx}(\theta)$, $A_{xz}(\theta)$, and polarization transfer coefficients $K_{zz}^{z'z'}(\theta)$, $K_{zz}^{x'z'}(\theta)$, $K_z^{y'z'}(\theta)$ for the elastic deuteron scattering from ^{16}O and ^{12}C nuclei at 700 MeV. The solid and dot-dashed curves were calculated in the TB approximation with the D and S waves included [see relations (35)–(39)], respectively, and the dashed curves were obtained in the RP approximation and coincide with the solid curves in Fig. 1. The experimental data are from Refs. [43,44].

approximation we have used the relations (28)–(32) with the amplitudes $\hat{f}_p(\mathbf{q}) = \hat{f}_n(\mathbf{q}) \equiv \hat{f}_{N\alpha}(\mathbf{q})$ in the form (34). In the RP approximation we have employed the amplitude $f_{d\alpha}(\mathbf{q})$ in the form (25) with the parameters given in Table I.

Figure 4 shows the predicted behavior of the d-^{4}He elastic scattering observables calculated in these two approaches. The notation of the curves is the same as in Figs. 2 and 3.

As is seen from Fig. 4, in the case of the d-^{4}He elastic scattering the RP and TB approaches lead to more evident discrepancies between the calculated observables as compared to those for the d-^{12}C and d-^{16}O scattering. These differences presumably arise for the same reasons as discussed above.

We should also note that the elementary amplitudes, which are used in the MDST calculations under consideration, can be modified by possible effects of the nuclear medium, as compared with the projectile scattering on the free α particles. The character of these effects requires a special investigation. However, in our previous works (see, for example, [35]) it was shown that the use of free N-α amplitudes for analyzing the proton elastic scattering on light α-cluster nuclei on the basis of the analogous approach yielded a fairly good description of the complete set of experimental observables at energies of several hundreds of MeV, which justifies neglecting the medium effects at such energies. Moreover, the effective elementary d-α amplitude with adjustable parameters, which is used in the RP approximation, can implicitly take account of some of the medium effects.

It should be stressed that the MDST variants based on cluster models seem to be rather attractive. Such approaches include the internal nucleon-nucleon correlations both in the incident and target nuclei, essentially reduce the maximum multiplicity of integration, and in many cases (for example, in the RP approximation) permit an accurate analytical calculation of the MDST series. A substantial problem of the RP approach is the lack of complete set of nucleus-nucleus scattering observables, which complicates a reliable determination of the "effective" amplitudes used in such a model. On the other hand, the TB approximation, being a more consistent microscopic approach than the RP approximation, does not use additional fitting parameters, but implies numerical integration, which does not allow obtaining analytical expressions for the resulting amplitudes.

