

Precision Q_{EC} -value measurement of ^{23}Mg for testing the Cabibbo-Kobayashi-Maskawa matrix unitarity

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We report a new direct measurement of the transition energy Q_{EC} of the ^{23}Mg β^+ decay, using the TITAN Penning-trap mass spectrometer. This value is found to agree with the latest atomic mass evaluation while being four times more precise. The increase in precision changes the uncertainty contribution of the Q_{EC} value on the statistical rate function f_v from 11% to 0.6%, an improvement by a factor of 18. This enables a more robust determination of the corrected $\mathcal{F}t$ value of this mirror transition to the required precision, making possible further test of the Cabibbo-Kobayashi-Maskawa matrix unitarity.

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The recent discovery of the Higgs boson at the Large Hadron Collider (LHC) [1,2], the long-awaited missing link of the standard model (SM) of the electroweak interaction [3], was one more success of what is now one of the most carefully tested theories in physics [4]. However, due to quadratic divergences in the Higgs mass, the SM is unable to explain the measured Higgs mass [5], which may require the existence of supersymmetric particles [6]. Consequently, there are large experimental efforts not only at the high-energy frontier [7] but also at the high-precision frontier [8,9] to search for potentially new physics beyond the SM.

Possible signatures of an extra quark generations, extra Z bosons, supersymmetry, or other new physics can be probed by a weak deviation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix from unitarity [10]. This matrix, which relates the weak interaction and mass eigenstates of the quarks, is usually tested for unitarity by adding the magnitude square of its top-row matrix elements. The largest and most precise element of this row is the V_{ud} coefficient [11]. This term can be derived from four different strangeness-conserving processes (by order of precision) [12]: pure superallowed Fermi $0^+ \rightarrow 0^+$ transitions, mixed superallowed transitions in mirror nuclei, neutron decay, and pion decay. Large interest in the second process has grown since its recent proposal [13]. These mirror transitions not only allow for an independent determination of V_{ud} that could confirm the high-precision value derived from Fermi transitions but also provide a second test of the conserved vector current (CVC) hypothesis [13].

Since the decay of mirror nuclei are mixed Fermi Gamow-Teller transitions, the axial-vector current is not conserved

in the decay, requiring the additional measurement of the mixing ratio between the two transitions. This mixing ratio can be determined from the measurement of one of three correlation coefficients [13]: the β asymmetry parameter A_β , the neutrino asymmetry parameter B_ν , and the β -neutrino angular correlation $a_{\beta\nu}$. This extra parameter, and the inherent difficulty in measuring these correlation coefficients, explains the smaller number of transitions being used to test the CVC hypothesis and calculate the corrected $\mathcal{F}t$ values from which the V_{ud} coefficient is extracted [12].

In this context, there are several dedicated experimental efforts to measure these parameters in mirror transitions. A group at Laboratoire de Physique Corpusculaire de Caen is preparing measurements of the $a_{\beta\nu}$ coefficient for ^{19}Ne and ^{35}Ar using a Paul trap [14]. This coefficient has been measured in ^{21}Na , at Berkeley, using a magneto-optical trap (MOT) [15]. There are plans to measure A_β for ^{21}Na , ^{23}Mg , ^{29}P , ^{35}Ar , and ^{37}K at the National Superconducting Cyclotron Laboratory's Beam Cooler and Laser spectroscopy experimental setup end station [16]. Finally, a precise measurement of B_ν for ^{37}K [17] was performed using the TRINAT MOT at TRIUMF.

In parallel to the experimental efforts aimed at determining the mixing ratio, corrected $\mathcal{F}t$ -value calculations require precise and accurate measurements of transition energies Q_{EC} , branching ratios (BR), and half-lives $t_{1/2}$ of mirror transitions [12]. Because of the high sensitivity of the statistical rate function to Q_{EC} , precise Q_{EC} , to the level of a few hundred eV, are required [13]. The mirror nucleus with the most uncertain Q_{EC} is ^{23}Mg [12], with an uncertainty of 700 eV [18]. Therefore, given the experimental efforts [16,19] to measure the mixing ratio of ^{23}Mg and to obtain a more robust average $\mathcal{F}t$ value, we have performed a high-precision measurement of the ^{23}Mg Q_{EC} value.

