

New mass-table parameters for nuclei $Z \leq 16$

C. Thibault and R. Klapisch

Laboratoire René-Bernas du Centre de Spectrométrie, Nucléaire et de Spectrométrie de Masse, 91-Orsay, France

(Received 4 October 1973)

Recent measurements of the masses of ^{11}Li and ^{14}B and a better fitting procedure result in a new set of parameters for the Garvey *et al.* mass table. A better agreement is obtained for the limit of stability as ^{14}Be and ^{19}C are now predicted to be bound.

NUCLEAR STRUCTURE Calculated mass excesses of nuclei $Z \leq 16$, $N \geq Z$.
Garvey-Kelson transverse relations.

I. INTRODUCTION

We have recently¹ recalculated the parameters of the mass table that had been derived by Garvey *et al.*² on the basis of local relations between masses.³ One of the conclusions of our work was the prediction that ^{17}B should be particle stable and the suggestion that it seemed possible to observe it experimentally. The experimental particle stability of ^{17}B has since been reported by Bowman *et al.*⁴ who also report evidence for the stability of ^{14}Be .

We had already pointed out¹ the special importance of the $T_\pi=2$, odd-odd nuclei as input data for this table. The basic reason for this is that one cannot calculate these using the local relations, because one would then have to use the $T_\pi=0$ odd-odd nuclei and this is forbidden.³ At the time of our last calculation,¹ ^{10}Li and ^{14}B were still unknown in that series and we had to make assumptions. Since then, new experimental developments have occurred allowing us to make a final readjustment of parameters.

II. RECALCULATION OF PARAMETERS

We start from the "transverse relation" (t):

$$M(N+2, Z-2) - M(N, Z) + M(N, Z-1) - M(N+1, Z-2) + M(N+1, Z) - M(N+2, Z-1) = 0,$$

which Garvey *et al.*² have shown to be equivalent to

$$M(N, Z) = g_1(N) + g_2(Z) + g_3(A).$$

g_1, g_2, g_3 are the parameters that we calculate using known masses of $T_\pi \leq 2$. Table I shows the result of our new calculation using the following changes:

(1) The experimental mass of ^{11}Li ⁵ is used in-

TABLE I. New parameters used to calculate the binding energies $B(N, Z) = 110.7824 + g_1(N) + g_2(Z) + g_3(A)$ MeV. Only the values which are different from the original ones (Ref. 2) are given.

$N, Z, \text{ or } A$	$g_1(N)$	$g_2(Z)$	$g_3(A)$
1
2	...	-98.279	...
3	...	-71.813	...
4	-61.368	-42.432	...
5	-45.322	-22.229	...
6	-28.777	-0.173	78.133
7	-19.301	11.016	61.614
8	-9.900	22.855	47.632
9	-7.700	25.564	35.139
10	-3.748	30.320	25.406
11	-5.182	28.530	16.432
12	-5.443	27.981	10.326
13	-10.757	20.792	5.804
14	-14.279	15.545	4.579
15	-23.355	3.307	3.598
16	-31.564		3.888
17	-44.570		5.829
18	-56.203		9.923
19			15.260
20			23.243
21			31.544
22			42.118
23			52.666
24			64.975
25			77.552
26			92.203
27			107.693
28			124.465
29			142.075
30			160.864
31			180.429
32			201.342
33			223.049
34			246.068
35			269.729

TABLE II. Mass excesses, neutron, and two-neutron binding energies calculated for nuclei $Z \leq 16$, $N \leq 2Z + 6$.