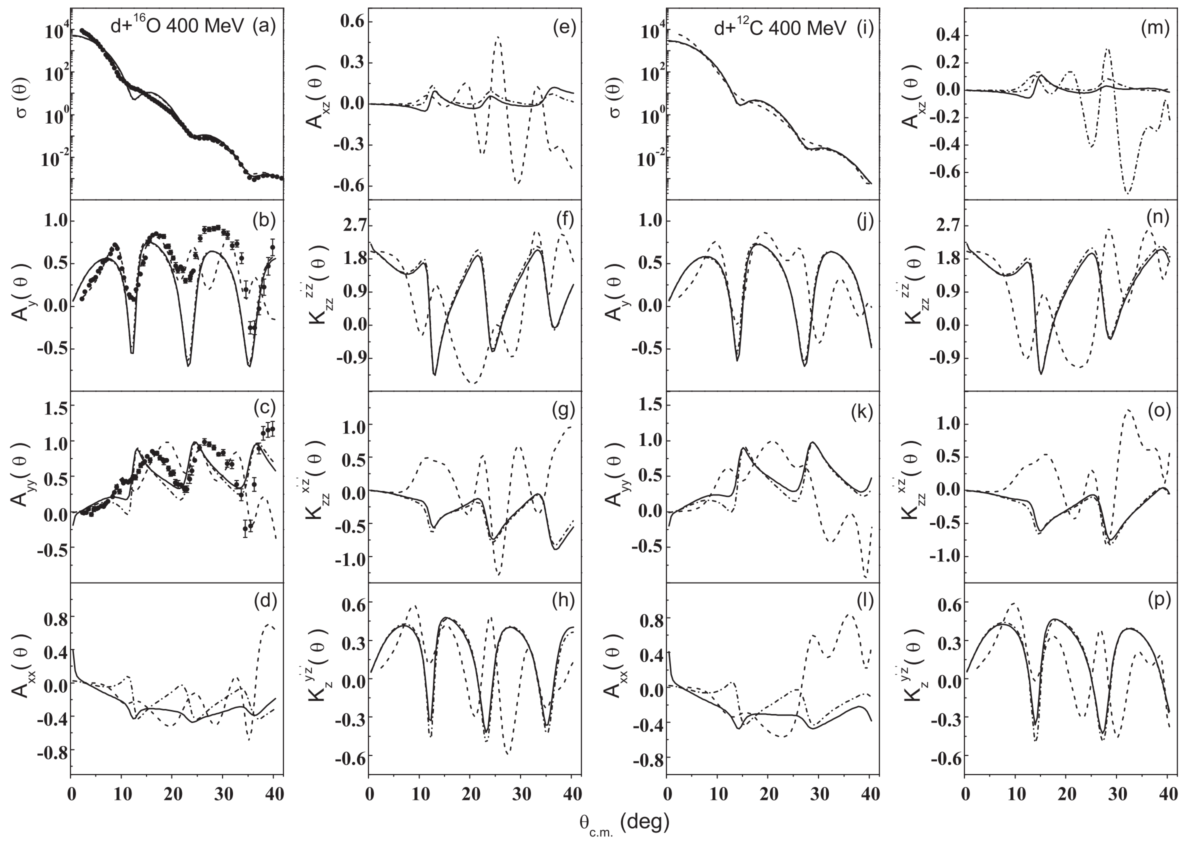

FIG. 3. The same as in Fig. 2, but for 400 MeV. The experimental data are from Ref. [43].

