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Transition intensities from projection-integral wave functions*

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Possible deviations from the Alaga rules based on the use of projection-integral wave functions are investigated. It is shown that while such deviations, in principle, exist, they identically vanish in the region of heavy deformed nuclei, if it is assumed that these nuclei can be described by "semirigid" intrinsic wave functions.

[NUCLEAR MOMENTS Deformed nuclei; calculated $B(E2)$ deviations. Projection integral wave functions.]

I. INTRODUCTION

When dealing with heavy deformed nuclei the adiabatic wave function, which treats the nucleus as a rigid rotator, is normally used; that is

$$|JMK\rangle = [(2J+1)^{1/2}/4\pi] \times [D^{J*}_{MK}(\Omega)|\phi_K\rangle + (-1)^{J+K}D^{J*}_{M-K}(\Omega)|\phi_{-K}\rangle], \tag{1}$$

where $|\phi_K\rangle$ is some axially deformed intrinsic state with J_Z projection K on the body-fixed Z axis. An alternate description utilizes a microscopic intrinsic wave function from which is projected a laboratory wave function with a good angular momentum quantum number. The projection-integral wave function can be written, as follows:

$$|JMK\rangle = [(2J+1)/16\pi^2 C(JK)] \times \int d\Omega\, R(\Omega)[D^{J*}_{MK}(\Omega)\,|\phi_K\rangle_\Omega + (-1)^{J+K}D^{J*}_{M-K}(\Omega)|\phi_{-K}\rangle_\Omega]\,. \tag{2}$$

The projection-integral wave function has the advantage of avoiding the problem of redundant variables which are implicit in the adiabatic wave function, and, in addition, does not require an *ad hoc* separation of collective and intrinsic motion. However, the general success of the adiabatic wave function in its various applications, along with the simplifications it introduces, argues strongly for its use. Many investigators, especially Alaga *et al.*,[1] have used the adiabatic wave functions to investigate electromagnetic transitions in deformed nuclei, and to derive formulas for the associated $B(E2, J_1K_1 \to J_2K_2)$'s. Much interest has been shown in deviations from these formulas. The deviations are generally attributed to band mixing; adjustments[2-6] to the Alaga formulas have been derived under that assumption for the effect of the band mixing. While differences in the definitions of important parameters in these various approaches make it difficult to compare them with each other, the results of these efforts are mutually consistent. The results in this paper will be compared with those reported by Mottelson.[6]

If projection-integral wave functions are used for estimating electromagnetic transition properties, nonadiabatic terms of the type produced by band mixing can result.[7] Thus it has been suggested[7] that the deviations from the Alaga rules are not symptomatic of band mixing, but instead, reflect the implicit simplifying assumptions about the nucleus which are made when adiabatic wave functions are used.

II. CHARACTERIZATIONS OF DEFORMED SEMIRIGID NUCLEI

The actual structure of the intrinsic state in a projection integral is determined by the type and number of nucleons present along with the effective particle-particle interaction which obtains. As a result, each intrinsic state depends in some detail on the particular nucleus being considered. In order to use projection-integral wave functions to furnish a general description of a wide range of nuclei, a way must be found to describe the projection-integral intrinsic state, which is both sufficiently typical in a broad sense of the nuclei being discussed and relatively independent of the precise structure of each given nucleus.

Fortunately, such a description exists. Many authors[8-10] have shown that projection-integral results can be obtained by approximating the intrinsic matrix element $\langle\phi_K|e^{-i\beta J_y}T^k_\mu|\phi_K\rangle$ as follows:

$$\langle \phi_K | e^{-i\beta J_y} T^k_\mu | \phi_K \rangle \approx r\beta^\mu \eta(\beta), \tag{3}$$

where r is a constant and $\eta(\beta)$ is essentially the overlap integral $\langle \phi_K | e^{-i\beta J_y} | \phi_K \rangle$. Based on general considerations [10,11] as well as detailed calculations[8,12] it is usually argued that the overlap integral will be closely approximated by the form:

$$\eta(\beta) = e^{-\beta^2 \langle J^2 \rangle / 4}. \tag{4}$$

For all but the very lightest nuclei this will be a sharply peaked function of β. It is well known[8-10] that as $\langle J^2 \rangle$ gets very large (i.e., for heavy nuclei) the results obtained by utilizing (3) and (4) will go over to the standard results obtained by using adiabatic wave functions. Thus, in effect nuclei with large $\langle J^2 \rangle$, which have intrinsic wave functions that satisfy Eqs. (3) and (4) can be said to be semirigid nuclei. It should be noted that for light nuclei for which the SU(3) model can be expected to be a good approximation, the correct expression (assuming $K=0$) for $\eta(\beta)$ is given by

$$\eta(\beta) = [\cos(\beta)]^{\langle J^2 \rangle / 2} \tag{5}$$

which is also a sharply peaked function of β differing from the result given in Eq. (4) around the point $\beta = 0$ only in terms on the order of β^4.

