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Predicted proton spectrum at forward angles for 29.4-GeV nitrogen on carbon*

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The proton spectrum over the angular interval of 0 to 10 mrad from the interactions of 29.4-GeV nitrogen on carbon has been calculated. The differential cross section for the emission of protons indicates that there is a significant number of emitted protons with energies greater than the energy per nucleon of the incident projectile (2.1 GeV). A description of the calculational method is included.

[NUCLEAR REACTIONS $^{14}N(^{12}C, p)$, $E = 29.4$ GeV; calculated $\sigma(E_p)$, $\theta = 0-10$ mrad.]

INTRODUCTION

An interesting result has been obtained from an exploratory approach to the calculation of heavy-ion reactions,[1] namely, that the predicted proton spectrum at small forward angles contains a significant number of protons emitted with energies greater than the incident energy per nucleon of the projectile. An early experimental test[2] of this theoretical prediction is important in that it will help guide this and other theoretical approaches.[3]

In carrying out the calculation, the basic concept employed was that heavy-ion reactions can be treated as the interaction of two Fermi gases. Approximations were made to facilitate the effort, and potentially important effects (such as depletion of local nuclear density and angular momentum transfers) were ignored. Due to the preliminary nature of the model, only the proton spectrum is discussed here while other results are reported elsewhere.[4] However, it is expected that the bulk of the high-energy proton spectrum in the forward direction will be relatively stable to changes in the model. The justification for this opinion is given in a subsequent section. Since the time delay will be significant for the completion and validation of the computer program with the approximations removed, publication of the proton results for subsequent experimental verification or nullification appears warranted at this time, particularly since the proton spectrum reflects the approach employed. An alternative theoretical approach, which is proceeding concurrently elsewhere,[3] will not yield the same shape for the spectrum.

MODEL AND CALCULATION

The method of carrying out the heavy-ion calculation is an extension of the method of intranuclear cascades. For incident nucleons, at least, it has been shown that the absolute value of experimental energy-angle-correlated nucleon spectra from continuum-state transitions involving nucleons (about 0.1 GeV or more) on complex nuclei can be reasonably reproduced by this method.[5-7] In this approach, the incident nucleon interacts initially with one of the bound nucleons of the nucleus in a "quasifree" interaction. The collision products from this interaction, which might include pions, move through the nucleus and interact with other bound nucleons in the same manner, and the process is repeated (building up a cascade of particles) until all of the collision products escape or are absorbed by the nucleus.

The heavy-ion reaction then is *envisioned* to take place as follows: During the passage of the incident heavy ion (projectile) through the target, those nucleons of the projectile that are in the region of overlap undergo quasifree reactions with the individual nucleons of the target. A cascade is thereby generated simultaneously in both target and projectile. The nucleons that have been jarred free of the binding forces in either the target or projectile and that also manage to survive capture during the development of the cascade escape from the target and projectile. They are emitted as free nucleons in various directions and with a variety of energies. After completion of the cascade, the remaining fragments of the projectile and of the target move off in highly excited states emitting evaporation particles until sufficient excitation energy is lost to stop the evaporation process.

The present version of the heavy-ion-collision model only approximates the feature of the simultaneous cascades in both projectile and target. To this end we first permit the projectile, moving with velocity $\vec{V}$, to impinge upon a target that is stationary in the laboratory frame of reference. Cascades are allowed to develop only in the target. This is called the "forward" reaction. Then the target, moving with velocity $-\vec{V}$, is made to impinge upon the projectile, which is taken to be stationary, and cascades are then permitted only in the projectile. The directions and energies of all particles thus calculated in this "inverse" reaction are transformed to their corresponding values in the frame of reference of the forward reaction, i.e., in the laboratory frame. The results from the forward and inverse reactions are then each weighted by one half.