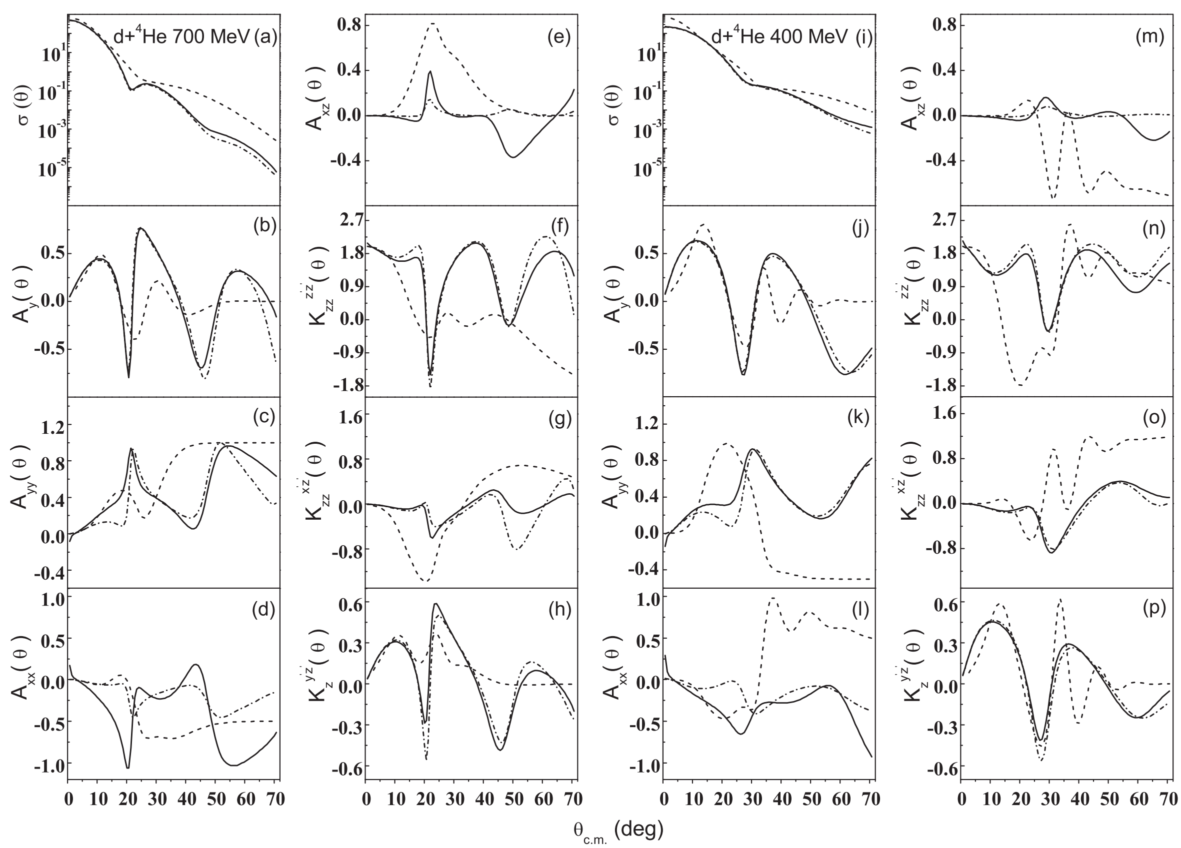

FIG. 4. The same as in Figs. 2 and 3, but for the d-^{4}He scattering.

APPENDIX: MULTIPLE SCATTERING AMPLITUDES IN THE RIGID PROJECTILE APPROXIMATION

In the RP approximation, using the α-cluster model with dispersion and MDST with the elementary d-α scattering amplitude in the form (25), the scattering amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$ (5) can be rewritten in the following convenient form:

$$\hat{\mathcal{F}}(\mathbf{q}) = \tilde{\mathcal{A}}_1(\mathbf{q}) + \tilde{\mathcal{A}}_2(\mathbf{q})(\mathbf{Sn}) + \tilde{\mathcal{A}}_3(\mathbf{q})(\mathbf{Sn})^2 + \tilde{\mathcal{A}}_4(\mathbf{q})(\mathbf{S\hat{q}})^2, \tag{A1}$$

where the invariant amplitudes $\tilde{\mathcal{A}}_l(\mathbf{q})$ for the projectile scattering from ^{12}C nuclei are determined by

$$\tilde{\mathcal{A}}_l(\mathbf{q}) = k_d\left\{3\hat{\sum_1}^{(l)}\hat{\mathcal{F}}_1^{(i)}(\mathbf{q}) + 3i\hat{\sum_2}^{(l)}\hat{\mathcal{F}}_2^{(i,j)}(\mathbf{q}) - \hat{\sum_3}^{(l)}\hat{\mathcal{F}}_3^{(i,j,l)}(\mathbf{q})\right\}, \tag{A2}$$

and those from ^{16}O nuclei are

$$\tilde{\mathcal{A}}_l(\mathbf{q}) = k_d\left\{4\hat{\sum_1}^{(l)}\hat{\mathcal{F}}_1^{(i)}(\mathbf{q}) + 6i\hat{\sum_2}^{(l)}\hat{\mathcal{F}}_2^{(i,j)}(\mathbf{q}) - 4\hat{\sum_3}^{(l)}\hat{\mathcal{F}}_3^{(i,j,l)}(\mathbf{q}) - i\hat{\sum_4}^{(l)}\hat{\mathcal{F}}_4^{(i,j,l,m)}(\mathbf{q})\right\}. \tag{A3}$$

