

Hindrance effects on the β moments for the $\frac{7}{2}^-$ (0.435-MeV β^-) $\frac{7}{2}^+$ transition from the decay of ^{141}Ce

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The β moments of the $\frac{7}{2}^-$ (β^-) $\frac{7}{2}^+$ transition from the decay of ^{141}Ce have been extracted from the available experimental data. We have obtained the following results (expressed in fm): $\langle \vec{\sigma} \cdot \vec{r} \rangle = 0.32 \pm 0.08$, $\langle \vec{r} \rangle = 0.029 \pm 0.012$, $\langle \vec{\sigma} \times \vec{r} \rangle = -0.20 \pm 0.07$, $\langle iB_{ij} \rangle = -0.43 \pm 0.57$, $\langle \gamma_5 \rangle / \xi = 0.98 \pm 0.10$, and $\langle \vec{\alpha} \rangle / \xi = 0.065 \pm 0.027$. These results show that the β -moments ratio $\Lambda (= \langle \vec{\alpha} \rangle / \xi \langle \vec{r} \rangle)$ agrees with the theoretical estimation of Fujita-Eichler $\Lambda = 2.36$, and that the β moments are strongly reduced with respect to a conventional pairing model. In addition, the value of the spin-flip γ moment $\langle 1g_{7/2} \| i\vec{r} \| 2f_{7/2} \rangle \gamma$, measured in the $(e, e'p)$ reaction through the isobaric-analog state in ^{139}La , is inconsistent with the present analysis.

I. INTRODUCTION

In several works¹⁻⁸ it was observed that the β moments extracted from the experimental data are smaller than the calculated values within the framework of the conventional models. These hindrance effects are an important feature of the β -decay studies. The importance arises in the fact that the β moments depend in a sensitive way on the particle-hole correlations (core polarization) induced by the isospin-dependent residual interactions, which have the same multipole structure as the β operators involved in the process. These charge-exchange p-h interactions are repulsive and tend to decrease the β moments.

The quenching of the β moments can be discussed in terms of the effective coupling constants $(g_{A,V}^{\text{eff}})_\lambda$ or, equivalently, in terms of the polarizabilities $(\chi_{A,V})_\lambda$ defined as

$$\frac{(g_{A,V}^{\text{eff}})_\lambda}{g_{A,V}} = 1 + (\chi_{A,V})_\lambda = \frac{\langle O_{A,V}(\lambda) \rangle_{\text{exp}}}{\langle O_{A,V}(\lambda) \rangle_{\text{cal}}}, \quad (1)$$

where $O_{A,V}(\lambda)$ represent axial (A) and vector (V) operators, respectively, λ being the tensorial rank.

One may attempt to estimate the effective charges for the β moments when the available

wave functions explain satisfactorily other properties of the involved states, which are not considerably disturbed by the charge-exchange interactions. Studies involving allowed operators have been performed for a large number of spherical nuclei,^{2,3} while the search for the hindrance phenomena in the first forbidden decays is mainly concentrated in the nuclei which lay near the closed shells.⁴⁻⁸

For instance, the low-lying states in the ^{141}Ce and ^{141}Pr nuclei are experimentally well known,⁹ and many of their static as well as dynamic properties are satisfactorily explained in the framework of the quasiparticle plus Q-Q phonon model (K-S).¹⁰⁻¹³ In a former paper⁸ the $\frac{7}{2}^-$ (0.580-MeV β^-) $\frac{5}{2}^+$ transition in ^{141}Ce was studied. In that work⁸ was also pointed out that the moments presented by Sunier and Berthier¹⁴ for the like transition $\frac{7}{2}^-$ (2.23-MeV β^-) $\frac{5}{2}^+$ from ^{139}Ba show equal features, as it is expected from nuclear-structure considerations.

From the above mentioned favorable situation, one could think of another transition in ^{141}Ce [i.e., $\frac{7}{2}^-$ (0.435-MeV β^-) $\frac{7}{2}^+$] where the quenching may also be observed. In a previous work this transition was analyzed.¹⁵ From that study no decisive information about the nuclear structure could be

obtained. However, it was pointed out that the experimental data presented¹⁵ agreed with the circular polarization datum of Deutsch, Grenacs, and Lipnik¹⁶ but disagreed with the measurement performed by Daniel *et al.*¹⁷ It was also mentioned that further measurements for both the circular and longitudinal polarizations might solve the conflicting situation. For the time being two new measurements of the circular polarization (van Rooijen *et al.*¹⁸ and Schmidt¹⁹) and one of the longitudinal polarization (Blinowska, Chojnacki, and Czyżewski²⁰) are available. Both experimental results for the circular polarization are consistent with the one reported by Daniel *et al.*¹⁷ Meanwhile another measurement for angular correlation was also published.²¹