The general physical properties that are simulated for the nuclei of every target and every projectile are as follows: The nucleons making up the nucleus are clustered closer together near the center of the nucleus than on the edges. The density distributions are thus approximations to measured Fermi-type charge-distribution functions.[8] Some of the distributions that were tested for applicability to this model are shown in Fig. 1. Details of these tests will be given subsequently in this paper. The nucleons bound in the nucleus are in constant motion with zero-temperature Fermi energy distributions, which are determined by their local densities. Attractive single-particle potentials are assumed to exist in the nucleus, and these are made to vary in strength with the Fermi energy. An approximation to account for exclusion effects in all reactions is incorporated. Further details on the nuclear properties are given elsewhere.[9]

Briefly, the calculation, which employs Monte Carlo techniques, proceeds as follows[4]:

(1.) A center-to-center impact parameter, modified crudely for Coulomb effects, is randomly selected from a uniform distribution over the area of a circle whose radius is the sum of the target and projectile radii. The coordinate system of the laboratory frame of reference is located at the center of the target with the z axis in the direction of the incident projectile. Approximate calculations are used to alter the direction of the incident heavy ion to account for Coulomb deflection. The deflection and impact parameter modifications to account for Coulomb effects are calculated as follows: A relativistic transformation is made to the center-of-mass system (C system). In this system, a nonrelativistic kinematics calculation for point charges acting only under the influence of their Coulomb forces is carried out using the initially selected center-to-center impact parameter. This calculation yields the distance of closest approach and the C-system scattering angle. The impact parameter used in the heavy-ion calculation is set equal to this distance of closest approach. The C-system Coulomb deflection for the heavy-ion reaction is taken to be one half the C-system scattering angle for the point charges. The value of one half is used because the ions are acting under the influence of a pure Coulomb force only until they overlap, at which time the forces become extremely complicated. A relativistic transformation back to the laboratory system gives the Coulomb deflection used in the heavy-ion reaction.

(2.) Each nucleon in the projectile is assigned a position inside the projectile and is given an internal energy. The positions and energies are randomly selected from the density and Fermi energy distributions described above. The internal energies do not change the incident kinetic energy (taken to be the kinetic energy per nucleon of the heavy-ion reaction) or the direction of each nucleon. They are used only in calculating the energy available to the reaction when the nucleons collide. For this purpose, the incident energy is reduced by the difference between the local well depth in the projec-

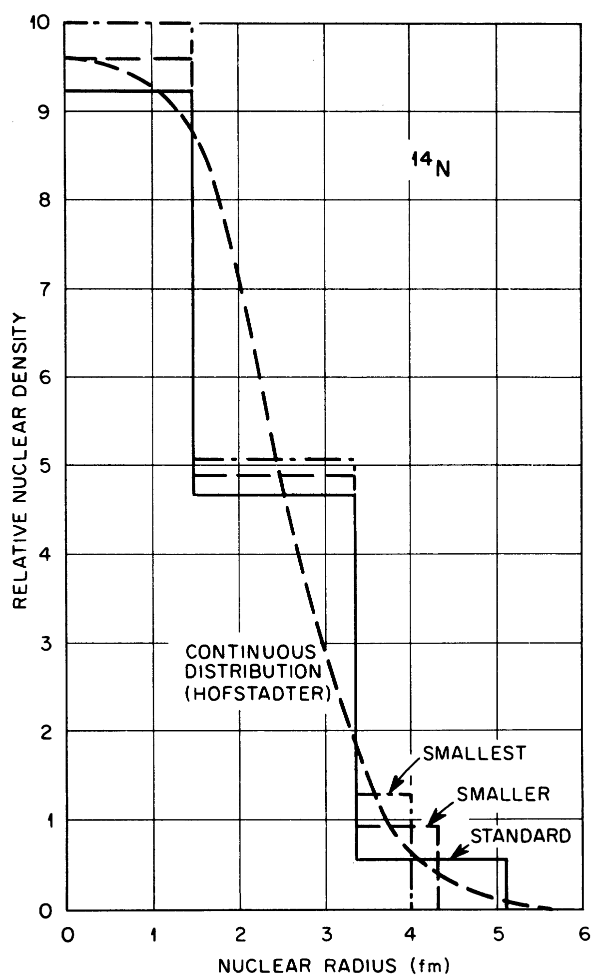

FIG. 1. Relative nuclear density vs nuclear radius for three nuclear configurations representing the ^{14}N nucleus. Solid line: "standard" configuration; dashed line: "smaller" configuration; dot-dashed line: "smallest" configuration; dashed curve: Hofstadter's Fermi-type distribution (Ref. 8).

tile and the internal kinetic energy of the nucleon; i.e., the incident energy is reduced by an amount which approximates the actual binding energy of the nucleon in the projectile in order to obtain the energy available to the reaction.

