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Spin and quasifree proton-proton scattering on ^{3}He and ^{4}He

R. D. Haracz and T. K. Lim
Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104
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Theoretical expressions for the differential cross section of symmetric, coplanar $(p, 2p)$ quasifree scattering on ^{3}He and ^{4}He are derived in the plane-wave impulse approximation with full account given to the spin effects of the nucleon-nucleon interaction and the nuclear bound states. The phenomenological M_{12} matrix is utilized as are correlated functions and Eckart parametrizations of the distribution functions for the light nuclei. Shape fits to experimental data for the Eckart parametrizations are found to be extremely good although theory overpredicts the magnitudes of the cross sections. It is shown that an intuitive extension of the attenuation model of Rogers and Saylor is able to remove this latter discrepancy in ^{3}He$(p, 2p)^2$H. We also indicate the possible usefulness of our matrix elements in polarization studies.

[NUCLEAR REACTIONS ^{2}H,^{3,4}He$(p, 2p)$, E =20–160 MeV; calculated σ.]

I. INTRODUCTION

Reactions involving the breakup of light nuclei by protons in the quasielastic region, e.g. $(p, 2p)$ reactions,[1–5] have been investigated extensively in recent years as an important source of information on the structure of such nuclei. Generally, in the theoretical description of the $(p, 2p)$ quasifree process, the plane-wave impulse approximation[3–9] (PWIA) is used for the analysis of data. Implicit in this approach is the assumption that the reaction mechanism is direct, wherein the incoming nucleon interacts strongly with only one of the protons in the nucleus while the rest behaves as a "spectator." Thus the process may be represented by the diagram shown in Fig. 1. Nucleon 1, with momentum $\vec{k}_1$, is incident on the nucleus A and interacts with nucleon 2. It scatters at an angle θ_1 with respect to the direction of incidence while nucleon 2 scatters at an angle θ_2, leaving the residual nucleus denoted by B. If S is the scattering operator, the probability for all time and space for the transition from the initial state Φ_i to the final state Φ_f is

$$P_{if} = \left|\Phi_f^\dagger \left(\frac{S-1}{2i}\right)\Phi_i\right|^2 = |\delta(\vec{k}_1+\vec{k}_A-\vec{k}_1'-\vec{k}_2'-\vec{k}_B')\langle f|M|i\rangle|^2, \quad (1)$$

where the δ function expresses conservation of energy and momentum, and $\langle f|M|i\rangle$ is the reduced scattering matrix element. This matrix element can be expressed more explicitly as

$$F=\langle\vec{k}_1', \vec{k}_2', \vec{k}_B', B|M_{12}|\vec{k}_1, A\rangle . \quad (2)$$

M_{12} is that part of the total scattering matrix which corresponds to the interaction of the incoming nucleon and nucleon 2 of the target. In PWIA, F is written as

$$F=\langle\vec{k}_1', \vec{k}_2' |M_{12}|\vec{k}_1, \vec{k}_B'\rangle\Phi, \quad (3)$$

where the first factor is the free nucleon-nucleon matrix element and Φ is defined by

$$\langle\vec{k}_2, \vec{k}_B', B|A\rangle = \delta(\vec{k}_2+\vec{k}_B')\Phi . \quad (4)$$

The δ function represents the fact that the two elements of the target have zero total momentum and Φ, the overlap integral of the target and residual nuclei, represents the effective momentum distribution function in the nucleus A of nucleon 2.

When Eq. (3) is substituted into Eq. (1), with the density-of-states factor added, the proper kinematical integrations performed, and off-shell effects neglected, the differential cross section for the $(p, 2p)$ process takes the factorized form

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = K\left(\frac{d\sigma}{d\Omega}\right)_{p-p} |\Phi|^2 . \quad (5)$$

The kinematic phase-space factor is denoted by K, while $(d\sigma/d\Omega)_{p-p}$ is the free p-p on-shell differential cross section. This latter factor is usually obtained from experiment for the appropriate final-state energy. Direct comparison of experimental cross sections for ^{3}He$(p, 2p)^2$H and ^{4}He$(p, 2p)^3$He and PWIA calculations with simple forms for Φ has led investigators[3,8] to the following conclusions:
(1) The simple pole mechanism dominates at small momentum transfers, and under such conditions the shape of the cross section is mainly sensitive to the asymptotic form of the coordinate-

space, target-nucleus, wave function. Attempted theoretical fits to the cross sections for a wide range of values of the momentum transfer are generally too broad.