It should be mentioned that the authors of Ref. 18 also extracted the ratios for the β moments taking into account the shape factor from Ref. 22, the angular correlation from Ref. 23, and their own circular polarization data. For that study¹⁸ Kotani's formulas²⁴ were used. It is well known, since the work of Newsome and Fischbeck,²⁵ that when the calculations for the observables are performed by means of Kotani's formulas and with the complete expressions both results could be different. This point is discussed with more details in Sec. III. Besides, the data from oriented nuclei experiments^{26, 27} were not considered in Ref. 18.

The most serious trouble for the extraction of the β moments occurs when the ξ approximation²⁴ is fulfilled. From the comparison of the end-point energy for this transition ($W_0 = 1.858$) with respect to

$$\xi = \frac{\alpha Z}{2r_0} = 13.23, \quad (2)$$

where α is the fine-structure constant, Z and r_0 are the charge and radius of the daughter nucleus, respectively, and since $\xi \gg W_0$, it seems that the ξ approximation should be fulfilled. Some years ago, Weidenmüller²⁸ pointed out that when the ξ approximation held, the extraction of the individual β moments was impossible and only the following linear combinations^{15, 24, 28}

$$V = v + \xi w \quad (3)$$

and

$$Y = y - \xi(x + u) \quad (4)$$

could be determined. However, in this case some of the experimental data show deviations from the ξ approximation. Moreover, it was recently shown in Refs. 29–31 that even though the ξ approximation is fulfilled when great numbers of accurate experiments are available, one is able

to extract meaningful β moments.

In order to add more information about the existence of hindrance effects in this region of spherical nuclei it is worthwhile to realize a new study of the $\frac{7}{2}^-(0.435\text{-MeV } \beta^-) \frac{7}{2}^+$ transition from the decay of ¹⁴¹Ce. The aim of the present work is to perform a complete analysis of this transition taking into account all available experimental data. In Sec. II the formulas for the observables are given. The analysis and the results are presented in Sec. III. Finally, Sec. IV is devoted to a detailed discussion.

II. FORMULAS FOR THE β -DECAY OBSERVABLES

When terms induced by the strong interactions³² and the other higher-order corrections^{33, 34} are neglected in the weak Hamiltonian, all the observables for the nonunique first-forbidden transitions can be written in terms of the β moments,^{31, 35} listed in Table I.

The relativistic β moments are usually related to the corresponding nonrelativistic ones by defining the ratios

$$\Lambda = \frac{\langle \vec{\alpha} \rangle}{\xi \langle i \vec{F} \rangle} \quad (5)$$

and

$$\Lambda_0 = \frac{\langle \gamma_5 \rangle}{\xi \langle i \vec{\sigma} \cdot \vec{F} \rangle} \quad (6)$$

The various approximations for these ratios were already extensively discussed in the former works.^{8, 31} In order to do this work more self-consistently, they are only mentioned. For the vector β moments Fujita³⁶ and Eichler³⁷ (FE), taking into account the conservation of the vector current (CVC), under the Ahrens-Feenberg³⁸ approximation for the Coulomb potential have obtained

$$\Lambda(\text{FE}) = 2.4 + (W_0 - 2.5)\xi^{-1}. \quad (7)$$

TABLE I. Definitions of the nuclear matrix parameters in terms of the Cartesian representation of the nuclear matrix elements (β moments).

Notation	$O(\lambda)$ ^a	Multipolarity λ
ηv	$-g_A \langle \gamma_5 \rangle$	0
ηw	$g_A \langle i \vec{\sigma} \cdot \vec{F} \rangle$	0
ηu	$-g_A \langle \vec{\sigma} \times \vec{F} \rangle$	1
ηx	$-g_V \langle i \vec{F} \rangle$	1
ηy	$-g_V \langle \vec{\alpha} \rangle$	1
ηz	$g_A \langle i B_{ij} \rangle$	2

^a The same sign conventions as Schopper (Ref. 35) are used. The only difference is the identification $\int O(\lambda) = \langle O(\lambda) \rangle \equiv \langle f \| O(\lambda) \| i \rangle$.

By means of a different approach for the Coulomb potential Damgaard and Winther³⁹ have arrived at the following formula:

$$\Lambda(DW) = (3 - \epsilon_1) + (W_0 - 2.5)\xi^{-1}, \quad (8)$$

where the parameter ϵ_1 is defined as

$$\epsilon_1 = \frac{\langle r^3 \rangle}{r_0^2 \langle r \rangle}. \quad (9)$$

When $\epsilon_1 = 0.6$ both Eqs. (7) and (8) are numerically equivalent.