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The transition Q value of ^{23}Mg was measured using the TRIUMF Ion Trap for Atomic and Nuclear science (TITAN) Penning trap mass spectrometer [20,21], located in the ISAC facility [22,23] of TRIUMF. The radioactive magnesium was produced by impinging a 480-MeV, 40- μA proton beam on a thick SiC target. The produced nuclei were then released by heating the target to high temperature, allowing them to diffuse as neutral Mg atoms into a radio-frequency quadrupole ion guide, where the ^{23}Mg was selectively laser ionized. As described in Ref. [24], the use of this ion-guide laser ion source (IG-LIS) suppressed surface-ionized contamination (^{23}Na) by a factor of 10^6 from several nA to count rates below 10^5 particles/s. This major increase in beam purity allowed to perform the experiment with a $^{23}\text{Mg}^+$ yield of 2.2×10^7 particles/s. The large ion beam intensity required an additional attenuation prior to injection into the TITAN experimental setup.

Penning traps are well suited for precise and accurate mass determinations [25]. TITAN currently consists of three ion traps: a radio-frequency quadrupole (RFQ) cooler and buncher [26], an electron beam ion trap (EBIT) [27] to charge breed the ions for increased precision [28,29] as well as to perform in-trap decay spectroscopy [30,31], and the mass measurement Penning trap (MPET) [32]. The system has been used for a series of precision mass measurements on singly [33–39] and highly charged [29,40–42] ions. For the ^{23}Mg measurement, the mass-selected radioactive beam from the IG-LIS was first accumulated in the RFQ, where it was cooled through collisions with a helium buffer gas and subsequently extracted as bunches. Then, the cooled bunches were sent directly to MPET as singly charged ions.

Once the ions have been injected into MPET, the $^{23}\text{Mg}^+$ ion cyclotron frequency $\nu_c = qB/(2\pi m)$ was determined via the time-of-flight ion cyclotron resonance technique [43,45] (TOF-ICR). The trapped ions are exposed to a quadrupole rf field with a frequency near $\nu_c = \nu_+ + \nu_-$, where ν_{\pm} are the reduced cyclotron and magnetron eigenfrequencies. In resonance, a beating between the initial magnetron and the reduced cyclotron eigenmotions occurs. The excitation time and rf amplitude are chosen such that the initial magnetron motion is entirely converted into reduced cyclotron motion. This conversion leads to a significant change in the ions' radial energy, which can be used to identify the cyclotron resonance when the ions are ejected from the trap. As they drift through the inhomogeneous magnetic field, the radial energy is converted into axial energy, which is then measured by the ions' time of flight to a downstream detector. A maximum change in time of flight is observed when the rf frequency equals the cyclotron frequency.

A typical quadrupole resonance is shown in Fig. 1(a) for $^{23}\text{Mg}^+$. Each data point in Fig. 1(a) is the combination of 20 individual cycles. The uncertainty on these data points is given by the standard deviation of the time of flight of the ions within a chosen time window. As indicated in Fig. 1(a), the TOF-ICR technique leads to high-precision measurements of the cyclotron frequency on the order of $\delta\nu_c/\nu_c = 10^{-8}$, where $\delta\nu_c$ is the uncertainty from the fit of the analytical line shape [45]. A narrower cyclotron resonance

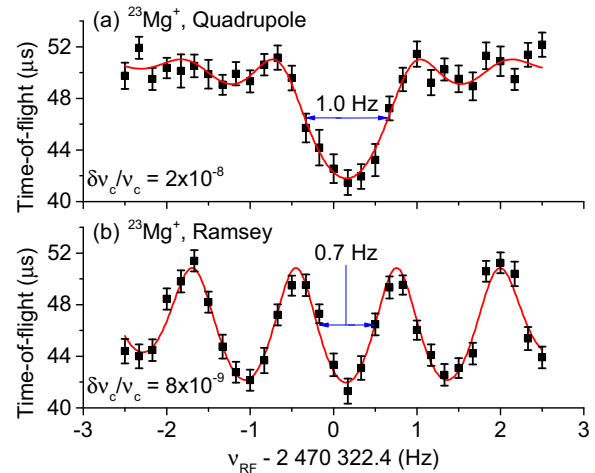


FIG. 1. (Color online) Typical quadrupole (a) and Ramsey (b) resonances of $^{23}\text{Mg}^+$ with the fitted theoretical line shapes [45,46]. The quadrupole excitation time is $T_{RF} = 977$ ms, while the Ramsey excitation scheme cycle ratio is 200:577:200 ms. The full width half maximum is indicated by the arrows.

line width, and therefore a lower $\delta\nu_c$, can be accomplished by applying time-separated oscillating fields of quadrupole excitation using the so-called Ramsey technique [44,46]. For the ^{23}Mg measurement, Ramsey excitation scheme with a cycle ratio of 200:577:200 ms was used. Figure 1(b) shows a Ramsey resonance for $^{23}\text{Mg}^+$.