(2) The magnitude of the PWIA cross section, as given by Eq. (5), is also too large, indicating the importance of multiple scattering and distortion effects.

Now, in the derivation and use of Eq. (5) a number of other approximations are also often assumed besides the impulse approximation. Firstly, spin effects are neglected so that the detailed spin structure of the transition amplitude is not obtained. Secondly, distortions of the incident and emerging waves and multiple scattering effects are taken to be absent. Thirdly, the influence on Φ of short-range correlations and admixed states in the nuclear wave functions is neglected. And fourthly, the free on-shell nucleon-nucleon cross section from experiment is used in the expectation that off-shell effects are small.

In this paper we will seek to test the quasifree scattering hypothesis in PWIA by removing, or at least weakening, some of the above approximations. First, all spin effects consistent with quasifree scattering will be included, and a simple asymmetry experiment is suggested. Moreover, multiple scattering effects will be included phenomenologically by using a simple extension of the attenuation model of Rogers and Saylor.[10] An estimate of the effects of short-ranged correlations is made by using Eckart forms for the nuclear overlap functions. We maintain the assumption of on-shell scattering.

II. SPIN EFFECTS OF THE FULL N-N INTERACTION AND NUCLEAR BOUND STATES

In this section, we reconsider the reduction of the matrix element F. We rewrite Eq. (2) as

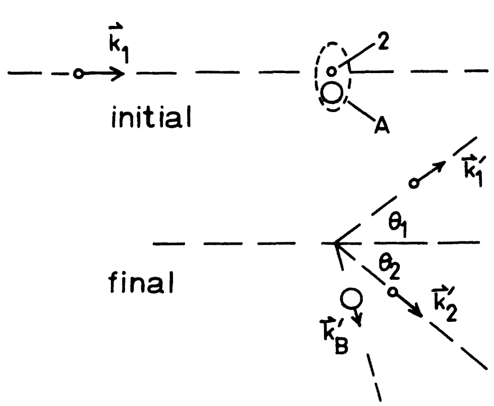

FIG. 1. Diagram showing the geometry of the $(p, 2p)$ quasifree reaction.

$$F=\sum_{\text{spin, isospin}}\langle \vec{k}_1', \vec{k}_2', \vec{k}_B, B_S \,|\, \chi_B^\dagger \chi_{1f}^\dagger \chi_{2f}^\dagger M_{12}\, \chi_{1\text{inc}}\, \chi_A \,|\, \vec{k}_1, A_S\rangle . \tag{6}$$

In this expression χ_A, $\chi_{1\text{inc}}$, χ_{1f}, χ_{2f}, and χ_B are representative spin and isospin states of the target nucleus, the incident nucleon, the scattered nucleon, the knocked-out nucleon and the residual nucleus.

Now, the spin matrix elements for two interacting nucleons are given by[11]

$$\begin{aligned} M &= \langle \vec{k}_1', \vec{k}_2' | M_{12} | \vec{k}_1, \vec{k}_2\rangle \\ &= \sum_{S M_S} \sum_{S' M_S'} {}^S\chi_{M_S}\, {}^S\chi_{M_S}^\dagger M^{S'} \chi_{M_{S'}}\, {}^{S'}\chi_{M_{S'}}^\dagger \\ &= {}^0M_{00}\, {}^0\chi_0\, {}^0\chi_0^\dagger + {}^1M_{00}\, {}^1\chi_0\, {}^1\chi_0^\dagger \\ &\quad + {}^1M_{01}({}^1\chi_0\, {}^1\chi_1^\dagger - {}^1\chi_0\, {}^1\chi_{-1}^\dagger) \\ &\quad + {}^1M_{10}({}^1\chi_1\, {}^1\chi_0^\dagger - {}^1\chi_{-1}\, {}^1\chi_0^\dagger) \\ &\quad + {}^1M_{11}({}^1\chi_1\, {}^1\chi_1^\dagger + {}^1\chi_{-1}\, {}^1\chi_{-1}^\dagger) \\ &\quad + {}^1M_{-11}({}^1\chi_{-1}\, {}^1\chi_1^\dagger + {}^1\chi_1\, {}^1\chi_{-1}^\dagger) , \end{aligned} \tag{7}$$

