Effect of short-range correlations on Coulomb matrix elements

Paul Goldhammer
Department of Physics, University of Kansas, Lawrence, Kansas 66044
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The normalization correction introduced by McCarthy and Walker is examined. It is concluded that it should be absent.

Recent publications have attempted to evaluate the significance of short range correlations, induced by nuclear forces, on Coulomb matrix elements in light nuclei.[1,2] McCarthy and Walker[3] raise some doubt on the importance of the effect. First they find that corrections due to the Pauli operator reduce the Coulomb matrix elements calculated previously. This is a logical result, and certainly represents an improvement over the treatment in Refs. 1 and 2.

In addition to the Pauli effect, Walker and McCarthy calculate a "normalization correction." The omission of this correction from Refs. 1 and 2 was not accidental. To see why it should be absent from the G matrix let us consider the following derivation.

A proper procedure that incorporates all two-body correlation effects would be to start with the Bethe-Goldstone equation:

$$\psi = \phi_0 + \frac{Q}{e}(V_N + V_C)\psi, \tag{1}$$

where ϕ_0 is the oscillator wave function, V_N the nuclear potential, V_C the Coulomb interaction, and

$$\frac{Q}{e} \equiv \sum_{n\neq 0} \frac{|n\rangle\langle n|}{W - H_0}. \tag{2}$$

The net potential energy (nuclear plus Coulomb) is then given in terms of the G matrix elements:

$$\langle\phi_0|G|\phi_0\rangle = \langle\phi_0|V_N + V_C|\psi\rangle = \left\langle\phi_0\left|V_N + V_C + (V_N + V_C)\frac{Q}{e}(V_N + V_C) + \cdots\right|\phi_0\right\rangle. \tag{3}$$

The procedure adopted in Refs. 1, 2, and 3 is to calculate only first-order terms in V_C. One starts by calculating a correlated wave function ψ' with only the nuclear force in the Bethe-Goldstone equation:

$$\psi' = \phi_0 + \frac{Q}{e} V_N \psi' = \phi_0 + \frac{Q}{e} G_N \phi_0. \tag{4}$$

The nuclear potential energy then is given by the elements:

$$\langle\phi_0|G_N|\phi_0\rangle = \langle\phi_0|V_N|\psi'\rangle. \tag{5}$$

The Coulomb correction can then be defined as:

$$\langle\phi_0|G_C|\phi_0\rangle \equiv \langle\phi_0|G - G_N|\phi_0\rangle. \tag{6}$$

Comparing Eqs. (3), (4), (5), and (6) one finds:

$$\langle\phi_0|G_C|\phi_0\rangle = \left\langle\phi_0\left|V_C + V_C\frac{Q}{e}G_N + G_N\frac{Q}{e}V_C + G_N\frac{Q}{e}V_C\frac{Q}{e}G_N\right|\phi_0\right\rangle + \cdots = \langle\psi'|V_C|\psi'\rangle + \cdots, \tag{7}$$

in which terms not linear in V_C have been omitted.

Thus to terms of only first order in V_C, the appropriate Coulomb correction to the G matrix is $\langle\psi'|V_C|\psi'\rangle$. McCarthy and Walker divide this factor by $\langle\psi'|\psi'\rangle$, inducing a substantial reduction in K_{SD} and K_{SP}. The point that the "normalization correction" is inappropriate in calculating the expectation value of a potential term omitted from the Bethe-Goldstone equation is not new. It has been made previously by Kuo,[4] with regard to refinements for the Moszkowski-Scott separation method (Kuo's derivation is entirely equivalent to the one given above). Dividing by $\langle\psi'|\psi'\rangle$ restores unlinked diagrams into the perturbation series.

[1] R. K. Anderson, M. R. Wilson, and P. Goldhammer, Phys. Rev. C 6, 136 (1972).
[2] R. K. Anderson and P. Goldhammer, Phys. Rev. Lett. 26, 978 (1971).
[3] R. J. McCarthy and G. E. Walker, Phys. Rev. C 9, 809 (1974), this issue.
[4] T. T. S. Kuo, Nucl. Phys. A103, 71 (1967).