To derive the transition energy Q_{EC} , the ratio of the two cyclotron frequencies, $R = \nu_{c,\text{ref}}/\nu_c$, where $\nu_{c,\text{ref}}$ is the linearly interpolated frequency of two calibrations bracketing the measurement of the ion of interest, needs to be taken. A total of two quadrupole and two Ramsey excitation frequency ratios were taken and their respective weighted average are given in Table I.

Most systematic uncertainties in the measured frequency ratios scale linearly with the mass difference between the ion of interest and the calibrant ion [32]. These systematic effects include magnetic field inhomogeneities, trap misalignment with the magnetic field, harmonic distortion of the electric potential, nonharmonic imperfections in the trapping potential, and relativistic effects when both species are injected in the trap under the same conditions [47] (which was the case for the presented measurement). The small mass difference between ^{23}Na and ^{23}Mg effectively eliminates all these sources of uncertainty at the level of interest. The remaining systematic effects stem from nonlinear time-dependent changes in the

TABLE I. Measured frequency ratios of $^{23}\text{Mg}^+$ to $^{23}\text{Na}^+$ utilizing the quadrupole and Ramsey excitation scheme. N indicates the number of measurements taken.

Method	Frequency Ratio \bar{R}	N
Quadrupole	1.000 189 454(21)	2
Ramsey	1.000 189 417 5(80)	2
Average	1.000 189 422 1(75)	4

magnetic field and ion-ion interaction in the trap [47]. First, the field produced by the superconducting magnet is affected by environmental variations of temperature and pressure. This was minimized at TITAN by using a pressure stabilization system, resulting in temporal magnetic field fluctuations that are typically below 0.15 ppb/h [47]. Because the calibration measurements were separated by at most 1 h, the effect of magnetic field variations on the cyclotron frequency are negligible compared to our achieved statistical uncertainty. Lastly, the cyclotron frequency of the probed ion can be affected by the presence of contaminant ions of different mass [48]. These ions, which are not affected during the excitation phase, do interact with the ion of interest, resulting in a reduction of the measured cyclotron frequency. Isobaric contamination ($^{23}\text{Na}^+$ in the case of $^{23}\text{Mg}^+$) was removed in the Penning trap through the application of a dipolar excitation at the reduced cyclotron frequency of the contaminant ion [49]. Nonisobaric contaminants from the off-line ion source or produced by charge exchange in the RFQ were removed using a Bradbury-Nelson gate [50] upstream of the Penning trap. Finally, undesirable species can be created from charge exchange with residual background gas molecules in the Penning trap itself. Hence, throughout the experiment, we kept the maximum number of recorded ions below five, and a count-rate analysis [51] was performed on each individual resonance. Using the results from the count-rate analysis, we scaled the individual measured cyclotron frequencies to the value corresponding to 0.7 ions in the trap, which is our MCP detection efficiency. On average, the cyclotron frequency changed by $-3(8)$ and $2(6)$ ppb/ion when quadrupole and Ramsey excitations were used, respectively. This results in an average systematic uncertainty of 9 and 5 ppb respectively, making it the largest systematic contribution.

Table I gives the average frequency ratio of all four measurements together with the final uncertainty. The total uncertainty in the individual frequency ratios presented in Table I includes the dominant systematic uncertainty from ion-ion interaction added in quadrature to the statistical uncertainty. The Q_{EC} value of ^{23}Mg is determined directly from the averaged cyclotron frequency ratio:

$$Q_{EC} = (\bar{R} - 1)(M_{\text{ref}} - m_e) = 4056.35(16) \text{ keV}, \quad (1)$$

where M_{ref} is the atomic mass of ^{23}Na and m_e is the electron mass. The new transition energy, calculated using the ^{23}Na mass excess from the 2012 Atomic Mass Evaluation (AME12) [18], is presented in Table II together with the AME12 value. Our Q_{EC} value agrees with the tabulated value within 1σ but is four times more precise.

TABLE II. Total transition energy Q_{EC} , statistical rate function f_v , relative uncertainty $\delta f_v/f_v$ and the $f_v t$ value of ^{23}Mg from this work (TITAN) and from the latest atomic mass evaluation (AME12) [18] assuming $S(Z, W) = 1$.