where ${}^S\chi_{M_S}$ are the usual spinor states for two-spin $\frac{1}{2}$ particles and

$${}^SM_{M_S M_S'} = {}^S\chi_{M_S}^\dagger M^{S'} \chi_{M_{S'}} . \tag{8}$$

These coefficients contain all the kinematical information about the scattering of the two nucleons. When Eq. (7) is used in the evaluation of Eq. (6), the spin dependence breaks the matrix element into a series of terms containing the ${}^SM_{M_S M_{S'}}$. Treatment of the kinematics involves transforming to the c.m. frame for the system, and the expression is simplified by the judicious choice of kinematical parameters. Following Redish *et al.*,[7] we find that Eq. (6) takes the form

$$F=\sum_{\text{spin, isospin}} \chi_B^\dagger \chi_{1f}^\dagger \chi_{2f}^\dagger M \chi_{1\text{inc}}\, \chi_A \langle B_S | -\vec{k}_B, A_S\rangle , \tag{9}$$

with

$$M=\langle \vec{P}_f | M_{12}(E_f) | \vec{P}_i\rangle . \tag{10}$$

In this expression, $\vec{P}_f = \frac{1}{2}(\vec{k}_1' - \vec{k}_2')$, $\vec{P}_i = \frac{1}{2}(\vec{k}_1 + \vec{k}_B)$, and E_f is the relative energy of the two protons in the final state. Noting that $\vec{P}_f = \vec{k}_1' - \frac{1}{2}\vec{k}_1 + \frac{1}{2}\vec{k}_B \cong \vec{k}_1'(\text{c.m.})$ and $\vec{P}_i \cong \vec{k}_1$ (c.m.) in the nonrelativistic limit when $\vec{k}_B \cong 0$, M can be approximated by the on-shell c.m. nucleon-nucleon scattering matrix element. As we are mainly interested in spin effects, this assumption is made. The spin coefficients ${}^SM_{M_S M_{S'}}$ can then be determined directly from the p-p phase shifts. To consider Eq. (9) further, we need the spin and isospin treatments for the light nuclei involved. We use the classification schemes of Refs. 12 and 13 for the few-nucleon states.

For example, the three-nucleon ground state has quantum numbers $J^\pi = \frac{1}{2}^+$ and $T=\frac{1}{2}$. Its most general wave function is a linear superposition of ${}^2S_{1/2}$, ${}^2P_{1/2}$, ${}^4P_{1/2}$, and ${}^4D_{1/2}$ functions. This can be written as

$$|\Psi_3\rangle = |\Psi_3({}^2S_{1/2})\rangle + |\Psi_3({}^2P_{1/2})\rangle + |\Psi_3({}^4P_{1/2})\rangle + |\Psi_3({}^4D_{1/2})\rangle . \quad (11)$$

Each state is represented by products of functions in spatial, spin and isospin coordinates. Likewise, the ground-state wave functions for the deuteron and the α particle are conveniently written as

$$|\Psi_2\rangle = |\Psi_2({}^3S_1)\rangle + |\Psi_2({}^3D_1)\rangle \quad (12)$$

and

$$|\Psi_4\rangle = |\Psi_4({}^1S_0)\rangle + |\Psi_4({}^3P_0)\rangle + |\Psi_4({}^5D_0)\rangle . \quad (13)$$

We can now evaluate Eq. (9) for the two reactions under study.