	N	Z	A	Mass excess		Binding energy		N	Z	A	Mass excess		Binding energy		
				Calc (MeV)	Exp-calc (MeV)	Neutron (MeV)	2 neutron (MeV)				Calc (MeV)	Exp-calc (MeV)	Neutron (MeV)	2 neutron (MeV)	
He	4	2	6	17.60	0.00	0.00	0.00	N	11	7	18	13.27	0.00	2.66	8.55
	5	2	7	26.11	-0.00	-0.44	0.00		12	7	19	16.27		5.08	7.74
	6	2	8	31.65	-0.00	2.53	2.09		13	7	20	21.67		2.67	7.75
	7	2	9	42.74		-3.02	-0.48		14	7	21	24.96		4.78	7.45
	8	2	10	51.14		-0.33	-3.35		15	7	22	31.54		1.50	6.28
	9	2	11	65.99		-6.77	-7.11		16	7	23	37.27		2.34	3.84
	10	2	12	76.21		-2.15	-8.93	17	7	24	46.04		-0.70	1.64	
Li	4	3	7	14.91	-0.00	0.00	0.00	18	7	25	53.17		0.94	0.25	
	5	3	8	20.95	0.00	2.03	0.00	19	7	26	63.30		-2.06	-1.11	
	6	3	9	24.97	-0.00	4.05	6.09	20	7	27	71.22		0.14	-1.91	
	7	3	10	33.30		-0.26	3.79	O	8	8	16	-4.74	0.00	0.00	0.00
	8	3	11	40.94	0.00	0.43	0.17		9	8	17	-0.81	0.00	4.14	0.00
	9	3	12	52.92		-3.91	-3.48		10	8	18	-0.78	-0.00	8.05	12.19
10	3	13	61.56		-0.57	-4.48	11		8	19	3.39	-0.05	3.90	11.95	
11	3	14	72.29		-2.66	-3.23	12		8	20	3.74	0.06	7.72	11.63	
12	3	15	81.60		-1.24	-3.90	13		8	21	8.82	0.48	2.99	10.71	
Be	4	4	8	6.83	-1.89	0.00	0.00	14	8	22	9.84	1.66	7.05	10.04	
	5	4	9	11.35	0.00	3.55	0.00	15	8	23	16.44		1.47	8.52	
	6	4	10	12.61	0.00	6.81	10.37	16	8	24	20.41		4.10	5.57	
	7	4	11	20.18	-0.00	0.50	7.31	17	8	25	28.91		-0.43	3.67	
	8	4	12	24.95	-0.00	3.30	3.80	18	8	26	33.97		3.02	2.59	
	9	4	13	35.35		-2.32	0.97	19	8	27	43.26		-1.22	1.80	
	10	4	14	40.69		2.73	0.41	20	8	28	49.90		1.43	0.21	
	11	4	15	51.18		-2.42	0.31	21	8	29	61.59		-3.61	-2.19	
	12	4	16	59.22		0.03	-2.39	22	8	30	70.36		-0.70	-4.31	
	13	4	17	70.67		-3.37	-3.34	F	10	9	19	-1.54	0.05	0.00	0.00
	14	4	18	78.17		0.57	-2.80		11	9	20	-0.02	-0.00	6.55	0.00
B	6	5	11	8.67	0.00	0.00	0.00		12	9	21	0.02	-0.06	8.04	14.59
	7	5	12	13.37	0.00	3.37	0.00		13	9	22	2.83	0.00	5.26	13.30
	8	5	13	16.56	-0.00	4.88	8.25		14	9	23	3.87		7.03	12.29
	9	5	14	23.66	0.00	0.98	5.85		15	9	24	8.71		3.23	10.26
	10	5	15	28.76		2.97	3.95	16	9	25	12.42		4.37	7.60	
	11	5	16	37.98		-1.14	1.83	17	9	26	18.84		1.65	6.01	
	12	5	17	44.37		1.68	0.54	18	9	27	23.06		3.86	5.50	
	13	5	18	53.66		-1.22	0.46	19	9	28	31.06		0.06	3.92	
	14	5	19	59.92		1.81	0.59	20	9	29	36.87		2.26	2.33	
	15	5	20	69.08		-1.09	0.72	21	9	30	47.38		-2.43	-0.17	
	16	5	21	77.06		0.09	-1.00	22	9	31	55.37		0.08	-2.36	
C	6	6	12	0.01	-0.01	0.00	0.00	23	9	32	65.96		-2.52	-2.44	
	7	6	13	3.12	0.00	4.95	0.00	24	9	33	73.98		0.05	-2.47	
	8	6	14	3.02	-0.00	8.18	13.13	Ne	10	10	20	-6.99	-0.05	0.00	0.00
	9	6	15	9.87	-0.00	1.22	9.40		11	10	21	-5.78	0.05	6.87	0.00
	10	6	16	13.70	-0.01	4.24	5.46		12	10	22	-8.02	-0.00	10.31	17.18
	11	6	17	21.27		0.51	4.75		13	10	23	-5.19	0.04	5.23	15.55
12	6	18	25.51		3.83	4.34	14		10	24	-5.90	-0.05	8.79	14.02	
13	6	19	33.55		0.02	3.86	15		10	25	-1.33	-0.84	3.50	12.29	
	14	6	20	37.17		4.46	4.48	16	10	26	0.30		6.44	9.94	
	15	6	21	46.01		-0.77	3.69	17	10	27	5.89		2.48	8.93	
	16	6	22	51.72		2.36	1.59	18	10	28	8.82		5.14	7.62	
	17	6	23	62.25		-2.46	-0.09	19	10	29	15.99		0.90	6.04	
	18	6	24	69.64		0.68	-1.78	20	10	30	20.62		3.44	4.35	
N	8	7	15	0.10	0.00	0.00	0.00	21	10	31	30.35		-1.66	1.79	
	9	7	16	5.68	-0.00	2.49	0.00	22	10	32	36.99		1.42	-0.23	
	10	7	17	7.86	0.01	5.89	8.38								

TABLE II (Continued)