Reference	Q_{EC} (keV)	f_v (s^{-1})	$\delta f_v/f_v$	$f_v t$
[18]	4056.6(7)	376.58(39)	1.0×10^{-3}	4650(15)
This work	4056.35(16)	376.44(9)	2.4×10^{-4}	4648(14)

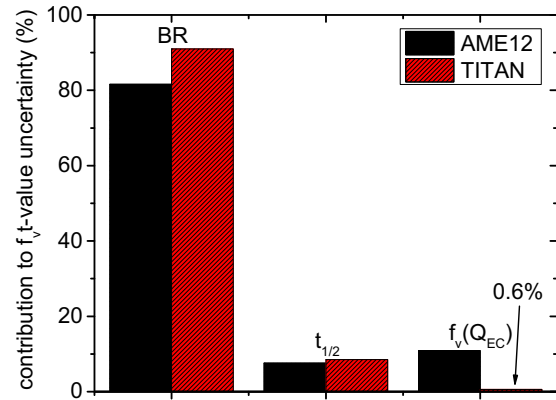


FIG. 2. (Color online) Respective relative contribution to the $f_v t$ value of the branching ratio BR, half-life $t_{1/2}$, and statistical rate function f_v in the case where either the AME12 [18] or TITAN Q_{EC} of ^{23}Mg used to calculate f_v . Since the f_v -value uncertainty is significantly reduced, the relative contributions of the BR and $t_{1/2}$ uncertainties are increased.

The transition energy enters in the determination of the uncorrected statistical rate function f_v [52]:

$$f_v(Z, W) = \int_1^{W_0} F(-Z, W) S(-Z, W) (W - W_0)^2 p W dW, \quad (2)$$

where W is the total positron energy in units of electron mass, $W_0 = Q_{EC}/m_e - 1$, $p = \sqrt{W^2 - 1}$, $S(-Z, W)$ is the shape-correction function, which is approximately one for the ^{23}Mg decay, and $F(-Z, W)$ is the Fermi function [53].

Table II gives the statistical rate function f_v for the β^+ decay of ^{23}Mg using the Q_{EC} value from the presented measurement and assuming a shape-correction function $S(Z, W) = 1$. Note that this last assumption results in a small underprediction of the f_v value of 0.6% [54]. Our new total transition energy yields a reduction of the statistical rate function relative uncertainty $\delta f_v/f_v$ by a factor of four.

The three known experimental quantities (Q_{EC} , $t_{1/2}$, and BR) relevant to the corrected $\mathcal{F}t$ value can be used to calculate the $f_v t$ value:

$$f_v t = f_v(Q_{EC}) t_{1/2} \left(\frac{1 + P_{EC}}{\text{BR}} \right), \quad (3)$$

where $P_{EC} = 0.073\%$ is the electron capture fraction, $t_{1/2} = 11.3243(98)$ s, and $\text{BR} = 91.78(26)\%$ for the decay of ^{23}Mg [52]. Figure 2 shows that with the new Q_{EC} value, the relative contribution of the statistical rate function uncertainty to the total uncertainty of the $f_v t$ value was reduced from 11% to 0.6%. This is a consequence of the Q_{EC}^5 dependence of the statistical rate function. The β^+ -decay branching ratio remains the largest (91%) source of experimental uncertainty in the $f_v t$ value.

In summary, we measured the transition energy of the β^+ decay of ^{23}Mg , confirming the 2012 atomic mass evaluation value. The new Q_{EC} value results in a four times more precise statistical rate function, reducing the relative contribution of

f_v to the uncertainty of the decay f_{vt} value by a factor of 18 down to less than 1%. Our measurement renders the second largest source of uncertainty to the f_{vt} value to a negligible value, solidifying it in preparation for the awaited measurements of the mixing ratio of ^{23}Mg [16,19]. This ratio is the last remaining piece to calculate the corrected $\mathcal{F}t$ values necessary for the evaluation of the V_{ud} CKM matrix element from mixed transitions. With our new Q_{EC} and assuming that precision of 0.5% could be achieved for $a_{\beta\nu}$ [55], a precision of 0.0023 could be obtained for the V_{ud} matrix element. This is only a factor of 10 larger than the achieved precision for the

entire sample of superallowed Fermi transitions [11]. While superallowed mixed transitions might not reach the precision achieved by pure superallowed Fermi transitions, they will be an important independent way of testing the conserved vector current hypothesis, testing theoretical corrections, and determining the V_{ud} matrix element in the future.

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