${}^3\mathrm{He}(p,2p){}^2\mathrm{H}$

We delay consideration of D-state contributions to the quasifree scattering cross section to a future publication. Thus the ^{3}He bound state is represented by its spatially symmetric (S) and mixed symmetric (S') components of the ${}^2S_{1/2}$ state. For $\langle S_z\rangle = \frac{1}{2}\hbar$,

$$|A_S\rangle = |{}^3\mathrm{He}\rangle_1 = \chi_0|u\rangle + \chi'|v_2\rangle - \chi''|v_1\rangle , \quad (14)$$

where the detailed forms of the spatial functions $|u\rangle$, $|v_1\rangle$, and $|v_2\rangle$ will be presented in the next section, and

$$\begin{aligned} \chi_0 &= (1/\sqrt{2})(\xi''\eta' - \xi'\eta'') \\ \chi' &= (1/\sqrt{2})(\xi''\eta'' - \xi'\eta') , \\ \chi'' &= (1/\sqrt{2})(\xi''\eta' + \xi'\eta'') , \end{aligned} \quad (15)$$

with the doublet spin and isospin functions

$$\begin{aligned} \xi' &= (1/\sqrt{6})(\alpha_2\alpha_3\beta_4 + \alpha_2\beta_3\alpha_4 - 2\beta_2\alpha_3\alpha_4), \\ \xi'' &= (1/\sqrt{2})(\alpha_2\alpha_3\beta_4 - \alpha_2\beta_3\alpha_4) , \end{aligned} \quad (16)$$

$$\begin{aligned} \eta' &= (1/\sqrt{6})(\nu_2\nu_3\delta_4 + \nu_2\delta_3\nu_4 - 2\delta_2\nu_3\nu_4) , \\ \eta'' &= (1/\sqrt{2})(\nu_2\nu_3\delta_4 - \nu_2\delta_3\nu_4) . \end{aligned} \quad (17)$$

Here α and β are the Pauli-basis spinors and ν and δ are the isospin-basis spinors for the proton and neutron states, respectively.

The reversed state is

$$|{}^3\mathrm{He}\rangle_2 = \bar{\chi}_0|u\rangle + \bar{\chi}'|v_2\rangle - \bar{\chi}''|v_1\rangle , \quad (18)$$

where $\bar{\chi}_0$, $\bar{\chi}'$, and $\bar{\chi}''$ are obtained from χ_0, χ', and χ'' respectively, by interchanging α and β. The deuteron exists in three spin-1 states, namely,

$$\begin{aligned} |{}^2\mathrm{H}\rangle_1 &= {}^1\chi_0(3,4){}^0X_0(3,4)|w\rangle , \\ |{}^2\mathrm{H}\rangle_2 &= {}^1\chi_1(3,4){}^0X_0(3,4)|w\rangle , \\ |{}^2\mathrm{H}\rangle_3 &= {}^1\chi_1(3,4){}^0X_0(3,4)|w\rangle . \end{aligned} \quad (19)$$

Here ${}^1\chi_{M_S}(3,4)$ are the triplet spinor states for nucleons 3 and 4, and ${}^0X_0(3,4)$ is the corresponding isosinglet spinor state. It follows that Eq. (9) takes the form

$$F_{ij} = {}_i\langle{}^2\mathrm{H}|\chi_{1f}^\dagger\chi_{2f}^\dagger M\chi_{1\,\mathrm{inc}}|{}^3\mathrm{He}\rangle_j , \quad (20)$$

where $i = 1-3$ and $j = 1, 2$. It should be noted that the antisymmetrized M matrix is used for the $(p, 2p)$ reaction to ensure that the identity of the final-state protons is taken into account.

If the spin orientations of the outgoing protons are not measured, the helium target is unpolarized, and the deuteron states not distinguished, Eq. (20) contributes the following to the differential cross section:

$$\mathrm{A} = \sum_{i=1}^{3}\sum_{j=1}^{2} |{}_i\langle{}^2\mathrm{H}|M\chi_{1\,\mathrm{inc}}|{}^3\mathrm{He}\rangle_j|^2 . \quad (21)$$