Using the "effective deformation" approximation [45], the general form for the amplitude operators $\hat{\mathcal{F}}_l(\mathbf{q})$ is given as

$$\hat{\mathcal{F}}_l(\mathbf{q}) = \Gamma^{(n)}\exp\left(-\chi_q^{(n)}q^2 - \chi_d^{(n)}d^2\right)j_0(qd\sqrt{\Phi^{(n)}}), \tag{A4}$$

where the coefficients Γ, χ, and Φ for the corresponding scattering order n are as follows:

(i) the single scattering, $\Gamma^{(1)} = G_i$:

$$\begin{aligned} &\chi_q^{(1)} = \beta_i + \tfrac{1}{6}\Delta^2, \quad \chi_d^{(1)} = 0, \quad \Phi^{(1)} = \tfrac{1}{3}, \quad \Rightarrow {}^{12}\mathrm{C},\\ &\chi_q^{(1)} = \beta_i + \tfrac{3}{16}\Delta^2, \quad \chi_d^{(1)} = 0, \quad \Phi^{(1)} = \tfrac{3}{8}, \quad \Rightarrow {}^{16}\mathrm{O}; \end{aligned} \tag{A5}$$

(ii) the double scattering, $\Gamma^{(2)} = G_iG_j/\gamma_1$:

$$\begin{aligned} &\chi_q^{(2)} = \frac{1}{24}\Delta^2 + \frac{1}{4}\alpha_1 - \frac{\alpha^2}{2\gamma_1}, \quad \chi_d^{(2)} = \frac{1}{3\gamma_1}, \quad \Phi^{(2)} = \varphi_1^2 + \frac{1}{12}, \quad \Rightarrow {}^{12}\mathrm{C},\\ &\chi_q^{(2)} = \frac{1}{16}\Delta^2 + \frac{1}{4}\alpha_1 - \frac{\alpha^2}{2\gamma_1}, \quad \chi_d^{(2)} = \frac{1}{3\gamma_1}, \quad \Phi^{(2)} = \varphi_1^2 + \frac{1}{8}, \quad \Rightarrow {}^{16}\mathrm{O}; \end{aligned} \tag{A6}$$

(iii) the triple scattering, $\Gamma^{(3)} = 4G_iG_jG_l/(3\gamma_1\gamma_2)$:

$$\begin{aligned} &\chi_q^{(3)} = \frac{1}{9}(\beta_i+\beta_j+\beta_l) - \frac{2\alpha_5^2}{9\gamma_1} - \frac{3\alpha_4^2}{2\gamma_2}, \quad \chi_d^{(3)} = \frac{1}{3\gamma_1} + \frac{1}{3\gamma_2} + \frac{4\alpha^2}{9\gamma_1^2\gamma_2}, \quad \Phi^{(3)} = \varphi_2^2 + \frac{3}{4}\varphi_3^2, \quad \Rightarrow {}^{12}\mathrm{C}\\ &\chi_q^{(3)} = \frac{1}{16}\Delta^2 + \frac{1}{9}(\beta_i+\beta_j+\beta_l) - \frac{2\alpha_5^2}{9\gamma_1} - \frac{3\alpha_4^2}{2\gamma_2}, \quad \chi_d^{(3)} = \frac{1}{3\gamma_1} + \frac{1}{3\gamma_2} + \frac{4\alpha^2}{9\gamma_1^2\gamma_2}, \quad \Phi^{(3)} = \varphi_2^2 + \frac{3}{4}\varphi_3^2 + \frac{1}{24}; \quad \Rightarrow {}^{16}\mathrm{O} \end{aligned} \tag{A7}$$