(3.) Trajectories for each nucleon of the projectile are calculated, all initially parallel to each other, with x and y positions determined by steps 1 and 2. Depending on the impact parameter selected, some of these nucleons will miss the target completely, while some will pass through the target but will not collide (because of target transparency). The remaining fragment of the projectile consists of all of these uncollided nucleons and it also consists of holes interspersed throughout the fragment. The holes result from the removal of those nucleons of the incident projectile that have collided.

(4.) For each nucleon that collides, an independent cascade is developed in the target. Some of the cascade neutrons and protons (and also π mesons) escape with various energies and directions.

(5.) The excitation energy remaining in the projectile fragment is calculated from the holes therein, while that for the target is calculated from a kinematic energy balance.

(6.) Conservation of total energy and momentum is invoked in carrying out the relativistic kinematics calculations that determine the directions and kinetic energies of the fragments of the projectile and target. An angular distribution for the projectile fragment suggested by experimental evidence is employed[10]; i.e., it is assumed that the projectile fragment is isotropically distributed in a system moving with velocity $\overline{V}$ of the initial projectile.

(7.) As the excited projectile and target fragments move away from the interaction site, they are made to evaporate particles until their excitation energy is lost. The evaporation of each particle from the fragment is calculated in the rest system of the fragment, where isotropic emission of the particles is assumed. The laboratory energy of each particle is calculated by a transformation back to the laboratory system. The change in momentum of the fragment following each evaporation is taken into account.

(8.) Steps 1–7 are repeated several hundred times in order to sample properly from the distributions employed.

(9.) The inverse reaction described above is calculated by repeating steps 1–8, while interchanging the roles of target and projectile, and the results are transformed back to the laboratory frame of reference. However, the fragment of the stationary projectile in the inverse reaction is given the same angular distribution as it has in the forward reaction.

DISCUSSION

The sensitivity of the results to the assumptions deserves discussion. The nuclear radius was varied from that normally used in the intranuclear-cascade calculation.[9] The nuclear configurations, designated as "standard," "smaller," and "smallest," and the radii used in this variation are shown in Fig. 1. The configurations correspond to those whose outer radii are equal to the radius of the Hofstadter Fermi-type charge-distribution function when the value of this function reaches 1, 4, and 7%, respectively, of its value at zero radius. A comparison of the results using these radii for the emission of high-energy fragments at 0° with the data of Heckman *et al.*[4,10] indicated that the sensitivity of data of this type to the radius is about the same as the change in the geometric cross section with the radius. The configuration designated as "smaller" is the one adopted for this paper.

The angular distribution (which is not determined from the model) of the excited target and projectile fragments that emerge from the reaction prior to evaporation was varied. In one case it was assumed that the target recoiled isotropically in the center-of-mass system and in another it was assumed that the projectile recoiled with a momentum equal and opposite to the momenta of the nucleons removed from the projectile. The data of Heckman *et al.*[10] were again used as a standard for comparison. For the first assumption the predicted cross sections for the yield of various high-energy fragments emitted into the appropriate angular and energy ranges (corresponding to the measurement) were all zero. For the second assumption, the high-mass isotopes appeared too frequently and there was a deficiency of low-mass isotopes. The bulk of the high-energy proton spectra is insensitive to the assumed angular distribution of the fragments.