Each process is uncorrelated and assumed equally likely. If, in addition, the incident proton is polarized in a plane perpendicular to the direction of incidence, the initial state is

$$\chi_{1\,\mathrm{inc}}|{}^3\mathrm{He}\rangle = (a\alpha_1 + b\beta_1)|{}^3\mathrm{He}\rangle ,$$

where $|a|^2 + |b|^2 = 1$. Equation (21) becomes

$$A = [I_0 + 2\,\mathrm{Im}(a^*b)\,\mathrm{Im}(M_1)] \times |\langle w|(|-\vec{k}_d, u\rangle + |-\vec{k}_d, v_1\rangle)|^2 , \quad (22)$$

where I_0 is the proton-proton differential cross section, Im denotes "the imaginary part of," and

$$M_1(\theta,\phi) = (1/\sqrt{2})[{}^1M_{00}{}^1M_{01}^* + {}^1M_{10}({}^1M_{11}^* - {}^1M_{-11}^*)] , \quad (23)$$

where θ and ϕ are the c.m. polar and azimuthal scattering angles for proton 1. For an unpolarized incident beam, $a = b = 1/\sqrt{2}$ and $A = I_0$ times the overlap integral, which agrees with Eq. (5). It should be noted that $M_1(90°, \phi) = 0$, in which case $A = I_0$ times the overlap integral.

Explicit forms for the M matrix element are given in Ref. 11, and the azimuthal dependence of M_1 is $\sin\phi$. The polar axis is along the direction of incidence, and the y axis is along $\vec{k}_1 \times \vec{k}_1'$. Thus, the second term of Eq. (22) introduces an asymmetry in the scattering cross section when the incident proton beam is polarized. For example, if the incident beam is polarized along the y axis, then $a=(1/\sqrt{2})$ and $b=(i/\sqrt{2})$. In this case, measurements made with the scattering plane oriented at $\phi=90$ and $270°$ yield

$$e=\frac{A(90°)-A(270°)}{A(90°)+A(270°)}=\frac{\mathrm{Im}[M_1(\theta,90°)]}{I_0}. \tag{24}$$

This result is independent of the nuclear bound-state contributions, and hence it is a direct test of the quasifree scattering hypothesis. In the special case when the laboratory energy of the incoming proton is 150 MeV and the c.m. scattering angle is $\theta=60°$, the asymmetry is $e=0.184$.[14]

^{4}He$(p, 2p)^3$H

As will be noted subsequently, the S'-state contribution is a small correction to that of the S state in the cross section. For this reason and for simplicity, it will be omitted in this discussion. The ^{4}He bound state is taken to be

$$|A_S\rangle=|^4\mathrm{He}\rangle=(1/\sqrt{2})(\xi_4''\eta_4'-\xi_4'\eta_4'')|y\rangle, \tag{25}$$

where

$$\xi_4'=(1/2\sqrt{3})(\alpha_2\beta_3\alpha_4\beta_5-2\beta_2\beta_3\alpha_4\alpha_5-2\alpha_2\alpha_3\beta_4\beta_5+\beta_2\alpha_3\beta_4\alpha_5+\beta_2\alpha_3\alpha_4\beta_5+\alpha_2\beta_3\beta_4\alpha_5), \tag{26}$$

$$\xi_4''=\tfrac{1}{2}(\alpha_2\beta_3-\beta_2\alpha_3)(\alpha_4\beta_5-\beta_4\alpha_5),$$

and η_4', η_4'' are defined similarly, with ν and δ replacing α and β, respectively.

The residual nucleus is $|B_S\rangle=|^3\mathrm{H}\rangle_{1,2}$, where $|^3\mathrm{H}\rangle_{1,2}$ is of the same form as $|^3\mathrm{He}\rangle_{1,2}$ given in Eqs. (14) and (18), but the isospin part contains ν and δ in exchanged order. If there are no polarization measurements in the final state, we need to evaluate

$$F_i={}_i\langle{}^3\mathrm{H}|M\chi_{1\,\mathrm{inc}}|^4\mathrm{He}\rangle,$$

and the differential cross section is proportional to

$$B=\sum_{i=1}^{2}|F_i|^2. \tag{27}$$

For a beam polarized in the plane perpendicular to the direction of incidence, B is the same as A of Eq. (22) with $\langle u|-\vec{k}_3,y\rangle$ now the overlap integral. Thus, an asymmetry measured as indicated by Eq. (24) would yield the same result as in the case of ^{3}He target.