(iv) the quadruple scattering, $\Gamma^{(4)} = 3G_iG_jG_lG_m/(8\gamma_1\gamma_2\gamma_3)$:

$$\begin{aligned} &\chi_q^{(4)} = \frac{1}{16}(\beta_i+\beta_j+\beta_l+\beta_m) - \frac{\alpha_5^2}{8\gamma_1} - \frac{3\alpha_{10}^2}{2\gamma_2} - \frac{\alpha_{11}^2}{4\gamma_3}, \quad \chi_d^{(4)} = \frac{1}{3\gamma_1} + \frac{1}{3\gamma_2} + \frac{1}{16\gamma_3} + \frac{4\alpha^2}{9\gamma_1^2\gamma_2} + \frac{\alpha_{12}^2}{6\gamma_3} + \frac{\alpha_9^2}{2\gamma_2^2\gamma_3},\\ &\Phi^{(4)} = \varphi_4^2 + \frac{3}{4}\varphi_5^2 + \frac{2}{3}\varphi_6^2. \end{aligned} \tag{A8}$$

In (A2) and (A3) the summation operators $\hat{\sum_n}^{(l)}$ for various scattering orders are equal to to following:

(i) the single scattering:

$$\hat{\sum_1}^{(1)} = \sum_{i=1}^{2}, \quad \hat{\sum_1}^{(2)} = q\sum_{i=3}^{4}, \quad \hat{\sum_1}^{(3)} = q^2\sum_{i=5}^{6}, \quad \hat{\sum_1}^{(4)} = q^2\sum_{i=7}^{8}; \tag{A9}$$

(ii) the double scattering:

$$\begin{aligned} &\hat{\sum_2}^{(1)} = \sum_{i,j=1}^{2}, \quad \hat{\sum_2}^{(2)} = 2\sum_{i=1}^{2}\sum_{j=3}^{4}\hat{D}_1 + 2\sum_{i=3}^{4}\sum_{i=5}^{6}\hat{D}_1\hat{D}_2, \quad \hat{\sum_2}^{(3)} = 2\sum_{i=1}^{2}\sum_{j=5}^{6}\hat{D}_1^2 + \sum_{i,j=3}^{4}\hat{D}_5 + \sum_{i,j=5}^{6}\hat{D}_1^2\hat{D}_3,\\ &\hat{\sum_2}^{(4)} = \sum_{i,j=3}^{4}\hat{D}_4 + 2\sum_{i=1}^{2}\sum_{j=7}^{8}\hat{D}_1^2 + \sum_{i,j=7}^{8}\hat{D}_1^2\hat{D}_3; \end{aligned} \tag{A10}$$

(iii) the triple scattering:

$$\hat{\sum}_3^{(1)} = \sum_{i,j,l=1}^{2}, \quad \hat{\sum}_3^{(2)} = 3\sum_{i,j=1}^{2}\sum_{l=3}^{4}\hat{D}_6 + 6\sum_{i=1}^{2}\sum_{j=3}^{4}\sum_{l=5}^{6}\hat{D}_6\hat{D}_9 + \sum_{i,j,l=3}^{4}\hat{D}_6\hat{D}_{10} + 3\sum_{i=3}^{4}\sum_{j,l=5}^{6}\hat{D}_6\hat{D}_9\hat{D}_{10},$$

$$\hat{\sum}_3^{(3)} = 3\sum_{i,j=3}^{4}\sum_{l=1}^{2}\hat{D}_8 + 3\sum_{i,j=1}^{2}\sum_{l=5}^{6}\hat{D}_6^2 + 3\sum_{i=1}^{2}\sum_{j,l=5}^{6}\hat{D}_6^2\hat{D}_{11} + 3\sum_{i,j=3}^{4}\sum_{l=5}^{6}\hat{D}_6^2\hat{D}_{10} + \sum_{i,j,l=5}^{6}\hat{D}_6^2\hat{D}_{11}\hat{D}_{12}, \tag{A11}$$

$$\hat{\sum}_3^{(4)} = 3\sum_{i,j=3}^{4}\sum_{l=1}^{2}\hat{D}_7 + 3\sum_{i,j=1}^{2}\sum_{l=7}^{8}\hat{D}_6^2 + 3\sum_{i=1}^{2}\sum_{j,l=7}^{8}\hat{D}_6^2\hat{D}_{11} + \sum_{i,j,l=7}^{8}\hat{D}_6^2\hat{D}_{11}\hat{D}_{12};$$