Equation (3) is equivalent to assuming that

$$T^k_{\pm\mu} = \sum_n C_n J^{2n} (J_{\pm 1})^\mu \tag{6}$$

or, in particular, for the case of interest here it follows that

$$Q^2_{\pm 1} = a_0 J_{\pm 1} + a_1 J^2 J_{\pm 1} + \cdots, \tag{7}$$

and

$$Q^2_{\pm 2} = b_0 (J_{\pm 1})^2 + b_1 J^2 (J_{\pm 1})^2 + \cdots. \tag{8}$$

For example, to the second order, we find using Eq. (7) in the left-hand side of Eq. (3) that

$$\langle \phi | e^{-i\beta J_y} Q^2_{+1} | \phi \rangle = -\frac{\beta \langle J^2 \rangle}{2\sqrt{2}} \left(a_0 e^{-\beta^2 \langle J^2 \rangle / 4} + 2a_1 \langle J^2 \rangle e^{-\beta^2 (3 \langle J^2 \rangle / 8)} \right). \tag{9}$$

III. DEVIATIONS FROM THE ALAGA RULES FOR $E2$ TRANSITIONS IN THE GROUND-STATE ROTATION BAND

It is well known that, in general, the action of all the components of the tensor operator Q^2_q acting in the body-fixed system contribute to the evaluation of the operator Q^2_0 in the laboratory reference frame. In fact, a straightforward application of projection-integral techniques establishes that

$$B(J_1 \to J_2) = \left[\frac{C'(J_2)}{C'(J_1)} \right]^2 D(J_2; J_1)^2, \tag{10}$$

where $B(J_1 \to J_2)$ is the ratio of the derived $B(E2)$ to that given by Alaga *et al.*,[1] and $C'(J)$ is the projection coefficient $C(J, K=0)$ divided by $(2J+1)^{1/2}$, further

$$D(J_2; J_1) = \left[1 + H(J_2) \frac{\langle J_2\, 2\, 1\, -1 | J_1 0 \rangle}{\langle J_2\, 2\, 0\, 0 | J_1 0 \rangle} + F(J_2) \frac{\langle J_2\, 2\, 2\, -2 | J_1 0 \rangle}{\langle J_2\, 2\, 0\, 0 | J_1 0 \rangle} \right]. \tag{11}$$

In Eq. (11), the form of $H(J)$ is determined by the effect of the Q^2_1 operator on the intrinsic state in question and similarly $F(J)$ is determined by the action of Q^2_2 on the intrinsic state. By utilizing the Hermiticity of Q^2_0 operator, it follows that,

$$\left[\frac{C'(J_2)}{C'(J_1)} \right]^2 = \frac{D(J_1; J_2)}{D(J_2; J_1)} \tag{12}$$

and thus we find that

$$B(J_1 \to J_2) = D(J_2; J_1) D(J_1; J_2). \tag{13}$$

To evaluate Eq. (13) we utilize formulas (7) and (8). If only coefficients through the second order (i.e., a_0, a_1, and b_0) are assumed to be nonzero then it follows that

$$B(J_1 \to J_2) = [1 + 12A_0 - 16A_0^2(J^2 + 3J) - 4A_1(5J^2 + 11J + 6) + 2B_0(J^2 + 3J + 6)], \tag{14}$$

where

$$A_0 = \frac{a_0}{2\sqrt{3}\, q_0}, \quad A_1 = \frac{a_1}{2\sqrt{3}\, q_0}, \quad B_0 = \frac{b_0}{2\sqrt{6}\, q_0}, \tag{15}$$

and q_0 is the intrinsic quadrupole moment. Similarly, the same technique can be used to evaluate Eq. (12) which yields the relationship

$$\left[\frac{C'(J+2)}{C'(J)} \right]^2 \approx 1 + (4J+6)\{2A_0 + 2[A_1(J^2 + 3J + 6) - B_0 + A_0^2(4J + 12)]\}. \tag{16}$$

The coefficients A_0, A_1, and B_0 can be evaluated by utilizing either Eq. (4) or Eq. (5) along with the definition of the projection coefficient, i.e.,

$$[C'(J)]^2 = \int_0^{\pi/2} \sin(\beta) d^J_{00}(\beta) \eta(\beta) d\beta, \tag{17}$$

for the case $K=0$. If Eq. (4) is used, the result is that

$$\left[\frac{C(J+2)}{C(J)}\right]^2 = 1+(4J+6)\left[\frac{-1}{\langle J^2\rangle}+\frac{(2J+2)}{\langle J^2\rangle^2}\right], \qquad (18)$$

from which it follows that

$$A_0=\frac{-1}{2\langle J^2\rangle}, \quad A_1=0, \quad B_0=8A_0^2 . \qquad (19)$$

If Eq. (5) is used, the result is that

$$\left[\frac{C(J+2)}{C(J)}\right]^2 = 1+(4J+6)\left[\frac{-1}{\langle J^2\rangle}+\frac{2(J+3)}{\langle J^2\rangle^2}\right], \qquad (20)$$

from which it follows that

$$A_0=\frac{-1}{2\langle J^2\rangle}, \quad A_1=0, \quad \text{and } B_0=0 . \qquad (21)$$

The results given in Eqs. (21) are identical to those that are known to exist for the SU(3) model, and when they are substituted in Eq. (14) will produce results that differ from the band-mixing result only in a static term; and therefore, will produce identical branching ratios. Thus it can be said that deviations from the Alaga rules of the sort discussed in the literature do not, in themselves, indicate a deviation from a pure rotational band. However, in the region of heavy deformed nuclei of interest in the aforementioned papers, it is usually assumed that the projection-integral wave functions are in the semirigid limit; and therefore, the "correct" set of values for A_0, A_1, and B_0 are given by Eqs. (19).

IV. CONCLUSION

The results given in Eqs. (19) when substituted into Eq. (14) will cause the significant part of the projection-integral corrections to the Alaga rules to identically cancel. Thus, one must conclude that if heavy deformed nuclei are fairly described by semirigid projection-integral wave functions as defined herein, no deviations from the Alaga rules should be expected for intraband $E2$ transitions. However, it must be emphasized that, as shown, even a small deviation from the Gaussian overlap shape can produce results which will mimic the band-mixing results.

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