III. SPATIAL WAVE FUNCTIONS OF ^{2}H, ^{3}He, ^{3}H, ^{4}He AND THE OVERLAP INTEGRAL

For our initial calculations we use explicit forms for $|w\rangle$, $|u\rangle$, $|v_1\rangle$, and $|v_2\rangle$.[15] The deuteron function is relatively well known. We describe it by the Hulthén wave function

$$\langle r|w\rangle=N_d(e^{-\alpha r}-e^{-\beta r})/r, \tag{28}$$

where

$$\alpha=0.232\ \mathrm{fm}^{-1},\quad \beta=1.434\ \mathrm{fm}^{-1},$$

and N_d is the normalization constant. The spatially symmetric $\langle r|u\rangle$ is defined by

$$\langle r|u\rangle=A\prod_{i<j=1}^{3}f(r_{ij}), \tag{29}$$

with

$$f(r_{ij})=e^{-ar_{ij}}-ce^{-br_{ij}}, \tag{30}$$

where

$$a=0.40\ \mathrm{fm}^{-1},\quad b=2.00\ \mathrm{fm}^{-1},$$

and

$$c=0.4.$$

The coordinate functions $\langle r|v_1\rangle$ and $\langle r|v_2\rangle$ can be written in terms of a single function $g(ij,k)$ which is symmetric in its first two arguments:

$$\langle r|v_1\rangle=(1/\sqrt{6})[g(12,3)+g(13,2)-2g(23,1)],$$
$$\langle r|v_2\rangle=(1/\sqrt{2})[g(12,3)-g(13,2)]. \tag{31}$$

Numerical calculations for the reaction ^{3}He$(p,2p)$-^{2}H are carried out with

$$g(ij,k)=B\exp(-\alpha' r_{ik}-\alpha' r_{jk}-\beta' r_{ij}), \tag{32}$$

where $\alpha'=0.37\ \mathrm{fm}^{-1}$ and β' is determined by α' and $P_{S'}$, the probability of the S' state. These correlated functions have reasonable asymptotic behavior and are utilized here to extract a reliable estimate of the S' state contribution.

As we shall report, the contribution of the S' state is relatively small and influences only the magnitude of the cross section. We have therefore also used recently developed Eckart forms[16] for the overlap integrals $\langle w|-\vec{k}_d,u\rangle$ and $\langle u|-\vec{k}_3,y\rangle$. For example, in the Eckart parametrization

$$\langle w|-\vec{k}_d,u\rangle=\int N_E\frac{e^{-\mu\rho}(1-e^{-\nu\rho})^4}{\rho}e^{-i\vec{k}_d\cdot\vec{\rho}}\,d\vec{\rho}, \tag{33}$$

where the parameters μ and ν have values 0.418 fm^{-1} and 1.90 fm^{-1}, respectively. The corresponding Eckart parameters for ^{4}He are 0.843 fm^{-1} and 1.20 fm^{-1}. These Eckart functions interpolate smoothly between the expected short-range behavior and the correct asymptotic form of the spatial distribution function. They have been manifestly successful in reproducing the

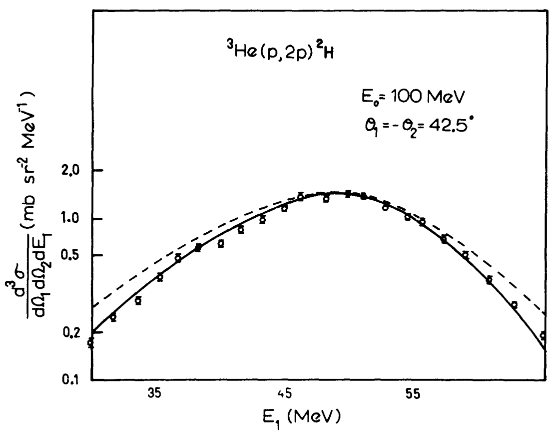

FIG. 2. Symmetric, coplanar $^3He(p,2p)^2H$ cross section at 100-MeV incident-proton energy. The solid curve is the Eckart result while the dashed curve is that of the correlated-exponential function. The data are from Ref. 5.