Direct tests of the sensitivity of the results to the other assumptions in the model have not been made, but conjectures regarding this sensitivity warrant a brief discussion. The neglect of the Fermi motion of the nucleons in the projectile with respect to the energy and the direction of the incoming nucleon should be small. The laboratory energy of a bound nucleon will vary from 1.4 to 2.9 GeV if it has a kinetic energy of 30 MeV with respect to the center of mass of a projectile whose laboratory energy is 2.1 GeV/nucleon. However, the total cross sections are relatively constant over this energy range of variation,[11] and the differential scattering cross sections are all peaked forward.[12] Therefore, one would expect that the inclusion of the Fermi motion of the nucleons in the projectile would give rise to second-order effects only. At the energies under consideration (~1 GeV/nucleon) the changes in the directions of these nucleons due to the Fermi motion would be extremely small.

The representation of the development of simultaneous cascades in both projectile and target (while they are interacting) by the separated forward-inverse reactions should lead to changes in the mass distribution for the light fragments and leave that for the heavy fragments (masses close to the target or projectile) relatively unaltered. The reactions leading to the formation of heavy fragments are simple in that they are generally peripheral interactions in which only a few nucleons are involved, and hence they should be better represented by the forward-inverse approximation; i.e., their residual mass and excitation energy distributions (which affect the evaporation phase) should be better represented.

The neglect of local nuclear-density depletion as the interaction proceeds is expected to have a significantly greater effect on the results that are due primarily to central collisions (and hence more complicated reactions) than on those from peripheral collisions.

The neglect of angular momentum transfer could affect both peripheral and central collisions because there is such an enormous amount of angular momentum involved in these high-energy heavy-ion reactions. For example, the incident angular momentum for a 1-GeV/nucleon Al-on-Al reaction is typically $1000\hbar$. Even if relatively small fractions of this are transferred to either the target or projectile fragments, their evaporation characteristics will be greatly altered.

The bulk of the proton spectrum at high energies comes from peripheral quasifree (cascade) interactions, and hence the effects of the forward-inverse approximation and the neglect of both nuclear depletion and angular momentum should not affect this portion of the spectrum significantly. At the maximum energies, however, the evaporation contribution dominates, and although the cross section is relatively small, its effect is quite interesting and is discussed in the next section.

NUCLEON SPECTRUM IN FORWARD DIRECTION

The predicted proton spectrum at forward angles from 29.4-GeV N-on-C collisions is shown in Fig. 2 for the "smaller" nuclear configuration. There are two peaks predicted at high energies. The peak centered at 1.7 GeV is made up primarily of cascade or direct-interaction protons from the forward reaction (84%) with the remainder consisting of cascade protons from the inverse reaction (10%) and evaporation protons from the projectile in the inverse reactions (6%). The peak centered at 2.3 GeV consists mainly of protons evaporated from the projectile in the forward reaction (56%), with cascade and projectile evaporation protons from the inverse reaction contributing the remainder (16 and 28%, respectively).

The peak at 1.7 GeV is the "quasifree" peak from the direct interactions of the nucleons of the projectile with those of the target. The peak is located at a smaller energy than expected (expected at ~2.1 GeV at these angles) because of exclusion effects.

It is both interesting and important to note that the the cross section for the emission of protons with energies greater than the energy per nucleon of the incident projectile (2.1 GeV) is quite significant. Experimental verification of the shape of the proton spectra is important in order to guide the theoretical approaches. Other attempts at heavy-ion calculations, which are similar to these but which do not include the inverse reaction, yield proton spectra whose ratio of cascade-to-evaporation contributions is very different.[3] This parti-