(iv) the quadruple scattering:

$$\hat{\sum}_4^{(1)} = \sum_{i,j,l,m=1}^{2},$$

$$\hat{\sum}_4^{(2)} = 4\sum_{i,j,l=1}^{2}\sum_{m=3}^{4}\hat{D}_{13} + 12\sum_{i,j=1}^{2}\sum_{l=3}^{4}\sum_{m=5}^{6}\hat{D}_{13}\hat{D}_{16} + 4\sum_{i,j,m=3}^{4}\sum_{l=1}^{2}\hat{D}_{13}\hat{D}_{17} + 12\sum_{i=1}^{2}\sum_{j=3}^{4}\sum_{l,m=5}^{6}\hat{D}_{13}\hat{D}_{16}\hat{D}_{19}$$

$$+ 4\sum_{i,j,l=3}^{4}\sum_{m=5}^{6}\hat{D}_{13}\hat{D}_{16}\hat{D}_{17} + 4\sum_{i=3}^{4}\sum_{j,l,m=5}^{6}\hat{D}_{13}\hat{D}_{16}\hat{D}_{17}\hat{D}_{19},$$

$$\hat{\sum}_4^{(3)} = 6\sum_{i,j=3}^{4}\sum_{l,m=1}^{2}\hat{D}_{15} + \sum_{i,j,l,m=3}^{4}\hat{D}_{16}\hat{D}_{15} + 6\sum_{i,j=1}^{2}\sum_{l,m=5}^{6}\hat{D}_{13}^2\hat{D}_{18} + 12\sum_{i,j=3}^{4}\sum_{l=1}^{2}\sum_{m=5}^{6}\hat{D}_{13}^2\hat{D}_{17} \tag{A12}$$

$$+ 4\sum_{i,j,l=1}^{2}\sum_{m=5}^{6}\hat{D}_{13}^2 + 4\sum_{i,j,m=5}^{6}\sum_{l=1}^{2}\hat{D}_{17}^2\hat{D}_{13}^2 + 6\sum_{i,j=3}^{4}\sum_{l,m=5}^{6}\hat{D}_{15}\hat{D}_{16}\hat{D}_{17} + \sum_{i,j,l,m=5}^{6}\hat{D}_{15}\hat{D}_{16}^2\hat{D}_{17}$$

$$\hat{\sum}_4^{(4)} = 6\sum_{i,j=3}^{4}\sum_{l,m=1}^{2}\hat{D}_{14} + \sum_{i,j,l,m=3}^{4}\hat{D}_{16}\hat{D}_{14} + 4\sum_{i,j,l=1}^{2}\sum_{m=7}^{8}\hat{D}_{13}^2 + 6\sum_{i,j=1}^{2}\sum_{l,m=7}^{8}\hat{D}_{13}^2\hat{D}_{18}$$

$$+ 4\sum_{i,j,m=7}^{8}\sum_{l=1}^{2}\hat{D}_{17}^2\hat{D}_{13}^2 + \sum_{i,j,l,m=7}^{8}\hat{D}_{14}\hat{D}_{16}^2\hat{D}_{17}.$$

Here the differential operators $\hat{D}_i$ are defined as follows:

$$\hat{D}_i = \lambda q^2 + \sum_{j=1}^{n}\mu_j\frac{\partial}{\partial\alpha_j} + \sum_{k=1}^{m}\nu_k\frac{\partial^2}{\partial\beta_k\partial\gamma_k} \equiv \hat{D}(\lambda;\mu_1,\alpha_1;\cdots;\mu_n,\alpha_n;\nu_1,\beta_1,\gamma_1;\cdots;\nu_m,\beta_m,\gamma_m). \tag{A13}$$

For example, using the notation (A13) we have

$$\hat{D}(1;1,\alpha) = q^2 + \frac{\partial}{\partial\alpha}, \tag{A14}$$

and the relation for other operators $\hat{D}_i$ can be obtained in a similar way.