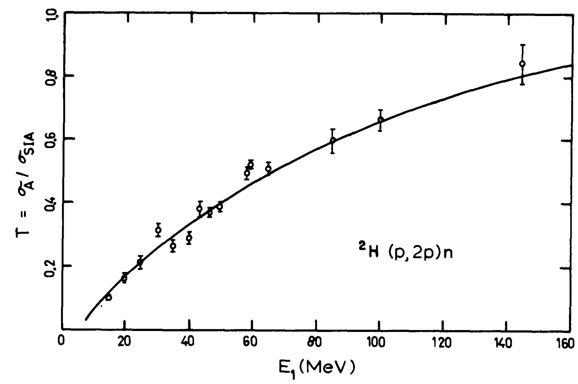

FIG. 4. Experimentally determined values of the transmission factor for symmetric, coplanar $^2H(p,2p)n$ quasifree scattering, plotted as a function of laboratory bombarding energy. The solid curve is the visual, best-fit curve used in the computation of T and $^3He(p,2p)^2H$.

charge form factors and charge radii of ^3He and ^4He.

IV. RESULTS AND DISCUSSION

Data for the symmetric, coplanar $^3He(p,2p)^2H$ reaction are available for incident proton energy ranging from 35 to 155 MeV. Representative data from Refs. 3 and 5 are shown in Figs. 2 and 3. We find that theoretical curves obtained using the correlated three-body wave function, though giving fair fits to experiment, are broader and consistently larger than the data. The S' state contribution, assuming $P_{S'}$ to be 2%, only influences the over-all magnitude of the curves and, in each case, decreases the result by about 9%, a relatively small effect. Our result agrees with that of Hu *et al.*[9] who used a variational wave function for ^3He extracted from binding-energy calculations with the Hamada-Johnston potential. They predicted a decrease of about 10%. It is obvious that the correlated function is not altogether adequate. Nevertheless, its estimate of the S' state contribution should be reliable and suggests that dropping the S' state should not affect our main conclusion.

We note that the Eckart curves improve considerably the shape fit to data, emphasizing the accuracy and usefulness of the Eckart parametrization. We believe the narrowing of the theo-

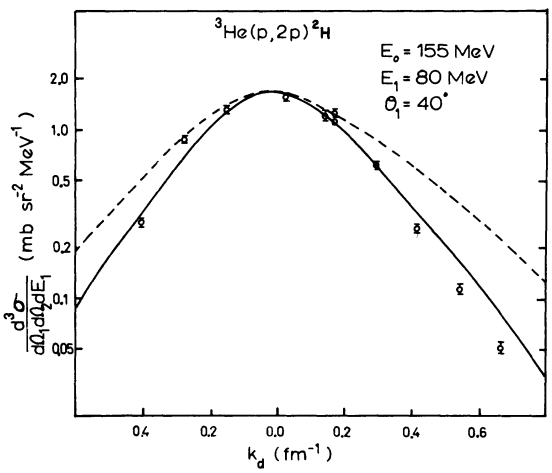

FIG. 3. Symmetric, coplanar $^3He(p,2p)^2H$ cross section at 155-MeV incident-proton energy. The solid curve is the Eckart result while the dashed curve is that of the correlated-exponential function. The data are from Ref. 3.

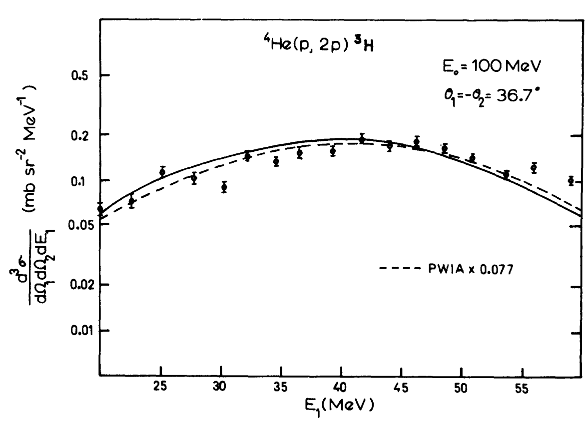

FIG. 5. Symmetric, coplanar $^4He(p,2p)^3H$ cross section at 100-MeV incident-proton energy. The solid curve is the Eckart result normalized to the experimental quasifree peak. The dashed curve, which is the PWIA result, and the data are from Ref. 5.