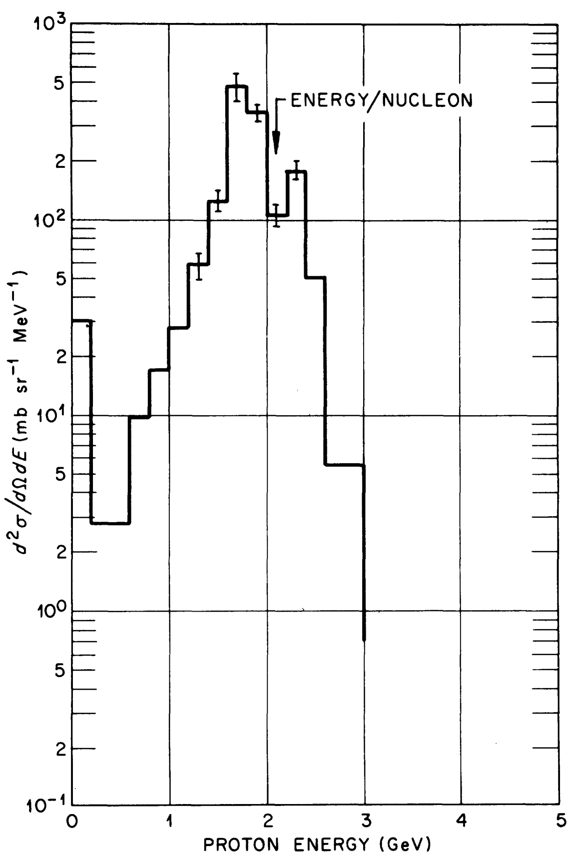

FIG. 2. Proton spectrum for the angular interval of 0 – 10 mrad from 29.4-GeV N on C. The energy per nucleon of the incident projectile is indicated by the arrow.

cular spectrum was selected because it is amenable to early experimental verification,[2] and it clearly distinguishes the cascade and evaporation protons at the high energies.

Statistically, the peak at the highest energy is significant in that the same case was repeated using the "standard" and "smallest" nuclear configurations, and essentially all of the high-energy results were within statistics of each other (i.e., their error bars overlapped), and all results indicated two similarly located peaks at high energy.

Physically, however, the second peak may not be manifest as such because its existence, theoretically, is dependent mainly on the evaporation phase of the calculation, particularly evaporation from the projectile fragment. As was discussed in the previous section, the forward-backward approximation and the neglect of simultaneous cascades and angular momentum transfers will have a direct bearing on the evaporation process, and they will therefore affect the magnitude and, to some extent, the shape of the second peak.

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[1]Introductory descriptions of heavy-ion reactions in general and their significance in the energy range under investigation are published in several articles in Science 174, 1121–1131 (1971).

[2]An experimental attempt to verify the prediction is in progress. D. E. Greiner, Lawrence Berkeley Laboratory, Berkeley, California, private communication.

[3]V. D. Toneev, Joint Institute for Nuclear Research, Dubna, USSR, private communication.

[4]Details of the results and of the model are given in Oak Ridge National Laboratory Report No. ORNL-TM-4134 by H. W. Bertini, T. A. Gabriel, R. T. Santoro, O. W. Hermann, N. M. Larson, and J. D. Hunt, to be published.

[5]V. S. Barashenkov *et al.*, Nucl. Phys. 14, 522 (1959/60).

[6]H. W. Bertini, Phys. Rev. 188, 1711 (1969).

[7]H. W. Bertini, Phys. Rev. C 6, 631 (1972).

[8]R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

[9]H. W. Bertini, Phys. Rev. 131, 1801 (1963); 138, AB2(E) (1965).

[10]H. H. Heckman, D. E. Greiner, P. J. Lindstrom, and F. S. Bieser, Phys. Rev. Lett. 28, 926 (1972).

[11]D. V. Bugg *et al.*, Phys. Rev. 146, 980 (1966).

[12]L. W. Smith *et al.*, Phys. Rev. 97, 1186 (1955); B. Cork *et al.*, *ibid.* 107, 859 (1957); M. M. Block *et al.*, *ibid.* 103, 1484 (1956); T. Fuji *et al.*, *ibid.* 128, 1836 (1962); S. P. Kruchinin *et al.*, Yad. Fiz. 1, 225 (1965) [transl.: Sov. J. Nucl. Phys. 1, 225 (1965)]; S. A. Azimov *et al.*, Zh. Eksperim. Teor. Fiz. 42, 430 (1962) [transl.; Sov. Phys. — JETP 15, 299 (1962)]. These references are for *p-p* scattering. For *n-p* scattering, refer to H. Palevsky *et al.*, Phys. Rev. Lett. 9, 509 (1962); J. L. Friedes *et al.*, *ibid.* 15, 38 (1965); M. N. Kriesler *et al.*, *ibid.* 16, 1217 (1966).