In (A9)-(A12) we use the following operators:

$$\hat{D}_1 = \frac{1}{q}\hat{D}\left(\frac{1}{2};1,\alpha\right), \quad \hat{D}_2 = \hat{D}\left(\frac{1}{4};2,\gamma_1\right), \quad \hat{D}_3 = \hat{D}\left(\frac{1}{4};-2,\gamma_1;-1,\alpha\right),$$

$$\hat{D}_4 = \hat{D}\left(0;2,\gamma_1;\frac{1}{q^2},\alpha,\alpha\right), \quad \hat{D}_5 = \hat{D}\left(\frac{1}{4};-\frac{1}{q^2},\alpha,\alpha\right),$$

$$\hat{D}_6 = \frac{1}{3q}\hat{D}(1;2,\alpha_4), \quad \hat{D}_7 = \hat{D}\left(0;2,\gamma_1;-\frac{2}{3},\gamma_2;\frac{9}{4q^2},\alpha_5,\alpha_5;-\frac{1}{9q^2},\alpha_4,\alpha_4\right),$$

$$\hat{D}_8 = \hat{D}\left(\frac{1}{9};-\frac{2}{9},\alpha_4;-\frac{9}{4q^2},\alpha_5,\alpha_5;\frac{1}{9q^2},\alpha_4,\alpha_4\right), \quad \hat{D}_9 = \hat{D}\left(\frac{1}{9};-1,\alpha;\frac{1}{2},\alpha_5;\frac{4}{3},\gamma_2;\frac{1}{9},\alpha_4\right),$$

$$\hat{D}_{10}=\hat{D}\left(\frac{1}{9};2,\gamma_1;-\frac{2}{3},\gamma_2;-\frac{2}{9},\alpha_4\right),\quad \hat{D}_{11}=\hat{D}\left(\frac{1}{9};-2,\gamma_1;-\frac{2}{3},\gamma_2;1,\alpha;1,\alpha_5;-\frac{2}{9},\alpha_4\right),$$
$$\hat{D}_{12}=\hat{D}\left(\frac{1}{9};-2,\gamma_1;-\frac{2}{3},\gamma_2;-1,\alpha;-1,\alpha_5;-\frac{2}{9},\alpha_4\right),$$
$$\hat{D}_{13}=\frac{1}{q}\hat{D}\left(\frac{1}{4};\frac{3}{4},\alpha_{11}\right),$$
$$\hat{D}_{14}=\hat{D}\left(0;\frac{1}{6},\alpha_9;2,\gamma_1;-\frac{2}{3},\gamma_2;-\frac{1}{16},\gamma_3;\frac{4}{q^2},\alpha_5,\alpha_5;-\frac{1}{9q^2},\alpha_{10},\alpha_{10};-\frac{1}{16q^2},\alpha_{11},\alpha_{11};-\frac{1}{6q^2},\alpha_{10},\alpha_{11}\right),$$
$$\hat{D}_{15}=\hat{D}\left(\frac{1}{16};-\frac{1}{6},\alpha_{10};-\frac{1}{8},\alpha_{11};-\frac{4}{q^2},\alpha_5,\alpha_5;\frac{1}{9q^2},\alpha_{10},\alpha_{10};\frac{1}{16q^2},\alpha_{11},\alpha_{11};\frac{1}{6q^2},\alpha_{10},\alpha_{11}\right),$$
$$\hat{D}_{16}=\hat{D}\left(\frac{1}{16};-1,\alpha;\frac{1}{2},\alpha_9;\frac{1}{6},\alpha_{10};\frac{1}{8},\alpha_{11};\frac{3}{16},\gamma_3\right),$$
$$\hat{D}_{17}=\hat{D}\left(\frac{1}{16};\frac{1}{6},\alpha_9;-\frac{1}{8},\alpha_{10};-\frac{1}{8},\alpha_{11};2,\gamma_1;-\frac{2}{3},\gamma_2;-\frac{1}{16},\gamma_3\right),$$
$$\hat{D}_{18}=\hat{D}\left(\frac{1}{16};-\frac{1}{3},\alpha_9;\frac{1}{3},\alpha_{10};-\frac{1}{8},\alpha_{11};-\frac{8}{3},\gamma_2;-\frac{1}{16},\gamma_3\right),$$
$$\hat{D}_{19}=\hat{D}\left(\frac{1}{16};-1,\alpha;\frac{1}{2},\alpha_5;\frac{1}{2},\alpha_8;-\frac{1}{12},\alpha_9;\frac{1}{12},\alpha_{10};-\frac{1}{8},\alpha_{11};\frac{4}{3},\gamma_2;-\frac{1}{16},\gamma_3\right). \tag{A15}$$