TABLE I. The transmission factor T at the quasifree peak for $^3He(p,2p)^2H$ at various energies. The experimental values are quoted from Refs. 3 and 5.

Incident proton energy (MeV)	T_{expt}	T_{theor}
35	0.16	0.14
65	0.25	0.27
85	0.33	0.36
100	0.38	0.40
155	0.60	0.62

retical curves at the quasifree peak can be attributed to the correct representation of short-range effects by the Eckart functions. In agreement with other authors, we observe that the PWIA calculations predict cross sections that are larger than experiment. The variation of the absolute normalization factor with energy at the quasifree peak can be illustrated if one writes

$$\left(\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}\right)_{\text{expt}} = T\left(\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}\right)_{\text{PWIA}} \tag{34}$$

The transmission factor T has been shown to assume values ranging from 0.16 at 35 MeV to 0.60 at 155 MeV.[3,5] Our values of T at 100 and 155 MeV are close to those obtained by the Maryland group. These results suggest that multiple scattering corrections to the PWIA must be made. In this connection, Rogers and Saylor[10] have developed a simple attenuation model for the evaluation of T in $^2H(p,2p)n$ reactions. We have applied their rescattering model to the corresponding quasifree reaction in ^3He. We altered their T vs σ_T (the total spectator cross section) curve to better fit the $^2H(p,2p)n$ data up to 100 MeV and projected it to 160 MeV. This is shown in Fig. 4. In determining T for the trinucleon reaction, we made the physically plausible assumption that $\sigma_T^{\,^3He} \cong 2\sigma_T^d$ for the same incident proton energy; i.e., we assumed that the final-state deuteron behaves as two isolated nucleons in its interaction with the final-state protons. Using this assumption and Fig. 4, we extracted T. In Table I, we show a comparison between the experimental values of T from Ref. 5 and the theoretical magnitudes based on this intuitive extension of the Rogers-Saylor model. Considering the simplicity of the model, the good agreement is slightly embarassing. As one has reason to believe that off-

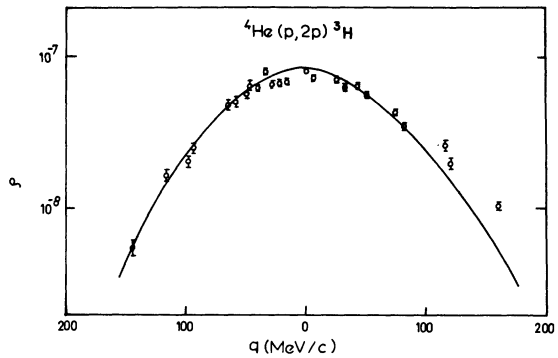

FIG. 6. p-^3H momentum distribution at 600-MeV incident-proton energy. The solid curve is the Eckart result normalized to the experimental quasifree peak. The data are from Ref. 2.

shell and distortion effects are likely to be small at incident proton energies exceeding about 50 MeV, one is tempted to conclude from this that the $(p,2p)$ reaction on ^3He is indeed a simple direct process whose description is mainly afforded by PWIA and the attenuation model.

The experimental data for $^4He(p,2p)^3H$ are sparse and featureless. Figures 5 and 6 show that our Eckart curve gives reasonable fits to data. No extension of the attenuation model is made for this case as it is obvious that we cannot treat ^3H, a more compact nucleus than the deuteron, as three isolated nucleons. We are studying a more detailed derivation of T based on the ideas of Rogers and Saylor. The results of this study will be reported in a subsequent publication.

In summary, it may be concluded that $(p,2p)$ reactions on ^3He and ^4He can be explained by PWIA when suitable overlap functions are used and attenuation effects are included. Good fits to the shape and magnitude of the cross sections can be derived. If our good results are not fortuitous, it appears that the attenuation model accurately accounts for multiple scattering effects. A further and sensitive test of the quasifree model is provided by an asymmetry measurement in which only nucleon-nucleon effects contribute. If the model should survive even this kind of scrutiny, further studies on the nuclear bound states could be made by merely analyzing the overlap integrals of the target and residual nuclei. This is a far simpler procedure than would be necessary if a many-body treatment is required.

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