For the axial β -moments relation Pursey⁴⁰ has obtained $\Lambda_0 = 2$ and Ahrens and Feenberg³⁸ have obtained $\Lambda_0 \approx 1$. Recently, by using the concept of the conserved second-class axial vector current (CSAC), Eman and Tadić⁴¹ have deduced the expression

$$\Lambda_0(ET) = 2.4 + (W_0 - 2.5)\xi^{-1}, \quad (10)$$

which is analogous to the Eq. (7) for Λ .

The formulas for the β observables presented in Ref. 31 were used in this analysis.⁴²

A special mention is worthwhile for the observables from oriented nuclei. The directional distribution of the γ ray following the β decay of oriented nuclei is given by

$$W_{\beta\gamma}(\theta) = \sum_k U_k F'_k B_k P_k(\cos\theta), \quad (11)$$

where θ is the angle of the emission of the γ rays with respect to the axis of quantization. The factors U_k represent the degree of orientation of the parent nucleus. The factors F'_k are functions determined by, for pure transitions, the multipolarity and the initial and final spins of the observed γ transition; for mixed transitions involving multiplicities L and L' , the functions are also determined by the amplitude mixing ratio $\delta(L'L)$. The factors B_k are the functions which give the realignment of the nuclei due to the β ray preceding the γ rays. Finally, the functions $P_k(\cos\theta)$ are Legendre polynomials of order k , where k is an even integer.

We are interested in B_2 ; its expression for this transition is

$$B_2 = (a_0 + \frac{17}{21}a_1 + \frac{7}{15}a_2)/(a_0 + a_1 + a_2), \quad (12)$$

where the coefficients a_i are those of Ref. 31.

The angular distribution of β rays from oriented nuclei is given by

$$N_e(W, \theta) = 1 + N_1(W)(p/W)f_1P_1(\cos\theta). \quad (13)$$

The variables W and p are the electron energy and momentum, respectively. The factor f_1 is the orientation parameter. The coefficient $N_1(W)$ con-

tains information on the β moments according to

$$N_1(W) = \frac{\sqrt{7}}{3} b_{01}^{(1)} - \frac{\sqrt{2}}{9} b_{11}^{(1)} - \frac{2\sqrt{6}}{9} b_{12}^{(1)} + \frac{\sqrt{10}}{15} b_{22}^{(1)}, \quad (14)$$

where the particle parameters $b_{LL'}^{(1)}$ are those defined in the Appendix of Ref. 31.

III. ANALYSIS AND RESULTS

The following experimental data were analyzed:

(a) Shape factor $C_\beta(W)$. The results from the work of Beekhuis and van Duinen²² were taken into account.

(b) β - γ directional-correlation coefficient $\epsilon(W)$. The datum reported by Wohn and Wilkinson²¹ was considered. We have chosen the measurement of Ref. 21 instead of that presented in Ref. 23 because of its better statistics.

(c) β - γ circular polarization factor $\omega_{\beta\gamma}(W, \theta)$. Its energy dependence for the angle $\theta = 150^\circ$ was picked up from the paper of van Rooijen *et al.*¹⁸ while its angular dependence averaged over the energy $\omega_{\beta\gamma}(\theta)$ was also taken from Ref. 18 and, besides, from the works of Daniel *et al.*¹⁷ and Schmidt.¹⁹

(d) Longitudinal polarization of β rays $P_L(W)$. The experimental result of Blinowska, Chojnacki, and Czyżewski²⁰ was included.

(e) Directional correlation of γ rays from oriented nuclei B_2 . The datum reported by Haag, Shirley, and Templeton²⁸ was considered.

(f) Angular distribution of β rays from oriented nuclei. The experimental results of Hoppes²⁷ was taken into account.

(g) Partial half-life $t_{1/2}$. It was extracted from *Tables of Isotopes*.⁹ The mixing ratio $\delta = \langle E2 \rangle / \langle M1 \rangle$ was taken from the work of Haag, Shirley, and Templeton.²⁸ This value $\delta = 0.066 \pm 0.022$ is consistent with the older one $\delta = 0.08 \pm 0.02$ presented by Cacho *et al.*⁴³ and with the $E2$ admixtures published by Schooley, Hoppes, and Hirshfeld⁴⁴ and Geiger *et al.*⁴⁵

In order to obtain the β moments, the above mentioned experimental data were fitted to the theoretical values by means of a computer program which uses the package of subroutines MINUTS—kindly provided to us by the library of Centre d'Etudes Recherche Nucléaires (CERN).