In the above formulas we introduce the notation

$$\alpha=\beta_i-\beta_j,\quad \alpha_1=\beta_i+\beta_j,\quad \alpha_2=\beta_i+\beta_j+4\beta_l,\quad \alpha_3=\beta_i+\beta_j-2\beta_l,$$
$$\alpha_4=\frac{2}{9}\alpha_3-\frac{4\alpha\alpha_5}{9\gamma_1},\quad \alpha_5=\alpha_8=\alpha,\quad \alpha_6=\beta_i+\beta_j+\beta_l+9\beta_m,$$
$$\alpha_7=\beta_i+\beta_j+\beta_l-3\beta_m,\quad \alpha_9=-\frac{1}{6}\alpha_3+\frac{\alpha\alpha_8}{3\gamma_1},\quad \alpha_{10}=\frac{1}{6}\alpha_3-\frac{\alpha\alpha_5}{3\gamma_1},$$
$$\alpha_{11}=\frac{1}{8}\alpha_7-\frac{\alpha_5\alpha_8}{4\gamma_1}+\frac{3\alpha_9\alpha_{10}}{\gamma_2},\quad \alpha_{12}=\frac{\alpha_8}{2\gamma_1}+\frac{2\alpha\alpha_9}{\gamma_1\gamma_2}, \tag{A16}$$
$$\gamma_1=2\alpha_1+\Delta^2,\quad \gamma_2=\frac{2}{3}\alpha_2-\frac{4\alpha^2}{3\gamma_1}+\Delta^2,\quad \gamma_3=\frac{1}{16}\alpha_6-\frac{\alpha_8^2}{8\gamma_1}-\frac{3\alpha_9}{2\gamma_2}+\frac{3}{16}\Delta^2,$$
$$\varphi_1=\frac{\alpha}{\gamma_1},\quad \varphi_2=\frac{2\alpha_5}{3\gamma_1}-\frac{2\alpha\alpha_4}{\gamma_1\gamma_2},\quad \varphi_3=\frac{2\alpha_4}{\gamma_2},\quad \varphi_4=\frac{\alpha}{2\gamma_1}-\frac{2\alpha\alpha_{10}}{\gamma_1\gamma_2}-\frac{\alpha_{11}\alpha_{12}}{2\gamma_3},$$
$$\varphi_5=\frac{2\alpha_{10}}{\gamma_2}+\frac{\alpha_9\alpha_{11}}{\gamma_2\gamma_3},\quad \varphi_6=\frac{3\alpha_{11}}{8\gamma_3}.$$

Taking into account the relation

$$(\mathbf{S}\hat{\mathbf{q}})^2=2\cos^2\frac{\theta}{2}+\frac{i}{2}\sin\theta(\mathbf{S}\hat{\mathbf{n}})-\cos^2\frac{\theta}{2}(\mathbf{S}\hat{\mathbf{n}})^2-(\mathbf{S}\hat{\mathbf{k}})(\mathbf{S}\hat{\mathbf{k}}'), \tag{A17}$$

after some algebraic transformations we can easily represent the amplitude operator $\hat{\mathcal{F}}(\mathbf{q})$ (A1) in the form (5).

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