N	Z	A	Mass excess		Binding energy		N	Z	A	Mass excess		Binding energy	
			Calc (MeV)	Exp-calc (MeV)	Neutron (MeV)	2 neutron (MeV)				Calc (MeV)	Exp-calc (MeV)	Neutron (MeV)	2 neutron (MeV)
S	28	16	44	-11.77	7.21	10.77	S	34	16	50	22.90	2.43	2.22
	29	16	45	-5.92	2.22	9.43		35	16	51	32.37	-1.40	1.03
	30	16	46	-2.51	4.67	6.88		36	16	52	39.01	1.42	0.02
	31	16	47	4.44	1.12	5.78		37	16	53	49.13	-2.05	-0.62
	32	16	48	8.97	3.54	4.66		38	16	54	56.39	0.82	-1.23
	33	16	49	17.25	-0.21	3.33							

stead of an estimated value for ^{10}Li in the last calculation;

(2) the experimental mass of $^{14}\text{B}^6$ is used instead of our value¹ assuming that it is just bound;

(3) parameters have also been readjusted for slight changes in the masses of $^{26}\text{Na}^7$ and $^{34}\text{P}^8$.

Another improvement has been introduced in the evaluation of a consistent set of calculated mass values. When the mass table is restricted to light nuclei there are very few cases of redundancy and one given mass M_i is calculated by just one relation.⁹ A least-squares fit can thus be done exactly and we have done it as follows: Let there be a t relation between six masses M_i which is not satisfied exactly:

$$\sum_i M_i = \Delta \neq 0.$$

The difference between experiment and calculation for mass M_i is taken to be

$$\delta_i = \pm \left(\frac{\Delta - \sum_{i=1}^6 \sigma_i}{6} + \sigma_i \right),$$

where σ_i is the experimental uncertainty on M_i and where $\sum_{i=1}^6 \sigma_i \ll \Delta$.

This formula is slightly different from the one used by Garvey *et al.*² where $\sigma_i = 0$, but the main difference comes from the fact that in this simple case the fit can be done without any approximation or iterative procedure.

III. RESULTS

Table I gives the values of the parameters g_1, g_2, g_3 when they are different from the values published by Garvey *et al.*² Table II gives the mass excesses and neutron binding energies calculated for nuclei $Z \leq 16$ and $N \leq (2Z + 6)$. Nuclei with $Z > 16$ are unchanged. Some new consequences for the limit of stability have to be pointed out: Five nuclei (^{14}Be , ^{19}C , ^{25}N , ^{28}O , ^{28}F) which were predicted unstable in our last table are now calculated to be bound. This is in better agreement with experiment as two of them have already been observed: ^{19}C , which was first reported by Raisbeck *et al.*¹⁰ and recently confirmed by Bowman *et al.*,¹¹ and ^{14}Be .⁴ The only case of discrepancy for the limit of particle stability is thus ^{32}Na which is calculated unbound by 310 keV while it is found experimentally bound.^{5,12} This is probably within the limits of uncertainty of the mass predictions.

IV. CONCLUSION

This table is now the best fit which can be done, based on the assumptions of Garvey *et al.*² and using as input only $T_\pi \leq 2$ known masses. It will thus be very interesting to carefully compare the predictions to the new experimental masses such as the masses of $T_\pi = \frac{5}{2}$ nuclei or of the sodium isotopes with $T_\pi > 2$, to see if the differences reveal some systematic deviations which were not taken into account or some abrupt changes which could reflect changes in the structure of the nucleus.

¹C. Thibault and R. Klapisch, *Phys. Rev. C* **6**, 1509 (1972).

²G. T. Garvey, W. J. Gerace, R. L. Jaffe, I. Talmi, and I. Kelson, *Rev. Mod. Phys.* **41**, S1 (1969).

³G. T. Garvey and I. Kelson, *Phys. Rev. Lett.* **16**, 197 (1966).

⁴J. D. Bowman, A. M. Poskanzer, R. G. Korteling, and G. W. Butler, *Phys. Rev. Lett.* **31**, 614 (1973).

⁵R. Klapisch, C. Thibault, C. Rigaud, A. M. Poskanzer, L. Lessard, and W. Reisdorf, in *Proceedings of the International Conference on Nuclear Physics, Munich, Germany, Aug. 27-Sept. 1*, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1973), p. 325.

⁶G. C. Ball, G. J. Costa, W. G. Davies, J. S. Forster, J. C. Hardy, and A. B. McDonald, *Phys. Rev. Lett.* **31**, 395 (1973).

⁷E. R. Flynn and J. D. Garrett, private communication to A. M. Poskanzer.

⁸D. R. Goosman, C. N. Davids, and D. E. Alburger, Phys. Rev. C 8, 1324 (1973).

⁹The only cases of redundancy are $T_{\pi}=2$ even-even nuclei.

¹⁰G. M. Raisbeck, P. Boerstling, P. Roesenfeldt, T. D.

Thomas, R. Klapisch, and G. T. Garvey, in *High Energy Physics and Nuclear Structure*, edited by S. Devons (Plenum, New York, 1970), pp. 341-345.

¹¹J. D. Bowman, A. M. Poskanzer, R. G. Korteling, and G. W. Butler, Phys. Rev. (to be published).

¹²¹¹Li is used as input in this calculation as a substitute for ¹⁰Li. One thus cannot test its predicted value.