The method for searching the values of the β moments is based on the minimization of the χ^2_T function, defined as

$$\chi^2_T = \sum_{k=1}^n \chi^2(k), \quad (15)$$

with

$$\chi^2(k) = \sum_{i=1}^{N(k)} \{ [Q_{\text{th}}^k(i) - Q_{\text{exp}}^k(i)] / \Delta Q_{\text{exp}}^k(i) \}^2, \quad (16)$$

where n is the number of the β observables taken into account (for example $C_\beta(W)$, $P_L(W)$, $\epsilon(W)$, $\omega_{\beta\gamma}(W)$, $\omega_{\beta\gamma}(\theta)$, B_2 , and $N_1(W)$); $N(k)$ is the total number of experimental values of the observable k ; $Q_{\text{exp}}^k(i)$ and $\Delta Q_{\text{exp}}^k(i)$ are the experimental values of the observable k and its error, at a given energy $W(i)$, respectively; $Q_{\text{th}}^k(i)$ is the theoretical value for the observable k at the energy $W(i)$. The criterion adopted for accepting some particular minimum is to satisfy the condition

$$\frac{\chi^2(k)}{N(k)} \leq 1, \quad (17)$$

for all the k .

The errors on the β moments were estimated by considering 1 standard deviation. This means the χ^2_T function increased by unity from its absolute minimum.

For the extraction of the β moments $u=1$ was taken, so the scaling factor was

$$\eta = -g_A \langle \vec{\sigma} \times \vec{F} \rangle. \quad (18)$$

The physical limits set for the free parameters were:

$$-10 \leq w \leq 10, \quad -10 \leq x \leq 10, \quad -10 \leq z \leq 10, \quad (19)$$

and

$$0 \leq \Lambda_0 \leq 4, \quad 0 \leq \Lambda \leq 4. \quad (20)$$

The limits for the ratios Λ_0 and Λ were determined in accordance with theoretical estimates.³⁶⁻⁴¹

The scaling factor η was determined from the experimental partial half-life⁹ $t_{1/2}$ and the inte-

TABLE II. The initial minima and the results for the "average longitudinal polarization."

	Set I	Set II
w	1.65 (± 0.20)	-4.79
x	0.123 (± 0.051)	0.35
u	1.0	1.0
z	-2.15 (± 2.90)	0.95
Λ_0	2.94 (± 0.24)	1.07
Λ	2.36 (± 0.63)	3.90
P_L ($\frac{1}{2}^- \rightarrow \frac{1}{2}^+$)	-0.990 (v/c)	-0.843 (v/c)
\bar{P}_L (theory)	-0.907 (v/c)	-0.807 (v/c)
\bar{P}_L (exp)	(-0.90 \pm 0.04) (v/c)	
χ^2 (\bar{P}_L)	0.03	5.39

TABLE III. Results for the $\frac{1}{2}^- (0.435\text{-MeV } \beta^-) \frac{1}{2}^+$ transition from the decay of ^{141}Ce .

Quantity ^a	Value
$\langle i \vec{\sigma} \cdot \vec{F} \rangle$	0.32 \pm 0.08
$\langle i \vec{F} \rangle$	0.029 \pm 0.012
$\langle \vec{\sigma} \times \vec{F} \rangle$	-0.20 \pm 0.07
$\langle i B_{ij} \rangle$	-0.43 \pm 0.57
$\xi^{-1} \langle \gamma_5 \rangle$	0.98 \pm 0.10
$\xi^{-1} \langle \vec{\alpha} \rangle$	0.065 \pm 0.027
$ \eta /r_0$	0.038 \pm 0.013
$ \eta V/r_0$	-1.64 \pm 0.12
$ \eta Y/r_0$	-0.42 \pm 0.20
$\log[f_c t / (f t)_{\text{exp}}]$	1.22

^a The β moments are given in Fermi units. Besides, for the evaluation of the β moments the scaling factor η was considered with negative sign in order to obtain compatible results with the theoretical values calculated taking into account the $i^l Y_{lm}$ choice of phases for single-particle wave functions.

grated spectrum shape $C_\beta(W)$:

$$\eta^2 = \frac{6222}{t_{1/2} f_c} \quad (21)$$

with

$$f_c = \int_1^{W_0} C_\beta(W) p W q^2 F(Z, W) dW. \quad (22)$$

Here the variable q is the neutrino momentum. The Fermi function $F(Z, W)$ is defined in work of Alaga.⁴⁶ The other variables have usual meaning.

The datum for the longitudinal polarization²⁰ was not included in the minimization procedure because it contains information about the two β branches together.

All the β moments ratios w , x , z , Λ_0 , and Λ were considered as free parameters. Two minima were found and the results are presented in Table II. The second minimum was rejected due to con-

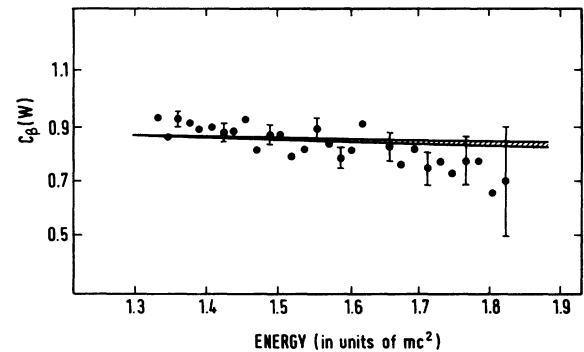


FIG. 1. Experimental shape factor from Ref. 22, with theoretical values calculated from the β moments.

TABLE IV. Fitting of the nuclear orientation and longitudinal polarization data.

Observable	Present work	Ref. 18 ^a	Experimental value
B_2	0.99 ± 0.01	0.85	0.96 ± 0.04
N_1	-0.33 ± 0.05	0.33	-0.37 ± 0.10
$\bar{P}_L/(v/c)$	-0.907 ± 0.010	-0.833	-0.90 ± 0.04

^a Calculated by means of complete formalism with the ratios for the β moments presented by van Rooijen *et al.* (Ref. 18).

siderations discussed below.

Let us define the "average longitudinal polarization" $\bar{P}_L(W)$, which contains information about the two branches, as follows

$$\bar{P}_L(W) = \frac{\sum_{I_f} [P_L(W)N(W)]_{I_f}}{\sum_{I_f} [N(W)]_{I_f}}, \quad (23)$$

where the summation goes on the different final states $I_f = \frac{7}{2}^+$ and $I_f = \frac{5}{2}^+$; and the quantity $N(W)$ is given by

$$N(W) = \frac{g_\beta^2}{2\pi^3} \eta^2 C_\beta(W) p W q^2 F(Z, W). \quad (24)$$

Here the symbol g_β is the semileptonic weak-coupling constant.

The experimental datum for the observable $\bar{P}_L(W)$ is given for an energy $W = 1.49 mc^2$. The theoretical values of $\bar{P}_L(W = 1.49)$ corresponding to both minima were evaluated. The quantities $[P_L(W)]_{I_f = 5/2}$ and $[N(W)]_{I_f = 5/2}$ were taken from the former work.⁸ The calculated results confronted with the experimental one, as well as the $\chi^2(\bar{P}_L)$ function are presented in Table II. As only the Set I of β -moments ratios reproduces satisfactorily

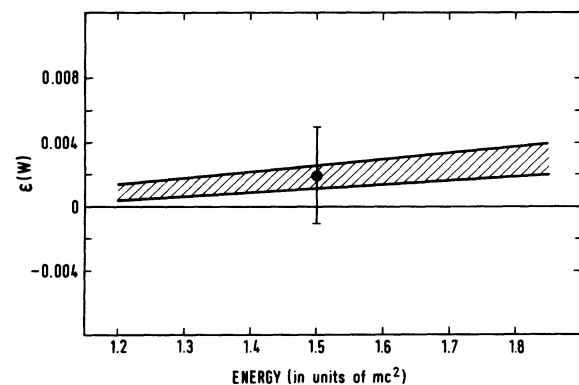


FIG. 2. Experimental angular correlation and the theoretical predictions.

the experimental datum and, in addition, the value of the ratio Λ for Set II deviates strongly from the estimation based on the CVC theory, we discarded the second solution.

The errors of the β -moments ratios for the true minimum (Set I) are also presented in Table II. The β moments together with other results of interest are listed in Table III.

The fits of the experimental data of the shape factor, β - γ directional correlation and circular polarization both as function of energy and angle, to the theoretical values calculated from the β -moments ratios are shown in Figs. 1-4, respectively. The theoretical band for the observables are due to the variation of the β -moments ratios within their errors. The corresponding comparison between the experimental and theoretical values for the coefficients of the directional distribution from oriented nuclei, for both the γ rays and β particles, and the "average longitudinal polarization" is given in Table IV.

The fitting for the observables is quite good; it could be mentioned that $\chi^2(k)/N(k)$ for the shape factor is slightly larger than 1. It is interesting to note that the prediction for the angular correlation, see Fig. 2, does not exhibit any important dependence on the electron energy outside of the experimental uncertainty in agreement with the comments of Wohn and Wilkinson.²¹ This result disagrees with the energy dependence found by Rao *et al.*,²³ but it is also worthwhile to mention that their integral datum $\bar{\tau} = -0.002 \pm 0.008$ (cf. Ref. 18) is in accordance with the present prediction.

We have also performed a calculation of the β -decay observables for the β -moments ratios reported by van Rooijen *et al.*,¹⁸ but using the complete formalism instead of Kotani's formulas.²⁴

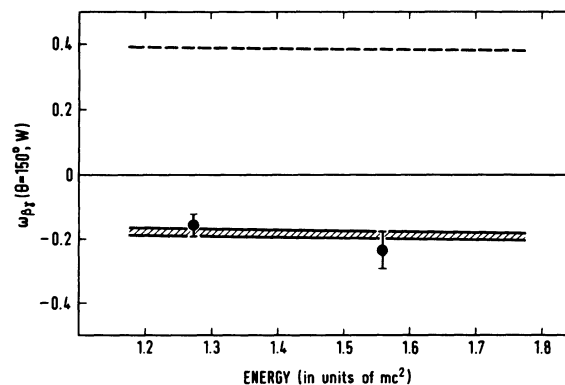


FIG. 3. The experimental circular polarization for $\theta_{\beta\gamma} = 150^\circ$ as function of energy. The band and the dashed line correspond to the predictions due to the minima from the present analysis and Ref. 18, respectively.

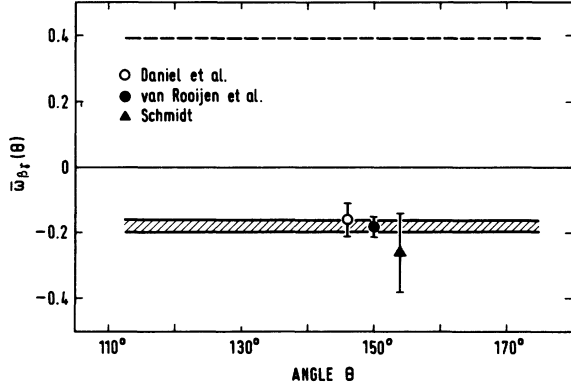


FIG. 4. The experimental average circular polarization as a function of angle. The band and the dashed line correspond to the predictions due to the minima from the present analysis and Ref. 18, respectively.

Only the shape factor and the angular correlation coefficient are satisfactorily reproduced, in this way. The other observables are compared in Figs. 3 and 4 and in Table IV.

Let us note that Kotani's formalism changes the sign of the calculated $\omega_{\beta\gamma}(W, \theta)$ and $N_1(W)$. Mainly because of the cancellation in the leading combination of tensorial rank-zero $V(|\eta|V/r_0 = -0.14$ while $|\eta|Y/r_0 = 0.79$) the approximate expressions give wrong predictions for these observables. The influence of this cancellation is essentially important through the particle parameter $b_{01}^{(1)}$.

IV. SUMMARY AND DISCUSSION

As both nuclear states involved by the transition are in good approximation pure one quasiparticle states,¹¹⁻¹³ namely $2f_{7/2}$ (initial) and $1g_{7/2}$ (final), the corresponding β moments are those of single

particle corrected by superconducting effects

$$\langle O_{A, \nu}(\lambda) \rangle_{\text{cal}} = \langle (1g_{7/2})_p \| O_{A, \nu}(\lambda) \| (2f_{7/2})_n \rangle \times U_p(1g_{7/2})U_n(2f_{7/2}). \quad (25)$$

Considering for the radial integral the wave functions corresponding to the harmonic-oscillator potential and taking the vacancies from Ref. 11 the results for the β moments are the following:

$$\begin{aligned} \langle i\vec{\sigma} \cdot \vec{r} \rangle &= 1.15 \text{ fm}, \\ \langle i\vec{r} \rangle &= 0.145 \text{ fm}, \\ \langle \vec{\sigma} \times \vec{r} \rangle &= -1.16 \text{ fm}, \\ \langle iB_{ij} \rangle &= -0.92 \text{ fm}, \\ \epsilon_1(\text{HO}) &= 1.40. \end{aligned} \quad (26)$$

If the Woods-Saxon potential is considered none of the conclusions of the present work change.

Let us comment first on the results for the ratios Λ and Λ_0 . The value obtained for Λ from the present analysis is in very good agreement with the theoretical estimation $\Lambda(\text{FE}) = 2.36$. It should be mentioned that, since for this transition $\epsilon_1 = 1.40$, a value $\Lambda(\text{DW}) = 1.56$ follows and it is outside of the error.

From the result for the ratio Λ_0 it seems that the estimation of Eman and Tadic⁴¹ is the more realistic one.

The calculated polarizabilities are presented in Table V and compared with the similar studies performed previously.^{4, 7, 8} Since the error in the determination of $\langle iB_{ij} \rangle$ is large, it is better to speak of the corresponding polarizability as an approximated value (taken at the minimum). For ¹³⁹Ba the polarizabilities were calculated taking into consideration the β moments presented by Sunier and Berthier.¹⁴ There is available another analysis for this transition performed by Agarwal, Baba, and Mitra.⁴⁷ In both works the conclusions are practically the same.

TABLE V. The polarizabilities for spherical nuclides extracted from experimental data.

	$N = 82$		^{139}Ba	Around $N = 50$ ^d		
	^{141}Ce			Unique transitions	Lead region ^e	
	$\frac{7}{2}^- \rightarrow \frac{7}{2}^+$ a	$\frac{7}{2}^- \rightarrow \frac{5}{2}^+$ b	$\frac{7}{2}^- \rightarrow \frac{5}{2}^+$ c		Sol. 1	Sol. 2
$\langle \chi_A \rangle_{\lambda=0}$	-0.72 ± 0.07
$\langle \chi_V \rangle_{\lambda=1}$	-0.80 ± 0.08	-0.63 ± 0.08	-0.72 ± 0.06	...	-0.5	-0.8
$\langle \chi_A \rangle_{\lambda=1}$	-0.83 ± 0.07	-0.59 ± 0.11	-0.38 ± 0.10	...	-0.6	-0.4
$\langle \chi_A \rangle_{\lambda=2}$	~ -0.54	-0.74 ± 0.10	-0.64 ± 0.25	-0.64 ± 0.13

^a Present work. In the preliminary report (Ref. 48) in Table II there are some misprints.

^b From Ref. 8.

^c Calculated from the results presented in Ref. 14.

^d From Ref. 7.

^e From Ref. 6.

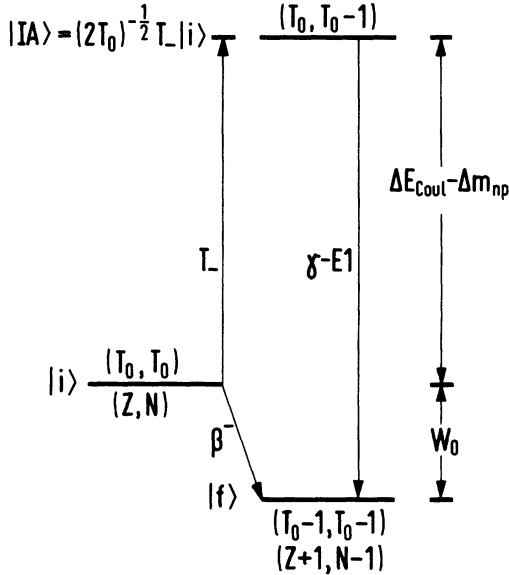


FIG. 5. Schematic diagram of β^- decay and the isobaric-analog (IA) charge-exchange $E1$ γ transition. The quantities ΔE_c and Δm_{np} are the difference in the Coulomb energies of the initial and final nuclei and the mass difference between the neutron and the proton, respectively.

A glance at the Table V indicates that the quenching of the β moments may be the same everywhere in the Periodic Table. It is very interesting to observe the consistency of the results for $\langle i\vec{r} \rangle$ in the $N=82$ region. This feature was pointed out in a preliminary report on part of the present work.⁴⁸

The β moment $\langle i\vec{r} \rangle$ for the transition $|i\rangle \rightarrow |f\rangle$ (see Fig. 5) can be related to the corresponding one for the charge-exchange $E1$ γ transition from the isobaric-analog state $|IA\rangle$ of $|i\rangle$ to the low-lying $|f\rangle$ state. Under the assumptions that (a) the isospin T_0 of the state $|i\rangle$ is equal to T_3 for this state, and (b) the $|IA\rangle$ is a pure isospin state, the following relation is obtained

$$\langle i\vec{r} \rangle_\beta = (2T_0)^{1/2} \langle i\vec{r} \rangle_\gamma. \quad (27)$$

The Eq. (27) implies that the same hindrance effects as for β moments may be observed for the charge-exchange $E1$ γ transitions. Several studies in this direction were done.⁴⁹⁻⁵³

The results, which correspond to the phase difference between the isobaric analog and the giant electric dipole resonance $\Delta\Phi = 0^\circ$, are summarized in Table VI. It can be noted that the general features are similar to those of β decay. The exceptions are the spin-flip transitions $2f_{7/2} \rightarrow 1g_{7/2}$ in ^{139}Ba and $2g_{9/2} \rightarrow 1h_{9/2}$ in ^{209}Bi , where the enhancement phenomena, instead of the hindrance effects, were reported by Shoda *et al.*^{50, 52}

Fujita, Hirata, and Shoda⁵⁴ have suggested that

TABLE VI. Results from the study of charge-exchange $E1$ γ -transitions.

Nucleus	Transition	e_{eff}/e^a	$(\chi_V)_{\lambda=1}$	Reference
^{139}La	$2f_{7/2} \rightarrow 1g_{7/2}$	5.5 ± 1.0	?	50
^{141}Pr	$2f_{7/2} \rightarrow 2d_{5/2}$	0.26 ± 0.02	-0.74 ± 0.02	50
		0.26 ± 0.06	-0.74 ± 0.06	51
^{207}Pb	$3s_{1/2} \rightarrow 3p_{1/2}$	0.56 ± 0.08	-0.44 ± 0.08	52
^{209}Bi	$2g_{9/2} \rightarrow 1h_{9/2}$	21 ± 1	?	52
		~ 0.57	~ -0.43	53
	$1i_{11/2} \rightarrow 1h_{9/2}$	0.46 ± 0.03	-0.64 ± 0.03	52
		0.45	-0.55	53
	$2g_{9/2} \rightarrow 2f_{7/2}$	0.55	-0.45	53
	$3d_{5/2} \rightarrow 2f_{7/2}$	0.63	-0.37	53
	$3d_{5/2} \rightarrow 3p_{3/2}$	~ 0.37	~ -0.63	53

^a The ratio e_{eff}/e is defined as $\langle i\vec{r} \rangle_{\gamma \text{exp}} / \langle i\vec{r} \rangle_{\gamma \text{cal}}$.

the above mentioned enhanced $E1$ γ transition could be explained due to the coupling interaction between the fields \vec{r} and $\vec{\sigma} \times \vec{r}$. This possibility was also discussed by Bohr and Mottelson.¹

Quite recently, Tanaka and Ikeda⁵⁵ have shown that, for nuclei near some double closed shells, there may be a considerable coupling between spin-flip and non-spin-flip vibrational modes, induced by the tensor force working coherently with the central charge-exchange force

$$H = \frac{K_T}{2} \sum_{i>j} (\vec{r}_i \cdot \vec{r}_j) (\vec{r}_i \cdot \vec{r}_j) + \frac{K_{T\sigma}}{2} \sum_{i>j} (\vec{r}_i \cdot \vec{r}_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) (\vec{r}_i \cdot \vec{r}_j). \quad (28)$$

Nevertheless, they were not able to explain the anomalously large $E1$ radiative width of the isobaric-analog state in ^{209}Bi , which was deduced from the $^{209}\text{Bi}(e, e'p)^{208}\text{Pb}$ reaction study.⁵² In addition, the more recent measurement of the γ moment $\langle 2g_{9/2} \| i\vec{r} \| 1h_{9/2} \rangle_\gamma$, obtained by Snover *et al.*⁵³ from the analysis of the $^{208}\text{Pb}(p, \gamma)^{209}\text{Bi}$ reaction data, is inconsistent with the results reported in Ref. 52 and indicates that the previous measurement of a strongly enhanced spin-flip transition in ^{139}La may be also in error. This possibility is also corroborated by the present study. Bearing in mind the similar nuclear structure of ^{139}Ba and ^{141}Ce and the results for the $\frac{7}{2}^-(\beta^-)^{\frac{5}{2}^+}$ transitions (see Table V) one can assume that the β moments of the $\frac{7}{2}^-(\beta^-)^{\frac{7}{2}^+}$ transition in ^{139}Ba are nearly the same as in ^{141}Ce . Consequently, one should expect the ratio e_{eff}/e for the $\frac{7}{2}^- \rightarrow \frac{7}{2}^+$ isobaric analog transition in ^{139}La to be close to the ratio $(g_V^{\text{eff}})_{\lambda=1}/g_V$ for the like transition in ^{141}Ce , which is not verified.

We can conclude that the hindrance of the first-forbidden β moments, as well as of the charge-exchange $E1$ γ moment $\langle i\vec{r} \rangle_\gamma$, is a regular phenomenon over a large mass region.

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